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J Soc 1316.10.5 (1)

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May 5, 1703.

AT A

MEETING

OF THE

ROYAL SOCIETY,

Sir JOHN HOSKYNS, V. P. *in the Chair*,

Mr. LOWTHORP *preſented a* Propoſal *for Printing an* Abridgment *of the* Philoſophical Tranſactions. *This* Deſign *was approv'd by the* Society, *and he was deſired to* proceed *therein.*

Imprimatur,

May 12, 1705.

Iſ. Newton, R. S. Pr.

THE PHILOSOPHICAL TRANSACTIONS AND COLLECTIONS,

To the End of the Year MDCC,

ABRIDGED,

AND

Diſpoſed under GENERAL HEADS.

London, Eng. – Royal Society.

In THREE VOLUMES.

By *JOHN LOWTHORP*, M.A. and F.R.S.

The FIFTH EDITION, Corrected,
In which the LATIN Papers are now firſt tranſlated into ENGLISH.

LONDON,
Printed for W. Innys, R. Ware, J. and P. Knapton, S. Birt, E. Comyns, D. Browne, T. Longman, H. Whitridge, C. Hitch, J. Hodges, S. Austen, R. Manby and H. S. Cox, A. Millar, J. and J. Rivington, and J. Ward.
MDCCXLIX.

LSoc 1816.10.5 (1)

HARVARD
UNIVERSITY
LIBRARY

TO HIS
Royal Highneſs
THE
PRINCE,
Lord High Admiral of England.

SIR,

YOUR *Royal Highneſs's* great Condeſcenſion to ſubſcribe the Statutes of the *Royal Society*, as one of their *Fellows*, commands ſome Tribute from every Member.

This, Sir, I humbly hope, will in ſome meaſure excuſe the Preſumption of laying theſe *Papers* at Your *Royal Highneſs's* Feet. They have unavoidably, in this Abridgment, loſt much of their Original Beauty: But the *Order*, wherein the remaining Subſtance is diſpoſed, will give Your *Royal Highneſs* a nearer Proſpect of the Courſe of thoſe Studies you have the Goodneſs to *protect*.

It was a Noble Deſign, worthy of their *Royal Founder*, by *Incorporating* this *Society* to perpetuate a *Succeſſion* of *Uſeful Inventions:* But the diſcouraging Neglect of the Great, the impetuous Contradiction of the Ignorant, and the Reproaches of the Unreaſonable, have unhappily retarded them in their Purſuit of thoſe great Ends. To *reſtore* them therefore to their firſt Vigour, is a Glory reſerved for Your *Royal Highneſs:* And already, Sir, we feel the chearful Influence of a returning Spring.

The Commands Your *Royal Highneſs* has given, for Publiſhing, at Your own Expence, a moſt magnificent *Uraniography* (far exceeding that of all the *Arabian Princes*, the Noble *Tycho Brahe*, and the Induſtrious *Hevelius)* cannot fail of ſurpriſing Effects. All *Art* and *Nature* will exert their Powers upon this Occaſion, to keep pace with *Aſtronomy*; particularly, *Navigation* (being under Your *Royal Highneſs's* immediate *Care)* will induſtriouſly apply thoſe *accurate Obſervations* to all *Nautical* Purpoſes, and by ſome familiar Method deliver the anxious *Seamen* from the *fatal Accidents* that frequently attend their *miſtaken Longitude.*

Thus, SIR, the *Munificence* of the *Prince*, and the *Vigilance* of the *Lord High Adimral*, will be equally a Bleſſing to the preſent, and to all future Ages.

May Your *Royal Highneſs* ever meet with Returns of Gratitude, ſuitable to the univerſal *Beneficence.* So wiſhes, with great Sincerity,

May it pleaſe Your ROYAL HIGHNESS,

Your ROYAL HIGHNESS'S

Moſt Faithful, *and*

Moſt Obedient Servant,

John Lowthorp.

THE

PHILOSOPHICAL TRANSACTIONS

AND

COLLECTIONS,

To the End of the Year MDCC,

ABRIDGED,

AND

Diſpoſed under GENERAL HEADS.

VOLUME I.

Containing all the

MATHEMATICAL PAPERS.

By *JOHN LOWTHORP*, M. A. and F.R.S.

The FIFTH EDITION, Corrected,
n which the LATIN Papers are now firſt tranſlated into ENGLISH.

LONDON,
Printed for W. Innys, R. Ware, J. and P. Knapton, S. Birt, E. Comyns,
D. Browne, T. Longman, H. Whitridge, C. Hitch, J.
Hodges, S. Austen, R. Manby and H. S. Cox, A. Millar,
J. and J. Rivington, and J. Ward.
MDCCXLIX.

To the HONOURABLE
Sir *ISAAC NEWTON*, Kt:
PRESIDENT;

And to the
COUNCIL and FELLOWS
OF THE
Royal Society of London
For the Advancement of
NATURAL KNOWLEDGE,
THESE
MATHEMATICAL PAPERS,
Abridged and Diſpoſed under GENERAL HEADS,

Are moſt humbly Dedicated, by

John Lowthorp.

THE

PREFACE.

T*HE* Philoſophical Tranſactions *having met with* general Applauſe *and* Encouragement *for many Years, it would be a* needleſs Trouble *to give any* Hiſtory *of them: 'Tis enough to ſay, that many of the* Diſcourſes *were* compoſed, *and all of them* collected *and* publiſhed *by* particular Members *of the* Royal Society. *I ſhall therefore employ theſe very few Pages, only to acquaint the* Reader *with my own* Conduct *in this* Abridgment *of them.*

When I firſt reſolved *upon this* Undertaking, *I had two Sorts of* Readers *in view, whom I was* deſirous *to ſerve;* thoſe who make uſe of Books for their private Inſtruction or Entertainment, and thoſe who conſult them in order to publiſh ſomething of their own. *To a* Reader *of the former Claſs, I thought it ſufficient to give him the Subſtance of ſo many* curious Papers, *in ſuch* Order *as would beſt ſuit with the* Courſe *of thoſe* Studies *that might denominate him a* general Scholar. *But, for the Sake of the latter, I have, in the Margin, given the* Title *and* Author *of each Paper, and directed to the* Number *and* Page *of the* Tranſactions *or* Collections, *where he may meet with the* Original *itſelf. To the former, I* deſigned *this* Abridgment *to be as uſeful as the* Volumes *at large; and to ſerve the latter, inſtead of a not* inconvenient Repertorium: *And in the Proſecution of this Deſign, I have generally* confined *myſelf to theſe Rules.*

I. *I have not only* retained *the Eſſential Parts of the* Diſcourſes, *but I have kept in many Places to the very* Words *of their own* Authors, *(except where I was* forced *to vary them a little, to preſerve the* Connection:) *For, I thought it very unwarrantable to* obtrude *any thing of mine under the Name of another Perſon.*

II. *But to* ſhorten *the* whole Work, *wherever I found any* Perſonal Addreſſes, *long and unneceſſary* Excurſions, *or* pompous Citations *of* Books, *I have taken the Liberty to* ſuppreſs *them; yet, I hope, without injuring the* Force *of the* Author's Reaſoning.

III. *I have* omitted *all* Accounts *and* Extracts *of* Books, *which now, after ſo many Years* Publication, *ſeem almoſt uſeleſs: Yet, to put the* Readers in mind *of them, eſpecially ſuch as are about to furniſh or enlarge their* Libraries, *I have* added *a* Catalogue *at the* End *of each* Chapter *to which they chiefly belong; and I have alſo* directed *them to ſuch* Additions, Emendations, *or* Refutations, *as ought to be* conſulted, *when thoſe Books fall under their* Examination.

IV. *I have alſo* omitted *all* Heads *of* Inquiries *and* Experiments *ſimply propoſed, without farther* Proſecution; *believing that the* Anſwers *already given to many of them, and other* Diſcourſes *upon the ſame, or the like* Subjects, *will ſufficiently direct the* Notice *of an* Inquiſitive Reader.

V. *The previous* Calculations *of* Eclipſes, Lunar Appulſes, *and* Satellite Eclipſes *and* Occultations; *alſo* Tide-Tables, *and many other curious* Papers *of that kind, have long ago outlived the Reaſon of their* Publication.

VI. *All Simple* Catalogues *of* Natural Curioſities *(as of* Shells, Minerals, Plants, Animals, *&c.) without particular* Deſcriptions *of them, are little* inſtructive: *and chiefly* *ſerve to* enlarge *the* Hiſtory *of the* Muſeum, *where they are* depoſited: *Which is no Part of the* Deſign *of theſe* Volumes.

VII. *I have commonly* omitted *ſuch Papers as have been* collected *into juſt* Volumes *by their own* Authors. *For this Reaſon I have* omitted *ſome of thoſe ſurprizing* Microſcopial Diſcoveries *by the* famous *M.* Leeuwenhoeck: *But I farther confeſs, I was alſo leſs inclined to inſert them here, becauſe moſt of them treat of* Subjects *not at all convenient* (in my Opinion) *for* common Readers.

VIII. *But, to do all the* Right *I could to the* ingenious Authors *of thoſe* Papers, *which the* Limits *of this* Abridgment *obliged me to* omit, *I have, at the* End *of each* Chapter, *annexed their* Titles, *and ſometimes a ſhort* Account *of them.*

Theſe are the Rules *I have carefully obſerved thro' the whole* Conduct *of this* tedious Work: *Wherein I have faithfully aimed at the* General Good *of all ſorts of* Readers; *if I have failed in the* Performance, *'tis for want of* Judgment *to do it* better: *But I am bold to ſay, That if a kind* Reception *of this ſhall encourage a like* Abridgment *of the* Foreign Philoſophical Journals, *in the* ſame *or a* better Order, *it will much facilitate the many* Diſcoveries *ſtill ready to* reward *the* Labours *and* Expences *of all* induſtrious Promoters *of* Natural Knowledge.

Adver-

Advertiſement of the Bookſellers

TO THE

FIFTH EDITION.

THE *Philoſophical Tranſactions*, ſince the Year 1700, having been abridged, and diſpoſed under General Heads, (after the ſame Method with this Work) by Mr. *JONES*, and by him publiſhed in Two Volumes: We thought it would tend to make the whole complete, if the ſeveral *Indices* of the Five Volumes were made into One; by which means the Reader will have but one Trouble in ſeeking any Particular he has occaſion for. To the ſame Purpoſe we have thrown the *Contents* of the ſeveral Volumes together, and prefixed them to the Firſt.

THE

PREFACE.

THESE two Volumes being only a Continuation of a Work, which has been long ſince very well received in the World, it will be needleſs to give a minute and particular Account of my Conduct herein: It is ſufficient for me to obſerve, that I have generally followed Mr. LOWTHORP's *Method*; not only as it is a Method the Reader is already acquainted with, but as it is a very good one; that I have endeavoured to diſpoſe of the *Papers* in as tolerable an *Order*, as ſuch a variety of independent Subjects would well admit of; that I have looked upon myſelf as, in ſome Meaſure, anſwerable for the Reputations of the ſeveral *Authors*, and do believe there is nothing omitted, but what the *Authors* themſelves would have omitted, if it had been their Buſineſs to have abridged them; and that upon the whole I have taken an uncommon Care and Pains in correcting the Work.

I ſhould not have troubled the Reader with any thing more, if another Undertaking of the ſame Kind had not lately appeared in the World, performed by Mr. *Motte* a *Printer*, and recommended by the learned Dr. *Halley*. I think it not improper to obſerve to thoſe, whom the Name and Character of Dr. *Halley* may influence, that his Recommendation is drawn up in general Terms, *that he had* inſpected *that Epitome, and* believes *it is done with due Care and Judgment*; and that the Recommendation is dated *July* 26, 1720. *before* the Work was *printed*; ſo that the *Doctor* has *no* Share in thoſe Remarks, which I may make on the *Deficiencies*, *Omiſſions*, and *groſs Errors* of that Performance.

As I have *omitted* ſome Papers, which Mr. *Motte* has *inſerted*, I ſhall firſt mention what thoſe are, with my Reaſons for *omitting* them; and ſhall take Notice of thoſe which he has *omitted*, which are *inſerted* by me.

I have *omitted* Mr. *Cotes*'s *Logometria*, becauſe it is now in the Preſs with ſome other Pieces of the ſame Author.

Dr. *Halley*'s Synopſis of the *Aſtronomy of Comets* has been often printed, by itſelf, at the End of Mr. *Whiſton*'s *Prælectiones Phyſico-Mathematicæ*, with a Comment, and with the *Engliſh* Tranſlation of Dr. *Gregory*'s Aſtronomy.

Dr. *Keill*'s Theorems relating to the *Infinite Diviſibility of Matter* are printed in the *Third Edition* of his *Introductio ad veram Phyſicam*.

The Account of the *Contagious Diſtemper amongſt the black Cattle* in *Italy* is only an *Extract* from a *printed* Treatiſe of Dr. *Ramazzini*'s.

Dr. *Drake*'s Diſcourſe on the *Influence of Reſpiration on the Motion of the Heart* is the Subſtance of a Chapter in the *Doctor*'s *Anatomy* of *Human Bodies*.

Dr. *Muſgrave*'s Diſſertations, on *Britain*'s having been formerly a *Peninſula*, on the *Roman* Legions, on the *Roman* Eagles, and his *Commentary* on an *Inſcription* found at *Arragon*, are printed amongſt other Pieces in the *Doctor*'s *Collection* of *Antiquities*, in four Volumes in *Octavo*.

Mr. *De Moivre*'s Treatiſe *De Menſura Sortis* has been publiſhed in *Engliſh*, with conſiderable Additions and Improvements: And thoſe *Experiments* of the late Mr. *Haukſbee*, which I have omitted, are printed in the *ſecond* Edition of his *Phyſico-Mechanical Experiments*. Theſe are all the Papers *omitted* by me, which Mr. *Motte* has *inſerted*.

I proceed now to inform the Reader of thoſe Papers *omitted* by him, which I have *inſerted*; moſt of which are extant no where elſe, except in the Original *Tranſactions*.

Mr. *Craig*'s Solution of a *Problem* (on *Curves*) propoſed by Mr. *John Bernoulli*.

A *Letter* from Monſieur *L'Abbé Conti* to Mr. *Leibnitz* (concerning the *Invention* of the *Method* of *Fluxions*) with Mr. *Leibnitz*'s Anſwer.

Dr. *Gregory*'s Vindication of Mr. *J. Gregory*.

Dr. *Taylor*'s Apology for himſelf againſt *Bernoulli*.

Theſe Papers, Mr. *Motte* ſays in his Preface, *were intended rather for an honeſt Repreſentation of Matter of Fact to Poſterity, than for the immediate Inſtruction of the Studious in thoſe Sciences:* Whereas I apprehend, that they were deſigned too for the *preſent*

Age, and deſerved a Place in his Work, as much at leaſt as thoſe other Papers which he has inſerted, which have been ſo often *Reprinted.* But to proceed with Mr. *Motte*'s Omiſſions.

The Account of Mr. *Wilſon*'s *Pocket-Microſcope.*

A Paper on the Uſefulneſs of the *Obſervations* of the *Occultations* of the *fixt Stars* by the *Moon*, towards finding the *Longitude.*

The *Invention* of making *Clocks* to keep *Time* with the *Sun*'s apparent *Motion*, aſſerted by Mr. *Williamſon.*

The two *Experiments* of Mr. *Haukſbee*'s of *firing Gunpowder* in *Vacuo*, with the next of the Deſcent of *Malt-Duſt* in *Vacuo*, are left out in *both* the *Editions* of his *Experiments.*

Dr. *Wallis*'s Paper on the *Invention* and *Improvements* of the *Mariner*'s Compaſs.

Mr. *Haukſbee*'s Experiments of the *Power* of the *Loadſtone* at *different Diſtances* are only in the *Second* Edition of his Book of *Experiments*, and not in the *firſt*; and are *omitted* by Mr. *Motte*; I *inſerted* them becauſe they are *referred* to in the Paper which follows them.

A Letter of Dr. *Mather*'s from *New-England*; confirming an Account formerly given of the *Direction* of *Ship Compaſſes* being *changed* in a *Storm* of *Thunder* and *Lightning*; this I have placed in the *Chapter* of *Voyages*, amongſt ſome other *Letters* of the ſame *Author* from *New-England*; the Particulars of what I have given there in *one* Paper, are *diſperſed* by Mr. *Motte* in *different* Parts of his Work; he might have done the ſame (if he had pleaſed) with ſome other Papers, which he himſelf has placed in the *Chapter* of *Voyages.*

The next that occurs, is a Paper of Mr. *Derham*'s, on the *Migration* of *Birds.*

The large *Anatomical Account* of the *Elephant*, though it has been printed (as Mr. *Motte* ſays) by itſelf; yet, as it is ſome Years ſince it was printed, and on a Subject of which ſo few have wrote, it will not, I hope, be thought a Burden to the Reader: To that Account I have added Mr. *Lewenhoeck*'s Obſervations on the *Skin* of the *Elephant*, &c.

A Paper of Dr. *Liſter*'s on an Experiment of his of *powdered Blue* paſſing the *Lacteals*, is omitted by Mr. *Motte*, with this Title, A Letter from Dr. *Liſter* to Dr. *Tancred Robinſon*, commending the *Doctor's* Book on *Fevers*; whereas Dr. *Robinſon* never wrote any Book on *Fevers.* Blunders of this kind are frequent with Mr. *Motte*, and I take Notice of this only by the by.

Mr.

Mr. *Lewenhoeck*'s Obſervations on the *Fibres* of the *Muſcles*, and Mr. *Muys*'s Obſervations on the *Texture* of the *Muſcles*.

Remarks on a MS. in *Phyſick* at *Florence*.

The Account of an *Iſland* thrown up out of the *Sea* in the *Archipelago*, by Monſieur *Bourguignon*. Mr. *Motte* ſays, moſt of the Particulars of this Paper are contained in Father *Goree*'s Account of the ſame: whether they are, or not, I leave the Reader to judge.

Mr. *Lewenhoeck*'s Obſervations on the *Animalcula* found on *green Weeds* growing in *Water*, are inſerted by me, becauſe they are *referred* to in the *Paper* which follows them. I am ſorry that I have been forced to be ſo tedious on this Head; but it will ſave thoſe, who are deſirous of ſeeing the Difference of our Performances in this Particular, a much greater Trouble.

And here it may not be improper to remark, that in the *Accounts* of *Books* (which are often given by the *Authors* themſelves) there are ſometimes *Corrections* and *Additions* to thoſe *Books*, extant no where but in the Original *Tranſactions*: where any thing of this Kind has occurred, I have *inſerted* it: theſe Mr. *Motte* has thought fit entirely to *omit*: the Reader need go no further than to the *End* of the *Firſt* and *Second* Chapters for Inſtances of this Kind.

As to the *Order*, into which Mr. *Motte* has *thrown* the Papers, though I have many Objections to make to it, I ſhall trouble the Reader only with a few; ſufficient be it to mention, that he has placed Dr. *Keill*'s Paper on the *Laws* of *Attraction*, and another on the *Infinite Diviſibility* of *Matter*, &c. under the firſt Chapter of *Geometry* and *Algebra*.——A Paper on the *Spleen* in the Chapter of the *Thorax*, which all Anatomiſts have thought fit to place in the *Abdomen*.—An Account (of Dr. *Freind*'s) of an uncommon Kind of *Convulſions*, and another of Dr. *Leigh*'s on *Epileptic* Fits (which I have placed in the *Chapter* belonging to the *Diſeaſes* of the *Head*) Mr. *Motte* has placed in the *Chapter* of *General Affections of the Body, and Caſes not reducible to any Heads*. So amongſt *Antiquities* he has placed an Account of an *Impreſſion* of a *Skeleton* in *Stone*, and ſome other Papers relating to the Changes our Earth has received; which belong to *Natural Hiſtory*, and might be placed properly enough in the Chapter of *Mineralogy*. It would be tedious to recount the many *Abſurdities* on this Head, and might perhaps to ſome ſeem too minute and trifling; who will themſelves eaſily ſee, at how *wide* a *Diſtance* he has placed many Diſcourſes, which have a *near* Relation to each other; and might have been induced to excuſe the

Want of *Method*, if the Papers had not *abounded* with *Miſtakes* of *Conſequence*; and if the very *Errors* of the *Originals* had not been faithfully *tranſprinted* in this *Edition*.

As to the *Errors* in the *Mathematical* Part, I ſhall not mention either thoſe, which he has *copied* from the *Originals*, or thoſe of his *own* making, as it may be the Occaſion of a Diſpute, which may be carried on in an *improper* Manner, and of which every one will not be a Judge: I ſhall, therefore, at preſent only mention (what is full as *material*) the Errors which occur in the *Figures* belonging to that Part of the Work.

In the *firſt* Plate, and in the *ſixth* Figure, F is wanting, and c is put for *C*.——In the *ninth* Figure E is wanting.——In the *twenty third* Figure S is wanting, and T is wanting.——In the *twenty fourth* Figure I is put for T.——In the *thirty fifth* Figure O is wanting.—In the *thirty ninth*, l is put for i.——In the *fortieth*, l is put for i, and *B* is wanting.——In the *forty firſt*, Φ is put for F.——In the *forty ſecond*, f is put for c, and R is wanting.——In the *Fiftieth* Figure, O, m, n, and the *Triangle* belonging to thoſe three Letters is left out.——In the *fifty ſecond*, k is left out, and v is put for L.——In the *fifty ſixth* A is left out.——In the *fifty eighth*, (to which *Figure* he makes no Reference in the Diſcourſe belonging to it, but refers to *Fig*. 56. by a Miſtake) B is wanting.—In the Figure marked C (in the ſame Plate) F is wanting. Theſe are *ſome* of the Faults in the *firſt* Plate only; other Plates have *as many* Faults; it is a common Thing with Mr. *Motte*, to leave out *Letters* of *Reference*, to place others at *wrong* Angles, and to put *great* Letters for the *little* ones of the ſame Signification; ſo that there are often in the *ſame* Figure two *great* Letters of the ſame Signification, when one of them ought to have been a *little* one; and ſometimes even *Lines* and *Angles* are left out.

I chuſe rather to mention theſe *Faults* of the *Figures* becauſe it has been *advertiſed* (though it is not affirmed in the *Preface* to the Book) that that Edition contained *above ſixty Original Copper-Plates*; which, if it be meant (as I ſuppoſe it muſt be) of the *Plates* uſed in the *Original Tranſactions*, is *falſe* in *Fact*.

I ſhall therefore conſider *one* of theſe *Original Plates*, which ſhall be the *fifth* Plate in the *Second Part* of the Work, which conſiſts of

Diſcourſes in *Anatomy* and *Phyſic*: The *Firſt*, *Second*, and *Third* Figures are thoſe, which were engraved for the *Tranſactions*; the other Figures (which I am to conſider) were new-engraved on the ſame Plate for the preſent Occaſion. In Page 101. of the *Second* Part there is an Account *of a Ball which contained a Plumb-ſtone within it extracted from the Anus*; three Figures are to be referred to, to explain this; one to ſhew the *Shape* of the *Ball* itſelf, another the *inſide* of it when it was cut into *two* Parts, and the other *Figure* the *Plumb-ſtone* which was *contained*: In the *Margin* Mr. *Motte* refers to *Fig.* 4, 5, 6. *when there is no Figure, which is numbered* 6 *in the* Plate: In another Diſcourſe *(Page* 104.) to which the ſame *Plate* belongs, there is an Account of a *Stone* which was voided by *Stool*, which was ſuppoſed to have obſtructed the *Ductus communis Bilarius*; in the *Paper* itſelf he tells you, that Y Z *ſhew the Proportions exactly drawn*, when there are no ſuch Letters as Y Z in the Plate, and in the *Margin* refers to *Fig*. 8. which *Figure* belongs to another Paper, and is much *larger* than this *Stone*; and the *Figure*, which in this Caſe ſhould be referred to, has no Number at all, and is, beſides, much *leſs* than the *Stone*, whoſe *Proportions*, he ſays, *are exactly drawn*. All this *Confuſion* ariſes from two or three Figures being not *numbered* rightly. There is another Miſtake too in this *ſame* Plate, *Page* 103, he refers to the *ſeventh* Figure, when there is no *Figure* which is *numbered* 7 in the *Plate*; the *Figure*, he ſhould have referred to, is numbered 10.——I ſhall take Notice only of *one* more Fault, which is in the *Plate* belonging to *Page* 248. in the Chapter of *Aſtronomy*, where the Letter *H* is left out; as this belongs to a Paper of Dr. *Halley*'s, Mr. *Motte*, I think, ought in *Gratitude* for his *Recommendation* to have taken more Care of it: He may ſay, this *Letter* was left out in the *Original Plate*; but this is not the only *Letter* of *Reference*, which has been *wanting* in the *Original* Plates, which I have *ſupplied*.

From the *Miſtakes* of the *Plates*, I come to conſider the *Errors* Mr. *Motte* has committed in the *Papers* themſelves; and as in the former Article my Remarks were not extended beyond two or three *Plates*, ſo here (how *large* a *Multitude* ſoever preſent themſelves to my View) I ſhall confine myſelf to a very *few* Pages; and I ſhall more particularly chuſe to inſtance in a *ſmall* Number of *thoſe* Miſtakes, which happened to be ſo in the *Tranſactions* themſelves, which Mr. *Motte* has copied with great *Fidelity* and *Care*. And happy would it be for him, if *thoſe* were the *only ones* in his Performance:

ſtance: *His* Edition might be even *ſaid* to be *correct*, if the *Errors* of the *Originals* were the *ſmalleſt Number* in it.——The firſt *Paper* I ſhall pitch upon, is one of Sir *John Floyer*'s (in *Part* II.) in which he has *copied* above *twenty* Errors of the *Originals*; *ſome* of which are as follow. *P.* 207. *Granum Caninum* for *Gramen Caninum.* P. 208. *Fenugreke*——*Zed-Oary* for *Zedoary*—— *have a volatile Oyl*, for *have not a volatile Oyl*——*Fænil*——Page 210. for *a Ranunculus* he puts it *aranunculus*, and in the next Page *Rapunculus.*——Page 213. *Sylveſtre Nardum* for *Sylveſtris Nardus.*—— *Xyrs* for *Xyris.*——Page 214. Line 24. for *Fumitory*, he has *Fumaterry*, and Line 27. *Fumiterry.*——Page 215. *Violatri Color* for *Viola Tricolor.*——So in the ſame *Part* of the Work *Page* 181. *Line* 23. he has *in ſpaſmos inordinatos fieri* for *ferri.*——In *Part* III. Page 211. Line 22. *Vial* for *Wall.*——In the ſame *Part*, Page 227. Line 32. *Eidem opinioni jovet* for *favet.*——Page 260. Line 42. *Exortam Turbinem.*——In *Part* IV. Page 16. *Tuque Teſtudo reſonare Septem Calida Nervis*; and a little lower *Cordas* for *Chordas.* Page 247. Line 31. *Glandulæ Muſcoſæ* for *Mucoſæ.* I had not taken notice of *theſe* Errors, if they had not been *all* of them (amongſt a very large Number of *greater* Conſequence) *Errors* in the *Originals* themſelves: And I entirely ſubmit it to the Judgment of the Reader, how *capable* Mr. *Motte* muſt be of publiſhing *ſuch* a Work *correct*, who has *copied* Errors ſo *groſs* and *glaring*, and ſo eaſily to be *diſtinguiſhed.* No one can think me under any *Obligations* to correct any *more* of his Miſtakes: By what I have already mentioned, I have given him an Opportunity of correcting *ſome*, as particularly thoſe of the *Plates*; which, as they may be *ſoon* corrected and altered, I deſire the Reader (who has thoſe Books already) to take Notice, that thoſe Faults are there at *preſent.* Mr. *Motte* in his *Preface* (as if conſcious of his Errors) in order to excuſe *the Imperfections of his Work*, pleads the *Number of Papers, and Variety of Subjects*; I am very ready to admit of his Excuſe: And (as I have found the *Latin* Papers more faulty than the reſt) I beg leave to offer another, The *Variety of Languages.*——Nor is even the *low* Work of his *Indexes* free from *Miſtakes:* In the *Index* of the *Authors Names*, as he had given to ſome *wrong Titles*, ſo he has left out near *twenty* Names entirely.

But that Part of his *Management* of this Work, to which I could make the *moſt* Objections, is what I muſt not give myſelf leave to make *any* Remarks upon; which is, the *unwarrantable Liberty* he has taken in *ſome* Papers of *omitting* ſome Things, which, I think, deſerved more Regard than he has paid to them. But, as it might give Offence to others, I muſt deſiſt from being particular on that Head: Though I can with great Conſiſtence at the ſame Time affirm, that I *might* have taken the Liberty of *omitting* ſome Things which I have *inſerted*, if I had not thought that I had met with an Adverſary, who would place his chief Merit in *reprinting* all that came to his Hands.

And now I think it is high Time to come to a Concluſion, and to make an End of this *ungrateful* Taſk of *Criticiſing:* And as in making thoſe Remarks, which I thought *neceſſary*, I have abſtained from all *perſonal Reflections*, and have confined myſelf to *thoſe*, which more *immediately* ariſe from the *Performance* itſelf; ſo I ſhould not have been ſo *free* as I have been in my *Animadverſions*, if Mr. *Motte* had not ſpoke of me in his *Preface* in ſuch Terms, as, I think, I have a juſt Exception to. I have ſaid, I think, enough, to ſatisfy thoſe, who have not ſeen his Performance; and to thoſe, who have but *looked* into it, all that I have ſaid is needleſs: And as the World has been *beforehand* with me in their *Cenſure* of this Work, I ſhall leave him to enjoy thoſe imaginary *Advantages, which may accrue to himſelf and the Public from his Performance* (Preface, *p.* 5.) Neither ſhall I any farther indulge the ill-natured Pleaſure of triumphing over an expired *Author.*

THE
Mathematical Papers
PUBLISH'D and DISPERS'D
IN THE
Philoſophical Tranſactions
AND
COLLECTIONS,
ABRIDG'D,
AND
Diſpos'd under GENERAL HEADS.

CHAP. I.
Geometry, *Algebra*, *Arithmetick*, *Logarithmotechny*.

I. 1. AS to what I formerly conſider'd, about the Improvement of the Mathematical Sciences, the Reſult was chiefly this. While Men are deſtitute of Inclination, Genius, Aſſiſtances, and Leiſure neceſſary for theſe Studies, no wonder if they make no greater Progreſs in them. Therefore it ſeems probable to me, that by the Help of the following Means, a tolerable good Remedy may be found for this Evil. That is, if,

Dr. J. Pell's *Idea of the Mathematicks. An.* 1638. *Phil. Col.* N. 5. *p.* 127.

§. 1. A Mathematical Monitor (as we may call it) be compoſ'd, which may give proper Anſwers to theſe three Queſtions. 1. What Advantages and of what Kind, may be expected from the Study of Mathematicks? 2. What Helps are now in being for attaining ſo advantageous a Knowledge? 3. What Order is to be obſerved in making uſe of thoſe Aſſiſtances? Therefore this Monitor ſhould contain,

1. An eaſy and perſpicuous Diſcourſe upon the Limits or Extent of the Mathematical Arts, and of the conſiderable Advantages that will accrue, not only to the Perſons themſelves that ſtudy them, but likewiſe to a Nation that abounds in ſkilful Mathematicians.

2. A Catalogue of Mathematicians, and of Works publiſhed by them; which is to exhibit, 1. A Synopſis of all Kinds of Mathematical Books, whether ſuch as are already publiſh'd, or ſuch as are yet unpubliſh'd, and, being in Manuſcript, lie concealed in publick Libraries; proper Numbers or References being affix'd to every Kind. 2. A Chronological Catalogue of all the celebrated Mathematicians, diſpoſed according to the Ages in which they flouriſhed; always ſubjoining the Year of our Lord in which their Works were firſt printed. 3. A Catalogue of the ſame Works, according to the Series of Years in which they were printed in any Language. In digeſting of which I would proceed in ſuch Manner, that marking the Year of our Lord, I would add (as in common Catalogues) the Names of all the Mathematical Books that were publiſh'd that Year, in any Country or any Language; 1. Shewing in each how much the Volume contain'd, by marking not only whether it was in Folio, Quarto, &c. but the whole Number of Pages, ſo that the Bulk of the Work might eaſily be known. 2. Before the Title mentioning the Year, to which any one might turn back, who ſhould deſire to know when the Book was wrote, and when it was laſt publiſhed in any Language. 3. Marking in the Margin after the Title, 1. The Year in which any Work was laſt printed. 2. The Number referring the Reader to the Synopſis, which was given in the firſt Page of the Catalogue. Now by the Help of theſe Numbers any one might eaſily and readily run through all the Mathematical Books belonging to one Subject.

3. ~~An Admonition to the Studious, which are the beſt Books~~ in every Kind, in what Order and Method they are to be read, what is to be choſen, and what omitted in reading ſome of the minor Mathematicians, how we are to proceed ſo as to retain every Thing in Memory.

4. An Exhortation and Encouragement to all thoſe, who are ſufficiently provided with Wealth, Opportunity, and Ingenuity for the Purſuit of theſe Studies; that, 1. Having Regard to the great Advantages that redound from hence not only to themſelves but to all Mankind in general. 2. As likewiſe to that pure and ſincere Pleaſure which ariſes from the Search of hidden Truths, and from ſtriving with difficult Problems and the Conqueſt of them; that they may ſeriouſly apply themſelves to the Advancement of Science, and ſo much the rather, as, 3. More expeditious Methods are now found out than were known to our Anceſtors, which ſave us much Labour, Time, and Expence. Then an Exhortation to all ſuch as are eminent for ſetting a right Value on theſe Studies, and are likewiſe diſtinguiſhed for Power and Wealth (which ſurely may be made inſtrumental to perpetual Fame, if prudently diſpenſed) that they

may become Patrons to ingenious Men of this Kind, by propoſing handſome Rewards to the moſt deſerving of them, to encourage them to complete ſuch Diſcoveries as their own Genius's may prompt them to. Laſtly, to all Princes and Commonwealths, who cannot eaſily procure a greater Ornament to their Dominions, than by making it their Endeavour, 1. That they may abound with Perſons ſkilled in theſe Arts. 2. That the Way leading to them may be made as little laborious and expenſive as poſſible. 3. That Mathematical Genius's may be more publickly known, and meet with ſuch Encouragement as they ſhall deſerve.

For this end it with be very neceſſary that, will

§. 2. A Publick Library may be founded, which may be furniſh'd with all the Books abovementioned, and with one Inſtrument of every Sort that has been yet invented; and beſides may have an Endowment ſufficient, 1. To purchaſe Copies of all the mathematical Books that ſhall be yearly publiſh'd any where abroad. 2. To maintain a Library-keeper, whoſe Buſineſs it ſhould be,

1. To read over all the Books of this kind, which are publiſh'd in his own Country. 1. Suppreſſing thoſe which are not wrote according to the Rules of Art, that their Miſtakes may not lead their Readers into Error. 2. To admoniſh Authors, leſt they ſhould only republiſh Things already known, and treated of by others.

2. On Peril of their Reputation that they ſhould approve of notable Inventions, and heartily recommend the Inventors to proper Patrons.

3. To receive, to enter into their Catalogue, and diſpoſe in their proper Repoſirories, one Copy of the Books ſo read over; when preſented to the Library well bound up at the Charge of the Author or Bookſeller.

4. To give a civil and ready Anſwer to any ſtudious Perſon, who ſhall conſult him about any Problem, whether it is already ſolved or no; leſt he ſhould attempt any Thing that is well done already, or on the contrary ſuppreſs his Diſcoveries, out of Fear they may be already known, and perhaps diſcuſſed in ſome of the Books of the Library.

5. To receive, &c. all Manuſcripts that may be preſented to the Library, or bequeathed to it by Legacy.

6. To keep a conſtant Literary Correſpondence with all Perſons of this Kind, that reſide in foreign Countries; leſt he ſhould be ignorant of what Books are publiſh'd there.

7. To take Notice among his Countrymen, who are fitteſt and moſt expert in inſtructing others in theſe Arts.

8. To have an Acquaintance with all kinds of Artificers, who excel in the conſtructing of Mathematical Inſtruments and Contrivances, whether they work in Wood, Loadſtones, Metal, Glaſs, &c.

9. After a fair Trial to give their Teſtimony, both of ſpeculative Knowledge and practical Dexterity, to practical Men of all Kinds, whether Maſters of Ships, Surveyors, Accomptants, &c. that ſuch as have Occaſion for this Kind of Men may not be impoſed on by ignorant Pretenders, to their great Loſs.

The Catalogue will eaſily inform, which in ſuch a Multitude of Books that almoſt overwhelm the World, belong only to this Kind of Study. The Library will exhibit a Copy of every ſuch Book, and inform where more Copies may be bought. It will alſo be a Kind of Storehouſe both to Natives and Foreigners, whence they may eaſily learn, what Aſſiſtances that Country can ſupply to theſe Studies.

And this, in my Opinion, is the readieſt Way of making uſe of the Helps we are already in Poſſeſſion of. If more are wanting, it will be neceſſary, that by the Aſſiſtance of ſkilful Artiſts,

§. 3. The three following new Treatiſes may be compoſed and publiſhed.

1. Mathematical Pandects, containing, as perſpicuouſly, methodically, compendiouſly, and ingeniouſly as can be done, whatever may be collected, or deduced by Way of Corollary, from the Mathematical Books or Diſcoveries made before our Time; quoting the moſt ancient Authors in which they are found, at the End of every Period or Propoſition; and ſo marking in all the following Authors where they have been caught in a Theft; or where they have borrow'd without making any Acknowledgment, or (what is worſt of all) have boldly claimed to themſelves the Inventions of others. By this means that large Library would be contracted into a much narrower Compaſs, to a great ſaving of Labour, Time, and Expence for thoſe that come after; and this much more than any one would imagine at preſent. But now ſince this Work would hardly make a portable Volume, there ſhould be prepared alſo,

2. A Mathematical Companion, containing in a Manual (and therefore as conciſely as may be) all the moſt uſeful Tables, with Precepts to ſhew their Application to ſolving of Problems, whether of pure Mathematicks, or applied to other Subjects.

Finally, that we may not always be confined to Books in this Kind of Learning, there ſhould be contrived,

3. The Self-ſufficient Mathematician, or an Inſtruction to ſhew how any Mathematician, who is no Enemy to Labour, may acquire ſo much Skill, that without the Aſſiſtance of Books or Inſtruments he may attain the Solution of any Mathematical Problem, and that as eaſily as another would ſolve it by turning over Books.

And this is that Idea of Mathematicks which, in my Manner, I have long ago figured to myſelf; being always firmly perſuaded, that then only we can hope for Aſſiſtance in great Undertakings, when we have conceived an exact Idea of them in our Minds, and of the moſt appoſite Means of putting them in Execution. And if we cannot expreſs this Idea in fact, yet it is ſomething to come as near it as may be. I imagine this is ſo far from being above human Power, that I think the Induſtry of one Man alone to be equal to it, who is not hindered by his own domeſtick Affairs, or immerſed in a Multitude of buſy Cares. For it is evident that the Library and Catalogue may eaſily be provided, if Money is not wanting. And as to the Pandects above deſcribed, if the Taſk of compoſing them were committed to me, I ſhould impoſe upon myſelf much ſeverer Conditions than I have mention'd there. For firſt I would delineate the infallible Proceſs of human Reaſon in the Inveſtigation of whatever it propoſes to itſelf, by ſhewing how it proceeds from the firſt Principles or Rudiments, by an uninterrupted Chain, to the moſt ſublime as well as the loweſt Application of them. Which Art perhaps Men would not be long without, if hereafter they ſhould carefully examine, by what Means ſuch Thoughts have aroſe in the Minds of certain Men whom they admire, how ſuch apt Means have been found out to attain ſuch an End. How theſe Pandects may be abridg'd into a Manual, ſuch as may be fit for common uſe, may not be difficult to underſtand. But ſo to fix them in their Minds, that they ſhall have no farther Need of Books (which is what is aimed at by our ſelf-ſufficient Mathematician) will be thought by moſt to exceed the Power of the human Mind; ſince no one that know of has yet ventured to conceive ſuch a Thing in his Mind. Yet I believe that Men will diſmiſs ſomething of their Incredulity, when they conſider ſeriouſly with themſelves what Arts have been found out for ſtrengthening the Imagination, for aſſiſting the Memory, and for directing the Reaſoning Faculty, and what wonderful Effects may be produced by their Conjunction and conſtant Exerciſe.

2. It ſeems to me much more adviſeable, if inſtead of all that Apparatus, which the Author of the Idea propoſes, about collecting the various Writings of the Mathematicians, only the beſt and moſt deſerving were ſelected. And firſt thoſe ancient Authors ſhould be choſen, whoſe Works are ſtill extant; as *Euclid*, *Apollonius*, *Archimedes*, *Theodoſius*, *Pappus*, *Ptolemy*, with their other Fragments and Manuſcripts which have not yet ſeen the Light, ſome of which are in *Golius's* Cuſtody at *Leyden*, and ſome are preſerved at *Rome*. Then the more modern may be added to theſe, as *Vieta*, *Clavius*, and our *Herigon*. In like manner among the Opticians, ſhould be choſen *Vitellio*, *Kepler*, *Aquilonius*, and *Dom. de Villes*. Among Arithmeticians after *Diophantus*, the beſt are *Cardan*, *Tartaglea*, and your Countryman *Nepper*. For Spherical Triangles, and their Computation by Logarithms, you have *Briggs*, *Gordan*, *Pitiſcus*, *Snellius*, and our *Morinus*. For Aſtronomical Matters, after *Ptolemy* and ſome *Arabians*, all thoſe ſhould be procured who have compoſed Tables, as *Alphonſus*, *John Regiomontanus*, *Kepler*, and our *Duretus*. And to be brief, for Fortification and Muſick, eight or ten Authors might be ſelected, who have moſt excell'd in the Practice. In like manner for Mechanical Affairs, the Forces of Motion, Machines, and Water-works; ſo that ten or twelve Authors being peruſed, thoſe that are curious in ſuch Matters may be eaſily ſatisfied. And if twelve Men of good Underſtanding, having a friendly Correſpondence with each other, would take this Affair upon them, ſo that each of them might compoſe a Treatiſe upon a Science, in one convenient and perſpicuous Volume; ſuch Things being ſupplied as are wanting in others, and unneceſſary Things being pared away; without doubt we might have in twelve Volumes, in a ſhort and nervous Manner, all that could be deſired in this Matter. And in my Opinion, whatever belongs to Mathematicks, either pure or mixt, might be comprehended in thoſe twelve Volumes. So one might deliver all the neater Part of Philoſophy in three Books, and all the liberal Arts or mechanic Arts in three more;

Conſider'd, Oct. 1639, by Merſennus, ibid. p. 135.

ſo that Learning might be attained at a ſmall Price. Now as to Mathematical Inſtruments, it would be of little Conſequence to have an Apparatus of all that have hitherto been invented. It would be better to have four or five which are beſt in their Kind, and which ſhall be judged to be more convenient than any other.

Anſwer'd by Dr. Pell, *ibid. p.* 137. 3. If I underſtand you right, learned Sir, you approve of all I have ſaid, only you think I require a greater Apparatus than is neceſſary. You are of Opinion, that not all the Mathematical Books and Inſtruments ſhould be collected, which I contend for, but only the beſt ſhould be ſelected. I ſhould not oppoſe this your Opinion, if it was agreed on by all, in ſuch a Multitude of Authors which was the beſt, which ſhould have the Preference before all others; and if I had this only in view, that ſomething of Labour and Expence might be ſaved to the Studious in theſe Arts. But ſince I made it my Wiſh, to have the moſt perfect Foundation for the whole Ambit of the Mathematicks, I ought to delineate no other Scheme, than what might completely anſwer ſuch a Deſign. The principal Part of ſuch a Deſign, as I perſwade myſelf, muſt conſiſt in ſuch an univerſal Library, as I have deſcribed. I ought not to deſpiſe any Attempt, much leſs to condemn any one unheard, that ſhall throw in his Mite, and endeavour to promote this Undertaking. And if I may give my Judgment, the moſt trivial Writing or Mathematical Inſtrument ſhould be preſerved, one Copy at leaſt even for its Errors, in ſome certain acceſſible Place of every Country. For we ſee many Things that have been ingeniouſly invented, in the ruder Inſtruments of former Ages, which are now not only worthy of obſerving, but even of being imitated: As ſome Writers of a lower Claſs may give very good Hints, and aſſiſt the Invention of thoſe of a happier Genius; for we can often point at an Excellence which we cannot arrive at ourſelves. We ſee many Lemmata that have been well demonſtrated by this Kind of Writers; yet becauſe of ſome one fundamental Fallacy, their whole Superſtructure has come to the Ground. If you think many are to be rejected, as well for their Trifling and Verboſity, as for Error and falſe Concluſions; you ſhould conſider how different are the Notions and Taſte of Mankind, nor ſhould you eſtimate others by your own Sagacity. For there are ſome who can underſtand nothing, unleſs repeated to them an hundred Times, and that almoſt in the ſame Words; thoſe Tautologies therefore are adapted to this Sort of People. And becauſe we muſt always begin from Things more known, but the ſame Things are not more known to all; we muſt make very different Beginnings: So that you can hardly find a Learner, but who may be aſſiſted by a rude Inſtrument or unpoliſhed Author, and therefore he that undertakes the Office of a Mathematical Monitor ſhould not be ignorant even of theſe. So that the complete Collection of Books before-mention'd ſeems to me to be quite neceſſary.

Now the more I am diſpleaſed with thoſe minute Mathematicians, the more I ſhould wiſh for a Library of this Kind, as being the only Method of curing that licentious Itch of Scribbling. For thoſe prating Pretenders, ever trifling in a childiſh Manner, while they would ſeem to accommodate themſelves to the Capacity of Youth; may ſee that there are already more than enough, who have compiled Rudiments of this Kind. And they who fondly aim to advance the Mathematical Sciences by an Infinity of new Diſcoveries, when they ſee ſo many empty Paradoxes which have been condemned and laughed at by the Publick, may take Warning by the Miſcarriages of others. But eſpecially the Plagiaries, thoſe Peſts of all good Literature, will not have the Impudence to vend as their own any old Books, or any Part of them, which perhaps have not been printed more than once. On the other hand, Men of Candour and Ingenuity, able to deliver their Thoughts in a handſome Manner, when they ſee ſo many have gone before them, on almoſt every Subject, will be caution'd not to produce any Thing to the Publick but what is new and their own. Now what has been treated of already may be eaſily known, either by conſulting ſuch an ample Library, or if they will not be at this Trouble, they may be inform'd by the Library-keeper himſelf, to whoſe Cuſtody I have committed it. And theſe are the Reaſons in general, which I cannot retract, why I ſhould prefer ſuch an Univerſal Library as is above deſcribed.

To the Satiſfaction of Merſennus, *Dec.* 10, 1639. *Ibid. p.* 143.

4. I had no ſooner read your Letter, learned Sir, but I became wholly yours, and was ready to ſubſcribe to your Opinion, which I intirely approve: And likewiſe an unuſual Ardour of Mind hurried me on: ſo that I would recommend this Undertaking of yours, great as it is, to the great ones of the World, if I could have free Acceſs to them. But where is the King, that will make a Beginning? For I cannot but call it a truly Royal Deſign.

The Judgment and Approbation of Des Cartes, *Feb.* 1640. *Ibid. p.* 144.

5. I inſpected the Mathematical Idea only by the bye, and now only remember, that I found nothing in it from which I ſhould diſſent; and I much approved that, firſt, an Inventory of the whole Mathematical Furniture was there exhibited, and then the Self-ſufficient Mathematician was deſcribed, as containing every Thing in himſelf. Almoſt in the ſame Senſe I am uſed to diſtinguiſh two Things in the Mathematicks, the Hiſtory and the Science. By the Hiſtory, I mean all that is already found, and is committed to Books. And by the Science, the Skill of reſolving all Queſtions, and therefore of finding by one's own Induſtry whatever can be found by human Ingenuity in that Science. He that poſſeſſes this Faculty does not much want Foreign Aſſiſtance, and therefore may very properly be call'd ſelf-ſufficient. Now it is much to be wiſh'd, that this Mathematical Hiſtory, which lies diſperſed in many Volumes, and is not yet intire and complete, were to be all collected into one Book. Nor for this Purpoſe would there be any Occaſion to be at the Charge of ſeeking or purchaſing Books. For ſince Authors tranſcribe many Things from one another, nothing is extant any where which may not be ſomewhere found in any Library that is but moderately furniſh'd. Nor is Diligence in collecting all Things ſo neceſſary, as Judgment to reject what is ſuperfluous, and Knowledge to ſupply ſuch Things as are not yet found out. Now if ſuch a Book were at hand, from thence any one might learn the whole Mathematical Hiſtory, and a good Part of the Science alſo. But if any one ſhould deſire to have the whole that belongs to the Practice, as Inſtruments, Machines, Engines, &c. if

he was a King, and had the Wealth of the whole World at Command, he could not ſupply the neceſſary Expences. Neither indeed is there any Occaſion for them. It is enough if he can deſcribe them all, and either knows how to make ſuch as are wanting himſelf, or can ſet Artificers to work upon them.

Some of Euclid's *Propoſitions, demonſtrated independently from the reſt, by Mr.* Aſh. N. 162. *p.* 672. 32. 1. *E.* II. THE Propoſitions which I ſhall endeavour to demonſtrate independently from all others, ſhall be theſe; the 32d and 47th of the *Firſt Book*; moſt of the *Second* and *Fifth Books*; the 1ſt and 16th of the *Sixth*; with their Corollaries. In order to demonſtrate the 32d; I ſuppoſe it known what is meant by an Angle, Triangle, Circle, External Angle, Parallels, and that the Meaſure of an Angle is the Arch of a Circle intercepted between its Sides; that a Right Angle is meaſur'd by a Quadrant, and two Right Angles by a Semicircle. *Fig.* 1. I ſay then, that in the Triangle A B C, the External Angle B C E is equal to the two oppoſite Internal ones A B C, B A C; for let a Circle be drawn, C being the Center, and B C the Radius; and let C D be drawn parallel to A B, thoſe two Lines being always equidiſtant, will both have the ſame Inclination to any third Line falling upon them; that is, (by the Definition of an Angle) they will make Equal Angles with it: For if any Part of C D (for Inſtance) did incline more to B C than to A B, upon that very Account they would not be parallel; it follows therefore that the Angles A B C, B C D, are equal: Alſo B A C = D C E, becauſe A E falls upon two Parallels; but the External Angle B C E = B C D + D C E, which was before prov'd to be equal to A B C, B A C. *Q. E. D.* Hence may be inferr'd as a Corollary, That the three Angles of every Triangle are equal to two Right ones; for the Angles A C B + B C E, are meaſur'd by a Semicircle, and therefore equal to two Right Angles: Corollaries alſo from 20, 22, 31. 3. E. hence are the 20th, 22d, and 31ſt of the *Third* Book, which contain the Properties of Circles; whoſe Deduction from hence is moſt natural and obvious.

47. 1. E. In order to demonſtrate the 47th, I ſuppoſe the Meaning of the Terms made uſe of, to be known; and that two Angles or Superficies are equal, when one being put on the other, it neither exceeds, nor is exceeded. This being allow'd, I ſay, the Sides about the Right Angle are either equal or unequal; if equal, let all the Squares be deſcrib'd; the whole Figure exceeds *Fig.* 2. the Square of the Hypotenuſe B C, by the two Triangles M, V, and exceeds alſo the Squares of the other two Sides, A B, A C, by the two Triangles, A B C, and S; which Exceſſes are equal, for M is equal to A B C, the two Sides about the Right Angle, being two Sides of a Square upon A B, by Suppoſition equal to A C, and the third Side equal to B C; therefore the whole Triangles are equal. After the ſame manner, S and V are proved to be equal; therefore the Square of C B is equal to the Squares of the two other Sides. *Q. E. D.*

Fig. 3. But if the Sides be unequal, let the Squares be deſcribed, and the Parallelogram L Q compleated; the whole Figure exceeds the Square upon B C, by three Triangles, X, R, Z; and exceeds alſo the Square L A, A D, by the Triangle A B C, and the Parallelogram P Q: Which Exceſſes, I ſay, are

equal; for Z is equal to ABC, the Side $OC = BC$, $CD = AC$, the
Angle $D = A$, and $OCD = BCA$; which is manifeſt, by taking the
common Angle ACO, out of the two Right Angles BCO, ACD; therefore
by Superimpoſition the whole Triangles are equal. In like manner X
is proved equal to ABC, alſo R; and the Parallelogram PQ to be double
of the Triangle ABC: Thus the Exceſſes being proved equal, the Remainders
alſo will be equal; *viz.* the Square of BC to the Square of AB, AC.
Q. E. D. Manifeſt Corollaries from hence are the 35th and 36th of the *Third* 35, 36, 3. E.
Book; alſo the 12th and 13th of the *Second.*

The firſt Ten Propoſitions of the *Second* Book are evidently demonſtrated, 12, 13. 2. E.
only by ſubſtituting Species or Letters inſtead of Lines, and multiplying 1, 2, &c. 2 E.
them according to the Tenor of the Propoſition: Thus, to inſtance in one or
two, Call the whole Line A, and its Parts B and C, therefore $A = B + C$,
and conſequently $AA = BB + CC + 2BC$, which is the very Senſe of Fig. 4.
the *Fourth* of the *Second* Book. Thus alſo, Let a Line be cut into equal Parts
F, F, and let another Line, S, be added thereto; 'tis manifeſt, that $4FF$ 4. 2. E.
$+ 4SF + 2SS = 2FF + 2FF + 2SS + 4SF$; which is the *Tenth* Fig. 5.
Propoſition of the ſame Book. 10. 2. E.

Almoſt the whole Doctrine of Proportionals, *viz.* Permutation, Inverſion,
Converſion, Compoſition, Diviſion of Ratio's, and Proportion *ex æquo*; and
conſequently the moſt uſeful Propoſitions of the *Fifth* Book, are clearly
demonſtrated by one Definition, and that is of Similar or Like Parts, which are
ſaid to be ſuch as are after the ſame manner, or equally contain'd in their
Wholes: Thus the Antecedents A and C, are either equal to their
Conſequents, or greater, or leſs; if equal, the thing is manifeſt; if leſs, then (by Fig. 6.
the Definition of Proportionals) A and C are Like Parts of B and E; therefore
what Proportions the whole, B and E, have to one another, the ſame will A
and C have; which is Permutation: Likewiſe $E : C :: B : A$, which is
Inverſion. So alſo if from Proportionals you take Like Parts, the Remainders will
be proportional; whence Converſion and Diviſion are demonſtrated: And if to
Proportionals you add Like Parts, the Wholes will ſtill be proportional; which
is Compoſition, &c. If the Antecedents be greater than the Conſequents, the
Conſequents will be Like Parts of them, and the Demonſtration exactly the
ſame with the former.

The firſt of the *Sixth* Book is prov'd, by conſidering the Generations of 1. 6. E.
the Parallelograms, which are produced by drawing or multiplying the
Perpendicular upon the Baſis; that is, taking it ſo often as there be Parts and
Diviſions in the Baſe: Therefore the ſame Proportion that RX ſingle, hath to Fig. 7.
NX ſingle, the ſame hath RX multiplied by XZ, that is, repeated a certain
number of times, to NX multiplied by XZ, that is, repeated the ſame
number of times; which is as much as to ſay, $RX : NX ::$ Paral. RZ : Paral.
NZ. Now that this Propoſition is alſo true in Oblique-angled Parallelograms,
is proved, becauſe they are equal to the Rectangled ones upon the ſame Baſis,
and between the ſame Parallels, as does thus independently appear; the
Triangles RQX, and MPZ, are equal; for $RX = MZ$, $QX = PZ$, RM Fig. 7.
$= QP$; therefore adding to both MQ, $RQ = MP$; if therefore from theſe

equal Triangles you take what is Common, *viz.* M L Q, the Remainders will be equal, R X L M = Q L Z P; to both which add X L Z, and the whole Parallelograms will be equal, R Z = Q Z. *Q. E. D.* That Triangles alſo having a common Baſis, are in the Proportion of their Altitudes, does hence follow; becauſe they are the Halves of Parallelograms upon the ſame Baſis. This alſo is true, and the Demonſtration exactly the ſame, in Priſms, Pyramids, Cylinders, and Cones, having the ſame Baſis.

16. 6. E. Fig. 6. To prove the 16th of the *Sixth*, I ſuppoſe the 4 Lines, A, B, C, E, to be proportional, that is, granting A and C to be the leſſer Terms, the ſame way that A is contained in B, ſo is C in E; and that D is the Denominator of the Ratio; 'twill follow then, that B is made up of A, multiplied by D, and E of C, multiplied by D; ſo that A D = B, and C D = E; draw therefore the Extremes upon one another, that is, A upon C D, and the Means, that is, C upon A D; the Factors being the ſame, I ſay, the Products A C D, and C A D, are the ſame, and conſequently equal. *Q. E. D.*

The Problem propoſed by M. *Comier*, (with Oſtentation enough) as if it contain'd ſomething New, though in reality it be nothing but the Old Buſineſs of doubling the Cube, a little diſguiſed, is eaſily ſolved Algebraically, as follows,

Fig. 8.

8. 6. E.	1	$a : 2x :: x : \frac{2x^2}{a} = p$
47. 1. E.	2	$aa + 2ax - \frac{2x^3}{a} = \frac{4x^4}{aa}$
2 × by aa	3	$a^4 + 2a^3x = 4x^4 + 2x^3a$
3 ÷ $\overline{a + 2x}$	4	$a^3 = 2x^3$; that is, The Cube upon x is half the Cube upon a.

The Squaring of the Hyperbola, by the Lord Viſcount Brounker. N. 34. *p.* 645. *Fig.* 9.

III. Let A B be one Aſymptote of the Hyperbola E d C; and let A E, and B C, be parallel to the other: Let alſo A E be to B C, as 2 to 1; and let the Parallelogram A B D E be equal to 1.

Suppoſing the Reader knows, that E A, $\alpha\zeta$, K H, $\beta\eta$, $d\theta$, $\gamma\kappa$, $\delta\lambda$, $\epsilon\mu$, C B, *&c.* are in an Harmonick Series, or a Series *Reciproca-Primanorum, ſeu Arithmetice Proportionalium*, (otherwiſe he is referr'd for Satisfaction to *Arithm. Infinitor. Walliſii*, Prop. 87, 88, 89, *&c.*) I ſay,

$$ABCdEA = \frac{1}{1\times2} + \frac{1}{3\times4} + \frac{1}{5\times6} + \frac{1}{7\times8} + \frac{1}{9\times10}, \text{ \&c.}$$

$$EdDE = \frac{1}{2\times3} + \frac{1}{4\times5} + \frac{1}{6\times7} + \frac{1}{8\times9} + \frac{1}{10\times11}, \text{ \&c.}$$

$$EdCyB = \frac{1}{2\times3\times4} + \frac{1}{4\times5\times6} + \frac{1}{6\times7\times8} + \frac{1}{8\times9\times10}, \text{ \&c.}$$

Fig. 10, 11, 12.

For (in *Fig.* 10. and 11.) the *Parallelogram*, And (in *Fig.* 12.) the *Triangle*,

	$CA = \frac{1}{1\times2}$	$EdC = \frac{1}{2\times3\times4} = \frac{\square dD - \square Fd}{2}$
$dD = \frac{1}{2\times3}$	$dF = \frac{1}{3\times4}$	$Ebd = \frac{1}{4\times5\times6} = \frac{\square br - \square bn}{2}$
$br = \frac{1}{4\times5}$	$bn = \frac{1}{5\times6}$	$dfC = \frac{1}{6\times7\times8} = \frac{\square fG - \square fk}{2}$
$fG = \frac{1}{6\times7}$	$fk = \frac{1}{7\times8}$	$Eab = \frac{1}{8\times9\times10} = \frac{\square aq - \square ap}{2}$
$aq = \frac{1}{8\times9}$	$ap = \frac{1}{9\times10}$	$bcd = \frac{1}{10\times11\times12} = \frac{\square cs - \square cm}{2}$
$cs = \frac{1}{10\times11}$	$cm = \frac{1}{11\times12}$	$def = \frac{1}{12\times13\times14} = \frac{\square et - \square el}{2}$
$et = \frac{1}{12\times13}$	$el = \frac{1}{13\times14}$	$fgC = \frac{1}{14\times15\times16} = \frac{\square gu - \square gb}{2}$
$gu = \frac{1}{14\times15}$	$gb = \frac{1}{15\times16}$	&c.
&c.	&c.	

Note, $\frac{1}{2}CA = dD + dF$
$\frac{1}{2}dD = br + bn$
$\frac{1}{2}dF = fG + fk$
$\frac{1}{2}br = aq + ap$
$\frac{1}{2}bn = cs + cm$
$\frac{1}{2}fG = et + el$
$\frac{1}{2}fk = gu + gb$, &c.

And that therefore in the firſt Series half the firſt Term is greater than the Sum of the two next, and half this Sum of the Second and Third greater than the Sum of the four next; and half the Sum of thoſe Four greater than the Sum of the next Eight, *&c. in infinitum.* For $\frac{1}{2}\,dD = br + bn$; but $bn > fG$, therefore $\frac{1}{2}\,dD > br + fG$, *&c.* And in the Second Series, half the ſecond Term is leſs than the Sum of the two next, and half this Sum leſs than the Sum of the four next, *&c. in infinitum.*

That the firſt Series are the even Terms, *viz.* the 2d, 4th, 6th, 8th, 10th, *&c.* and the ſecond the odd, *viz.* the 1ſt, 3d, 5th, 7th, 9th, *&c.* of the following Series, *&c.*

$$\frac{1}{1\times2},\ \frac{1}{2\times3},\ \frac{1}{3\times4},\ \frac{1}{4\times5},\ \frac{1}{5\times6},\ \frac{1}{6\times7},\ \text{\&c. } \textit{in infinitum.} = 9.$$

Whereof a, being put for the Number of Terms taken at Pleaſure, $\frac{1}{a^2+a}$ is the laſt, $\frac{a}{a+1}$ is the Sum of all thoſe Terms from the Beginning, and $\frac{1}{a+1}$ the Sum of the reſt to the End.

That $\frac{1}{4}$ of the firſt Term in the third Series is leſs than the Sum of the two next, and $\frac{1}{4}$ of this Sum leſs than the Sum of the four next, and $\frac{1}{4}$ of this laſt Sum leſs than the next eight, I thus demonſtrate:

Let a be equal to the third or laſt Number of any Term of the firſt Column, *viz.* of Diviſors;

$$\frac{1}{a\times\overline{a-1}\times\overline{a-2}} = \frac{1}{a^3-3a^2+2a} = \frac{16a^3-48a^2+56a-24}{16a^6-96a^5+232a^4-288a^3+184a^2-48a} = A.$$

$$\left.\begin{aligned}\frac{1}{2a\times\overline{2a-1}\times\overline{2a-2}} &= \frac{1}{8a^3-12a^2+4a}\\ \frac{1}{\overline{2a-2}\times\overline{2a-3}\times\overline{2a-4}} &= \frac{1}{8a^3-36a^2+52a-24}\end{aligned}\right\} =$$

$$\frac{16a^3 - 48a^2 + 56a - 24}{64a^6 - 384a^5 + 880a^4 - 960a^3 + 496a^2 - 96a} = \text{B}.$$

$$\frac{64a^6 - 384a^5 + 928a^4 - 1152a^3 + 736a^2 - 192a}{64a^6 - 384a^5 + 880a^4 - 960a^3 + 496a^2 - 96a} \times \tfrac{1}{4}\text{A} = \text{B}.$$

And $48a^4 - 192a^3 + 240a^2 - 96a$ = Exceſs of the Numerator above the Denominator.

$$\left.\begin{array}{rrcrr} & \text{But the Affirmat.} & > & \text{the Negat.} & \\ \text{That is,} & 48a^4 + 240a^2 & > & 192a^3 + 96a & \\ \text{Becauſe} & a^4 + 5a^2 & > & 4a^3 + 2a & \\ & a^3 + 5a & > & 4a^2 + 2 & \\ \text{Therefore} & \text{B} & > & \tfrac{1}{4}\text{A}. & \end{array}\right\} \text{if } a > 2.$$

Therefore one fourth of any Number of A, or Terms, is leſs than their ſo many reſpective B; that is, than twice ſo many of the next Terms. *Q. E. D.*

By any one of which three Series it is not hard to calculate, as near as you pleaſe, theſe and the other like Hyperbolic Spaces, whatever be the rational Proportion of A E to B C; as for Example, When A E is to B C, as 5 is to 4. (Whereof the Calculation follows, after that where the Proportion is, as 2 to 1; and both by the *Third* Series.)

Firſt then, when (in *Fig.* 9.) A E : B C :: 2 : 1. *Fig.* 9.

2× 3× 4) 1.	(0. 0416666666	0. 0416666666
4× 5× 6) 1.	(0. 0083333333	0. 0113095237
6× 7× 8) 1.	(0. 0029761904	
8× 9×10) 1.	(0. 0013888888	0. 0029019589
10×11×12) 1.	(0. 0007575757	
12×13×14) 1.	(0. 0004578754	
14×15×16) 1.	(0. 0002976190	
16×17×18) 1.	(0. 0002042484	0. 0007306482
18×19×20) 1.	(0. 0001461988	
20×21×22) 1.	(0. 0001082251	
22×23×24) 1.	(0. 0000823452	
24×25×26) 1.	(0. 0000641026	
26×27×28) 1.	(0. 0000508758	
28×29×30) 1.	(0. 0000410509	
30×31×32) 1.	(0. 0000336021	

32×33×34) 1. (0. 0000278520
34×35×36) 1. (0. 0000233426
36×37×38) 1. (0. 0000197566
38×39×40) 1. (0. ~~0000168691~~
40×41×42) 1. (0. 0000145180
42×43×44) 1. (0. 0000125843
44×45×46) 1. (0. 0000109793
46×47×48) 1. (0. 0000096361 } 0. 0001829939
48×49×50) 1. (0. 0000085034
50×51×52) 1. (0. 0000075415
52×53×54) 1. (0. 0000067193
54×55×56) 1. (0. 0000060125
56×57×58) 1. (0. 0000054014
58×59×60) 1. (0. 0000048704
60×61×62) 1. (0. 0000044068
62×63×64) 1. (0. 0000040002

0. 0416666666
0. 0110095237
0. 0029019589
0. 0007306482
3)0. 0001829939 (0. 0000609980

0. 05679179
+0. 00006100

0. 05685279 < E *d* C *y*

But 0. 0007306482
0. 0001829939 } ÷
0. 0000458315

Therefore 0. 05679179
+0. 00004583
+0. 00001528

0. 05685290 > E *d* C *y*.

For it has been demonſtrated, That ¼ of any Term in the laſt Column is leſs than the Term next after it; and therefore that ⅓ of the laſt Term, at which you ſtop, is leſs than the remaining Terms; and that the Total of theſe is leſs than ¼ of a third Proportional to the two laſt.

And therefore A B C *y* E being = 0· 75 ———————— 0. 57
and E *d* C *y* $>$ 0. 05685279 —— and $<$ 0. 05685290

And A B C *d* E is $<$ 0. 69314720 —— and $>$ 0. 69314709

But when A E : B C :: 5 : 4, or as E A to K H; then will the Space ABCE, or now, the Space A H K E (A H = ¼ A B) be found as follows. *Fig. 9.*

8× 9×10) 1. (0. 0013888888] 0. 0013888888

$$\left.\begin{array}{l}16\times17\times18)\ 1.\ (0.\ 0002042484\\ 18\times19\times20)\ 1.\ (0.\ 0001461988\end{array}\right\}\ 0.\ 0003504472$$

$$\left.\begin{array}{l}32\times33\times34)\ 1.\ (0.\ 0000278520\\ 34\times35\times36)\ 1.\ (0.\ 0000233426\\ 36\times37\times38)\ 1.\ (0.\ 0000197566\\ 38\times39\times40)\ 1.\ (0.\ 0000168691\end{array}\right\}\ 0.\ 0000878204$$

0. 0013888888
0. 0003504472
3) 0. 0000878204 (0. 0000292735

0. 0018271564
+0. 0000292735

0. 0018564299 $<$ E *a b*

$$\text{But}\ \left.\begin{array}{l}0.\ 0003504472\\ 0.\ 0000878204\\ 0.\ 0000220737\end{array}\right\}\ \div$$

Therefore 0. 0018271564
+0. 0000220074
+0. 0000073358

0. 0018564296 $>$ E *a b*

Therefore E M *b* (*Fig.* 12.) being = 9. 025 ———————— 0. 025
E *a b* $>$ 0. 0018564292 and $<$ 0. 0018564996

E M *b a* (*Fig.* 12.) or E K M (*Fig.* 9.) $>$ 0. 02685643 $<$ 0. 02685650
A H K E $<$ 0. 22314356 $<$ 0. 22314349

Therefore 3 ABC*d*E = 2. 37944154
and AHKE = 0. 2231435

ABC*d*E (when AE : $\bar{\text{BC}}$ :: 10 : 1.) = 2. 3025850

Therefore,
The Logar. of 10,
is to the Log. of 2,
as 2. 302585,
to 0. 693147.

The Quadrature of a Circle, by M. Leibnitz, *Phil. Coll.* N. 7. *p.* 204.

IV. The Quadrature of the Circle, or the turning it into an equal Share, or any other Right-lin'd Figure, (which depends upon the Ratio of the Circle to the Square of its Diameter, or of the Circumference to its Diameter) may be underſtood to be fourfold, to wit, either by Calculation, or by Linear Conſtruction; and each of them again may be either perfectly exact, or elſe almoſt, or pretty near. The Quadrature by accurate Calculation, I call the Analytical; That which is done by accurate Conſtruction, I call the Geometrical: That which is done by Calculation pretty near, I call the Approach; that which is by Conſtruction pretty near, I call Mechanical.

The Approaches have been furtheſt carried on by *Ludolph van Ceulen*: *Vieta*, *Hugenius*, and others, have given ſeveral Mechanical. The accurate Geometrical Conſtruction may be had, by which not only an entire Circle may be meaſured, but any Section or Arch of it alſo; which is by an exact and ordinate Motion, but ſuch notwithſtanding as ſuits with Tranſcendental Curves, which erroneouſly are accounted Mechanical, though in truth they are as Geometrical as thoſe which are commonly ſo eſteemed; though they are not Algebraical, nor can be reduced to Equations Algebraical, or of certain Degrees; they having Degrees proper to themſelves, which though they be not Algebraical, are yet nevertheleſs Analytical.

The Analytical Quadrature, or that which is made by accurate Calculation, may be again ſubdivided into three Kinds; namely, into the Analytical Tranſcendent, the Algebraical, and the Arithmetical. The Analytical Tranſcendent is to be obtained, amongſt others, by Equations of Degrees indefinite, hitherto conſidered by none: As if $X^x + X$ be equal to 30, and X be ſought, it will be found to be 3; becauſe $3^3 + 3$, is 27 + 3, or 30.

The Algebraical is done by Vulgar Numbers, though irrationally, or by the Roots of common Equations; which for the general Quadrature of the Circle, or its Sectors, is indeed impoſſible. Now there remains the Arithmetical Quadrature, which is performed by certain Series exhibiting the Quantity of the Circle exact by a Progreſſion of Terms (firſt) Rational, ſuch as I ſhall here propound.

I have found therefore, that if the Square of the Diameter be put 1, the Area of the Circle will be $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \frac{1}{15} + \frac{1}{17}$, &c. to wit, the entire Square of the Diameter being diminiſhed (that it may not be too big) by a third Part; and again (becauſe hereby too much is taken away) being augmented by one fifth; and again (becauſe by this laſt too much is added) diminiſhed by one ſeventh; and ſo onward, continually.

And the firſt Quantity will be too great, *viz.* 1. but the Error will be leſs than ——————————————— } $\frac{1}{3}$

The next too little, *viz.* $1-\frac{1}{3}$, but the Error will be leſs than $\frac{1}{5}$

The 3d, too much, *viz.* $1-\frac{1}{3}+\frac{1}{5}$, but, *&c.* - - - - - - - $\frac{1}{7}$

The 4th too little, *viz.* $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}$, but, *&c.* - - - - $\frac{1}{9}$ *&c.*

&c.

The whole Series contains all the Approaches together, or the Values both greater than they ought to be, and leſs than they ought to be.

So that by continuing the Series, the Errors may be made leſs than any Fraction given, and conſequently leſs than any aſſignable Quantity. Whence it follows, that the whole Series muſt give the true Value. And though the Sum of the whole Series cannot be expreſſed by one Number, and that the Series be infinitely continued; yet becauſe it conſiſteth of one regular Method of Progreſſion, the whole may ſufficiently enough be conceived by the Mind. And if *Van Ceulen* could have given a Rule by which his Numbers 314159, *&c.* could have been continued *in infinitum*, he would have given us the Arithmetical Quadrature exact in whole Numbers, which we have here done in Fractions.

There are ſeveral things relating to this Quadrature which might be taken notice of, eſpecially one, *viz.* That the Terms of this our Series, $\frac{1}{1}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{9}$, *&c.* are of Harmonical Progreſſion, or in a continued Harmonical Progreſſion; as will be evident to any one that ſhall examine them. And a Series made by ſkipping, as $\frac{1}{1}$, $\frac{1}{5}$, $\frac{1}{9}$, $\frac{1}{13}$, $\frac{1}{17}$, *&c.* is alſo of Harmonical Progreſſion. And $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{11}$, $\frac{1}{15}$, $\frac{1}{19}$, *&c.* is alſo a Series of Harmonical Proportionals. Wherefore ſince the Circle is $\frac{1}{1}+\frac{1}{5}+\frac{1}{9}+\frac{1}{13}+\frac{1}{17}$, *&c.* $-\frac{1}{3}-\frac{1}{7}-\frac{1}{11}-\frac{1}{15}-\frac{1}{19}$, *&c.* by ſubſtracting the latter partial Series from the former partial Series, the Circle will be the Difference of two Series in Harmonical Progreſſion. And becauſe the Sum of any Number of Terms in Harmonical Progreſſion, how many ſoever, may by ſome Compendium be obtained; hence Compendious Approaches (if after *Van Ceulen* there be any need of them) may be deduced; if one would in this our Series take out the Terms affected with the Sign —, by adding the two next into one, +

$\frac{1}{1}-\frac{1}{3}$, and $+\frac{1}{5}-\frac{1}{7}$, and $+\frac{1}{9}-\frac{1}{11}$, and $+\frac{1}{13}-\frac{1}{15}$, and $+\frac{1}{17}-\frac{1}{19}$, and ſo onward, he will have a new Series for the Magnitude of the Circle, namely, $\frac{2}{3}$ (that is, $\frac{1}{1}-\frac{1}{3}$) $+\frac{2}{35}$ (that is, $\frac{1}{5}-\frac{1}{7}$) $+\frac{2}{99}$ (that is $\frac{1}{9}-\frac{1}{11}$) &c. Wherefore

The Square inſcrib'd being $\frac{1}{4}$

The Area of the Circle ſhall be $\frac{1}{3}+\frac{1}{35}+\frac{1}{99}+\frac{1}{195}+\frac{1}{323}$, &c.

But the Numbers 3, 35, 99, 195, 323, &c. by Skipping are taken out of the Series of Square Numbers, 4, 9, 16, 25, &c. diminiſhed by an Unite, and ſo made the Series 3, 8, 15, 24, &c. out of the Members of which Series, every fourth after the firſt, is a Number of this our Series. But I have found (which is worth nothing) the Sum of this infinite Series, $\frac{1}{3}+\frac{1}{8}+\frac{1}{15}+$ &c. to be $\frac{3}{4}$. Nay, and by culling out by ſingle Skipping, as $\frac{1}{3}+\frac{1}{15}+\frac{1}{35}$, &c. the Sum of this infinite Series maketh $\frac{2}{4}$ or $\frac{1}{2}$. But if out of this again another Progreſſion be culled by ſingle Skipping, as $\frac{1}{5}+\frac{1}{35}+\frac{1}{99}$, &c. the Sum of that infinite Series ſhall be the Semicircle, the Square of the Diameter being 1. Now becauſe by the ſame Means the Arithmetical Quadrature of the *Hyperbola* is obtain'd, I thought it not amiſs to repreſent to View the whole Harmony.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	&c.	
1	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400	&c.	
0	3	8	15	24	35	48	63	80	99	120	143	168	195	224	255	288	323	360	399	&c.	
	$\frac{1}{3}$	$\frac{1}{8}$	$\frac{1}{15}$	$\frac{1}{24}$	$\frac{1}{35}$	$\frac{1}{48}$	$\frac{1}{63}$	$\frac{1}{80}$	$\frac{1}{99}$	$\frac{1}{120}$	$\frac{1}{143}$	$\frac{1}{168}$	$\frac{1}{195}$	$\frac{1}{224}$	$\frac{1}{255}$	$\frac{1}{288}$	$\frac{1}{323}$	$\frac{1}{360}$	$\frac{1}{399}$	&c.	$=\frac{3}{4}$
	$\frac{1}{3}$		$\frac{1}{15}$		$\frac{1}{35}$		$\frac{1}{63}$		$\frac{1}{99}$		$\frac{1}{143}$		$\frac{1}{195}$		$\frac{1}{255}$		$\frac{1}{323}$		$\frac{1}{399}$	&c.	$=\frac{2}{4}$
		$\frac{1}{8}$		$\frac{1}{24}$		$\frac{1}{48}$		$\frac{1}{80}$		$\frac{1}{120}$		$\frac{1}{168}$		$\frac{1}{224}$		$\frac{1}{288}$		$\frac{1}{360}$		&c.	$=\frac{1}{4}$
	$\frac{1}{3}$				$\frac{1}{35}$				$\frac{1}{99}$				$\frac{1}{195}$				$\frac{1}{323}$			&c.	= to
		$\frac{1}{8}$				$\frac{1}{48}$				$\frac{1}{120}$				$\frac{1}{224}$				$\frac{1}{360}$		&c.	= to

the Circle A B C D, whoſe Inſcribed Square is
the Hyperbola CBEHC, whoſe Power, A B C D, is $\Big\}\ \frac{1}{4}$

Fig. 13.

To the Aſymptotes A F, A E, at Right Angles to each other, let there be deſcribed the Curve Line of an Hyperbola G C H, whoſe Vertex is C; and A B C D, the Power or Square, to which every Rectangle made of the Ordinate,

ate, as E H, and the intercepted Part, A E, is always equal. About this Square let a Circle be drawn, and let the Hyperbola be continued from C to H, ſo that A E be double to A B. Then putting A E to be 1, A B ſhall be $\frac{1}{2}$, and its Square, ABCD, ſhall be $\frac{1}{4}$, and the Circle, (whoſe Power A B C D is inſcrib'd) ſhall be $\frac{1}{3}+\frac{1}{35}+\frac{1}{99}$, *&c.* but the Portion of the Hyperbola CBEHC (whoſe Power inſcrib'd is the ſame Square $\frac{1}{4}$) which repreſents the Logarithm of the Ratio of A E to A B, (or of 2 to 1) ſhall be $\frac{1}{8}+\frac{1}{48}+\frac{1}{120}$, *&c.*

Tangents to all Geometrical Curves; by Renatus Fran. Sluſius. n. 90. p. 513. Jan. An. 1673. *Fig.* 14.

V. 1. Let any Curve D Q be given, all the Points of which may be referred to any given Right Line E A B, by the Right Line D A; whether E A B be the Diameter, or any other Line, or any other Lines are given at the ſame time, any of which, or their Powers, enter the Equation of the Curve, it matters little.

In the Analytical Equation, for the more eaſy Explanation, let D A be always called v, and B A y. And let E B, and all the other given Quantities, be denoted by Conſonants.

Then ſuppoſe the Line D C to be drawn, touching the Curve in D, and meeting E B (produced if neceſſary) in the Point C; and let C A be always called a. Now to find A C, or a, this will be the general Rule.

1. Rejecting thoſe Parts of the Equation in which y or v are not found, let all thoſe be ſet on one Side of the Equation in which is y, and thoſe on the other Side in which is v, with their Signs + or —. For Diſtinction Sake we will call this the Right Side, and that the Left.

2. On the Right Side, let there be prefixt the Exponent of the Power of v to all the Parts, or, which is the ſame thing, let the Parts be multiplied by it.

3. Let the ſame thing be done on the Left Side, that is, prefix to each Part the Exponent of the Power of y: And farther, in each of the Parts let one y be changed into a.

I ſay that the Equation ſo form'd will ſhew the Manner of drawing the Tangent at the given Point D. For when it is given, at the ſame time are given y and v, and the other Quantities which are expreſſed by Conſonants; and then a will be known alſo.

If the Rule ſhall ſeem to be any thing obſcure, it may be thus illuſtrated by ſome Examples. Let this Equation be given, $by - yy = vv$; in which let E B be b; B A, y; D A, v; and let a or A C be ſought, ſuch as D C being joined may touch the Curve D Q in D. By the Rule nothing is here to be rejected, becauſe either y or v is found in every Part of the Equation. Alſo it is ſo diſpoſed, that all the Parts in which y is found are on one Side, and all in which there is v are on the other. Therefore we need only prefix to each the Exponents of the Powers of y and v; and on the Left Side one y is to be changed into a. Therefore it will be $ba - 2ya = 2vv$. I ſay now, that this Equation will ſhew the Manner of drawing the Tangent to the Point D; that is, $a = \frac{2vv}{b - 2y} = AC$.

Thus if the Equation $qq + by - yy = vv$ were given, the Equation for the Tangent would be the ſame as the former; for we muſt reject qq as the Rule preſcribes.

Thus from the Equation $2byy - y^3 = v^3$, we have $4bya - 3yya = 3v^3$, or $a = \frac{3v^3}{4by - 3yy}$. And from the Equation $bby + zyy + y^3 = qvv$, we ſhall have $bba + 2zya + 3yya = 2qvv$, and thence $a = \frac{2qvv}{bb + 2zy + 3yy}$. And from $b^4 + by^3 - y^4 = qqvv + zv^3$, we ſhall have $3byya - 4y^3a = 2qqvv + 3zv^3$, and then $a = \frac{2qqvv + 3zv^3}{3byy - 4y^3}$.

And in all other Equations of this kind I think no Difficulty can ariſe. Perhaps there may be ſome in ſuch Equations; ſome Parts of which conſiſt of Products of y into v. As yv, yyv, y^3v^2, and ſuch like. But this Difficulty will be but a ſlight one, as will appear from theſe Examples. For let there be given $y^3 = bv^2 - yv^2$. Now there is nothing to be rejected from hence, ſince y or v is found in all the Parts or Terms.

But that they may be diſpoſed as the Rule preſcribes, the Term yv^2 is to be aſſumed twice, and placed as well on the Right Side of the Equation, in which are theſe Terms that have v, as on the Left Side, where are the Terms that have y; ſince yv^2 contains both y and v. Therefore we muſt make $y^3 + v^2y = bv^2 - yv^2$. Then changing, as before, this Equation into another $3yya + vva = 2bvv - 2yvv$, we ſhall have $a = \frac{2bvv - 2yvv}{3yy + vv}$.

For the Rule is to be ſo underſtood, that on the Left Side the Power of v is not conſidered, and therefore the Exponent of v^2 is not to be prefixed to yv^2, but only that of y. And on the contrary on the other Side, the Power of y is not to be conſidered in yvv, but only that of v, and to this its Exponent is to be prefixed. Thus if the Equation were $y^5 + by^4 = 2qqy^3 - y^2v^3$, we muſt make $y^5 + by^4 + v^3y^2 = 2qqv^3 - y^2v^3$, and we ſhould have the Equation for the Tangent $5y^4a + 4by^3a + 2v^3ya = 6q^2v^3 - 3y^2v^3$, and thence $a = \frac{6q^2v^3 - 3y^2v^3}{5y^4 + 4by^3 + 2v^3y}$.

And in theſe Examples I think I have comprehended all the Variety of Caſes that can be propoſed. But perhaps it may be of ſome uſe to apply what I have delivered in general, to ſome particular Curve. Therefore let the Curve BD be given, which has this Property, that taking any Point in it D, if BD be joined, and DE be raiſed perpendicular to it, meeting the Fig. 15.

Right Line B E in E, the Right Line D E may always be equal to the given Right Line B F.

That we may have an Equation in Analytical Terms, make D A $= v$, B A $= y$, B F or D E $= q$. Therefore it will be E A $= \frac{vv}{y}$. And ſince the Square of D E is equal to the two Squares of D A and A E, we ſhall have the Equation $qq = \frac{v^4}{y^2} + v^2$; or $q^2y^2 = v^4 + y^2v^2$; which for the Tangent, as preſcribed by the Rule, is to be reform'd to $qqy^2 - v^2y^2 = v^4 + y^2v^2$, and $2qqya - 2v^2ya = 4v^4 + 2y^2v^2$, whence $a = \frac{2v^4 + y^2v^2}{q^2y - v^2y}$.

Now how ſuch Equations are to be reduced to eaſier Terms, for the Sake of the Conſtruction, will be no Myſtery to ſkilful Mathematicians. As in this Example, becauſe the Rectangle B A E is ſuppoſed equal to the Square of A D, if E A be called e, it will be $vv = ye$, and $v^4 = y^2e^2$, and $qq = ye + ee$. Therefore inſtead of them ſubſtituting their Values in the Equation, it becomes $a = \frac{2y^2e^2 + y^2e}{e^2y + ey^2 - ey^2}$, or $a = \frac{2ey + yy}{e}$, that is, $ae = 2ey + yy$, and adding ee on both Sides, $ae + ee = ee + 2ey + yy = \overline{e+y}|^2$. So that the three Quantities e, $e + y$; $a + e$, or E A, E B, E C, are in continual Proportion; whence the Conſtruction becomes very eaſy.

But whereas we ſeem to have ſuppoſed hitherto, that the Tangent is always to be drawn towards B, when it may happen from the given Quantities, that it ſhould be either parallel to A B, or ſhould be drawn the contrary Way; we muſt now determine how this Diverſity of Caſes is to be diſtinguiſhed in Equations. A Fraction therefore being made for the Value of a, as in the foregoing Examples, the Parts of the Numerator and the Denominator are to be conſidered, as alſo their Signs.

1. For in both if either all the Parts have the Sign +, or at leaſt if the Affirmative prevail over the Negative, the Tangent muſt be drawn towards B.

2. If in the Numerator the Affirmative prevail over the Negative, but are equal in the Denominator, a Line drawn through D parallel to A B will touch the Curve in D: For in this caſe a will be of an infinite Length.

3. If as well in the Denominator as in the Numerator the affirmative Parts are leſs than the Negative; then all the Signs being changed, the Tangent muſt ſtill be drawn towards B: For this caſe coincides with the firſt.

4. If in the Denominator they prevail, but in the Numerator are leſs; the Tangent muſt be drawn on the contrary Side, that is, A C muſt be taken towards E.

5. Laſtly, if in the Numerator the affirmative Parts are equal to the Negative, however they may be in the Denominator, then a will become nothing. Therefore either A D will be the Tangent, or E A, or parallel to it, which is eaſily known from the given Quantities. Now all the Variety of theſe Caſes may be ſhewn by the Equation to the Circle.

Fig. 16. For let there be a Semicircle whoſe Diameter is E B, and let D be any given Point in it, from whence let fall the perpendicular $DA = v$. Make $BA = y$, $BE = b$; the Equation will be $by - yy = vv$. And drawing the Tangent D C, it will be A C or $a = \frac{2vv}{b - 2y}$. Now if b be greater than $2y$; the Tangent muſt be drawn towards B; if equal, it is parallel to E B; if leſs, it muſt be drawn towards E. See 1, 2, 4.

Fig. 17. Again, let there be given another Semicircle inverted, whoſe Points are to be conceived as referred to a Right Line parallel to the Diameter, and equal to the ſame, as in the Scheme. The Parts being denominated as before, and making $NB = d$, the Equation will be $by - yy = dd + vv - 2dv$. Therefore A C or $a = \frac{2vv - 2dv}{b - 2y}$. Now ſince by the Example we have ſuppoſed v always to be leſs than d; if b is greater than $2y$, the Tangent muſt be drawn towards E; if equal, it will be parallel; if leſs, then changing all the Signs, it muſt be drawn towards B; as 4, 5, 3. But no Tangent would be drawn, or E B itſelf would be the Tangent, if we ſhould ſuppoſe N B to be equal to the Semidiameter, or $2d = b$ as at *n.* 5.

Fig. 18. Laſtly, let there be another Semicircle, whoſe Diameter N B is perpendicular to the Right Line B E, to which its Points are ſuppoſed to be referred. Let N B be called b, and let the other Parts be named as before. The Equation will be $yy = bv - vv$, and thence $a = \frac{bv - 2vv}{2y}$. Now if b is greater than $2v$, the Tangent muſt be drawn towards B; if leſs, towards E; if equal, D A itſelf will be the Tangent. As by 1, 4, 5.

And this, I think, is all the Variety of Caſes, which can be obſerved from the Conſideration of Equations.

The Lemmata, *whereby the preceding Method is demonſtrated, by* M. Sluſius. n. 95. *p.* 6059. n. 97. *p.* 6126. June, An. 1673.

(1) The Difference of two Dignities of the ſame Degree, applied to the Difference of the Sides, will give the ſeveral Parts of the inferior Degree of a Binomial of the Sides. Thus $\frac{y^3 - x^3}{y - x} = y^2 + yx + x^2$; as is eaſy to prove.

(2.) There are ſo many Parts ariſing from a Binomial of any Degree, as the Exponent of the Dignity next above has Units. That is, three in the Square, four in the Cube, *&c.* And this is generally known.

(3.) If the ſame Quantity is applied to two others, whoſe Ratio is given, the Quotients will be in the ſame Ratio reciprocally.

By theſe Lemmata my Method may eaſily be demonſtrated, they being here diſpoſed in ſuch an Order, as will naturally lead to the Demonſtration.

The Teſtudo Veliformis Quadrabilis *Ænigmatically propos'd by* V. V. *n.* 196. *p.* 584. Jan. An. 1693.

VI. 1. *A Geometrical ÆNIGMA, concerning the wonderful Structure of a Hemiſpherical Cupola which is quadrable. By D.* Pio Liſci, *a minute Mathematician.*

Among the venerable Monuments of ancient learned *Greece*, there is ſtill in Being, and for ever will endure, a moſt Auguſt Temple dedicated to *Divine Geometry*, of a Circular Ichnography, which is cover'd by a Cupola perfectly Hemiſpherical within. But in this there are the equal Areas of four Windows, (diſpoſed about and upon the Baſe of the Hemiſphere) of ſuch a Configuration, Amplitude, and of ſuch Ingenuity, that theſe being taken away, the remaining curved Superficies of the Cupola, adorned with curious Moſaick Work, is capable of a true Geometrical Quadrature.

Now it is inquired, which is that quadrable Part of the curved Hemiſpherical Superficies, which is like a diſtended Nautical Sail, and by what Art or Method the Geometrical Architect attained to it; and laſtly, what quadrable Geometrical Plain is it equal to?

2. A Day or two ago, juſt as I was going to Bed, I received (learned Sir) your Letter, which being very buſy I could not anſwer Yeſterday. Within was incloſed a little printed Paper, which, you ſay, you received from *Florence*, to be tranſmitted to me. *Solved by Dr. Wallis, ibid. p. 587.*

That Paper contains a Geometrical Ænigma, which when ſtript of its verbal Diſguiſe ſeems to inſinuate the following Problem. From the curved Superficies of a Hemiſphere to cut off four equal Segments, ſo that what remains ſhall be capable of Quadrature.

And at the ſame Time it gives us a Hint, that ſomething yet remains among the *Grecian* Monuments, by means of which this may be done.

I imagine this to be the Quadrature of the Lunula of *Hippocrates* of *Chius*.

For whereas *Archimedes* has demonſtrated, that the curved Superficies of an Hemiſphere is equal to two great Circles of the ſame Sphere, that is, to four Semicircles; and *Hippocrates* of *Chius* has taught us how to ſquare a certain Lunula: If from every Quarter of this Hemiſpherical Cupola ſo much be taken away as that Lunula is deficient from a Semicircle, the Remainder will be equal to the Square that may be inſcribed in the great Circle of the Sphere, on which the Hemiſpherical Cupola inſiſts.

Now if beſides the Ænigmatical Involution of the Problem, any Hiſtorical Fact about a Temple ſhould be intended, I ſhould imagine the Temple here meant is that of *Sancta Sophia* at *Conſtantinople*.

SCHOLIUM.] By the Quadrature of the Lunula of *Hippocrates* of *Chius*, (mentioned in the firſt Book of *Ariſtotle's* Phyſicks, and in the Commentaries of *Simplicius* upon that Place) the Semicircle A B D being divided into two Quadrants, A C D, B C D, if the Subtenſe of the Quadrantal Arch A D be adapted, biſected in H by the Radius C E, and if Center H the Semicircle A D F is deſcribed: Becauſe of the Square of the Right Line A D being ſubduple of the Square of the Right Line A B, the Semicircle A D F will be ſubduple of the Semicircle A B D, and therefore will be equal to the Quadrant A C D. Now taking away the common Segment A D E from each, the remaining Lunula A E D F will be equal to the remaining Triangle A D C. And four ſuch *Lunulæ* will be equal to four ſuch Triangles, that is, to the whole Square A D B G inſcribed in the Circle. *Fig. 19.*

Again, by what *Archimedes* has demonſtrated, the Surface of the Sphere is equal to four of its great Circles; and therefore the curved Superficies of an Hemiſphere is equal to four ſuch Semicircles, and a Quadrant of ſuch an Hemiſpherical Surface is equal to one Semicircle.

Fig. 20. Let the Circle A D B G be now the Baſe of a curved Hemiſpherical Surface, whoſe Pole is P, its Axis C P perpendicular to the Plain of the Baſe, and one of its Quadrants D P A, which is biſected by a Plain E P C paſſing through the Axis.

Alſo for the Convenience of Calculation, let the Radius of the Circle be call'd R, the Diameter $D = 2R$, the Periphery P, and the Arch exhibited *a*.

And putting the Quadrantal Arch $DEA = a = \frac{1}{4}P$, the Semicircle A B D is $aR = \frac{1}{4}RP$: The Triangle $ADC = \frac{1}{2}R^2 = \frac{1}{4}RD$. And the Remainder of the Semicircle, when this Triangle is taken away, will be $\frac{1}{4}RP - RD$; an equal to which is to be taken away from D P A, a Quadrant of the curved Hemiſpherical Surface, equal to the Semicircle A B D, that the Remainder may be equal to the exhibited Triangle A D C.

Now ſince this may be done various Ways, by what we have ſhew'd long ago, *An.* 1659. (at the End of the Treatiſe of the *Cycloid* then publiſh'd, p. 122. at § 68.) and again *An.* 1670, (in the Treatiſe concerning Motion, C. V. P. 24.) concerning a Plain Figure, which is equal to any Figure in a Spherical Surface, that is terminated by any Circles greater or leſſer. It may be thus done in the ſimpleſt Manner.

Fig. 21. Since the Segments of a Spherical Surface, cut off by parallel Plains, are proportional to the Segments of the Axis; (which obtains likewiſe in the exhibited Superficies D P A of the Quadrantal Cuneus) if in the Axis C P there be taken, as the Semicircle $\frac{1}{4}RP$ to the Semicircle except the Triangle $\frac{1}{4}RP - \frac{1}{4}RD$, that is, as P to $P - D$, ſo C P to C Y: (or, which is the ſame thing, as P to D, ſo C P to P Y:) A Plain through Y Z, parallel to the Baſe, will cut off a Portion of this curved Superficies, adjoining to the Pole, which will be equal to the Triangle A D C. And the ſame being done in the other Quadrants of the curved Surface, the whole ſo cut off, conterminous to the Pole, will be equal to the whole Square inſcribed in the Baſe, and it will be diſtended as was required. *Q. E. F.*

Or thus more conciſely. The curved Surface of the Hemiſphere, (being equal to two great Circles) will be equal to R P. And the Square inſcribed in a great Circle $= 2RR = RD$: And that is to this as P to D. And therefore, becauſe of the Segments of the Surface cut off by the parallel Plains, (being proportional to the Segments of the Axis) taking C P to P Y as P to D, not only the whole Superficies will be equal to R P, but alſo the Portion at the Pole, cut off by the Plain Z Y, will be equal to R D, or to the Square inſcribed in the Baſe. *Q. E. F.*

If it be ſaid that we proceed here upon the Preſumption of the Quadrature of the Circle, or of the Ratio of the Perimeter of the Circle to its Diameter; this is true indeed, but not to be objected here. Becauſe the propoſed Ænigma does not require, that the Portions of the Hemiſpherical Surface which are cut off, or which are call'd the Windows, ſhould be quadruble, but only the remaining Portion. And indeed if both were required, the abſolute

Geometrical Quadrature of the Circle would be required at the ſame time; which it is plain is not yet found.

As to the manual Conſtruction of the Cupola; upon a plain Baſe, ſituate Fig. 19. out of the Baſe of the Hemiſphere, but contiguous to it, whoſe two Sides meet in an Angle at A, within the protracted Right Lines D A, G A, (that there may be a free Proſpect from the Windows on each Side) let a good ſtrong Foundation be laid; ſo that as the Superſtructure riſes higher, its Edge may ſtretch out, ſupported by an Angle, making an Arch of a Circle as D Z, ariſing to the Height Y. And let the ſame be done at the other Angles D, B, G. Laſtly upon theſe Structures, as it were ſo many Columns raiſed to the ſame Height, let the Cupola be placed, ſo excavated within, as the Surface of an Hemiſphere requires. And thus the whole Work is finiſhed as was required.

Otherwiſe. The ſame thing may be done, if any Square Q Q be given, for the Square inſcribed in the Baſe, which is leſs than the curved Hemiſpherical Superficies. For it muſt be made, as R P the curved Hemiſpherical Superficies, to Q Q the given Square, ſo C P the Axis of the Hemiſphere, to P Y the Portion of the Axis that is adjacent to the Pole. Then the Plain Z Y, parallel to the Baſe, will cut off a Portion of the Spherical Superficies which is capable of Quadrature, as being really equal to the given Square Q Q.

The ſame may be perform'd thus, but with more Trouble.

Since a Quadrant of the curved Hemiſpherical Surface D P A, as is ſhewn Fig. 20. above, is equal to the Semicircle A B D, and its Segments cut off by Plains parallel to the Baſe are proportional to the Segments of the Axis; in the Quadrantal Arch D P let there be taken P Q an Arch of 60 Degrees; (which was ſuggeſted to me by Mr. *Caſwell.*) Then a Circle Q T S deſcribed with the Pole P will biſect the Axis, becauſe of the Verſed Sine of 60 Degrees being equal to half the Radius: So that it will divide the Quadrant D P A of the curved Hemiſpherical Superficies into two equal Segments. One of theſe, which is the Quadrilineum D Q T S A, will be equal to the Circular Quadrant B C D; and the remaining Trilineum P Q T S will be equal to the Quadrant A C D. Whence if the Bilineum Q R S T be alſo taken away, equal to the Segment of the Circle A D E; the remaining Trilineum P Q R S will be equal to the Triangle A D C. And four ſuch, in the four Quadrants of the Hemiſphere, will be equal to the Square inſcribed in the Baſe. But that Bilineum may be had by what we have formerly proved in the Places above cited.

The ſame may be thus done more univerſally.

Q being any where taken in the Arch D Z, ſo that D Q may not be greater than D Z; and how much the Quadrilineum D Q T S A is deficient from the whole to be taken away, ſo much let the Bilineum Q R S T ſupply. The Remainder will be equal to the Triangle A D C.

And therefore if Q be taken in D, ſo that the Quadrilineum may vaniſh; the Bilineum muſt be taken equal to the whole that is to be taken away.

But if Q is taken in Z, ſo that there is no Occaſion for the Bilineum, the Quadrilineum will be equal to the whole that is to be taken away.

And all the ſame Things, concerning the Quadrilineum and Bilineum, which together compleat the whole to be taken away, are alike to be accommodated, *mutatis mutandis*, if inſtead of the Square inſcribed in the Baſe, any other Square Q Q is ſubſtituted, which is not greater than the whole Surface of the Hemiſphere.

After this was wrote, and in the Preſs, I was informed, that the learned Mr. *Leibnitz* had given an Anſwer to this Problem, and that it was inſerted in the Journals of *Leipſick* for the Month of *June* 1692. Upon which I ſtopt the Preſs for ſome Weeks, that I might get a Sight of it, which with ſome Difficulty I at laſt obtained. There I ſaw that that great Man was of my Opinion, that the Problem is not of that Kind which is called Determinate, but may be ſolved in a great Variety of Manners, or rather infinite Ways.

The Propoſer's Solution demonſtrated, by Dr. D. Gregory. N. 207. *p.* 25. Jan. An. 1694.

The Author of this Ænigma has now given us a very ingenious and ready Conſtruction of his Problem, in an *Italian* Treatiſe on the Formation and Menſuration of all Vaults and Cupolas, dedicated to the Grand Duke of *Tuſcany*; wherein he has been pleaſed to give us his Name, by V V. the laſt Diſciple of *Galileus*: Whereas before he concealed it by a Tranſpoſition of the Letters, as in an Anagram, under the feigned Name of *D. Pio Liſci, puſillo Geometra.*

But now the Ænigma is converted by the Author into the following Problem: *Upon the Surface of an Hemiſphere to aſſign a Portion equal to a given Square.* Which he thus conſtructs.

Fig. 22.

Let the Sphere, whoſe Axis is equal to the Side of the given Square, be repreſented by the Circle A C B D, which is vertical in the propoſed Sphere, whoſe Horizontal Diameter is A B, and Center E. Let the Sphere be perforated by two upright Cylinders, whoſe common Sections with the Plain A C B D are the Circles B L E G, A H E I, deſcribed with the Diameters E B, E A. I ſay the Thing is done; that is, from every Hemiſphere, for Inſtance the upper A C B, four bilinear Figures are taken off by the perforating Cylinders, two on the anterior Side, and two on the poſterior, which are ſimilar and ſimilarly poſited, ſo that the remaining Hemiſpherical Superficies is equal to the Square of the Line A B. And becauſe the Hemiſpherical Superficies, when the four ſaid Bilinear Spaces are taken away, repreſents a Sail filled and extended by the Wind, and alſo an Hemiſpherical Cupola admitting Light by four Windows, which being conſtructed upon a Circular Baſe A E B, reſts upon it at the Points A, E, E, B. This he calls, according to a Right he has, *The Quadruple Florentine and Veliform Cupola.*

Then the Author in his Treatiſe delivers many Things which regard the Practice; how by the Aſſiſtance of the Lathe and Cylindrical Auger to make Models of this, as well as of five other Cupolas. And for this purpoſe, he conſtructs ſome other curious Problems; the Demonſtrations of all which are omitted by the Author, but will eaſily follow from what is here delivered.

It appears plainly, that the four Windows in the Hemiſphere, conſtructed as above, are Figures equal, ſimilar, and ſimilarly poſited. It only remains, that we ſhould prove the remaining Hemiſpherical Superficies is capable of a true Geometrical Quadrature.

In the Point E, equal to the Line E A, let a Line be ſuppoſed to be erected, perpendicular to the Plain C A D B; and upon the Periphery ACBD let there be an erect Cylindrical Superficies of the ſame Height. It is commonly known, that a Portion of the Spherical Surface, comprehended between two Plains parallel to the Circle A B C D, is equal to the Portion of the Cylindrical Superficies between the ſame Plains; and that like Portions of theſe Rings, cut off by Plains mutually interſecting one another in the Perpendicular erected at E, are equal alſo. Now by drawing innumerable Plains parallel to the Baſe A C B D, if in the Cylindrical Superficies Parts are conceived to be deſcribed in the aforeſaid Manner, equal to the correſpondent Spherical Parts; that which is repreſented by the Perforation of the Superficies, and taken away oppoſite to it, will be equal to it. So that it appears the remaining Superficies after the Perforation is equal to the remaining Cylindrical Superficies, excepting that which is determined by the ſaid innumerable Plains, and is oppoſite to that which is taken away. Let any Diameter P M be drawn, cutting the Periphery A H E any how in H. Join H A, and through H let R T be drawn perpendicular to A B, and parallel to D C drawn through E, meeting the Periphery A C B D in R and T, and the Periphery A I E in I. Upon the Diameter R T let a Semicircle be drawn, whoſe Periphery is cut by H S and I Q perpendicular to R T, in the Points S and Q. Let the Plain of this Semicircle be conceived to be perpendicular to the Circle A B C D. Whence the Periphery R S Q T will be in the Hemiſpherical Superficies, and the Right Line H S, now perpendicular to the Plain A C B D, will be the Height of the perforating Cylindrical Superficies above the Point of the Baſe H. And the ſame Thing obtains in every Point of the perforating Cylindrical Superficies, *viz.* that its Height to the Superficies of the Sphere above any Point H of the Baſe, is the Right Line H S produced as before. But H S is equal to H A, the Right Sine of the Arch M A, becauſe either of them is a mean Geometrical Proportional between P H and H M; one in the Circle M A P, the other in a great Circle of the Sphere, paſſing through the Points M, S, and P.

If in the Perpendicular to the Plain A C B D erected at E, from E be taken a Right Line equal to H S or H A, and from its Extremity parallel Lines are drawn to P M and U N; the Plain drawn through them will be parallel to the Plain A C B D, and theſe Lines will paſs through the Points S and Q, and being produced as far as the Cylindrical Surface conſcribed about the Hemiſphere, from the Sides of the Cylinder will cut off Right Lines, which will likewiſe be equal to H S or H A, and will include Arches equal and correſpondent to the Arches M N and V P. Now if another Plain Parallel to this at a very ſmall Diſtance be conceived to be drawn in like Manner, by what is already ſhewn, theſe two will mark out in the Cylindrical Surface a Portion of a Ring, equal to the Portion between the ſame Plains, which is taken away from the Hemiſpherical Superficies by its Perforation.

foration. Now if the ſame Conſtruction is ſuppoſed to be made at every Point in the Periphery A H E, all the Portions in the Cylindrical Superficies circumſcribed about the Hemiſphere, generated and marked out in the Manner aforeſaid, will be equal to the Spherical Superficies taken away by the Perforation. Therefore the remaining Hemiſpherical Surface will be equal to the remaining Cylindrical Surface, compoſed of all the Right Lines H A, erected at the reſpective Points M, N, V, and P, or to the Figure of the Right Sines of the Semiperipheries A C B, A D B; that is, by what has been long known to Geometricians, to four Times the Square of the Radius A E, or finally to the Square of the Diameter A B. And ſince the two intire Figures contained by the common Section of the aforeſaid perforating Cylindrical Surface with the Surface of the Sphere, are equal to the four Halves of the ſame; it is plain that the remaining Hemiſpherical Superficies A C B, taking away the four Bilinear Spaces as in the Conſtruction aforegoing, is equal to the Square of the Diameter A B. *Q. E. D.*

If the Semiperiphery A H E is ſo inflected, that it may coincide with the equal Quadrant of the Periphery A R C; the Point H will fall upon the Point M, becauſe of equal Arches A H, A M; and H S the Altitude at H of the Cylindrical Surface inſiſting upon A H E, will coincide with its equal H A, the Altitude at M of the Figure of Right Sines erected upon A M C; and the ſame Thing obtains in all other Points. Whence the Curve which is the common Interſection of the Spherical Surface with the Cylindrical Surface, inflected upon the Baſe A H E, though it does not lie in the ſame Plain, yet will coincide with it as ſaid before, and therefore is equal to the Curve that terminates the Figure of Right Sines; that is, to the common Section of the Cylindric Surface erected upon the Quadrantal Arch A R C, with the Plain cutting the Plain of the Baſe in the Right Line B A at half Right Angles; or to a Quarter of the Elliptical Curve whoſe leſſer Axis is A B, and its greater Axis is double in Power to the ſame. Therefore the Perimeter of the Quadrable *Florentine* Sail, conſiſting of four ſuch Arches, is equal to the Perimeter of the aforeſaid Ellipſis.

Moreover it will not be amiſs to add, that the Superficies of two perforating Cylinders within the Sphere, are equal to the Spherical Surface remaining after the Perforation, or to a double *Florentine* Sail, that is, to the double Square of the Diameter. And this appears from hence, that the *Florentine* Sail is equal to four Figures of Right Sines of the Quadrant, and the perforating Superficies is alſo equal to the ſame, becauſe it is congruous with them, if the Inflection is as above.

I ſhall only add one Word more, that the Conſideration of the Figure of Right Sines, (the Parts alſo of which are eaſily changed into Squares) are ſufficient for the Demonſtration of all thoſe Things, which are delivered concerning other Solids wrought by the Turning-Lathe, or perforated by a Cylinder, and their Superficies; by the moſt acute Geometrician V. V. (*Vincentio Viviani*, if I miſtake not) the very worthy Diſciple of *Galilæus*; when he inſtructs us in the Conſtruction and Menſuration of Vaults or Cupolas. Particularly the Surface of the Roman Boat-like Cupola [*Volta a Schifo alla Romana*] mena] conſiſts of eight Figures of the Right Sines of the Quadrantal Arch, and therefore is equal to the *Florentine* Veliform Cupola. Whence it appears how two Cupolas may be conſtituted upon equal Squares, one of which is ſhut up on all Sides, the other perforated by Windows, each of which is double to the Square of the Baſe.

VII. 1. *Drawing the ſtreight Lines E A, and E B (cutting the Arc A B in G,) and on A G, a Perpendicular E F, (which will therefore paſs to the Center C, beauſe biſecting A G at Right Angles;) the Right-lined Triangle A F E is equal to A D E, the propoſed Portion of the Lunula.*

The Quadrature of the Parts of the Lunula, *by Mr.* J. Perks, *a little varied, by Dr.* J. Wallis, N. 259. *p.* 411. Dec. An. 1699. *Fig.* 23.

The Demonſtration is to this purpoſe; *viz.* A D B being a Quadrantal Arc; the Angle A G B will be three Halves of a Right Angle; (and its conjunct Angle E G A, half a Right Angle) and that Angle (being external to the Triangle A G E) is equal to the two oppoſite Intervals G E A + E A G. Whereof G E A (becauſe an Angle in the Semicircle A E B) is a Right Angle, and therefore E A G is half a Right Angle, (as are alſo F E G, and F E A) and the three Triangles A F E, G F E and G E A, each of them half a Square. And A G to A E, as √ 2 to 1, (proportional to the reſpective Radii of the two Circles.) And the like Segments A D G, A E, in their reſpective Circles (as the Squares of their reſpective Radii) as 2 to 1. And therefore the Semiſegment A F D, equal to the Segment A E; and conſequently (one taking from the Triangle as much as the other adds to it) the Portion of the Lunula A D E, equal to the Triangle A F E. *Q. E. D.*

If the Point E chance to be in K (the middle of the Arc A E B), there will be no Interſection at G (the Points G, B, being then coincident, but without any Diſturbance to the Demonſtration): If it happen beyond it, toward B, then G will be on the other ſide; and what is here ſaid of E G B, muſt be accommodated to E G A.

The Ground of the whole Proceſs is plainly this: The Angle A C E, being an Angle at the Center of the greater Circle, but at the Circumference of the Leſſer, the Line C D E (as it paſſeth from C A to C B) doth, in the ſame Proportion, divide the Quadrantal Arc A D B, and the Semicircular A E B: Whence all the reſt doth naturally follow.

2. If you compleat the two Circles, whoſe Arcs contain the *Lunula* of *Hippocrates*, the ſame is true as well of the Points in the other Semicircle A C B, of thoſe in the Semicircle A E B, and for the ſame Reaſons; as appears in Scheme annexed, wherein I have mark'd the Points in the Semicircle ACB, rreſpondent to thoſe of Mr. *Perks*, in A E B) with the correſpondent ill Letters in the *Roman* and *Greek* Alphabets.

Improved by Dr. Gregory, *ibid. p.* 414. Dec. An. 1699. *Fig.* 24.

If Mr. *Perks* had made his Conſtruction univerſal, by making both E A and B meet with the greater Circle, (which he might have done by protracting ſe Lines and the greater Circle till they meet) he might have found that Portions of the Spaces A ε C M, B H C N, (ſuppoſing M C N parallel to B) are Quadrable as well as thoſe of *Hippocrates*'s *Lunula*, and that E A γ being

ing a ſtreight Line, the Portion A E D of *Hippocrates*'s *Lunula*, is to A ε δ (the Correſpondent of A ε C M) in duplicate Proportion of C ε to A ε; for E R ε (at R the Center of the leſſer Circle) is, in this Caſe, a Right Angle.

Moreover, If you take any Point ε, in the Semicircle A C B, and proceed according to Mr. *Perks*'s Conſtruction univerſaliz'd, as aboveſaid, you will find, on the one ſide, the Trilineum A ε δ (contain'd by the Arcs A ε, A δ, and the ſtreight Line ε δ) equal to the Rectilineal Triangle A ε φ; and on the other ſide, the Trilineum contained by the Arc B ε (the Complement of ε A to the Semicircumference) and the Arc B *d* (the Complement of A δ to the fourth Part of the Circumference) and the ſtreight Line ε *d*, (that is the Trilineum B H C *d* diminiſhed by the Segment C ε) to be equal to the Rectilineal Triangle B ε *f*; and that thoſe two Spaces A ε δ, and the Difference of B H C *d* from the Segment C ε (Parts of the *Lunula* A C B *g* γ A) taken together, are equal to the Triangle A C B, as well as to the two Spaces A E D and B E D, Parts of the *Lunula* of *Hippocrates*.

So that upon the whole it appears, that the two Circles (containing the *Lunula* of *Hippocrates*) being compleated, this *Lunula*, A E B G A, and the other, A C B *g* γ A, make up one Syſtem, and are conjugate Figures.

For, drawing a ſtreight Line C D E, or C ε δ, or ε C *d*, at pleaſure, thro' C, the Center of the greater Circle, and cutting thoſe two Circles, the Space contained within two Arcs of theſe two Circles, and Part of the ſaid ſtreight Line (as A E D, or A ε δ, or B H ε *d*) is equal to the rectilineal Triangle A E F or A ε φ, or B ε *f*, reſpectively.

And ſo it happens, that if this Line going out from C, be on the ſame ſide of the Diameter M N with the *Lunula* of *Hippocrates*, the aforeſaid Space (which receives a perfect Quadrature) is Solitary; ſuch as are the Parts of *Hippocrates*'s *Lunula*, and of the two Spaces, A ε C M, B H C N (which therefore are Parts of the *Lunula*, more nearly relating to one another.)

But if that Line going out from C be on the other ſide of M N, then the Space which is equal to the Rectilineal Triangle is the Difference of two Mixtilineal Figures (the one a Trilineum, the other a Segment of the leſſer Circle), as is aboveſaid; neither of which can be ſquared ſeverally.

All theſe Particulars are plain from Mr. *Perks*'s Demonſtration; which, with a little Variation (ſuch as is uſual in the different Caſes of the ſame Theorem), is applicable to them all; tho' perhaps he was not aware of it.

The like was done (without any Demonſtration) by M. *Tſchirnhauſe*, in the *Acta Lipſiæ* 1687, to this purpoſe: If from any Point E, in the Circumference of the leſſer Circle, we let fall on A B a perpendicular Line cutting it in L, and draw the Line C L; the Triangle C A L is equal to the Portion of the *Lunula* A E D (and conſequently the Triangle C B L equal to the Portion B E D); which I ſhall demonſtrate ſo as the Demonſtration may alſo reach the Portions of the conjugate Space A C B *g* γ A.

For the Triangles A C B, A E F, are Like Triangles, each being the Half of a Square; and therefore, by 19 *El.* 6. the Triangle A C B is to the Triangle A E F in the duplicate Proportion of B A to A E, that is, by 8 *El.* 6. as B A is to A L: But, by 1 *El.* 6. the Triangle A C B is to the Triangle

A C L

A C L as B A is to A L: Therefore, by 9 *El.* 5. the Triangles A C L and A E F are equal: But the Triangle A E F is (by Mr. *Perks*) proved equal to the Portion A E D; and therefore the Portion A E D is alſo equal to the Triangle A C L.

3. On the Center B, Mr. *Caſwell* draws by A a third Circle, which forms another *Lunula* than that of *Hippocrates*; and he doth (very dextrouſly) ſquare the Portions of this *Lunula*; and doth hereby let us into a new Syſtem, which may be purſued in like manner as Dr. *Gregory* hath done that of *Hippocrates*.

By Mr. Caſwell, *ib. p.* 417.

4. M. *Tſchirnhauſe*, letting fall from E (on A B) a Perpendicular E L, determines the Angle A L C equal to the Portion A D E; which being admitted, we may thus divide the *Lunula* in any given Proportion; if we divide A B at L in ſuch given Proportion, C L will, in the ſame Proportion (becauſe of the common Altitude) divide the Triangle A C B (which is equal to the whole *Lunula*), and L E (erected at Right Angles on A L B) will determine the Point E; from whence if we draw to C the ſtreight Line E C, this will, at D E, divide the *Lunula* in the ſame Proportion.

By Dr. Wallis, *ibid.* *Fig.* 25.

Mr. *Perks*, on E D C drawing the Perpendicular A F, determines the Semiquadrate A F E equal to the propoſed Portion A D E; which Semiquadrate is a Like Figure, and alike ſituate to A E as is A C B to A B.

Fig. 26.

And therefore (becauſe Like Figures are in duplicate Proportion of their reſpective Sides) if we ſo inſcribe A E, as that the Square of A E be to the Square of A B in ſuch given Proportion, the *Lunula* will, at D E, be ſo divided as is required.

And this will hold (if duly applied, according as the different Caſes may require) though E be taken (in the Continuation of the Semicircle) beyond A; for, ſtill like Figures will be in duplicate Proportion of their reſpective ſides, and C E = C D = D E; and the ſame is yet improveable much further.

VIII. If upon B C you take any two Points D, E, and draw the Perpendiculars D H, E M, meeting B A in I and L, and cutting a Portion FGMH, of the *Lunula*; the Solid generated by the Converſion of this Portion about the Axis B C, is equal to a Priſm, whoſe Baſe is I L M H, and Height the Circumference of a Circle whoſe Diameter is B C; and the Solid generated by the Semicircle B K A, is equal to a Priſm or Semicylinder, whoſe Baſe is the micircle B K A, and Height the Circumference of a Circle whoſe Diame- is B C.

The Dimenſion of Solids generated by the Converſion of Hippocrates's Lunula, *by M.* Ab. de Moivre, N. 265. *p.* 624. Jul. An. 1700. *Fig.* 27.

Having biſected B A in R, and B C in P, the Surface generated r the Converſion of the Arc H M about the Axis B C, is equal to $\overline{c\,BP \times HM + BR \times DE}$ (ſuppoſing the Ratio of the Radius to the Circ mference to be as *r* to *c*) and the Surface generated by the Semicircumfe- ice B K A is equal to a Rectangle whoſe Baſe is the Sum of that Semicir- mference and Diameter B A, and Height, the Circumference of a Circle, whoſe Diameter is B C. As for the Surface generated by the Arc G F, 'tis well known that it is equal to a Rectangle, whoſe Baſe is the Circumference of a Circle whoſe Radius is B C, and Height D E; therefore the Surface generated by the Converſion of the Portion M H F G is known.

Fig. 28. If upon B A, you take any two Points I, L, and draw I N, L V, perpendicular to it, cutting the Quadrant in O and T, and the Circumference in N and V; the Solid generated by the Converſion of the Portion O N V T about the Axis B A, is equal to a Priſm whoſe Baſe is I O T L, and Height the Circumference of a Circle whoſe Diameter is B A.

Having biſected B A in R, and drawn C R meeting the Quadrant in G, the Surface generated by the Converſion of the Arc O T about B A, is equal to $\frac{c}{r}\times\overline{CG\times IL - CR\times OT}$.

Fig. 27. Biſect D E in I; thro' the Center draw S Q, parallel to B C, meeting the Circumference B K A in S, B K parallel to A C in V, and the Lines D H, E M, in N and O; the Solid generated by the Converſion of the Portion F G M N about the Axis A C, is $\frac{c}{r}\times\overline{\tfrac{1}{3}MO^3 - \tfrac{1}{3}NH^3 + PC\times NOMH + CY\times DNOE - \tfrac{1}{3}EG^3 + \tfrac{1}{3}DF^3}$; and the Solid generated by the Segment K B S is $\frac{c}{r}\times\overline{\tfrac{2}{3}VK^3 + PC\times BVKS}$; therefore the Solid generated by the Semicircle B K A about A C, is $\frac{c}{r}\times\overline{PC\times VQAK + PC\times BCQV - \tfrac{1}{3}AC^3 + \tfrac{2}{3}VK^3 + PC\times BVKS}$, which by due Reduction, will be found equal to the Solid generated by the Converſion of the ſame Semicircle about the Axis B C.

Fig. 28. The Solid generated by the Portion O N V T, about the Axis C P, is equal to $\frac{c}{r}\times\overline{\tfrac{1}{3}LV^3 - \tfrac{1}{3}IN^3 - \tfrac{1}{3}QT^3 + \tfrac{1}{3}PO^3 + CS\times PQIL}$.

Fig. 27. From the Points M, H, drop the two Perpendiculars M Z, H W, upon C A prolonged, if need be; the Surface generated by the Converſion of the Arc H M, about the Axis C A, is equal to $\frac{c}{r}\times\overline{PC\times HM - RA\times WZ}$, when the Point Z is next to C; or $\frac{c}{r}\times\overline{PC\times HM + RA\times WZ}$, when the Point W is next to it.

Thoſe that will think it worth their while to beſtow ſome little Pains to find the Demonſtration of this, may ſolve the following Problem.

Any two Conic Sections being given, forming a Lunula by their Interſection, and a Right Line being given by Poſition, about which, as an Axis, this Lunula is imagined to turn, To find the Solid generated by the Converſion of any of its Parts, cut off by Lines perpendicular to that Axis, or parallel to it, or making any given Angle with it; as alſo the Surfaces made by that Converſion.

IX. Suppoſe D P V to be Half of an exterior Epicycloid, V B its Axis, V the Vertex, V L B Half the generant Circle, E its Center; D B the Baſe, C its Center: Biſect the Arc of the Semicircle V B in L, and on the Center C, thro' L, draw a Circle cutting the Epicycloid in P: Then, I ſay, the Curvilinear Triangle V L P will be $= BEq$ in $\frac{CE}{CB}$; that is, the Square of the Semidiameter of the generant Circle, will be to the Curvilinear Triangle V L P, as C B, the Semidiameter of the Baſe, to C E; which C E in the exterior Epicycloid is the Sum of the Semidiameters of the Baſe and Generant; but in the interior Epicycloid D *p u*, it is the Difference of the ſaid Semidiameters.

The Quadrature of a Portion of the Epicycloid, by Mr. Caſwell. N. 217. p. 113. Oct. An. 1695. *Fig.* 29.

COROLL. I.] In the interior Epicycloid, if C E is $\frac{1}{2}$ C B, the Epicycloid then degenerating into a Right Line, the Quadrature of the Triangle *l p u*, will be in effect the ſame with the Quadrature of *Hippocrates Chius.*

COROLL. II.] If the Semidiameter of the Baſe is ſuppoſed infinite, the Epicycloid then being the common Cycloid, the Area of the ſaid Triangle will be equal to the Square of the Radius of the Generant; and ſo it falls in with that *Theorem* which *Lalovera* found, and calls *Mirabile.*

The general Propoſition from whence I deduced the aboveſaid Quadrature, is this, *viz.* The Segments of the generant Circle are to the correſpondent Segments of the Epicycloid, as C B to 2 C E + C B. For Example; Suppoſe F *m* G, the Poſition of Part of the Generant, when the Point F of the exterior Epicycloid was deſigned, then the Segment F *m* G *n* is to the Segment D F *n* G, as C B to 2 C E + C B; and conſequently the whole Epicycloid to the whole Generant in the ſame Proportion; which is the only Caſe demonſtrated by M. *de la Hire.*

It follows alſo, that in the Vulgar Cycloid its Segments are triple of the correſpondent Sectors of the Generant; which was firſt ſhewn by Dr. *Wallis.*

X. *The Area of the Cycloid or Epicycloid, whether it be primitive, or contracted, or dilated, is to the Area of the generating Circle; and alſo the Areas of the generated Parts in the ſame Curves, to the Areas of the analogous Segments of the Circle; as the Sum of the double Velocity of the Center and the Velocity of the Circular Motion, to the Velocity of the Circular Motion.*

A general Propoſition for meaſuring all Cycloids and Epicycloids, by Mr. Edm. Halley, n. 218. p. 125.

Demonſtration. Let any Epicycloid Y P S Q V B be deſcribed, by the Re-ution of the Circle V L B, upon the Circular Baſe Y M N B. Let the

Fig. 30.

Center of the generating Circle be *c*, and drawing *c* M K, let the Circle inſiſt upon the Baſe in the Point M, and let S be the delineating Point. Now dividing the Motions, by the Circular Motion firſt let the Point S be tranſferred to R, that the Arch S M may be increaſed by the indiviſible Particle R S; then let the Center *c* go forward to C. By this Motion the Segment R S M being tranſlated to the Situation Q T N, the Point Q will reach the Curve. It is plain that the Triangle R S M is the Moment or Fluxion of the Area of the Circular Segment: But the Trapezium Q S M N is the Fluxion of the Area of the Curvilinear Space generated at the ſame Time. Now ſince S M, R M, Q M, are ſuppoſed to differ from one another only by a Point, conceive the little Area Q S M N to conſiſt of three Sectors R M S, R M Q, M Q N; and therefore the little Area R M S is to the little Area Q S M N, as the Angle R M S to the Sum of the three Angles R M S + R M Q + M Q N. But the Angles R M Q + M Q N are equal to the Angles M C N + M K N, or to the Angle *c* M C; becauſe of the Lines R M, Q N inclined to one another in an Angle equal to M K N, and becauſe of the Angle M Q N being half M C N, by *Eucl.* 20. III. Therefore the Angle R M S is to the Angles R M S + *c* M C, that is, (by the ſame) the Arch $\frac{1}{2}$ R S to the two Arches C *c* + $\frac{1}{2}$ R S, or R S to 2 C *c* + R S, as the little Area R S M to the little Area Q S M N; or as the Moment of the Circular Segment Q T N, to the Moment Q S Y M N of the Epicycloid generated at the ſame Time. And as theſe Moments are always in the ſame Ratio, wherever the Point Q is taken, it is plain that the Areas themſelves Q T N, Q S Y M N, generated by theſe Moments, have the ſame conſtant Ratio, or that of the Velocity of the Circular Motion R S, to the double Velocity of the Center adding the Circular Motion, or 2 C *c* + R S; alſo as the Area V B Z to the Area Q V B N, and therefore the Semicircle V L B to the Curvilinear Space V Q Y N B. So that the Propoſition is manifeſt. Now there is no other Difference in the Manner of demonſtrating, if the generating Circle moves upon an Arch of a concave Baſe, except that the Angle *c* M C in this Caſe is the Difference of the Angles M C N and M K N. But if the Baſe be a Right Line, then M K N vaniſhing, and becauſe of the Parallels R M, Q N, the Proof becomes eaſier. Now in all theſe Curves there are quadrable Portions, analogous to thoſe Portions in the primary Cycloid, which the learned Dr. *Wallis* has found to be capable of perfect Quadrature: Which eaſily follows from the Premiſes.

With Center K through the Point Q draw the Circular Arch Q Z, and draw Z B cutting off a Segment Z L B, equal to the Segment Q T N; then biſect the Semicircle V B in L, and through the Point L, with the ſame Center K deſcribe the Arch P L, cutting the Epicycloid in P, the generating Circle in T, and the Chords Q N, Z B, in *y* and X. Now let the Arch V Z $= a$, and its Sine $= s$, the generating Radius $= r$, and the Radius of the Baſe $=$ R. And make the Arch C E, or the Motion of the Center, $= m$. It is plain that the Sector C K E has the ſame Ratio to the Space X *y* N B, as the Square of K E has to the Difference of the Squares of K L and K B; or as $RR + 2Rr + rr$ to $2Rr + 2rr$; that is, as $R + r$ to $2r$, or K E to B V; and therefore the Rectangle B E × C E, or $r\,m$, is equal to the Space X y N B. But the Space V Z B is equal to the Rectangle $\frac{1}{2}ar + \frac{1}{2}sr$; and therefore according to our Propoſition it will be, as a to $2m$, ſo is $\frac{1}{2}ar + \frac{1}{2}sr$ to $\frac{mar + msr}{a}$, which is equal to the Curvilinear Space QVZLBNQ. From this ſubſtract the Space X y N B $= rm$, and there will remain the Space Q V Z X $y = \frac{mrs}{a}$. And ſince the Spaces Z X L, Q y T are equal to each other, the Space Q V L T Q will alſo be equal to $\frac{mrs}{a}$. Therefore whenever a to m, or the Circular Motion to the progreſſive Motion of the Center, ſhall be in a given Ratio; the perfect Quadrature of the Curvilinear Spaces QVLTQ will be given alſo. Alſo the whole Space V P L, to the Square of the Radius B E, will be in the ſame Ratio of the Motions m to a, that is, in every primary Epicycloid in the Ratio of the Radii K E to K B; which is the Propoſition of Mr. *Caſwell.* But the leſſer Spaces Q V L T Q will be to one another as the Sines of the Arches V Z; and the Triangular Spaces Q T P, by the ſame way of arguing, will be as the verſed Sines of the Arches Q T or Z L, and are therefore quadrable. In like manner it may be proved, that the Spaces P Λ ϒ, p L u, p λ ϒ, are always to the Square of the Radius B E (in all theſe Figures) in the aforeſaid Ratio of m to a; and their Portions $p\,q\,t$ as the verſed Sines of the intercepted Arches $q\,t$. But the remaining Segments, as $q\,t$ ϒ Λ, $q\,t$ ϒ λ, *&c.* will be as the Right Sines of the Complements of the ſame Arches $q\,t$.

Now the Ratio of the Velocities m to a is compounded of the Ratio of the Radii KE, BE, and the Ratio of the Angles that are equably deſcribed at the ſame Time CKE, VEZ; and therefore that Ratio of the Angles being given, all the foreſaid Epicycloidal Spaces will alſo be ſquared.

XI. 1. *A Curve is required with this Property, that the two Segments (of a Right Line drawn from a given Point through the Curve,) being raiſed to any given Power, and taken together, may every where make the ſame Sum. We leave it to Analyſts to exhibit a general Solution.*

A Problem propoſed by M. J. Bernouilli. n. 224. *p.* 387. Jan. An. 1697.

2. The Problem (if I rightly underſtand it) may be thus propoſed. A Curve KIL is required with this Condition, that if a Right Line PKL be ny how drawn from ſome given Point or Pole P, meeting the ſame Curve two Points K and L; the Powers of its two Segments PK and PL, awn from the given Point P to thoſe Points of meeting, if they are raiſed an equal Height, (that is, either Squares, or Cubes, or Biquadrates, *&c.*) every Poſition of that Right Line they may make the ſame Sum, P K q P L q, or P K$^{\text{cub.}}$ + P L$^{\text{cub.}}$ *&c.*

Solved by If. N. *ibid. p.* 389. *Fig.* 31.

olution. Through any given Point A let an infinite Right Line be drawn D B given in Poſition, meeting the moveable Right Line PKL in the int D; and call A D, x; and P K or P L, y; and let Q and R be Quanes any how compoſed of any given Quantities and the Quantity x; and t the Relation between x and y be denoted by this Equation, $yy + Qy +$

$R = 0$. And if R be a given Quantity, the Rectangle of the Segments PK and PL will be given. If Q be a given Quantity, the Sum of thoſe Segments (conjoined by their proper Signs) will be given. If $QQ - 2R$ is given, the Sum of the Squares ($PKq + PLq$) will be given. If $Q^3 - 3QR$ is a given Quantity, the Sum of the Cubes ($PK^{cub.} + PL^{cub.}$) will alſo be given. If $Q^4 - 4Q^2R + 2R^2$ be a given Quantity, then the Sum of the Biquadrates ($PKqq + PLqq$) will alſo be given. And ſo on *ad infinitum.* Therefore let it be provided, that R, Q, $Q^2 - 2R$, Q, $Q^2 - 2R$, $Q^3, - 3QR$, *&c.* may be given Quantities, and the Problem will be reſolved. *Q. E. F.*

In the ſame manner Curves may be found, which ſhall cut off three or more Segments having the like Properties. Let there be an Equation $y^3 + Qy^2 + Ry + S = 0$; where Q, R, S, denote Quantities compoſed of any given Quantities whatever, and of the Quantity x any how involved; in which Caſe the Curve will cut off three Segments. Now if S be a given Quantity, the Solid contained by thoſe three will be given. If Q be a given Quantity, the Sum of three ſuch will be given. If $QQ - 2R$ be a given Quantity, the Sum of the Squares of three ſuch will be given.

The Uſe of Fluxions *in the Solution of Geometrick Problems; by Mr.* Abr. de Moivre. n. 216. *p.* 52. Mar. An. 1695.

XII. Here you have a Method for the Quadratures of Curvilinear Figures; for the Dimenſion of Solids generated by the Rotation of a Plain, and of their Superficies; for the Rectification of Curves; and for the Calculation of their Centers of Gravity. But before I go any farther, I would have you underſtand, that I make uſe of what the great *Newton* has demonſtrated, in Pag. 251, 252, and 253 of his Philoſophical Principles, about the momentary Increments or Decrements of Quantities, which either increaſe or decreaſe by perpetual Flux; and eſpecially that the Moment of any Power $A^{\frac{n}{m}}$, is $\frac{n}{m} a A^{\frac{n}{m} - 1}$.

Therefore the Fluxion $\frac{n}{m} a A^{\frac{n}{m} - 1}$ being given, on the contrary we may find the flowing Quantity $A^{\frac{n}{m}}$, firſt by removing a from the Fluxion. Secondly by increaſing the Index of the Fluxion by Unity. And thirdly, by dividing the Fluxion by the Index ſo increaſed by Unity.

The Abſciſs of the Curve in what follows ſhall be denoted by x; its Fluxion by $\dot{x}$; the Ordinate by y, and its Fluxion by $\dot{y}$.

This ſuppoſed, to proceed now to Quadratures, firſt let the Value of the Ordinate be obtained, by means of the Equation expreſſing the Nature of the Curve. Secondly, let this Value be multiplied by the Fluxion of the Abſciſs, and the Rectangle hence ariſing will be the Fluxion of the Area. Laſtly, from this Fluxion of the Area let its Fluent be found, and we ſhall have the Area required.

Let the Equation $x^m = y^n$ be propoſed, expreſſing the Nature of any Paraboloid, in which the Value of the Ordinate will be $y = x^{\frac{m}{n}}$; which if multiplied by $\dot{x}$, the Rectangle $x^{\frac{m}{n}}\dot{x}$ will be the Fluxion of the Area. Therefore the Area required will be $\frac{n}{m+n} x^{\frac{m}{n}+1}$, or putting y for $x^{\frac{m}{n}}$, it will be $\frac{n}{m+n} xy$.

Again, let a Curve be propoſed whoſe Equation is $x^4 + a^2 x^2 = y^2$, (which is the firſt among Mr. *Craig*'s Examples) then aſſuming $x\sqrt{xx+aa} = y$, the Fluxion of the Area will be $x\dot{x}\sqrt{xx+aa}$. Now as it is involved in a Radical Sign, let us ſuppoſe $\sqrt{xx+aa} = z$; whence $xx+aa = z^2$, and therefore $x\dot{x} = z\dot{z}$; and putting $z\dot{z}$ and z for $x\dot{x}$ and $\sqrt{xx+aa}$, the Fluxion freed from Surds will be $z^2\dot{z}$. This if we bring back to its Original $\frac{1}{3}z^3$, and reſtore $\sqrt{xx+aa}$ for z, we ſhall have $\frac{1}{3} \times \overline{xx+aa}\sqrt{xx+aa}$ for the Area required.

But that it may farther appear with what Eaſe theſe Quadratures may be obtained, I will ſtill add one Example more. Let the Equation of the Curve be $\frac{x^2}{x+a} = y^2$; therefore $y = \frac{x}{\sqrt{x+a}}$, and then $\frac{x\dot{x}}{\sqrt{x+a}}$ will be the Fluxion of the Area. Let us ſuppoſe $\sqrt{x+a} = z$, whence $x = zz - a$, and $\dot{x} = 2z\dot{z}$. Therefore $\frac{x\dot{x}}{\sqrt{x+a}} = 2z^2\dot{z} - 2a\dot{z}$, and therefore $\frac{2}{3}z^3 - 2az$, or $\overline{\frac{2}{3}x - \frac{4}{3}a}\sqrt{x+a}$ will be the Area required.

But it often happens that ſome Curves, ſuch as the Circle, Ellipſis, or Hyperbola, are of ſuch a Nature, that it would be in vain to endeavour to free their Fluxions from Surds; and then the Value of the Ordinate muſt be reduced to an infinite Series; then every Term of this Series being multiplied by the Fluxion of the Abſciſs, as above, the Fluent of every Term muſt be ſeparately found, and the new Series thus ariſing will exhibit the adrature of the Curve propoſed.

ith the ſame Eaſe this Method may be accommodated to the Dimen- of Solids form'd by the Rotation of a Plain, by aſſuming for their ion the Product of the Circular Baſe into the Fluxion of the Abſciſs.

the Ratio of the Square to the inſcribed Circle be called $\frac{n}{1}$, the Equa- elonging to the Circle is $yy = dx - xx$, and therefore $4 \times \frac{dx\dot{x} - x^2\dot{x}}{n}$

is the Fluxion of a Portion of the Sphere, and conſequently $4\times\frac{\frac{1}{2}dx^2 = \frac{1}{3}x^3}{n}$ is the Portion itſelf. The Cylinder circumſcribed about this is $4\times\frac{dxx - x^3}{n}$, and therefore the Ratio of the Portion of the Sphere to the circumſcribed Cylinder is as $\frac{1}{2}d - \frac{1}{3}x$ to $d - x$.

The Rectification of Curves will be obtained, if the Hypothenuſe of the Right-angled Triangle, whoſe Sides are the Fluxions of the Abſciſs and Ordinate, is conſidered as the Fluxion of the Curve. But Care muſt be taken in the Expreſſion of that Hypothenuſe, that one of the Fluxions only may remain, and only one of the Indeterminate Quantities, which muſt be that whoſe Fluxion is retained. This will be plain from the Examples.

Fig. 32. From the given Right Sine C B, to find the Arch A C, making $AB = x$, $CB = y$, $OA = r$; let C E be the Fluxion of the Abſciſs, E D the Fluxion of the Ordinate, and C D the Fluxion of the Arch C A. The Property of the Circle is $2rx - xx = yy$, whence $2r\dot{x} - 2x\dot{x} = 2y\dot{y}$, and therefore $\dot{x} = \frac{y\dot{y}}{r-x}$. But $CDq = \dot{y}\dot{y} + \dot{x}\dot{x} = \dot{y}\dot{y} + \frac{y^2\dot{y}^2}{rr - 2rx + xx} = \dot{y}\dot{y} + \frac{yy\dot{y}\dot{y}}{rr - yy} = \frac{rr\dot{y}\dot{y}}{rr - yy}$. Therefore $CD = \frac{r\dot{y}}{\sqrt{rr - yy}}$. But $\frac{r\dot{y}}{\sqrt{rr - yy}}$ is the Product of $\frac{1}{\sqrt{rr - yy}}$ into $r\dot{y}$, or of $\overline{rr - yy}|^{-\frac{1}{2}}$ into $r\dot{y}$; ſo that if $\overline{rr - yy}|^{-\frac{1}{2}}$ be reduced to an infinite Series, and all its Terms multiplied by $r\dot{y}$; and if we find the Fluent of every Term, we ſhall have the Length of the Arch A C.

In a like manner the Arch may be found from the verſed Sine being given. Let us reſume the Equation found above $2r\dot{x} - 2x\dot{x} = 2y\dot{y}$, which becomes $\dot{y} = \frac{r\dot{x} - x\dot{x}}{y}$. But $CDq = \dot{x}\dot{x} + \dot{y}\dot{y} = \dot{x}\dot{x} + \frac{rr\dot{x}\dot{x} - 2rx\dot{x}\dot{x} + xx\dot{x}\dot{x}}{yy} = \dot{x}\dot{x} + \frac{rr\dot{x}\dot{x} - 2rx\dot{x}\dot{x} + xx\dot{x}\dot{x}}{2rx - xx}$, or reducing all to the ſame Denominator, and expunging thoſe which deſtroy one another, 'tis $CDq = \frac{rr\dot{x}\dot{x}}{2rx - xx}$, or $CD = \frac{r\dot{x}}{\sqrt{2rx - xx}}$. Therefore the Length of the Arch A C will eaſily be obtained, by what has been above delivered.

Sometimes the Fluxion of the Curve is found more eaſily by a Compariſon between the ſimilar Triangles C E D and C B O. For we ſhall have this Proportion, C B . C O :: C E . C D. That is, for the Circle, $\sqrt{2rx - xx} \,.\, r :: \dot{x} \,.\, \frac{r\dot{x}}{\sqrt{2rx - xx}}$.

The Curve-line of the Cycloid may be known in the ſame manner. Let A L K be a Semicyloid, whoſe generating Circle is A D L. Any Point B being aſſumed in the Diameter A L, let B I be drawn parallel to the Baſe L K, meeting the Periphery of the Circle in the Point D. Let the Rectangle A E I B be compleated, and draw F H parallel to E I, and infinitely near it, cutting B I produced in G, and the Curve A K in H. Make A L $= d$, A B $=$ E I $= x$, G H $= \dot{x}$. It is known that the Right Line B G is every where the Aggregate of the Arch A D, and of the Right Sine B D; hence it is manifeſt that the Fluxion I G is the Aggregate of the Fluxions of the Arch A D, and of the Right Sine B D. Now the Fluxion of the Arch A D is found to be $= \frac{\frac{1}{2} d\dot{x}}{\sqrt{dx - xx}}$, and the Fluxion of the Right Sine B D is $= \frac{d\dot{x} - 2x\dot{x}}{2\sqrt{dx - xx}}$. And therefore I G $= \frac{d\dot{x} - x\dot{x}}{\sqrt{dx - xx}}$, and I H q $=$ I G q $+$ G H q $= \frac{dd\dot{x}\dot{x} - dx\dot{x}\dot{x}}{dx - xx}$. Therefore I H $= \frac{\dot{x}\sqrt{dd - dx}}{\sqrt{dx - xx}} = \frac{\dot{x}\sqrt{d}}{\sqrt{x}} = d^{\frac{1}{2}} x^{-\frac{1}{2}} \dot{x}$; and therefore A I $= 2 d^{\frac{1}{2}} x^{\frac{1}{2}} = 2\sqrt{dx} =$ 2 A D. *Fig.* 33.

This Concluſion may be deduced with very little Trouble from the known Property of the Tangent. For ſince its Particle I H is always parallel to the Chord A D, it cauſes the Trangles I G H and A B D to be ſimilar. Whence A B . A D :: G H . I H. That is, $x . \sqrt{dx} :: \dot{x}$. I H $= \frac{\dot{x}\sqrt{dx}}{x} = d^{\frac{1}{2}} x^{-\frac{1}{2}} \dot{x}$.

Now by the Aſſiſtance of the Fluxion I H we may find the Area of the Cycloid. The Fluxion of the Area A E I is the Rectangle E I G $= \frac{dx\dot{x} - x^2\dot{x}}{\sqrt{dx - xx}}$ $= \dot{x}\sqrt{dx - xx}$. But the Fluxion of the Portion A B D does not differ from this; therefore the Area A E I and the correſpondent Portion of the Circle A B D are always equal.

Let A B be the Curve of a Parabola, whoſe Axis is A F, Parameter a. Make A E $= x$, E B $= y$, A B $= z$, B D $= \dot{x}$, D C $= \dot{y}$, B C $= \dot{z}$; and af-
ning an Equation expreſſing the Nature of the Parabola, ſuppoſe $ax = yy$, ill be $a\dot{x} = 2y\dot{y}$, whence $\dot{x} = \frac{2y\dot{y}}{a}$. But B C q $=$ B D q $+$ C D q, $\dot{z}\dot{z} = \dot{x}\dot{x} + \dot{y}\dot{y} = \frac{4y^2\dot{y}\dot{y}}{aa} + \dot{y}\dot{y} = \frac{4y^2\dot{y}\dot{y} + aa\dot{y}\dot{y}}{aa}$, and there- $= \dot{y}\frac{\sqrt{4yy + aa}}{a}$, or which is all one $\dot{z} = \dot{y}\frac{\sqrt{y^2 + \frac{1}{4}a^2}}{\frac{1}{2}a}$. If therefore *Fig.* 34.

this Quantity is reduced to an infinite Series, the Curve A B may thence be known.

Fig. 35. Now it eaſily appears, that if the Hyperbolical Space were given, this would be given alſo, and *vice verſâ.* For $\frac{1}{2}a\dot{z} = \dot{y}\sqrt{yy+\frac{1}{4}aa}$, and therefore $\frac{1}{2}az$ will be equal to the Space whoſe Fluxion is $\dot{y}\sqrt{yy+\frac{1}{4}aa}$. But this Space is nothing elſe but the exterior equilateral Hyperbola, whoſe Semiaxis A B $=\frac{1}{2}a$, the Abſciſſa A E $= y$, and the Ordinate E G $= x$.

For the Menſuration of the Superficies produced by the Converſion of a Curve about its Axis, there muſt be aſſumed for its Fluxion a Cylindrical Superficies, whoſe Altitude is the Fluxion of the Curve itſelf, and whoſe Diſtance from the Axis is the Ordinate belonging to this Fluxion.

Fig. 32. For Example, let A C be the Arch of a Circle, which by revolving about the Axis A B may generate a Spherical Superficies, which we undertake to meaſure. The Fluxion of the Arch D C is already found to be $\frac{r\dot{x}}{\sqrt{2rx-xx}}$. If we multiply this by the Circumference belonging to the Radius, B C, that is by $\frac{c}{r}\sqrt{2rx-xx}$, (ſuppoſing the Ratio of the Circumference to the Radius to be $=\frac{c}{r}$) we ſhall have the Fluxion of the Spherical Surface $= c\dot{x}$, and therefore the Surface itſelf is cx.

As to what belongs to Centers of Gravity, having found the Fluxion of the Superficies or Solid, and multiplying this into its Diſtance from the Vertex, we muſt then return back to the Fluent. This divided by the Superficies or Solid itſelf, will give the Diſtance of the Center of Gravity from the Vertex.

Let the Center of Gravity of all the Paraboloids be to be found. Their Fluxion is thus expreſſed in a general Manner $x^{\frac{m}{n}}\dot{x}$, which multiplied by x becomes $x^{\frac{m}{n}+1}\dot{x}$, whoſe flowing Quantity $\frac{n}{m+2n}x^{\frac{m}{n}+2}$ being divided by the Paraboloids Area $\frac{n}{m+n}x^{\frac{m}{n}+1}$ will give $\frac{m+n}{m+2n}x$ for the Diſtance of the Center of Gravity from the Vertex.

The Center of Gravity in a Portion of a Sphere is found much in the ſame Manner. For its Fluxion $4\,\frac{dx\dot{x}-xx\dot{x}}{n}$ being drawn into x becomes $4\,\frac{dx^2\dot{x}-x^3\dot{x}}{n}$, of which the Fluent is $4\,\frac{\frac{1}{3}dx^3-\frac{1}{4}x^4}{n}$, which divided by the

Solidity of the Portion, that is, by $4\,\frac{\frac{1}{2}dx^2 - \frac{1}{3}x^3}{n}$, produces $\frac{\frac{1}{3}d - \frac{1}{4}x}{\frac{1}{2}d - \frac{1}{3}x}x$, or $\frac{4d - 3x}{6d - 4x}x$, for the Diſtance of the Center of Gravity from the Vertex.

XIII. 1. Prop. I. Prob.] *To find the Relation between the Fluxion of the Axis, and the Fluxion of the Ordinate, in the Curve called Catenaria.*

The Catena; by Dr. Dav. Gregory. N. 231. *p.* 637. Aug. An. 1697. *Fig.* 36.

Let F A D be a Chain hanging by its Ends F and D, whoſe loweſt Point, or Vertex of the Curve, is A, its Axis A B perpendicular to the Horizon, and its Ordinate B D parallel to the ſame. We are to find the Relation between B *b*, or D *δ*, and *d δ*; ſuppoſing the Point *b* to be infinitely near B, and *b d* to be parallel to B D, as alſo D *δ* parallel to B A.

It appears from Mechanicks, that three Powers conſtituted in Æquilibrio have the ſame Ratio as three Right Lines that are parallel to their Directions, or which are inclined to them in a given Angle, being terminated by their mutual Concourſe. So that if D *d* denotes the abſolute Gravity of the Particle D *d*, (as would neceſſarily be in a Chain every where equally thick) then *d δ* will repreſent that Part of the Gravity that acts perpendicularly upon D *d*, by which it is brought about, (becauſe of the Flexibility of the Chain moving about *d*) that *d* D endeavours to reduce itſelf to a vertical Situation. Therefore if *δ d*, or the Fluxion of the Ordinate B D, be ſuppoſed conſtant, the Action of Gravity exerted perpendicularly upon the correſpondent Parts of the Chain *d* D, will alſo be conſtant, or every where the ſame. Let this be expounded by the Right Line *a*. Again, by the Mechanicks before cited, D *δ*, the Fluxion of the Axis A B, will denote the Force, which is exerted according to the Direction of *d* D, which is equivalent to the aforeſaid Endeavour of the heavy Line *d* D to reduce itſelf into a Vertical Situation, and which prevents its doing ſo. Now this Force ariſes from the heavy Line D A drawing according to the Direction *d* D, and therefore (*cæteris paribus*) is proportional to the Line D A. Therefore *δ d*, the Fluxion of the Ordinate, is to *δ* D, the Fluxion of the Abſciſs, as the conſtant Right Line *a* is to the Curve D A. *Q. E. F.*

COROL.] If the Right Line T D touches the Catenaria, and meets the Axis B A produced in T, it will be B D : B T : : (*d δ* : *δ* D : :) *a*. Curve D A.

Prop. II. Theor.] *If to the Perpendicular A B as an Axis, with Vertex A, an Equilateral Hyperbola A H be deſcribed, whoſe Semiaxis A C is equal to* a; *and to the ſame Axis and Vertex, a Parabola A P be drawn, whoſe Parameter is equal to four Times the Axis of the Hyperbola; and if the Ordinate H B of the Hyperbola be continually produced, till H F is equal to the Curve A P; I ſay the Curve F A D, in which the Points F and D are found, (ſuppoſing B D = B F) will be the Catenaria.*

Make $AB = x$, then $Bb = \dot{x}$, and $BH = \sqrt{2ax + xx}$. Whence by the Method of Fluxions, the Fluxion of $BH = \frac{a\dot{x} + x\dot{x}}{\sqrt{2ax + xx}} = mb$. Again, becauſe the Parameter of the Parabola A P is $= 8a$, 'tis $BP = \sqrt{8ax}$. Whence np the Fluxion of B P will be $\frac{2a\dot{x}}{\sqrt{2ax}}$. So that the Fluxion of the Curve $AP = Pp = \sqrt{np \times np + Pn \times Pn} = \sqrt{\frac{4a^2\dot{x}^2}{2ax} + \dot{x}^2} = \sqrt{\frac{2a\dot{x}^2 + x\dot{x}^2}{x}}$, which by multiplying both Numerator and Denominator into $\sqrt{2a + x}$ becomes $\frac{2a\dot{x} + x\dot{x}}{\sqrt{2ax + xx}}$. And ſince H E is every where equal to A P, the Fluxion of the Right Line H F, that is $mb + sf$, will be equal to $\frac{2a\dot{x} + x\dot{x}}{\sqrt{2ax + xx}}$. But it is already found that $mb = \frac{a\dot{x} + x\dot{x}}{\sqrt{2ax + xx}}$. Whence $sf = \frac{a\dot{x}}{\sqrt{2ax + xx}}$, which is the Fluxion of B F the Ordinate to the Axis of the Catenaria. Therefore the Fluxion of the Curve A F, or $Ff = \sqrt{sf^2 + Fs^2} = \sqrt{\frac{a^2\dot{x}^2}{2ax + xx} + \dot{x}^2} = \frac{a\dot{x} + x\dot{x}}{\sqrt{2ax + xx}}$, of which the Fluent is $\sqrt{2ax + xx}$, as juſt now found. Therefore $AF = \sqrt{2ax + xx}$. And it appears that the Fluxion of the Ordinate B F, or $\frac{a\dot{x}}{\sqrt{2ax + xx}}$, is to $\dot{x}$ the Fluxion of the Abſciſs A B, as the given Line a to the Curve A F; which is the Property of the Catenaria found above. Therefore the Points of the Catenaria are rightly determined by the foregoing Conſtruction. *Q. E. D.*

COROL. 1.] From the Conſtruction it appears, that B F, the Ordinate of the Catenaria, is equal to the Parabolical Curve A P, taking away B H the correſpondent Ordinate of the conterminate Hyperbola A H.

2. From the Demonſtration it appears, that the Curve of the Catenaria A F is equal to B H the correſpondent Ordinate of the conterminate equilateral Hyperbola. For ſince the Fluxions of theſe Lines are equal, and the Lines themſelves are naſcent at the ſame Time, it is plain they muſt be always equal. Whence the Chain being given, A C or a will be given alſo, as being equal to the Semiaxis of the Equilateral Hyperbola whoſe Vertex is A, and Ordinate equal to the Abſciſs A B of the Chain A D.

3. All Catenaria are ſimilar to one another, ſince they are generated by a like Conſtruction of like Figures ſimilarly poſited. Whence two Right Lines alike inclined to the Horizon, drawn through the Vertices of the Chains,

will cut off ſimilar Figures, and Portions of the Chains which are proportional to the Right Lines ſo cutting them off.

4. If the Chain Q A D is ſuſpended at the Points Q and D, which are at unequal Heights, the Part of the Curve F A D continues the ſame as if it had been ſuſpended at the Points F and D, which are equally high; becauſe it is all one whether the Point F be fixt to the Horizontal Plain or not.

5. If the Force of the Chain, drawing according to the Direction d D, be denoted by D d; let it be divided (as is commonly known) into the Force $d\,\delta$, according to a Horizontal Direction, and a Force δ D, according to a Vertical Direction. Therefore in the Extremity of the Chain, the Force of approaching directly to the Axis, is to the Force of perpendicular Deſcent in the ſame; or the Part of the ſuſtaining Force acting according to the Direction *B D*, is to a Part of the ſame acting according to the Direction D δ, as the Semiaxis of the conterminate Hyperbola A H, is to D A the Length of the Chain to the Vertex of the Curve. Whence when the Chain is given, this Ratio is given. And in the ſame Chain ſuſpended more or leſs looſely, that Horizontal Force is as the Axis of the conterminate Hyperbola, ſince D A remains the ſame, when the Extremes of the Chain are equally high.

6. In a Vertical Plain, but in an inverted Situation, the Chain will preſerve its Figure without falling, and therefore will conſtitute a very thin Arch or Fornix: That is, infinitely ſmall, rigid, and poliſh'd Spheres, diſpoſed in an inverted Curve of a Catenaria, will form an Arch, no Part of which will be thruſt outwards or inwards by other Parts, but the loweſt Parts remaining firm, it will ſupport itſelf by means of its Figure. For ſince the Situation of the Points of the Catenaria is the ſame, and the Inclination of the Parts to the Horizon, whether in the Situation F A D, or in an inverted Situation, ſo that the Curve may be in a Plain which is perpendicular to the Horizon; it is plain that it muſt keep its Figure unchanged as well in one Situation as the other. And on the contrary, none but the Catenaria is the Figure of a true and legitimate Arch or Fornix. And when Arches of other Figures are ſupported, it is becauſe in their Thickneſs ſome Catenaria is included. Neither would it be ſuſtained, if it were very thin, and compoſed of ſlippery Parts. From *Corol.* 5. before, it may be collected, by what Force an Arch or Buttreſs preſſes a Wall outwardly to which it is applied. For this is the ſame with that Part of the Force ſuſtaining the Chain, which draws according to a Horizontal Direction. For the Force which in the Chain draws inwards, in an Arch equal to the Chain drives outwards. All other Circumſtances, concerning the Strength of Walls to which Arches are applied, may be geometrically determined from this Theory, which are the chief Things in the Conſtruction of Edifices.

7. Inſtead of Gravity, if any other Power exerts its Force, acting in like manner upon a flexible Line, the ſame Curve will be produced. For Ex-

ample, if the Wind be ſuppoſed equable, and ſhould blow according to Right Lines parallel to a given Line; the Line thus inflated by the Wind would be the ſame as the Catenaria. For ſince all Things obtain in this other Force, as we have ſuppoſed in Gravity, it is evident the ſame Line muſt be produced.

Fig. 37. Prop. 3. Theor.] *The Hyperbola aforeſaid AH remaining, if through A a Right Line GAL be drawn perpendicular to the Axis AB, and a Curve KR be deſcribed of ſuch a Nature, that BK may be a third Proportional to the Right Lines BA and AC, and to AC be applied a Rectangle AV equal to the interminate Space ABKRLA; the Concourſe F of the Right Lines HB, VG, will be at a Catenaria.*

For by Conſtruction 'tis BK $= \frac{aa}{\sqrt{2ax+xx}}$. Therefore the Fluxion of the Space ABKRLA is BK*kb* = BK×B*b* $= \frac{aa\dot{x}}{\sqrt{2ax+xx}}$. And ſince BF $= \frac{\text{ABKRLA}}{\text{AC}}$, and AC is given; its Fluxion will be BF $= \frac{a\dot{x}}{\sqrt{2ax+xx}}$. But in the Conſtruction of the foregoing Propoſition, the Fluxion of the Ordinate BF $= \frac{a\dot{x}}{\sqrt{2ax+xx}}$. Wherefore this Conſtruction comes to the ſame as the Conſtruction of the foregoing Propoſition, and conſequently the Point F is at a Catenaria. *Q. E. D.*

COROL.] As in the foregoing Propoſition the Catenaria is deſcribed from the given Length of the Parabolical Curve; ſo in this its Deſcription depends on the Quadrature of the Space, in which $xxyy = a^4 - 2axyy$. For BK or $y = \frac{aa}{\sqrt{2ax+xx}}$.

Fig. 36. Prop. 4. Theor.] *The Space AGF contained by the Catenaria AF, and the Right Lines FG, AG, parallel to AB, BF, is equal to the Rectangle under the Semiaxis AC, and DH the Diſtance of the Ordinates in the Hyperbola and Catenaria.*

For $\dot{\text{D}}\text{H} = \dot{\text{B}}\text{H} - \dot{\text{B}}\text{D} =$ (by Prop. 2. of this) $\frac{a\dot{x}+x\dot{x}}{\sqrt{2ax+xx}} - \frac{a\dot{x}}{\sqrt{2ax+xx}} = \frac{x\dot{x}}{\sqrt{2ax+xx}}$. Wherefore the Fluxion of the Rectangle under the given Line AC and HD is $\frac{ax\dot{x}}{\sqrt{2ax+xx}} = x \times \frac{a\dot{x}}{\sqrt{2ax+xx}} = fs \times \text{FG} =$ the Fluxion of the Space AGF. And ſince theſe Figures are naſcent together, it follows that the Rectangle under A C and D H is equal to the Space AGF. *Q. E. D.*

COROLL.] Hence it follows that the Space F A D, comprehended by the Chain F A D and the Horizontal Right Line F D, is equal to the Rectangle under F D and B A, ſubſtracting the Rectangle under either Axis of the Hyperbola A H, and D H the Exceſs of the Right Line B H, or of the Curve A D, above the Ordinate B D.

Prop. 5. Theor.] *If the Rectangle L E, equal to the Hyperbolic Space A L H, be applied to the Right Line A L, the Point E will be the Center of Equilibrium of the Catenarian Curve A F D.* Fig. 36.

Let a heavy Curve F A be conceived to be poiſed upon the Axis G L. From the Doctrine of the Center of Gravity it follows, that the Moment of the Weight F A is expounded by the Surface of an upright Cylinder erected upon F A, and cut off by a Plain paſſing through G L, making half a Right Angle with the Plain of the Curve. And the Fluxion of this Surface, or $\dot{F}A \times FG$, is equal to the Fluxion of the Space A L H, or $\dot{B}H \times HL$; becauſe $\dot{F}A$ and $\dot{B}H$, as alſo F G and H L, are equal. And therefore, ſince they are naſcent at the ſame Time, the ſaid Superficies of the erect Cylinder is equal to the Hyperbolical Space A L H. Therefore this applied to the heavy Line itſelf A F, or to the Right Line A L which is equal to it, it produces a Breadth A E equal to the Diſtance of the Center of Gravity from the Axis of Libration G L. Hence E will be the Center of Equilibrium of the Curve F A D, lying equally on each Side of the Axis A B. *Q. E. D.*

COROL. 1.] The Spaces A B H L, B A H, and A G F, are in Arithmetical Proportion. For the Fluxion of the Space A L H $= \frac{a\dot{x} + x\dot{x}}{\sqrt{2ax+xx}} \times x = \frac{\overline{ax+xx} \times \dot{x}}{\sqrt{2ax+xx}} = \frac{\overline{2ax+xx-ax} \times \dot{x}}{\sqrt{2ax+xx}} = \dot{x}\sqrt{2ax+xx} - \frac{ax\dot{x}}{\sqrt{2ax+xx}} =$ Fluxion of the Space B A H, leſſened by the Fluxion of the Space A G F, by *Prop.* 4. of this. And as theſe three Figures are naſcent at the ſame Time, it will be B A H — A G F = (A L H =) B L — B A H. So that 2 B A H = B L + A G F. Whence it follows that the Spaces B L, B A H, and A G F, are in Arithmetical Proportion.

2. The Center of Gravity of the Catenaria deſcends lower, than that of any other Line of the ſame Length, and having the ſame Extremities. For every heavy Body deſcends as low as it may. And ſince a Figure deſcends juſt ſo much as its Center of Gravity deſcends, a heavy flexible Line will ſo diſpoſe itſelf, as that its Center of Gravity will be lower than if it aſſumes any other Figure. And from this Property of a heavy flexible Line, all its other Properties might be eaſily deduced.

3. If upon any Curves having the ſame Length, and the ſame Limits D and F as the Catenaria F A D, upright Cylinders were cut by a Plain paſſing through D F; of the Cylindrical Superficies, ſo cut off, the greateſt is that which inſiſts upon the Catenaria. For theſe Superficies, if the Angle made by the Plains is half a Right Angle, applied to the Curves themſelves, which in the preſent Caſe are of the ſame Length, produce Breadths equal to the Diſtances of the Centers of Gravity of the Curves from the Right Line D F. Now as in the Catenaria this Diſtance is the greateſt, becauſe of the greateſt Deſcent of the Center of Gravity, the Cylindric Surface to be applied will alſo be the greateſt. And becauſe there is the ſame Ratio of Cylindrical Surfaces cut off by a Plain, containing any Angle with the Plain of the Baſe, as when the ſaid Angle is half a Right Angle, the Propoſition obtains univerſally.

Fig. 36. LEMMA.] *If upon any Ordinate F B perpendicular to the Axis A B of any Curve A F Q, that is deſcribed by the Evolution of another Curve K V, from the correſpondent Point V in K V a perpendicular V R is let fall, meeting the Ordinate in R; if the Fluxion of the Axis A B remains the ſame, the Fluxion of the Fluxion of the Ordinate B F, the Fluxion of the Curve A F, and the Right Line F R, will be continual Proportionals.*

Let the little Right Line F *f* be produced, till it meets the next Ordinate W φ in *o*. And becauſe by the Hypotheſis $Fs = fW$, it will be $of = Ff$, and therefore $o\varphi$ will be the Fluxion of fs, that is, the Fluxion of the Fluxion of the Ordinate. Moreover the Triangles $o\varphi f$ and fFR are equiangular, becauſe $o\varphi f$ is equal to the Alternate φfr, and $fo\varphi = (Ffr =)$ FfR, becauſe their Difference Rfr vaniſhes in Reſpect of either of them, ſince Rr is nothing in Compariſon of fr. Therefore it is $o\varphi . \varphi f :: fF . FR$. But φf and fF are equal, ſince they only differ by the Fluxion of each. Therefore $o\varphi . fF :: fF . FR$. *Q. E. D.*

Fig. 36. Prop. 6. Probl.] *To find the Curve K V, by the Evolution of which the Catenaria A F Q is deſcribed.*

Make $AB = x$, and $BF = y$, as before. Then by *Prop.* 2. of this, 'tis $\dot{y} = \frac{a\dot{x}}{\sqrt{2ax + xx}}$, or $2ax\dot{y}\dot{y} + xx\dot{y}\dot{y} = aa\dot{x}\dot{x}$. Then by *Newton's* Method now in common uſe, 'tis $2a\dot{x}\dot{y}^2 + 4ax\dot{y}\ddot{y} + 2x\dot{x}\dot{y}^2 + 2x^2\dot{y}\ddot{y} = 2a^2\dot{x}\ddot{x} = 0$; (for $\ddot{x} = 0$, becauſe $\dot{x}$ is a conſtant Quantity.) Therefore $\ddot{y} = \frac{-a\dot{x}\dot{y} - x\dot{x}\dot{y}}{2ax + xx} = \frac{\overline{a+x} \times a\dot{x}^2}{2ax + xx \times \sqrt{2ax + xx}}$, by ſubſtituting inſtead of $\dot{y}$ its Value $\frac{a\dot{x}}{\sqrt{2ax + xx}}$; (for the Sign − prefixt to the Quantity $\ddot{y}$ only ſhews, that the Place of the Point R in reſpect of F, is oppoſite to the Place of the

Point F in reſpect of B, ſince the Curve A F Q is concave towards the Axis A B.) And by the ſecond *Prop.* of this, $Ff = \frac{\overline{a+x}\times\dot{x}}{\sqrt{2ax+xx}}$. Whereſore by the foregoing *Lemma*, $FR = \frac{Ffq}{\dot{y}} = \frac{\overline{a+x}^2\times\dot{x}^2}{2ax+xx}\times\frac{\overline{2ax+xx}\times\sqrt{2ax+xx}}{\overline{a+x}\times a\dot{x}^2} = \frac{\overline{a+x}\times\sqrt{2ax+xx}}{a}$. Again, becauſe of the Right-angled Triangles Fsf and FRV, having equal Angles fFs and VFR, becauſe VFs is the Complement of each to a Right Angle, it is $Fs : sf :: FR : VR$, or $\dot{x} : \frac{a\dot{x}}{\sqrt{2ax+xx}} :: \frac{\overline{a+x}\times\sqrt{2ax+xx}}{a}$. V R, which therefore is equal to $a+x$. Therefore this is the Nature of the Curve K V, that if A B be called x, it will be $FR = \frac{\overline{a+x}\times\sqrt{2ax+xx}}{a}$, and $VR = a+x$. *Q. E. I.*

COROL. 1.] $AC : CB :: BH : FR$. For this is the Property of the Right Line F R found above.

2. The Right Line C B is equal to B I or V R; for each of them is equal to $a+x$.

3. The evolving Right Line V F is a third Proportional to the Lines A C and C B. For becauſe of equiangled Triangles fFs and VFR, it is $sF : Ff :: FR : VF$. Or $\dot{x} : \frac{a\dot{x}+x\dot{x}}{\sqrt{2ax+xx}} :: \frac{\overline{a+x}\times\sqrt{2ax+xx}}{a}$. VF, which therefore is equal to $\frac{\overline{a+x}|^2}{a}$. Whence it is $a : a+x :: a+x$. V F, which alſo is the Radius of a Circle which is equicurved to the Catenaria in the Point F.

4. When the Point F falls in A, or when the Vertex is deſcribed by Evolution, that is, when $x = 0$, the Value of the evolving Right Line V F, which in this Caſe is K A, becomes $\frac{\overline{a+x}|^2}{a} = a$. That is, the Point K, where the Curve V K meets the Axis, is as much above the Vertex A of the Chain, as C is depreſſed below the ſame. Whence the Diameter of a Circle, equicurved to the Chain at its Vertex, is equal to the Axis of the conterminate Hyperbola A H. Therefore the Chain A D and the Hyperbola A H have the ſame Degree of Curvature at the Vertex A: For it is generally known that the aforeſaid Circle is equicurved to the Equilateral Hyperbola A H in the Vertex A. Alſo this appears from the Nature of the Chain itſelf, by what is demonſtrated *Prop.* 2. of this. For the naſcent Line $FH = AP =$ the naſcent $BP = \sqrt{8ax}$ is double to the naſcent Line B H or $\sqrt{2ax+xx}$ =

$=$ (when x vaniſhes) $\sqrt{2ax}$. And therefore the ſame Point is both in the naſcent Hyperbola and the naſcent Catenaria, That is, the naſcent Hyperbola A H coincides with the naſcent Catenaria A D, and therefore theſe Lines are equicurved at the Vertex A.

5. The Curve K V is a third Proportional to the Right Line A C, and the Curve A F, or the Right Line A L. For from the Nature of Evolution $KV = VKA - KA = VF - KA = \frac{\overline{a+x}^2}{a} - a = \frac{a^2+2ax+x^2}{a} - a = \frac{2ax+xx}{a}$. And therefore $a : \sqrt{2ax+xx} :: \sqrt{2ax+xx}.\ KV$. But $\sqrt{2ax+xx} = AF$, by *Cor.* 2. *Prop.* 2. Whence $AC : AF :: AF.\ KV$.

6. The Right Line K I is double to A B. For ſince $BI = BC = CA + AB$, it will be $AI = CA + 2AB$. But $AK = CA$, by *Corol.* 4. of this. Whence $KI = 2AB$.

7. The Rectangle of A C and B R is equal to twice the Hyperbolical Space B A H. For $FR \times AC = \frac{\overline{a+x} \times \sqrt{2ax+xx}}{a} \times a = \overline{a+x} \times \sqrt{2ax+xx} = x \times \sqrt{2ax+xx} + a \times \sqrt{2ax+xx} = AB \times BH + AC \times BH = AB \times BH + AC \times BD + AC \times DH$. Therefore $FR \times AC - BD \times AC = BR \times AC = AB \times BH + AC \times DH$. But by *Prop.* 4. of this, 'tis $AC \times DH =$ Space A G F. And therefore $BR \times AC = ABHL + AGF = 2BAH$, by *Cor.* 1. *Prop.* 5.

Fig. 38. Prop. 7. Theor.] *If in the Logarithmic Curve L A G, whoſe given Subtangent H S is equal to the Right Line* a, *(determined* Cor. 2. Prop. 2. *of this) a Point A be taken, whoſe Diſtance A C from the Aſymptote H P is equal to the Subtangent H S; and from the Points H and P, any how taken in the Aſymptote, equally diſtant from the Point C, if Ordinates H L, P G are erected to the Logarithmic Curve, to half the Sum of which H D or P F are made equal; the Points D and F will be in the Catenaria correſponding to the Right Line A C.*

Make $AB = x$, and therefore C B or D H, the half Sum of the Ordinates H L, P G, will be $a+x$; let the half Difference of the ſame be called y. Then $HL = a+x+y$, and $PG = a+x-y$. And ſince from the Nature of the Logarithmic Curve C A is a mean Proportional between theſe, it will be $aa+2ax+xx-yy = aa$, and therefore $y = \sqrt{2ax+xx}$. So that $HL = a+x+\sqrt{2ax+xx}$, and $PG = a+x-\sqrt{2ax+xx}$. Therefore the Fluxion of H L, or lm, is $\frac{a\dot{x}+x\dot{x}+\dot{x}\sqrt{2ax+xx}}{\sqrt{2ax+xx}}$.

And becauſe of ſimilar Triangles *lm* L and L H S, it is L H : H S :: *lm* : *m* L. Whence *m* L or $d\delta$, the Fluxion of B D, is equal to $\frac{a\dot{x}}{\sqrt{2ax+xx}}$. That is, the Curve A D derived from the Logarithmic Curve in the foregoing Manner, is of ſuch a Nature, that if its Axis is called x and its Fluxion $\dot{x}$, the Fluxion of the Ordinate B D will be $\frac{a\dot{x}}{\sqrt{2ax+xx}}$. But this is the very Property of the Catenaria to which a belongs, as demonſtrated in *Prop.* 1. of this. Therefore the Curve F A D above deſcribed is no other but the Catenaria. *Q. E. D.*

COROL. 1.] As by the Help of the Logarithms the Catenaria may be deſcribed, ſo on the contrary by means of the Catenaria, which is conſtructed by Nature herſelf, the Logarithm of a given Number, or rather of a given Ratio, may be found. As ſuppoſing C A to be Unity, whoſe Logarithm is equal to *o*, let us find the Logarithm of the Number C Q, or of the Ratio between C A and C Q. To the Right Lines C Q and C A let the third Proportional be C V, and let half the Sum of C Q and C V be C B. The Ordinate to the Catenaria from B, that is B D, is the Logarithm required. The Reaſon is plain from the Propoſition.

2. On the contrary, if from the Logarithm given C H or C P the correſpondent Number H L or P G were required, or the Ratio H L to C A, or P G to C A; from H or P let a Perpendicular be raiſed, meeting the Catenaria in D or F; and let C R be made equal to H D or P F, that is to C B, and let it be terminated at the Horizontal Line A R. Then will A R be the Semidifference of the Lines required L H, G P, as H D or C R is their Semiſum, by what is demonſtrated above about the Nature of the Catenaria. (For in three Quantities that are Geometrically proportional, ſuch as H L, C A, P G, the Square of the half-ſum of the Extreams leſſened by the Square of the Mean, is equal to the Square of the half-Difference of the Extreams.) And therefore C R + A R and C R — A R are the Numbers H L or G P, belonging to the given Logarithm C H or C P.

3. From the Demonſtration it is evident, that as H D the half Sum of the Ordinates H L, P G, of the Logarithmick Curve, applied perpendicularly to C H in H, is the Ordinate of the Catenaria; ſo the half Difference of the ſame H L, P G, applied perpendicularly to C A in B, is the Ordinate of the equilateral Hyperbola deſcribed with Center C and Vertex A; and therefore, by *Cor.* 2. *Prop.* 2. of this, is equal to the Catenaria A D: For $y = \sqrt{2ax+xx}$. And ſince it is ſhewn in the foregoing Corollary, that A R alſo is the half Difference of the Right Lines H L, P G, it is plain that A R is equal to the Portion of the Catenaria A D. Whence by the Way a Method is diſcovered, from the Chain A D being given to find C the Center of the conterminate Hyperbola, or that Point in the Aſymptote of the Loga-

rithmick Curve G L. For if A R is taken equal to the Chain A D, and from the middle Point of the Right Line B R a Perpendicular to it is raiſed, this will meet B A the Axis of the Chain in the Point required C, as very plainly appears. For thus C R will be equal to C B.

4. Hence alſo it follows, that if the Angle B D T be made equal to A C R, the Right Line D T will touch the Catenaria in D. For thus in the ſimilar Triangles D B T and C A R, it will be D B : B T :: C A : A R, or the Curve A D which is equal to it. And therefore by *Corol. Prop.* 1. of this, D T touches the Catenaria.

5. It alſo follows, that the Space A C H D is equal to the Rectangle of C A and A R. For becauſe A Y D, by *Prop.* 4. is equal to the Triangle under C A and A D—B D = (by *Corol.* 3. of this *Prop.*) A R—A Y = Y R, the Propoſition is plain. And becauſe C A is given, it is evident that the Space A C H D is as the Curve A D, or its Fluxion H *d* is as the Fluxion of this D *d*.

6. If through the Point K, where C R meets H D, a Line K Z is drawn parallel to P H, meeting the Right Line A C in Z, and C E be taken equal to half the Sum of B C, C Z; the Point E will be the Center of Equilibrium of the Curve F A D.

Upon F A D let it be conceived, that the upright Superficies of a Cylinder is erected, and cut by a Plain through P H, at half a Right Angle with the Plain of the Curve F A D. This Superficies will expound the Moment of the Curve F A D, when librated upon the Axis P H; and its Fluxion is

$$DH \times Dd + PF \times Ff = 2BC \times \dot{A}D = 2 \times \overline{a+x} \times \frac{a\dot{x}+x\dot{x}}{\sqrt{2ax+xx}} =$$

$$\frac{2aa\dot{x}+4ax\dot{x}+2xx\dot{x}}{\sqrt{2ax+xx}} = \frac{aa\dot{x}}{\sqrt{2ax+xx}} + \frac{aa\dot{x}+ax\dot{x}}{\sqrt{2ax+x}} + \frac{3ax\dot{x}+2xx\dot{x}}{\sqrt{2ax+xx}},$$

of which the Fluent is $a \times BD + a\sqrt{2ax+xx} + x\sqrt{2ax+xx} =$ $CA \times BD + CB \times AD$. Wherefore $CA \times BD + CB \times AD$ is equal to the aforeſaid Cylindrical Superficies, (for they are naſcent together) which is equal to the Moment of the Curve F A D, when poiſed upon the Axis P H. Whence the Diſtance of the Center of Gravity of the Curve F A D from the Point C is $\frac{CA \times BD + CB \times AD}{2AD}$, or $\frac{1}{2}\,\frac{CA \times BD}{AD} + \frac{1}{2}\,CB$. Moreover becauſe of Z K parallel to A R, it is A D : B D :: A R : Z K :: C A : C Z, whence $CZ = \frac{CA \times BD}{AD}$, and therefore C E, which by Conſtruction is equal to $\frac{1}{2}BC + \frac{1}{2}CZ$, will be equal to $\frac{1}{2}\,\frac{CA \times BD}{AD} + \frac{1}{2}BC$. That is, the Center of Gravity of the Curve F A D, and the Point E determined by the Conſtruction, will be equally diſtant from C; but being in the ſame Right Line, and ſituate the ſame Way, they muſt neceſſarily coincide.

The Coincidence of the Point E, as above determined, with the Center of Equilibrium determined *Prop.* 5. of this, may be thus ſhewn ſynthetically. By *Corol.* 1. *Prop.* 5. 'tis $2\,BAX = AYD + BA \times AR$. Whence $AH + 2\,BAX = ACHD + BA \times AR =$ (by the foregoing *Corol.*) $AR \times CA + BA \times AR$. That is, $BD \times AC + 2\,BAX = AR \times CB$, or $BD \times AC = AR \times CB - 2\,BAX$. Whence $BD \times AC + AD \times BC = AD \times BC + AR \times CB - 2\,BAX = 2\,AD \times BC - 2\,BAX = 2\,AD \times AC + 2\,AD \times AB - 2\,BAX$. And by applying to $2\,AD$, it will be $\frac{1}{2}\frac{BD \times AC}{AD} + \frac{1}{2}BC = AC + \frac{AB \times AD - BAX}{AD} = CA + \frac{ARX}{AR}$. But $\frac{ARX}{AR}$ is the Diſtance of the Center of Equilibrium of the Chain from the Vertex A, by *Prop.* 5. and therefore by the ſame $CA + \frac{ARX}{AR}$ is the Diſtance of the Point E from C, and $\frac{1}{2}\frac{BD \times AC}{AD} + \frac{1}{2}BC$ is the Diſtance of the ſame E from the ſame C; by this *Corol.* Whence it appears that theſe two Determinations of the Point E come to the ſame, becauſe $CA + \frac{ARX}{AR} = \frac{1}{2}\frac{BD \times AC}{AD} + \frac{1}{2}BC$.

7. The Center of Gravity of the Space PFADH is in I, the middle Point of the Right Line CE. For ſince the Center of Gravity of the Fluxion of AD, or Dd and Ff, is diſtant as far again from PH, as the Center of Gravity of the Fluxion of ACHD, or $DHbd$ and $FPpf$, and $\overline{Dd+Ff} \times AC$ is given, equal to $DdbH + FfpP$; it is plain that the Center of Gravity E of the Fluent FAD is as far again diſtant from PH, as the Center I of the Fluent PFADH. But I ſhall prove this otherwiſe, after the manner of the foregoing.

Let an erect Cylinder be ſuppoſed to be raiſed upon the Figure PFADH, and cut off by a Plain paſſing through PH, making half a Right Angle with the Plain of the Baſe; that Solid will repreſent the Moment of the Figure PFADH, when poiſed upon the Axis PH. The Fluxion of this Solid or of the aforeſaid Moment, (that is, the Solids erected upon $PFfp$ and $HDdb$) is produced, if the Moment of the Fluxion, or the Fluxion of the Moment of AD, is drawn into the given Line $\frac{1}{2}AC$. For by *Corol.* 5. of this Propoſition, $HDdb = Dd \times AC$. Wherefore the flowing Moment itſelf is produced by multiplying the Moment of the Curve FAD in reſpect of the Axis PH, determined by the foregoing *Corollary*, that is, $CA \times BD + CB \times AD$ into $\frac{1}{2}AC$; and therefore it will be $\frac{1}{2}AC \times AC \times$

BD + $\frac{1}{2}$ AC × CB × AD. Now if this be applied to the librated Figure PFADH, or 2 CA × AD, by *Cor.* 5. of this *Prop.* there will ariſe the Diſtance of the Center of Gravity of the Figure PFADH from the Axis PH, equal to $\frac{1}{2}\frac{CA \times BD}{AD} + \frac{1}{2} CB$, which is equal to half the Right Line CE, as above determined.

8. If through the Point N, where DT the Tangent to the Catenaria in D meets the Line AR, a Right Line be drawn parallel to BC, meeting a Right Line through E parallel to AR in the Point O; this Point O will be the Center of Gravity of the Curve AD. For by *Corol.* 6. the Center of Gravity of the Curve AD is in the Right Line EO. But it will be alſo demonſtrated, that it is in the Right Line NO, and therefore will be in the Point O. Let DA be conceived to be librated about the Axis HL; the Moment of this is the Curve DA, drawn into the Diſtance of the Center of Gravity from HL, and therefore its Fluxion is DA × H*b*, (H*b* is the Fluxion of the Diſtance of the Axis of Libration from the Center of Gravity) which is equal to $\sqrt{2ax+xx} \times \frac{a\dot{x}}{\sqrt{2ax+xx}} = a\dot{x}$. And therefore the Moment of the weighty Curve DA, librated about the Axis HL, is $a\dot{x}$. Therefore the Diſtance of the Center of Gravity from the ſame Axis is ax applied to AD, or $\frac{AC \times DY}{AR}$. But becauſe DT touches the Catenaria, by *Cor.* 4. of this *Prop.* the Angle BDT or DNY will be equal to ACR. And the Angles at A and Y are right; therefore in the equiangular Triangles RAC and DYN, 'tis RA : AC :: DY : YN. Whence YN = $\frac{AC \times DY}{RA}$, that is, YN is the Diſtance of the Center of Gravity of the Chain AD from the Axis HL; or the ſaid Center is in the Right Line NO.

9. If upon I a Right Line be drawn parallel to AR, meeting ON produced in W; the Point W will be the Center of Gravity of the Space ACHD. For by *Corol.* 7. this Center is in the Right Line IW; and it will be ſhewn preſently, that it is in NW, and therefore is the very Point W. For in the ſame manner as in the foregoing, the Fluxion of the Moment of the Space ACHD, librated about HL, is ſhewn to be ACHD × H*b* = AC × AD × H*b* = $ax\sqrt{2ax+xx} \times \frac{a\dot{x}}{\sqrt{2ax+xx}} = aa\dot{x}$. And therefore the Moment of the Space ACHD, librated about HL, is equal to the Fluent of the Fluxion $aa\dot{x}$, that is to aax. This therefore applied to the Space itſelf ACHD, or $a\sqrt{2ax+xx}$, gives the Diſtance of the Center of Gravity of the Space ACHD from HL, that is $\frac{ax}{\sqrt{2ax+xx}}$ =

AC × DY

$\frac{AC \times DY}{RA}$. But in the foregoing Corollary it is ſhewn, that $YN = \frac{AC \times DY}{RA}$. Therefore the Center of Gravity of the Space A C H D is in N W. And by theſe two laſt *Corollaries* the Center of Gravity is found, of any Portion of a Catena that does not reach to the Vertex A, or of any Space of a Catenaria, comprehended by any Portion together with Right Lines.

10. Hence are meaſured the Superficies and Solids produced by the Rotation of a Catenaria, or of any Space comprehended by that and Right Lines, revolving about a given Axis. For the Figure generated by Rotation, as is commonly known, is equal to the revolving Figure drawn into the Periphery that is deſcribed by the Center of Gravity in the Rotation; which Periphery is given, ſince its Radius is given, or the Diſtance of the Center of Gravity from the given Axis. Thus if the Catenaria A D revolves about the Axis A B, the Periphery deſcribed by the Center of Gravity O will be $\frac{\pi}{\varrho}$ A N, if $\frac{\pi}{\varrho}$ denotes the Ratio of the Periphery of a Circle to its Semidiameter; and therefore the Superficies produced by the Rotation of the Catena A D, will be $\frac{\pi}{\varrho} \times AN \times AD = \frac{\pi}{\varrho} \times AN \times AR$. That is, a Circle whoſe Radius is equal in Power to the double of the Rectangle R A N, will be equal to the Superficies produced by the Rotation of the Chain A D about the Axis A B. In like manner it may be ſhewn, that a Solid generated by the Rotation of the Space A C H D about A C, is equal to a Cylinder whoſe Baſe is the aforeſaid Circle, and its Height equal to A C. And ſo may the Superficies and Solids be meaſured, that are produced by the Rotation of theſe Figures about any other given Axis. For when the Centers of Gravity are known, the reſt will eaſily follow.

2. What has been objected by an Anonymous Author, in his Animadverſions upon our Demonſtrations concerning the Catenaria, is this, *Act. Lipſ. M. Feb. An.* 1699. That I have undertaken to demonſtrate, after my Manner, a Matter found out and publiſh'd by others ſeven Years ago. This is true, and I cannot find any thing in this that is Blame-worthy. Thoſe great Men *Huygens*, *Leibnitz*, and *Bernouilli*, have diſcovered and communicated many Properties of the Catenaria, but without Demonſtration. I have contrived Demonſtrations, which was the Thing I undertook to do.

The Animadverſions of . . . . . . *by Dr.* D. Gregory. N. 259. p. 419. Dec. An. 1699.

But was this Matter (that is, the Nature and primary Properties of the Catenaria) found out and publiſhed by others? Surely that Property of the Catenaria, in *Cor.* 6. *Prop.* 2. was not at all mentioned by others before the Publication of theſe Demonſtrations; although, if I am not miſtaken, it may be reckoned among its primary Properties, is the moſt uſeful of all, and moſt eaſily reduced to the common Purpoſes of Life. From all Ages Architects have made uſe of Arches in publick Buildings, as well for Strength as Beauty. Yet what was the true Geometrical Figure of an Arch, was not known before my Demonſtrations came out.

The firſt Thing he finds Fault with is, that I affirm ſome Things are plain from Mechanicks, which he thinks ſhould have been explained and applied more diſtinctly. As I undertook to demonſtrate ſome Theorems to Geometricians, I did not think it neceſſary to purſue every Thing very minutely. But that I may oblige the Animadverter, I will now demonſtrate that *Lemma*, *Prop.* 1. becauſe I cannot expreſs it more fully than I have already done, in the following Words:

“ Three Powers conſtituted in Equilibrio have the ſame Ratio as three “ Right Lines, which are parallel to the Directions of the Powers, or are “ inclined in a given Angle, and terminated by their mutual Concourſe.”

Fig. 39. As ſuppoſe three Powers are in Æquilibrio, that either draw, preſs, or any how act according to three Right Lines P A, P B, P C; and let the three Right Lines E F, F D, D E, be inclined to theſe Directions in any given Angle; that is, let the Angles E A P, F B P, D C P, be equal: I ſay the Powers A, B, C, are to one another as the Right Lines F E, F D, D E.

Let the Right Lines A P, B P, C P, be produced to G, H, K.

In the Quadrilaterum F A B P, becauſe by Hypotheſis the external Angle E A P is equal to the internal and oppoſite Angle P B F, the two internal oppoſite Angles F A P and F B P will be equal to two Right Angles; and ſince all the four internal Angles are equal to four Right Angles, the other two Angles F and A B P oppoſite in the ſame Quadrilaterum, will alſo be equal to two Right Angles. But A P B and B P G make two Right Ones; therefore the Angle F is equal to B P G. In like manner D and E may be ſhewn equal to B P K and A P K.

Now becauſe the three Powers are in Æquilibrio, they are immoveable, and therefore any one of them in reſpect of the two others that remain in Æquilibrio, may be conſidered as a Fulcrum. If B is the Fulcrum, by a moſt known Theorem in Mechanicks, the Power A is to the Power C, as the Sine of the Angle B P K to the Sine of the Angle B P G, that is, as the Sine of the Angle D to the Sine of the Angle F; that is, as the Right Line F E to the Right Line D E. Again, ſuppoſing C the Fulcrum, the Power A is to the Power B, as the Sine of the Angle C P H to the Sine of the Angle C P G, or the Sine of the Angle B P K to the Sine of the Angle A P K; that is, the Sine of the Angle D to the Sine of the Angle E, or as the Right Line F E to F D. Therefore the three Powers A, B, and C, are as the Right Lines F E, F D, and D E. *Q. E. D.*

Fig. 40. We muſt now ſay ſomething about the Application of this Mechanical *Lemma*. If the abſolute Gravity of the little Line dD, expounded by dD, as ſaid above in *Prop.* 1. is conceived to be collected in its Center of Gravity M, and this heavy Line, by Virtue of its Gravity, endeavours to deſcend according to the Direction M F perpendicular to dD; the Power drawing according to M D, which is in Æquilibrio with the ſaid heavy Line, by the

foregoing *Lemma* is to its Momentum or Power drawing according to M F, as δD is to δd. For the Angle δ D d, in which D δ is inclined to M D, is equal to the Angle $d\delta$ F, in which $d\delta$ is inclined to M F, for each is the Complement of the Angle d to a Right Angle. And this will obtain, as the Animadverter acknowledges, if the aforeſaid Weight (as in the vulgar Mechanicks) incumbing upon the Plain M F, is drawn by Help of a Pulley at M, by another Weight incumbing upon M D: Then this will be to that as D δ to $d\delta$.

Other Things remaining as before, if the Manner of Application of theſe Powers is changed, ſo that to the middle Point M of the flexible Line dD; whoſe Extremity d is fixt, a Weight be applied exerting its Force according to M F; (for in deſcending it would deſcribe an Arch with Center d and Radius d M) the Force of this Weight to bend the flexible Right Line at M, would be infinite in reſpect of the Force of its abſolute Gravity. And the Force drawing according to M D, which is required in order to prevent the aforeſaid bending, would alſo be infinite in reſpect of that which was before required, to ſupport the Weight M in the Plain M F. So that the Powers which in the former Manner of Application were expounded by $d\delta$ and δ D, muſt now be expounded by infinitely greater Lines, which are ſtill proportional to the former. For as before, the Weight M draws according to the Direction M F, and the Power ſuſtaining it according to M D; and that theſe two are in *Æquilibrio* appears from the Parts of the Chain being at reſt. Therefore the Ratio of theſe remains the ſame as before. But the Cauſe which extends into a Right Line the flexible Line d D, (whoſe Extremity d is immoveable, and to whoſe middle Point M a Weight is applied, which is indeed infinitely little, but whoſe Force by this Manner of Application is made infinitely greater, and therefore in the Language of the Animadverter becomes aſſignable) is the Weight of the Chain D A, which is proportional to its Length. This therefore to the conſtant and aſſignable Quantity a, (proportional to the conſtant but not aſſignable Quantity $d\delta$) is as D δ to δd. And thus I hope it will appear to the Animadverter, that I have proved the Concluſion to be true, without making any erroneous Poſitions.

XIV. Let A C F be a Semicircle, whoſe Diameter is A F; A D E is a Curve Geometrically irrational, whoſe Ordinate B D cuts the Semicircle in C. Now let the Quantities be thus repreſented. The Diameter A F $= 2a$, the Abſciſs A B $= y$, the Arch A C $= v$, and the Ordinate B D $= z$. Let $z = rvy^n$ be a general Equation, expreſſing the Natures of the Geometrically irrational Curves A D E, in which r denotes any given and determinate Quantity, and n the indefinite Exponent of the indeterminate Quantity y. I ſay the Area of the Curve will be

The Quadrature of Figures Geometrically irrational, by Mr. J. Craig. N. 232. p. 703. Sept. An. 1697. *Fig.* 41.

$$\text{ABD} = \frac{rvy^{n+1}}{n+1} = qv + \sqrt{ry-yy} \text{ into } \frac{-ra}{\overline{n+1}|^2}y^n + \frac{2nra^2+ra^2}{n\times\overline{n+1}|^2}$$

$$y^{n-1} + \frac{aA \times 2\overline{n-1}}{n-1} y^{n-2} + \frac{aB \times 2\overline{n-3}}{n-2} y^{n-3} + \frac{aC \times 2\overline{n-5}}{n-3} y^{n-4} + \frac{aD \times 2\overline{n-7}}{n-4} y^{n-5} + \frac{aE \times 2\overline{n-9}}{n-5} y^{n-6}, \text{ \&c.}$$

Concerning this infinite Series the following Things are to be obſerved. (1.) That the Capitals A, B, C, D, E, *&c.* denote the Coefficients of the Terms immediately preceding. Thus $A = \frac{ra^2 \times 2\overline{n+1}}{n \times \overline{n+1}|^2}$, $B = \frac{aA \times 2\overline{n-1}}{n-1}$, $C = \frac{aB \times 2\overline{n-3}}{n-2}$, and ſo on. (2.) That if the Exponent n repreſents any integer affirmative Number, or is $= 0$; or if $2n$ be an odd Number, then the Quadrature of the Space A B D is exhibited by a finite Quantity, becauſe of the Series breaking off in theſe Caſes. (3.) That q repreſents the laſt Term ſo breaking off. (4.) That all thoſe Figures in which the Series breaks off have a Portion Geometrically quadrable, which is eaſily aſſigned by the Series itſelf. For if the Abſciſs y is taken equal to $r^{\frac{1}{n+1}} \times \overline{nq+q}|^{\frac{1}{n+1}}$, the Area belonging to this Abſciſs will be Geometrically quadrable. (5.) That only the irrational Term $\sqrt{2ay-yy}$ is to be multiplied into the Terms that follow it.

Example 1.] Let $z = v$. Now becauſe in this Caſe 'tis $r = 1$, $n = 0$, therefore $\frac{ra}{\overline{n+1}|^2} y^n$ is the laſt Term breaking off. Therefore $q = a$, and $ABD = vy - av + a\sqrt{2ay-yy}$. Therefore by *Note* 4, if there be taken the Abſciſſa $y = a$, that is, if the Ordinate paſſes through the Center of the Circle, the Portion belonging to it will be Geometrically quadrable: For then Area $= aa$, or the Square of the Radius.

Example 2.] Let $z = \frac{vy}{a}$. Becauſe in this Caſe $r = \frac{1}{a}$, $n = 1$, therefore $\frac{2nraa + raa}{n \times \overline{n+1}|^2} y^{n-1}$ is the laſt Term breaking off, ſo that $q = \frac{3a}{4}$, whence $ABD = \frac{vy^2}{2a} - \frac{3av}{4} + \frac{y+3a}{4}$, $\sqrt{2ay-yy}$, wherefore by *Note* 4, if we take $y = \frac{\sqrt{3aa}}{2}$, the Geometrically quadrable Area belonging to this Abſciſs will be $\sqrt{\sqrt{6a^4} - \frac{3a^2}{2}} \times \frac{\sqrt{3a^2}}{32} \times \frac{3a}{4}$.

Example 3.] Let $z = \frac{vy^2}{aa}$, in this Caſe $r = \frac{1}{aa}$, $n = 2$, and therefore $\frac{aA \times \overline{2n-1}}{n-1} y^{n-2}$ is the laſt Term breaking off. Therefore $q = \frac{5a}{6}$; ſo that by the infinite Series it will be

$$ABD = \frac{6vy^3 - 15a^3v + \overline{2ay^2 + 5a^2y + 15a^3} \times \sqrt{2ay - yy}}{18a^2}.$$ And therefore by *Note* 4, if we take $y = \sqrt[3]{\frac{5a^3}{2}}$, the Area belonging to this Abſciſs will be Geometrically quadrable; that is, the Area $\frac{2ay^2 + 5a^2y + 15a^3}{18a}$ $\times \sqrt{2ay - yy}$.

Secondly, let A C F be a Parabola, whoſe Axis is A E, Vertex A, and Parameter B A. And let A D G be a Curve Geometrically irrational, whoſe Ordinate B D cuts the Parabola in C. Make the Abſciſs A B $= y$, the Ordinate B D $= z$, and the Arch of the Parabola A C $= v$. And let the general Equation, expreſſing the Natures of an Infinity of irrational Curves, be $z = rvy^n$, in which r denotes a given determinate Quantity, and n the indefinite Exponent of the indeterminate Quantity y. I ſay the Area *Fig.* 42.

$$ABD = \frac{ry^{n+1} \times v}{n+1} - qv + \sqrt{2ay + yy} \text{ into } \frac{r}{\overline{n+2} \times \overline{n+1}} y^{n+1}$$
$$- \frac{ra}{\overline{n+2} \times \overline{n+1}|^2} y^n + \frac{raa \times \overline{2n+1}}{n \times \overline{n+2} \times \overline{n+1}|^2} y^{n-1} - \frac{aA \times \overline{2n-1}}{n-1} y^{n-2}$$
$$+ \frac{aB \times \overline{2n-3}}{n-2} y^{n-3} - \frac{aC \times \overline{2n-5}}{n-3} y^{n-4}, \text{ \&c.}$$

Concerning this Series theſe Things are to be obſerved. (1.) That the great Letters A, B, C, &c. repreſent the Coefficients of the preceding Terms reſpectively. (2.) That if the Exponent n is an affirmative Integer, or equal to nothing, or likewiſe if $2n$ be an odd Number, then the Quadrature may be exhibited by a finite Number of Terms, the Series breaking off of itſelf. (3.) Then is q equal to the laſt Term breaking off. (4.) That of the Terms multiplying the Quantity $\sqrt{2ay + yy}$, the laſt breaking off is to be doubled. .) That all thoſe Figures in which n is an Integer poſitive and odd Numr, or more generally, all the Figures in which the final Term has the gn +, have a Portion Geometrically quadrable, which is eaſily aſſigned the Series, by taking the Abſciſs as in *Not.* 4. of the foregoing Series.

Example 1.] Make $z = v$. Becauſe in this Caſe $r = 1$, $n = 0$, theree the Term laſt breaking off is $- \frac{ra}{\overline{n+2} \times \overline{n+1}|^2} y^n$, whence $q = -\frac{a}{2}$

by *Not.* 3. And becauſe in this Caſe $-\frac{a}{2}$ is the final Term, therefore $-a$ is the laſt Term to be multiplied into $\sqrt{2ay+yy}$, by *Not.* 4. and therefore $ABD = vy + \frac{av}{2} + \sqrt{2ay+yy} \times \overline{-\frac{1}{2}y - a}$.

Example 2.] Let $z = \frac{vy}{a}$. Becauſe in this Caſe $r = \frac{1}{a}$, $n = 1$, therefore the Term breaking off is $\frac{raa + \overline{2n+1}}{n \times \overline{n+2} \times \overline{n+1}|^2} y^{n-1} = \frac{a}{4}$. Whence $q = \frac{a}{4}$, and $\frac{a}{2}$ is the laſt Term to be multiplied into $\sqrt{2ay+yy}$. Therefore $ABD = \frac{vyy}{2a} - \frac{av}{4} + \sqrt{2ay+yy} \times \overline{-\frac{yy}{6a} - \frac{y}{12} + \frac{a}{2}}$. And if we take $y = \sqrt{\frac{aa}{2}}$, the quadrable Area belonging to this Abſciſs will be $\frac{1}{12}\sqrt{\sqrt{2a^4} + \frac{aa}{2}} \times \overline{5a - \sqrt{\frac{aa}{2}}}$.

N. 235. p. 785. Fig. 41. *Thirdly.*] Let ACF be a Semicircle, ADE a Curve Geometrically irrational, whoſe Ordinate BD cuts the Semicircle in C. Let the Quantities be repreſented as before, that is, the Diameter $AF = 2a$, the Abſciſs $AB = y$, the Arch $AC = v$, the Ordinate $BD = z$. And let the Equation expreſſing the Natures of the Curves ADE be $z = rv^2y^n$, in which r denotes any given determinate Quantity, and n the indefinite Exponent of the indeterminate Quantity y. I ſay the Area is

$$ABD = rv^2y^{n+1} - qv^2 + v\sqrt{2ay-yy} \text{ into } \frac{2ra}{\overline{n+1}|^2}y^n + \frac{2ra^2 \times \overline{2n+1}}{n \times \overline{n+1}|^2}y^{n-1} + \frac{aA \times \overline{2n-1}}{n-1}y^{n-2} + \frac{aB \times \overline{2n-3}}{n-2}y^{n-3} + \frac{aC \times \overline{2n-5}}{n-3}y^{n-4} + \frac{aD \times \overline{2n-7}}{n-4}y^{n-5}, \&c. - \frac{2ra^2}{\overline{n+1}|^3}y^{n+1} - \frac{2ra^3 \times \overline{2n+1}}{n^2 \times \overline{n+1}|^2}y^n - \frac{a^2A \times \overline{2n-1}}{\overline{n-1}|^2}y^{n-1} - \frac{a^2B \times \overline{2n-3}}{\overline{n-1}|^2}y^{n-2}, \&c.$$

Of this Theorem theſe Things are to be obſerved; (1.) That it is compounded of two infinite Series, of which the firſt (connected by the Sign +) is multiplied into $v\sqrt{2ay-yy}$; but the Terms of the latter (affected with the Sign —) are abſolute. (2.) That in the former the great Letters A, B, C, D, &c. denote the Coefficients of the Terms that precede them: Alſo in the latter they have the ſame Values as in the former. (3.) That the

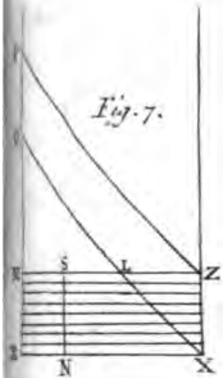
Fig. 7.
Z
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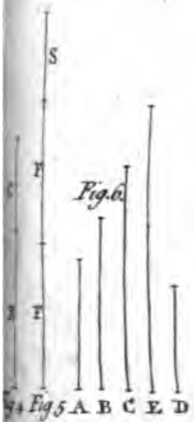
S
Fig. 6.
Fig 4
Fig 5
A
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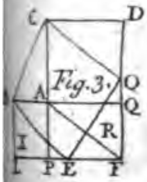
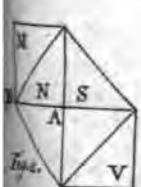
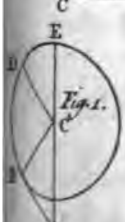
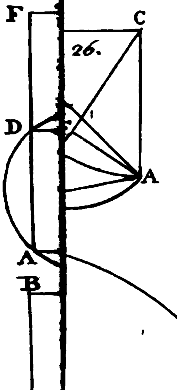
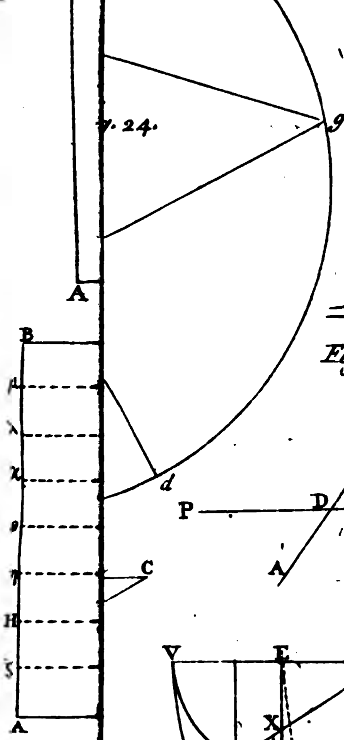

C
D
A
Fig. 3.
Q
Q
I
R
P
E
F


M
N
S
A
V
C
Fig. 2.


E
D
Fig. 1.
C
A

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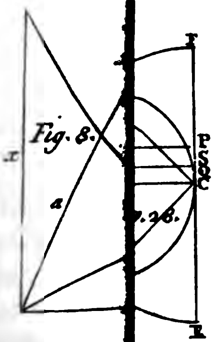
Fig. 8.
2 x
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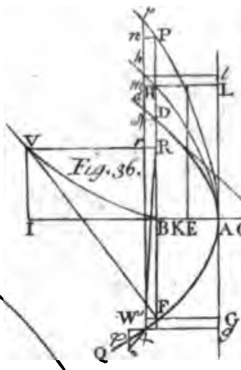
P
S
C
g.28.
E


P
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R
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Fig. 36.
D
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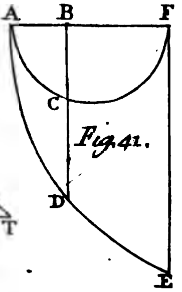
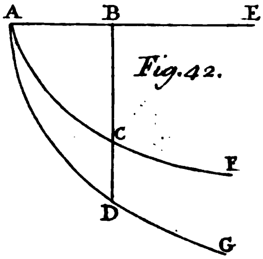
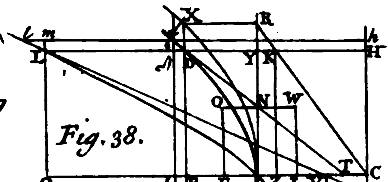
BKE
ACT
W
F
G
Q


A
B
F
C
Fig. 41.
D
E


A
B
E
Fig. 42.
C
F
D
G


X
R
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h
L
D
Y
K
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Fig. 38.
T
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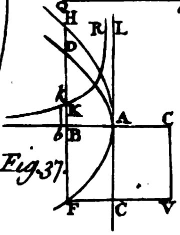
p


H
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A
C
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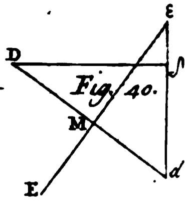
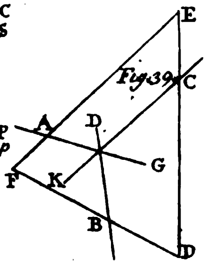
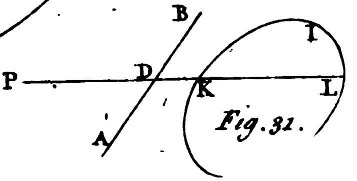
Fig. 37.
F
C
V


D
Fig. 40.
M
E
d


E
Fig. 39.
C
A
D
G
F
K
B
D


B
I
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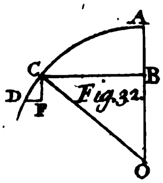
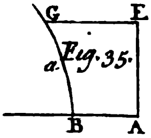
L
A
Fig. 31.


A
C
B
D
F
Fig. 32.
O


G
E
a
Fig. 35.
B
A

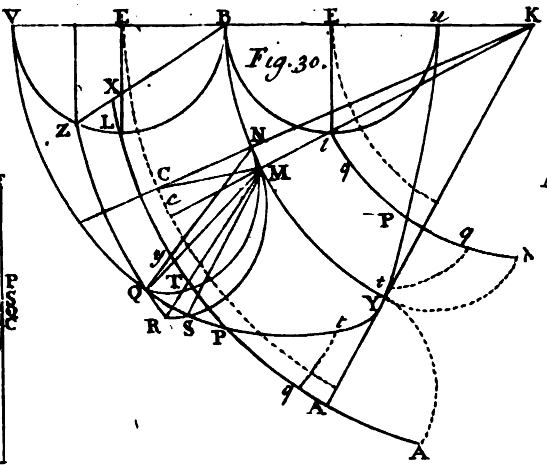

V
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Fig. 30.
X
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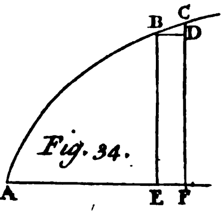
B
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Fig. 34.
A
E
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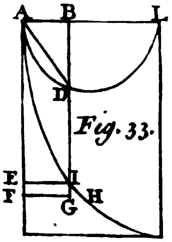
A
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Fig. 33.
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H
K

the Quadrature is exhibited by a finite Quantity, when n is an Integer poſitive Number, or equal to nothing, or when $2n$ is an odd Number. For in theſe Caſes both the Series break off. (4.) That $2q$ is equal to the laſt Term breaking off of the former Series.

Example 1.] Let $z = \frac{v^2}{a}$. Becauſe in this Caſe $n = 0$, $r = \frac{1}{a}$, therefore it will be Area A B D $= \frac{y v^2}{a} - v^2 + 2v\sqrt{2ay - yy} - 2ay$.

Corol. The intire Figure A F E is equal to a double Square, the Side of which is A C F, taking away the Square of the Diameter.

Example 2.] Let $z = \frac{y v^2}{a^2}$. Becauſe in this Caſe $n = 1$, $r = \frac{1}{a^2}$, therefore the Area will be A B D $= \frac{y^2 v^2}{2a^2} - \frac{3}{4}v^2 + v\sqrt{2ay - yy} \times \overline{\frac{v}{2a} + \frac{3}{2}} + \frac{1}{4}yy - \frac{3ay}{2}$.

Example 3.] Let $z = \frac{y^2 v^2}{3a^3}$. Becauſe in this Caſe $n = 2$, $r = \frac{1}{a^3}$, the Area will be A B D $= \frac{y^3 v^2}{a^3} - \frac{5}{6}v^2 + v\sqrt{2ay - yy} \times \overline{\frac{2y^2}{9a^2} + \frac{5y}{9a} + \frac{5}{3}} - \frac{2y^3}{27a} - \frac{5y^2}{18} - \frac{5ay}{3}$.

XV. Let O N F be a Logarithmick Curve, whoſe Aſymptote is A R, in which let ſuch a Point A be taken, that its firſt Ordinate A O may be equal to the Subtangent, or to Unity. The Curvilinear Space A O N M is required, comprehended by the two Ordinates A O, and M N, by the Abſciſs A M, and by the Logarithmic Curve O N.

The Quadrature of the Logarithmick Curve, by Mr. J. Craig. N. 245. *p.* 373. Oct. An. 1698. *Fig.* 43.

From O draw O E parallel to A M, cutting M N in E. I ſay that the Rectangle of the Segments M E, E N, is equal to the Space required.

Demonſtration.] Make the Ordinate M N $= z$, the Subtangent A O or M E $= s$, and to the Axis A R let another Curve H G Q be conſtructed, whoſe Equation is $2sz = xx$, where its Ordinate G M $= x$. I ſay it is the Quadratrix of the Logarithmic Curve, according to the Foundation of my Method: That is, its Subnormal is equal to the reſpective Ordinate of this, as according to the Calculation belonging to that Method may appear. Therefore according to what I have explained elſewhere, if to G be drawn G C perpendicular and equal to the Line G M, and alſo H D parallel to G C, meeting the Lines G M, C M, in B and D; it will be Trapezium

GBDC = AONM. But GBDC = GMC — BMD = $\frac{1}{2}xx - \frac{1}{2}$ BMq = $sz - \frac{1}{2}$HAq. But HA = $\sqrt{2}$ AOq by the Nature of the Curve HGQ. Therefore GBDC = sz — AOq = AO × $\overline{\text{MN} - \text{AO}}$ = ME × $\overline{\text{MN} - \text{ME}}$ = ME × EN. Therefore alſo AONM = ME × EN. *Q. E. D.*

A Quadratix to the Circle; being the Curve deſcrib'd by its Equable Evolution, by .... N. 260. *p.* 445. Jan. An. 1700. *Fig.* 44.

XVI. 1. By the Equable Evolution of a Circle, I mean ſuch a gradual Approach of its Periphery to Rectitude, as that all its Parts do together, and equally, evolve or unbend; or ſo that the ſame Line becomes ſucceſſively a leſs and leſs Arc of a reciprocally greater Circle.

2. Let A H K A be the Periphery of a Circle, A E a Tangent to the Point A. Let this circular Line be ſuppoſed cut or divided at A, and then to unbend (like a Spring,) its upper End remaining fixed to its Tangent A E, whilſt the other Parts do equally evolve or extend themſelves through all the Degrees of leſs Curvature (as in A B D, A M C, *&c.*) till they become ſtreight in Coincidence with the Tangent A E.

3. Let A M C be the evolving Curve in any middle Poſition between its firſt and laſt. Join the fix'd End A, and the moving End C, by the Chord-line A C, interſecting the firſt Circle at H; I ſay, that A M C is a like Segment to A n H, cut off in the firſt Circle by the Chord A H: For, by the Suppoſition, A M C is the Arc of a Circle, having A E a Tangent common both to it and A n H; and both Arcs are terminated in the ſame Right-Line A C.

4. Hence the Curve A D C E (deſcribed by the moving End of the Periphery in its Evolution) may be thus conſtructed: Let the Circle A H K A be by Biſections divided into any number of equal Parts; let H be one of the Points of ſuch Diviſion: Then ſay, As the Number of equal Parts in the Arc A n H, is to the Number of Parts in the whole Periphery A H K A; ſo is the Chord A H, to a fourth Line, which let be A C in A H produc'd. So is C a Point in the Curve A D C E.

5. *Dem.* Upon A C deſcribe A M C, an Arc like to the Arc A n H. Whence A H : A C :: A n H : A M C. But by Conſtruction, A H : A C :: A n H : Periph. A H K A; therefore is the Arc A M C equal to the whole Periphery A H K A, and like to the Arc A n H: Conſequently A M C repreſents the evolving Periphery, in a Poſition like to the Arc A n H, and C is the deſcribing Point.

6. After the ſame manner may be found other Points, thro' which the Curve may be drawn: But here (as in the old *Quadratrix* of *Dinoſtratus*) the Point E cannot be preciſely determined; but the Curve may be brought ſo near it, that its Flexure or Tendency will ſo lead to the Point E, that A E ſhall be near enough to the Truth for common Uſes.

7. Suppoſing the Point E found, a Tangent to any Point of the Curve may be drawn; and ſuppoſing a Tangent drawn, the Point E may be determined; the Property of the Tangent being this, that ſuppoſing R T a Tan-

gent to the Point C, and CA, CE, drawn from C to each End of the rectiſy'd Circle, the Angle ACT (the leſſer Angle that AC makes with the Tangent) is equal to ACE, the Angle made by the two Lines drawn from C.

8. Let c be a Point in the Quadratrix indefinitely near to C; and draw Ac interſecting $AHKA$ in b, and AMC in o. To Ac as a Chord, draw the Arc Amc, like unto the Arc Anb: To the Point C of the Arc AMC draw the Tangent $CL = AE$, and join LA; ſo is oC an indefinitely little Particle of the Arc coincident with its Tangent.

9. Becauſe of the like Segments $AnbA$, $AMoA$, $AmcA$, as Chord Ac to Chord Ao, ſo is Arc $Amc\,(=AMC)$ to Arc Amo: Or, $Ac : Ao :: Amc\,(=AMC) : AMo$; and dividing, $Ac - Ao\,(=co) : Ao :: Amc - AMo\,(=Co) : AMo$: That is, $co : Ao :: Co : AMo$; and alternately, $co : Co :: Ao : AMo$. Put AC for Ao, and AMC for AMo (as differing infinitely little) and then 'tis $co : Co :: AC : AMC$. But by *Conſtruction* $CL = AE = AMC$, whence $co : Co :: AC : CL$; and the Angle $LCA = Coc$, (oc being infinitely near to AC, is therefore parallel to it;) and therefore Coc, ACL, are like Triangles.

10. Becauſe of $CL = AE$, Angle $EAC = LCA$, (CL and EA being Tangents to the two ends of the ſame Circular Arc AMC, make equal Angles with its Chord AC) and AC common to both, the Triangles EAC, and ACL, are like and equal: therefore are all three Coc, ACL, EAC, like Triangles. Whence it follows, That the Angle ACE (in the Triangle EAC) is equal to the Angle ocC (in the Triangle coC) but $ocC = ACT$, becauſe oc and AC are parallel; therefore the Angle $ACE = ACT$. *Q. E. D.*

The Dimenſions of a Sphere and Cylinder compar'd; by D. Wallis. N. 263. *p.* 547. An. 1700.

XVII. In a certain Epiſtle of mine, in Vol. 3. of my Mathematical Works, among other Methods for Quadratures are to be found theſe two. One I call the Method of Convolution and Evolution, the other the Method of Complication and Explication. By Help of theſe I ſhew, which is the ſimpleſt Manner of meaſuring all Curve Figures, and particularly the Cycloid.

By a like Artifice may be ſhewn how to compare the Sphere and Cylinder, which *Archimedes* thought fit to chuſe for his Monument.

Fig. 45.

If to the Baſis P, equal to the Circumference of a Circle, a Height R be aſſumed equal to Radius, there will be made a Rectangular Parallelogram = P R. This may be conceived as compoſed of an infinite Number of ſmall allelograms, of the ſame Height, according to the received Method of ivisibles.

Fig. 46.

Now if the Vertices of all theſe are conceived to be contracted into one nt, ſo that of thoſe minute Parallelograms as many Triangles may be e, having the ſame Baſis and an equal Height; each will be half of each he others, and therefore all of all; and the Baſe being bent into the Cirference of a Circle, a Circle will be made whoſe Radius is R and Center which therefore is half the Parallelogram, or $\frac{1}{2}RP$.

This is *Archimedes*'s Dimenſion of a Circle, which is equal therefore to a Right-angled Triangle, one of whoſe Sides about the Right Angle is equal to the Periphery, and the other to the Radius of the propoſed Circle. For $\frac{1}{2}$ R, or half the Altitude of the Triangle, drawn into P the Baſe, exhibits the Magnitude of that Triangle $\frac{1}{2}$ R P, which is equal to the Circle. And the ſame may be accommodated to the Circular Sector, taking the Arch A inſtead of the Periphery P.

Fig. 47. Again, if to that Parallelogram = R P, as a Baſe, be taken in like manner an Altitude R, in order to a Hemiſphere; there will be made a Parallelepiped = R R P. This in the ſame manner may be conceived as compoſed of an infinite Number of ſmall Parallelepipeds of the ſame Height, inſiſting upon the minute Areas of that Plain; of all which the common Altitude is R, and the Aggregate of Baſes = R P. Now if this Parallelogram, the Magnitude R P continuing, be ſuppoſed to be bent into a Cylindrical Surface (whoſe Baſe P is now bent into the Periphery of a Circle, and whoſe Altitude is R) that thoſe minute Parallelepipeds may be changed into ſo many Wedges, or Priſms with Triangular Baſes, each of which are half their reſpective Parallelepipeds, and therefore all are half of all; having for their Vertices ſo many Points C, or minute Lines, in the Axis of the Cylinder, and thus filling it up; the Cylinder will become half the Parallelepiped, or $\frac{1}{2}$ R R P.

Or in order to come at the intire Sphere, if on each Side is taken the Altitude R, ſo that the whole Altitude may be D = 2 R, and if a Convolution be made in like manner, a Cylinder will be produced as before, conſiſting of Wedges or Priſms infinite in Number, having their Points or Vertices in the Axis of the Cylinder, which will be equal to R R P = $\frac{1}{2}$ R P x 2 R, equal to the Product of $\frac{1}{2}$ R P, or the Circular Baſe, into the Altitude 2 R: Or which is the ſame Thing, it will be equal $\frac{1}{2}$ R x 2 R P, or equal to the Product of $\frac{1}{2}$ R, the half of the common Altitude of the Wedges, into the Aggregate of the Baſes 2 R P.

Which Aggregate of the Baſes is the curved Cylindric Superficies itſelf, which is equal to P x 2 R, or to the Product of P the Periphery of the Circular Baſe drawn into the Altitude 2 R, or equal to $\frac{1}{2}$ R P x 4, four of the great Circles of the Sphere. To which if we add the two oppoſite Circular Baſes, there will be made the whole Superficies of the Cylinder circumſcribed to the Sphere, equal to ſix great Circles, $\frac{1}{2}$ R P x 6 = 3 R P. And the Magnitude of the Cylinder, = R R P = $\frac{1}{2}$ P P x 2 R, equal to the Product of the Circular Baſe $\frac{1}{2}$ R P drawn into the Altitude 2 R, as before.

Now if the Vertices of all theſe Wedges that conſtitute the Axis of the Cylinder, are conceived to be contracted into one Point, ſo that theſe Wedges or Priſms may now become ſo many Pyramids, being on the ſame Baſes and the ſame Height; each will be of each, and therefore all of all, in a Proportion ſubſeſqui-tertial or as $\frac{1}{3}$ to $\frac{1}{2}$; and the Superficies which before was curved Cylindrical, will now become Spherical becauſe of all its Points being equally remote from the Center, the Aggregate of the Baſes remaining as before = 2 R P, or equal to four great Circles; we ſhall then have the whole Superficies of the Sphere $= 2\,R\,P = \frac{1}{2}\,R\,P \times 4 =$ four great Circles; and equal to the whole curved Cylindrick Surface, and the Parts reſpectively equal to the Parts that belong to the ſame Parts of the Axis; alſo the Magnitude of the Sphere $\frac{2}{3}\,R\,R\,P = \frac{1}{3}\,R\,P \times 2\,R$, equal to the Product of $\frac{1}{3}\,R$, a third Part of the common Altitude of all the Priſms, drawn into $2\,R\,P$ the Aggregate of the Baſes, which is now become the Spherical Surface.

Therefore both the Superficies and Magnitude of the Cylinder circumſcribed to the Sphere, is ſeſqui-alter to the Superficies and Magnitude of the inſcribed Sphere, or as 3 to 2: There becauſe the Proportion is as ſix great Circles $= 3\,P\,P$ to four great Circles $= 2\,R\,P$; here becauſe the Proportion is as $R\,R\,P$ to $\frac{2}{3}\,R\,R\,P$: Which is the very Invention of *Archimedes* ſo much celebrated.

The ſame would be had a little ſhorter, if in the Parallelepiped upon the plain Baſe $2\,R\,P$ of the Altitude R, compoſed of minute Parallelepipeds, all their Vertices were immediately ſuppoſed to be contracted into one Point C: That the Aggregate of the Baſes continuing as before $= 2\,R\,P$, thoſe Parallelepipeds may be reduced to ſo many Pyramids, having their Vertices meeting at the Center of the Sphere, whoſe Radius R is the common Altitude of all the Pyramids, and the Spherical Superficies is the Aggregate of all the Baſes. For $\frac{1}{3}\,R$, a third Part of the common Altitude, drawn into $2\,R\,P$ the Aggregate of the Baſes, exhibits as before the Magnitude of the Sphere $\frac{2}{3}\,R\,R\,P$, and the Surface of the Sphere $= 2\,R\,P$.

In like manner this may be accommodated to the Spherical Sector, by drawing $\frac{1}{3}\,R$, a third Part of the common Altitude of all the Pyramids in it, into a Portion of the Spherical Surface cut off by a Plain: Which is to the whole Spherical Surface, as the Part cut off of the Diameter or Axis is to the whole Diameter; as was ſhewn above.

Now the Reaſon of this whole Proceſs depends on theſe Principles. That a Figure compoſed of Triangles is half the Figure compoſed of Parallelograms, upon the ſame Baſes and of equal Height. That I call a Convolute Figure, and this an Evolute. And that a Figure of Pyramids is a third Part of a Figure of Parallelepipeds, on the ſame Baſes and equal Height. That I call a Complicate Figure, and this an Explicate. Theſe Principles may be accommodated in a thouſand Manners to Curvilinear Figures, whether Superficial or Solid, however perplexed and intricate.

XVIII. 1. It hath been obſerved by divers of this Nation, that in any
E uation, howſoever affected, if you give a Root, and find the abſolute
mber or Reſolvend, (which *Vieta* calls *Homogeneum Comparationis*;) and
in give Roots and find more Reſolvends; that if theſe Roots, or rather
k of Roots, be aſſumed in Arithmetical Progreſſion, the Reſolvends, as
heir firſt, ſecond, or third Differences, *&c.* imitate the Laws of the pure
ers of an Arithmetical Progreſſion, of the ſame Degree, that the higheſt
ver, or firſt Term of the Equation is of. *E. g.* In this Equation $aaa -$
$a + 4a = N$.

Improvements in England *in the Reſolution of Equations in Numbers, by Mr.* J. Collins. N. 46. p. 929. Apr. An. 1669.

If *a* be = {10, 9, 8, 7, 6} Then N or the Abſolutes or Reſolvends will be found to be,

a	N	1 diff.	2 diff.	3 diff.
10	740	218		
9	522	170	48	
8	352	128	42	6
7	224	92	36	6
6	132			

To wit, the 3d Differences of thoſe Abſolutes are equal, as in the Cubes of an Arithmetical Progreſſion.

2. To find what Habitude thoſe Differences have to the Coefficient of the Equation, 'tis beſt to begin from an Unit.

3. In any Arithmetical Progreſſion, if you multiply Numbers by Pairs, you ſhall create a Rank of Numbers whoſe ſecond Differences are equal; and if by Ternaries, then the third Differences of thoſe Products ſhall be equal. And how to find the greateſt Product of an Arithmetical Progreſſion of any Number of Terms having any common Difference aſſigned, contained in any Number propoſed, is ſhewed by *Paſcal* in his Tract *Du Triangle Arithmetique*, where he applies it to the Extraction of the Roots of Simple Powers.

4. It appears, how this Rank may be carried eaſily by Addition, till you have a Reſolvend either equal or greater or leſs than that propoſed.

5. When you have a *Majus* and *Minus*, you may interpoſe as many more Terms in the Arithmetical Progreſſion as you will, that is to ſay, Subdivide the common Difference in the Arithmetical Progreſſion, and render it leſs; and then renew, and find the Reſolvends, which are eaſily obtained out of the Powers and their Coefficients, which are ſuppoſed known, and may be readily raiſed from a Table of Squares and Cubes, *&c*, with which kind the Reader may be furniſhed in *Guildini Centrobaryca*, and *Babington*'s *Fireworks*. By this means you may obtain divers Figures of the Root; and then the General Method of *Vieta* and *Harriot* runs away more eaſily, and is ſo far improved, that after any Figure is placed in the Root, moſt certain Characters are given to know, by Aid of the ſubſequent Dividend and Diviſor, whether the Figure before aſſumed be too great or too ſmall: Or, laſtly, it may be well concluded, that as in Logarithms, when you propoſe ſuch an one as is not abſolutely given in the Canon, you do, by proportional Work, uſing the Aid of their firſt Differences (when their abſolute Numbers differ by Unite) find the abſolute Numbers true to 5 or 6 Places farther than the Canon gives it (the Reaſon whereof is, that the firſt Differences do likewiſe agree to about the ſame Number of Places); that, I ſay, the like may be done in Equations, after divers of the firſt Figures of the Roots are found, provided there be the like Agreement in the firſt Differences of the Interpoled Reſolvends.

Moreover, we ought here to take notice of a more ſubtile kind of Interpolation, common to all Gradual Ranks or Progreſſions of Numbers, wherein Differences happen to be equal: Of which kind the Reader may find Examples in *Briggii Arithmetica Logarithmica*, & *Trigonometria Britannica*, relating to Logarithms, Sines, and the Powers of an Arithmetical Progreſſion:

But the Method there delivered may be rendered more eaſy and general, *viz.* by Aid of a Table of Figurate Numbers, by deriving Generating Differences ſought from thoſe given; a Doctrine that eaſily flows from *Mercator*'s *Logarithmotechnia*, and of uſe in the Caſe in hand, ſhould we ſuppoſe theſe Powers and their Coefficients unknown, or a Table of Squares and Cubes wanting, and give nothing more than a few Reſolvends belonging to equal Moments or Spaces. And this may likewiſe be of good Uſe in *Gauging*, when having the Contents of a Solid, for every three Inches more or leſs given, without knowing the Dimenſions of the Figure, and even in moſt Caſes, when the Differences are progreſſive of one Kind, without knowing the Figure itſelf, having nothing given but its Contents at ſeveral equal parallel Diſtances, each ſuch Diſtance may be ſub-divided, and made as many as you pleaſe, and the reſpective Contents found by this general Method of Interpolation.

After one Root is obtained, the Methods of *Huddenius* and others will depreſs the Equation ſo as to obtain more, and conſequently all of them.

6. It is eaſy by a Table of Figurate Numbers to give the Sum of any ſuch Rank, or any Term in it relating to a known Part of the Series of Equals or Roots; but *è converſo*, giving the Reſolvend to find the Root, comes to an Equation as difficult as that propoſed; as in Dr. *Wallis*'s Chapter of *Figurate Numbers*.

7. Some affirm, they can give good Approaches for the obtaining a Root of any pure Power, affected Equation, or for the finding any of the mean Proportionals in any Rank between the two Extremes given.

8. Others pretend to have found out the Method (incited thereto by an Example in *Albert Gerard*'s *Invention Nouvelle en Algebre, à Amſterdam* 1629) ſo much, by comparing of Equations, to increaſe or diminiſh the unknown Root of an Equation, as to render it a whole Number (or leſs differing therefrom than any Error aſſigned) and by *Albert Gerard*'s Method of Aliquot Parts to find the ſame, and thereby the Root ſought, altho' it be a mixt Number, Fraction, or Surd.

Probably this may ſympathize with what is promiſed by the learned *Huddenius in Annexis Geometriæ Carteſianæ*, where he ſaith he intended not then to publiſh certain Rules he had ready; whereof one was to find out all the Irrational Roots both of Literal and Numeral Equations: This muſt be underſtood, when ſuch Roots are poſſible; for 'tis certain there are Infinite Equations, whoſe Roots are no ways explicable, either in Whole or Mixed Numbers, Fractions or Surds, and can be no otherwiſe explained, but by a *quamproxime*.

9. The *Author of this Narrative* conſidering that the Conic Sections may be projected from leſſer Circles placed on the Sphere, and thence eaſily (otherwiſe than hitherto hath been handled) deſcribed by Points, and that by their Interſections, ſome Spherick Problem is determined; accordingly, he found that this following Problem, according to the various Situation of the Eye, and of the projecting Plan, would take in all Caſes. *Ibid. p.* 932.

The Diſtances of an unknown Star are given from two Stars of known Declination and Right Aſcenſion; the Declination and Right Aſcenſion of the unknown Star is required.

And ſaith, He hath obſerved, that, admitting the Mechaniſm of dividing the Periphery of a Circle into any Number of equal Parts, or (which is equivalent) the Uſe of a Line of Chords, that this Problem, wherever the Eye be placed, may be reſolved by plain *Geometry*, and yet the Eye ſhall be ſo placed, as to determine it by the Interſection of the Conic Sections; conſequently thoſe Points of Interſection (the Species and Poſition of the Figures being given) may be found without deſcribing any more Points than thoſe ſought; and the Lengths and Ordinates falling from thence on the Axes of either Figure calculated by Mixed *Trigonometry*, and hence likewiſe the Roots of all *Cubick* and *Biquadratic Equations* found by *Trigonometry*.

For giving, from the *Meſolabe* of *Sluſius*, the Scheme that finds theſe Roots, it will then be required to fit thoſe Sections into Cones, which have their Vertex either in the Center, or an aſſigned Point in the Surface of the Sphere, to which they relate as projected, and proceed to the Reſolution of the Problem propoſed; and how to fit in thoſe Sections, ſee the *ſeven Books of Apollonius*, *Mydorgius*, the 3d Volume of *Des Cartes*'s Letters, *Leotaudi Geometria Practica*, *Anderſonii Exercitat. Geometricæ*.

As to the Problem itſelf, it is determined on the Sphere by the Interſections of the two leſſer Circles of Diſtance, whoſe Poles are the known Stars. And this Problem hath divers Geometrick Ways of Reſolution.

1. By plain Geometry (in the Senſe before mentioned): Suppoſing a Plain to touch the Sphere or the North Pole; if the Eye be at the South Pole, projecting thoſe Circles into the ſaid Plane, they are ſtill Circles (by reaſon of the ſub-contrary Sections of the viſual Cones) whoſe Centers fall in the Sides of the Right-lined Angle, made by the projected Meridians, that paſs thro' the known Stars; and thus the Problem is eaſily ſolved in this Manner.

2. If it be required to be performed by Conic Geometry; In one Caſe it may be done, by placing the Eye at the Center of the Sphere, and projecting as before; to wit, when the longer Axes of the Figures being produced, concur above the Vertex: Here the Problem is determined by the Interſections of two Conic Sections (whereof a Circle cannot be one, unleſs its Center be in the Axis of the other Figure): And in this ſecond Caſe, theſe Points of Interſection fall in the ſame Right Line or projected Meridian they did before, but at a more remote Diſtance from the Pole Point, to wit, in the former Suppoſition the Polar Diſtance was meaſured by a Right Line, that was the double Tangent of half the Arch; here it is the Tangent of the whole Arch. Hence it is evident how one Projection may beget another, yea, infinite others, altering the Scale, and how the leſſer Circles in the *Stereographick* Projection help to deſcribe the Conic Sections in the *Gnomonick* Projection: But (to reduce the Matter to the common Radius) if we ſuppoſe two Spheres equal, and ſo placed about the ſame Axis, that the Pole Point of the one ſhall paſs thro' the Center of the other, and the Touch-Plain to paſs thro' the ſaid Center or Pole Point; and that a leſſer Circle hath the

ſame Poſition in the one as in the other: then, if the Eye be at the South Pole of the one, it is at the Center of the other; and any projected Meridian drawn from the projected Pole Point to paſs thro' both the Projections of theſe leſſer Circles, the Diſtances of the Points of Interſection are the Tangents of the half and the whole Arch of the Meridian ſo interſected. But as to the Points of Interſection, which determine the Problem propoſed, they may be found without the Aid of the former Way, from a *Gnomonick* and *Stereographick* Method of meaſuring and ſetting off the Sides and Angles of Spherical Triangles in thoſe Projections, which is neceſſary in what follows.

3. If the Problem is to be performed by mixed Geometry, as by a Circle, and either a Parabola, Hyperbola, or Ellipſis, the Circle may be conceived to be the ſub-contrary Section of a Cone projected by the Eye at the South Pole, and any of the reſt of the Sections by the Eye at the Center of the Sphere.

4. If by any of the Conick Sections however poſited, the projecting Plane may remain the ſame; but the Eye muſt be in ſome other Part of the Surface of the Sphere, and not in the Axis.

XIX. 1. The Conſtruction delivered by *Des Cartes*, which eaſily finds the Roots of all Cubic and Biquadratic Equations, wherein the ſecond Term is wanting, I may here ſuppoſe as known: Yet ſince the following depends upon it, I ſhall here ſubjoin the Rule taken from his Geometry, changing a few Things (as I hope) for the better.

The Conſtruction of Cubick and Biquadratic Equations, by a Parabola and Circle. By Mr. Edm. Halley. N. 188. *p.* 335. Jul. An. 1687. *Fig.* 48.

When the ſecond Term is wanting, all Cubic Equations may be reduced to this Form, $z^3 * apz : aaq = 0$. And all Biquadratics to this, $z^4 * apz^2 : aaqz : a^3r = 0$. Where a denotes the *latus rectum* of any given Parabola, which may be uſed in the Conſtruction; or making $a = 1$, the Equations will be $z^3 * pz : q = 0$, and $z^4 * pz^2 : qz : r = 0$.

Now the Parabola F A G being given, whoſe Axis is A C D K L, and its *latus rectum* a or Unity, let A C be its half, and let it always be placed from the Vertex A within the Figure. Then take $CD = \frac{1}{2}p$, in the Line A C continued towards C, if in the Equation it is $-p$, or the other Way if it ſhall be $+p$. Then from the Point D, or from C if $p = 0$, the Perpendicular to the Axis $DE = \frac{1}{2}q$ is to be raiſed, to the Right Hand if it ſhall be $-q$, but on the other Side of the Axis if it ſhall be $+q$. Then a Circle deſcribed with Center E, and Radius A E, if the Equation be only Cu-
., will cut the Parabola in ſo many Points F and G, as it has true Roots, which the Affirmative, as G K, will be to the Right Side of the Axis, d the Negative, as F L, will be on the Left Side.

But if the Equation is Biquadratic, the Radius of the Circle A E muſt be creaſed or diminiſhed; by adding if it is $-r$, or ſubtracting if it is $+r$, e Rectangle ar from its Square, or the Plain contained by the *latus rectum* d the given Quantity r; which is eaſily done Geometrically. Now the terſections of this Circle with the Parabola will exhibit all the true Roots the Biquadratic Equation, by letting fall Perpendiculars to the Axis;

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the Affirmatives to the Right Side of the Axis, and the Negatives to the Left Side. But I leave to *Des Cartes*, its Inventor, the Demonſtration of the whole.

Here it is to be obſerved, that I endeavour always to have the Affirmative Roots on the Right Side of the Axis, to avoid that Confuſion that will neceſſarily ariſe, from a Multitude of Cautions and Exceptions, the Reaſon of which would by no means be evident.

Theſe Things being premiſed, that we may come to the Conſtruction of ſuch Equations alſo, where the ſecond Term is preſent; the Rule for taking away the ſecond Term muſt be conſidered, and for reducing the Equation to another, which may be conſtructed by the foregoing Method. Now all the Cubic Equations of this Claſs may be reduced to this Form, $z^3 : bz^2 : apz : a^2q = 0$, or to this, $z^3 : bz^2 : * \; a^2q = 0$. And the Biquadraticks to this $z^4 : bz^3 : apz^2 : a^2qz : a^3r = 0$, or to this $z^4 : bz^3 : * : a^2qz : a^3r = 0$, or to this, $z^4 : bz^3 : apz^2 : * : a^3r = 0$, or laſtly to this, $z^4 : bz^3 : * : * : a^3r = 0$. Out of all theſe, as they may be differently connected by the Signs + and —, there ariſes a vaſt Variety. Whence a general Rule comprehending them all muſt needs be very difficult and obſcure, unleſs it be treated in the following Manner, and ſo delivered from its Perplexities.

In Biquadratics the ſecond Term is taken away, by making $x = z + \frac{1}{4}b$, if it be $+b$ in the Equation; or $x = z - \frac{1}{4}b$, if it be $-b$. Hence in the firſt Caſe $x - \frac{1}{4}b$, and in the ſecond $x + \frac{1}{4}b$, is equal to z; and in any Equation propoſed inſtead of z ſubſtituting its equal Quantity, there ariſes a new Equation in which the ſecond Term is wanting, all whoſe Roots x either exceed or are deficient from the Root ſought z, by a given Difference $\frac{1}{4}b$.

Example 1. $z^4 + bz^3 - apz^2 - a^2qz + a^3r = 0$. Make $x - \frac{1}{4}b = z$, then $x^2 - \frac{1}{2}bx + \frac{1}{16}b^2 = z^2$, $x^3 - \frac{3}{4}x^2b + \frac{3}{16}xb^2 - \frac{1}{64}b^3 = z^3$, and $x^4 - bx^3 + \frac{3}{8}b^2x^2 - \frac{1}{16}b^3x + \frac{1}{256}b^4 = z^4$. Hence by Subſtitution

$$\begin{array}{rrrrrl}
x^4 - bx^3 & + \frac{3}{8}b^2x^2 & - \frac{1}{16}b^3x & + \frac{1}{256}b^4 & & = +z^4 \\
+bx^3 & - \frac{3}{4}b^2x^2 & + \frac{3}{16}b^3x & - \frac{1}{64}b^4 & & = +bz^3 \\
& -apx^2 & + \frac{1}{2}abpx & - \frac{1}{16}apb^2 & & = -apz^2 \\
& & -a^2qx & + \frac{1}{4}a^2bq & & = -apqz \\
& & & + a^3r & & = +a^3r
\end{array}$$

The Sum of all theſe becomes a new Equation, in which the ſecond Term is wanting, and which therefore may be conſtructed by *Cartes*'s Rule; putting inſtead of $\frac{1}{2}p$ half the Coefficient of the third Term divided by a or the *latus*

tus rectum, that is, $-\frac{3bb}{16a}-\frac{1}{2}p$; and inſtead of $\frac{1}{2}q$ half the Coefficient of the fourth divided by aa, or $+\frac{b^3}{16a^2}+\frac{pb}{4a}-\frac{1}{2}q$. The Parts affected with the Sign $+$ are to be placed on the Left Side of the Axis, and thoſe affected with $-$ on the Right Side, that the Center of the Circle may be had, as is requiſite for the Conſtruction; ſo that the Interſections of this with the Parabola, by Perpendiculars let fall upon the Axis, may exhibit all the true Roots x, the Affirmatives on the Right Side of the Axis, and the Negatives on the Left Side. For ſince $x-\frac{1}{4}b=z$, by drawing a Line parallel to the Axis, on its Right Side and at the Diſtance $\frac{1}{4}b$, thoſe Perpendiculars terminated at this Parallel will exhibit all the Roots z, the Affirmatives to the Right Hand and the Negatives to the Left. As for the Radius of the Circle, it will be had by adding the negative Parts, and ſubtracting the affirmative Parts of the fifth Term divided by aa, from the Square of the Line A E drawn from the Center E (when found) to the Vertex of the Parabola A. And this may generally be done by taking inſtead of the Line A E the Line A O, which is terminated at O the Interſection of the Parabola and the aforeſaid Parallel. For its Square contains (as may eaſily be proved) all the Parts of the fifth Term accruing to the new Equation, by taking away the ſecond Term: And it only remains that the Square of E O may be increaſed, if there is $-r$ in the Equation, or diminiſhed if there is $+r$, by the Addition or Subtraction of the Rectangle ar; whence the Square of the Radius of the Circle required is compoſed.

This is Mr. *Baker*'s Method of finding the Central Rule, but free from all his Cautions, and eaſy enough. The only Difference ariſes from hence, that I determine the Center of the ſame Circle by the Axis, and he by the Parallel to the Axis; and that I always find the affirmative Roots on the Right Side of the Axis, which he places ſometimes on the Right and ſometimes on the Left.

As for what belongs to Cubic Equations, they muſt be reduced to Biquadratics, before they can be conſtructed by the ſame general Rule. And this is done by multiplying the Equation propoſed by its Root z, whence a Biquadratic Equation ariſes, in which the laſt Term, or r, is wanting. Wherefore taking away the ſecond Term, and the Center E being found, the Line E O is the Radius of the Circle; ſince $ar=0$, and in the new Equation the whole fifth Term ariſes, by only taking away the ſecond Term.

Example 2.] $z^3-bz^2+apz+aaq=0$. This multiplied by z comes $z^4-bz^3+apz^2+aaqz=0$. To take away the ſecond Term ake $x+\frac{1}{4}b=z$.

$$\begin{array}{llllllll}
\text{Then } x^4 & +bx^3 & +\frac{3}{8}b^2x^2 & +\frac{1}{16}b^3x & +\frac{1}{256}b^4 & = & z^4 \\
 & -bx^3 & -\frac{3}{4}b^2x^2 & -\frac{3}{16}b^3x & -\frac{1}{64}b^4 & = & -bz^3 \\
 & & & & & & +apx^2
\end{array}$$

$$+ \quad apx^2 + \frac{1}{2}abpx + \frac{1}{16}apb^2 = + apz^2$$

$$+ \quad a^2qx + \frac{1}{4}a^2bq = + a^2qz$$

In this new Equation, half the Coefficient of the third Term divided by a, that is $-\frac{3bb}{16a} + \frac{1}{2}p$, is to be uſed inſtead of $\frac{1}{2}p$; and half the Coefficient of the fourth Term, divided by aa or the Square of the *latus rectum*, that is $-\frac{b^3}{16a^2} + \frac{pb}{4a} + \frac{1}{2}q$, is to ſupply the Office of $\frac{1}{2}q$ in *Cartes*'s Conſtruction. Whence the Center E is determined. Then a Parallel being drawn to the Axis at the Diſtance of $\frac{1}{4}b$, on its Left Side, becauſe of $x + \frac{1}{4}b = z$, and let the Interſection of this with the Parabola be O. A Circle deſcribed with Center E, and Radius E O, will cut or touch the Parabola in ſo many Points at the Equation has true Roots; which Roots or Values of z are exhibited by Perpendiculars let fall from thoſe Points upon the Parallel to the Axis; the Affirmatives to the Right Hand, and the Negatives to the Left.

If in the Equation either the third or fourth Term, or both ſhall be wanting, there will be no Difference to be obſerved in inveſtigating the Central Rule, but the Quantities p or q vaniſhing, thoſe Parts of the Lines C D and D E ariſing from thoſe Quantities will be abſent; ſo that we muſt proceed with the remaining Coefficients of the third and fourth Terms in the new Equation, as is preſcribed in the foregoing Examples.

Hitherto we have purſued *Baker*'s general Method, than which we can expect none that will be eaſier or readier; whether a Parabola is aſſumed for the Conſtruction, or any other Curve, and when the Equation aſcends to the Biquadratic. And whilſt I am writing this there occurs to me a Geometrical Effection of the Central Rule, which is expeditious beyond Hope, and will abundantly ſatisfy the Curious in theſe Matters.

Fig. 49. The Parabola N A M being deſcribed, whoſe Vertex is A, its Axis A B C, and *latus rectum* a, let the Equation be reduced to this Form, $z^4 : bz^3 : apz^2 : a^2qz : a^3r = 0$; or to this, $z^3 : bz^2 : apz : a^2q = 0$, if it be a Cubic Equation. Then at the Diſtance of $BD = \frac{1}{4}b$ let the Line D H be drawn parallel to the Axis, to the Left Hand if it be $-b$, or to the Right if $+b$, meeting the Parabola in the Point D, from whence let fall B D perpendicular to the Axis. In the Line A B continued towards B make $BK = \frac{1}{2}a$, and draw the Line D K both ways indefinitely. Then make $KC = 2AB$ in the Axis always continued beyond K; and if the Quantity p is affected with the Sign —, take $CE = \frac{1}{2}p$ alſo towards the ſame Side, but contrariwiſe if it is $+p$; and from the Point E let there be raiſed E F perpendicular to the Axis, or from the Point C if the Quantity p is abſent, meeting in F the Line D R produced if need be; which Point F is the Center of the Circle required if q be abſent, but if it be preſent in F E, continued if need be, muſt be taken a Line $FG = \frac{1}{2}q$, to the Left if it is $+q$, but

to the Right if — q. And the Point G will be the Center of the Circle proper for the propoſed Conſtruction, and its Radius will be the Line G D, if r is wanting or if the Equation is only Cubical. In the Biquadratics the Square of this muſt be increaſed or leſſened, according as it is $-r$ or $+r$, by the Rectangle of r and the *latus rectum*. The Circle being thus deſcribed, a Perpendicular being let fall upon the Line D H from its Interſections with the Parabola, thoſe that are on the Left Hand, as N O, will always repreſent the negative Roots of the Equation, and thoſe on the Right Hand, as M L, the affirmative Roots.

Cubic Equations are conſtructed otherwiſe, and ſomething more ſimply; by a Rule of *Van Schootens*, by which alſo the Roots are referred to the Axis. But becauſe the Inventor himſelf does not deliver his Manner of finding it, nor the Demonſtration of his Invention, it may not be amiſs to ſhew here the Foundation of the ſame, and to make the Geometrical Effection ſomething more neat, and to free it from ſome Cautions in which it is involved.

This Rule is derived from hence, that every Cubic Equation may be reduced to a Biquadratic, in which the ſecond Term is wanting. This is done by multiplying the Equation propoſed into $z-b=0$, if there is $+b$ in the Equation; or into $z+b=0$, if it be $-b$. Then the new Equation ſo produced will have the ſame Roots with the Cubic, and another beſides equal to $-b$, if there is $-b$ in the Equation; or on the contrary.

Suppoſe we were to conſtruct $z^3-z^2b+apz+aaq=0$. Multiply this by $z+b$; it becomes

$$\left.\begin{array}{l} z^4-bz^3+apz^2+a^2qz \\ \quad\;\, +bz^3-b^2z^2+abpz+a^2bq \end{array}\right\}=0.$$

Here the ſecond Term is wanting, and the Coefficient of the third $-bb+ap$ gives $-\frac{bb}{2a}+\frac{1}{2}p$ to be ſubſtituted for $\frac{1}{2}p$ or C D in *Cartes*'s Conſtruction; and of half the Coefficient of the fourth Term is made $+\frac{1}{2}q+\frac{bp}{2a}$, to be uſed inſtead of $\frac{1}{2}q$ or D E, and ſo the Center of the Circle ſought is determined. Then becauſe one of the Roots of the new Equation is given, which is — or $+b$, a Point alſo in the Circumference will be given, and therefore the Radius. Laſtly, the Circle being deſcribed, Perpendiculars let fall upon the Axis from its Interſections with the Parabola will exhibit the oots of the Equation, the affirmative and negative as mentioned before.

Now the Center of the Circle is found by a very eaſy Conſtruction, which Fig. 59. preferable to all others in Cubic Equations. Let A be the Vertex and F the Axis of the deſcribed Parabola A M D. At a Diſtance equal to b D K be drawn parallel to the Axis, at the Right or Left Hand according it is $+b$ or $-b$, which meets the Parabola in the Point D. With Cen-
s D and A, and with equal Radius's, let occult Arches be deſcribed inter-
ting one another on both Sides, and through the Points of Interſection draw

draw the indefinite Right-Line P C, which may be perpendicular to the ſuppoſed Line A D at its middle, and meet the Axis at the Point E. From E, either below or above according as there is either $-p$ or $+p$ in the Equation, make $EF = \frac{1}{2}p$; and from F, or from E if p is wanting, draw the Perpendicular F G meeting the Line B C in the Point G; aud in G F produced make $GH = \frac{1}{2}q$, to the Right if there is $-q$ in the Equation, or otherwiſe to the Left. Then will H be the Center required, and H D the Radius of the Circle; which will ſhew all the Roots as before, by means of Perpendiculars let fall from its Interſections with the Parabola to its Axis.

The Number of Roots in ſuch Equations, with their Limits and Signs; by Mr. Edm. Halley. N. 190. *p.* 389. Nov. An. 1687.

It appears from *Des Cartes* and from what has been already ſaid, as well in Cubics as in Biquadratics, that the Roots may be repreſented by letting fall Perpendiculars upon the Axis, or a given Diameter of the given Parabola, from the Interſections of that Curve with a Circle. And ſince a Circle cutting a Parabola muſt cut it either in four or in two Points, it is plain that in Biquadraticks there muſt neceſſarily be two or four true Roots, either affirmative or negative; as alſo if perhaps the Circle may touch it, in which we may infer an Equality of two Roots of the ſame Sign. But in Cubicks, becauſe one of the Interſections is required for the Conſtruction, the one or the three remaining repreſent the one or the three Roots, as in the Caſe of Contact, whence it is plain that there are found two equal Roots, and that the Problem from whence the Equation reſults is truly a plane Problem.

Therefore all Cubic Equations however affected are explicable with one or with three Roots, as always poſſible; admitting negative Roots for true ones. So Biquadraticks, of which the laſt Term r is affected with the Sign
Fig. 4. $-$, are explicable by two or four. But if there is $+r$ in the Equation, and it is ſo great as that $\sqrt{GDq - ar}$ is leſs than that a Circle can reach the Parabala in any Point, when deſcribed with that Radius and the Center G; the given Equation is totally impoſſible, nor is it explicable by any Root affirmative or negative. But of this more afterwards.

Now whereas there is ſo great a Difference between the Caſes of Cubick and Biquadratick Equations, as that they cannot be comprehended together; I ſhall firſt treat of Cubicks, and then of the other. Now Cubicks may be conſtructed by an infinite Number of Circles in a given Parabola, but Biquadraticks with one only, at leaſt in this Method. And that becauſe by putting $z - e$, or any Indeterminate, equal to nothing, the Cubick Equation is reduced to a Biquadratick, having the ſame Roots with the Cubick, and another beſides which is equal to e. Whence it is that the Cubick may be conſtructed by ſo many different Circles as there are different Values of e, that is infinite. But among theſe Conſtructions that is by far the moſt eaſy, which I gave above in the laſt Paragraph. Yet there is another not much inferior to it, which ſeems more accommodated to the Diſcovery of the Number of the Roots and their Limits, and which derives its Original from taking away the ſecond Term, by ſuppoſing (as is commonly done) $x = z +$ or $-$ the third Part of the Coefficient of the ſecond Term. Here it follows:

Fig. 51. The Parabola A B Y being given, its Vertex A, its Axis A E, and its *latus rectum a*; let the Equation be reduced to the uſual Form $z^3 : bz^2 :$
$apz :$

$ap\,z : a^2 q = 0$. Then at the Diſtance $\frac{1}{2} b$ let B K be drawn parallel to the Axis, to the Right Hand if it be $+ b$, or elſe to the Left Hand, meeting the Parabola in B. And let an indefinite Perpendicular be erected D P on the middle of the ſuppoſed Line A B, meeting the Axis in the Point G. From B let fall B C perpendicular to the Axis, and let G E be always made equal to A C with its Direction downwards. From E make $EH = \frac{1}{2} p$ with its Direction upwards if it be $+ p$ in the Equation, but otherwiſe downwards, and from the Point H (or from E if p be wanting) let the Perpendicular H Q be raiſed meeting the indefinite Line D P in the Point O. Laſtly in the indefinite Line H Q make $OR = \frac{1}{2} q$, to the Right from O if it be $- q$, to the Left if $+ q$. Then a Circle deſcribed with Center R, and Radius R A, will cut the Parabola in ſo many Points as the Equation propoſed has true Roots, and theſe will be the Perpendiculars Z Y let fall from the Points of Interſection Y upon B K parallel to the Axis; of which thoſe on the Right Side of B K will be the affirmative Roots, and thoſe on the Left Side the Negative.

The Convenience of this Conſtruction conſiſts in this, becauſe it is performed by a Circle paſſing through the Vertex, juſt as if the ſecond Term were wanting. Therefore for determining the Number of Roots, it is enough to know the Properties of the Locus or the Curve-Line which diſtinguiſhes the Spaces, where if the Center of the Circle be placed that paſſes through the Vertex of the Parabola, its Circumference will cut it either in one or in three Points: That is, to define the Nature of the Curve, upon which ſhall fall the Centers of all the Circles, that paſs through the Vertex and then touch the Parabola.

But that Locus is the Paraboloid, which with the great *Wallis* we will call Semicubical, or in which the Cubes of the Ordinates are to one another as the Squares of the Portions of the Axis reſpectively. Its *latus rectum* is $\frac{27}{8}$ of the *latus rectum* of the given Parabola, and its Vertex is the Point V, A V being half the *latus rectum* of the ſame Parabola. That is, if Unity be aſſigned for the *latus rectum* of the given Parabola, $\frac{8}{27}$ of the Cube of the Ordinate will be equal to the Square of the Part of the Diameter, or the Cube of $\frac{2}{3}$ V H will be equal to the Square of H R, if R be the Center of the Circle that paſſes through the Vertex of the Parabola, and afterwards touches it. This is that Curve which firſt of all Mortals our Mr. *Neil* demonſtrated to be equal to a Right Line, who on this Account has been celebrated among the chief of Geometricians. And its Properties have been inquired into by the learned Dr. *Wallis*, at the End of his Book concerning the Ciſſoid, and *Huygens*, *Prop.* 8, 9. of the Evolution of Curve Lines, and other ingenious Men whoſe Writings the Reader may conſult. This Curve being deſcribed on each Side of the Axis of the Parabola, that is, V N L and V P X, will comprehend a Space, in which if the Center of the Circle be put, which paſſes through the Vertex A, that Circle will cut the Parabola in three other Points. But the Spaces that are more remote from the Axis will ſupply the Centers of Circles which will cut the Parabola but in one Point beſides the Vertex.

Theſe Things being rightly underſtood, we ſhall now proceed to determine the Number of the Roots. And firſt let the ſecond Term be wanting, and let the *latus rectum* be Unity, or $AV = \frac{1}{2}$. In the Conſtruction VH is $\frac{1}{2}p$, and $HR\ \frac{1}{2}q$. And as $\frac{1}{2}p$ is to be put upwards from V if it be $+p$, the Center of the Circle is always to be placed without the Space LVX, and therefore the Equation is explicable by one Root only, affirmative if it be $-q$, negative if $+q$, which Roots are inveſtigated by *Cardan's* Rules. But if it is $-p$, then $VH = \frac{1}{2}p$ muſt be taken below, and it may happen that HR may fall between the Axis and the Curve VX or VL, namely if the Cube of $2\,VH$ or $\frac{1}{3}p$ is greater than the Square of $\frac{1}{2}q$, or $\frac{1}{27}p^3$ greater than $\frac{1}{4}qq$, in which Caſe there are three Roots, two negative if it is $-q$, and one Affirmative equal to their Sum; or if it is $+q$, then two affirmative and one negative. But if $\frac{1}{27}p^3$ is leſs than $\frac{1}{4}qq$, then only one Root is found, affirmative if it is $-q$, but negative if $+q$. And theſe Things are commonly taught by thoſe who treat of this Part of Geometry.

Now let all the Terms be preſent, and firſt let this Equation be propoſed $z^3 - bz^2 + pz - q = 0$, to which we have adapted Figure 51. In the Conſtruction of this, $BC = \frac{1}{3}b$, $VG = \frac{1}{2}AC = \frac{1}{18}bb$, $VE = \frac{1}{6}bb$, $VH = \frac{1}{6}bb - \frac{1}{2}p$, $GH = \frac{1}{9}bb - \frac{1}{2}p$, or $\frac{1}{2}p - \frac{1}{9}bb$, hence $HO = \frac{1}{27}b^3 - \frac{1}{6}bp$, or $\frac{1}{6}bp - \frac{1}{27}b^3$, and HR, or the Diſtance of the Center of the Circle R from the Axis, is always the Difference between $\frac{1}{6}bp$ and $\frac{1}{27}b^3 + \frac{1}{2}q$; which if they are equal, the Center falls in the Axis; if $\frac{1}{6}bp$ is greater than $\frac{1}{27}b^3 + \frac{1}{2}q$, it falls to the Left of the Axis, but if leſs, to the Right. If the Square-root of the Cube of $\frac{2}{3}VH$, (that is, of $\frac{1}{9}bb - \frac{1}{3}p$, which we may call d,) that is, if $\sqrt{ddd}$ is greater than HR, or the Difference between $\frac{1}{27}b^3 + \frac{1}{2}q$ and $\frac{1}{6}bp$, the Center R is found within the Space HPV, which is circumſcribed by the Paraboloids VPX and VNL, and the indefinite Right Line DNP. And therefore the Circle will cut the Parabola in three Points Y, Y, Y, ſituate to the Right Hand of the Line BK, and therefore the Equation has three affirmative

Roots. But the Center being without this Space, N U P, it can be explained but by one affirmative Root. Here we may take Notice, that the Right Line D P that touches the Paraboloid V P X in the Point P, E P being $\frac{1}{27} b^3$, will cut the other V N L in the Point N, ſo that letting fall the Perpendicular N F upon the Axis, V F will be a fourth Part of E V, or $\frac{1}{24} bb$, but N F will be $\frac{1}{108} b^3$. But V W, which from the Point V being drawn perpendicular to the Axis, and meets the Line D P in W, is equal to $\frac{1}{54} b^3$, of $\frac{1}{2}$ E P.

Hence we may ſafely conclude, if in the Equation p is greater than $\frac{1}{3} bb$, or q greater than $\frac{1}{27} b^3$, there is only one Root to be had, and that affirmative. Therefore *Des Cartes*'s Rule is not true, (*pag*. 70. Edit. *Amſt*. 1659) wherein he aſſerts, that ſo many true Roots are to be had as in the Equation there are Changes of the Signs + and —; nor is it to any Purpoſe that *Schooten* in his Commentaries excuſes this Miſtake. For infinitely more Equations may be contrived of the foregoing Form, having three Changes of the Signs, which have but one Root, than which have three Roots. Alſo the fifth Propoſition of the fifth Section of our *Harriot's Ars Analytica*; as likewiſe the 18th Problem of *Vieta's Numeroſa Poteſtatum Reſolutio*, are not ſufficiently ſound, ſince from the Limitations they have there given, that ſhould belong to the whole Parallelogram P I V W, which we have now proved to belong only to the Space N V P; that is, that it ſhould afford a Center for a Circle, which cuts the Parabola in three other Points beſides the Vertex.

Now the Quantity q, or the laſt Term, when b and p are given in ſuch Conditions that p is leſs than $\frac{1}{3} bb$, is accurately limited by the foregoing Equation $\surd ddd = \frac{1}{27} b^3 + \frac{1}{2} q \backsim \frac{1}{6} bp$, when the Circle touches the Parabola. Therefore $\frac{1}{2} q$ ought to be leſs than $\frac{1}{6} bp - \frac{1}{27} b^3 + \surd ddd$. But if p is greater than $\frac{1}{4} bb$, it is neceſſary that $\frac{1}{2} q$ ſhould be greater than $\frac{1}{6} bp - \frac{1}{27} b^3 - \surd d^3$, leſt the Center ſhould fall within the little Space N V W. And with theſe Conditions the Equation is always explicable by three Roots, but otherwiſe by one only. But whether three or one, they are always affirmative, becauſe of the Poſition of the Center R to the Right Hand of the Line D P.

And this is the moſt difficult Caſe, ſo that he that well underſtands what goes before will eaſily underſtand the reſt. Now let be given the Equation $z^3 - bz^2 + pz + q = 0$. Here that three Roots may be had, it is neceſſary that the Center of the Circle may be ſomewhere within the Space P N Δ, included by the Right Lines P N, P Δ, and by the Curve of the Paraboloid N Δ. Wherefore ſince E F $= \frac{1}{8}bb$, p muſt be leſs than $\frac{1}{4}bb$. Now for the Determination of the Quantity q, making $d = \frac{1}{9}bb - \frac{1}{3}p$ as before, then $\sqrt{ddd + \frac{1}{27}bbb - \frac{1}{6}bp}$ muſt always be greater than $\frac{1}{2}q$, that the Center of the Circle may be found in the aforeſaid Space P N Δ: Which when it happens, ſuch an Equation has two affirmative Roots and one negative. But if p is greater than $\frac{1}{3}bb$, or $\frac{1}{2}q$ greater than $\sqrt{d^3 + \frac{1}{27}b^3 - \frac{1}{6}bp}$, the Equation is explicable but by one Root, and that will be negative.

Now let be propoſed the Equation $z^3 - bz^2 - pz - q = 0$. That this Equation may have three Roots, the Center of the Circle muſt be found ſomewhere in the indefinite Space, between the Right Line D P D and the Curve of the Paraboloid P X. Hence the Quantity p will be ſubject to no Limitations, but $\frac{1}{2}q$ muſt always be leſs than $\sqrt{d^3 - \frac{1}{27}b^3 - \frac{1}{6}bp}$ making $d = \frac{1}{9}bb + \frac{1}{3}p$. By this means there will be two negative Roots, and one affirmative; but otherwiſe if $\frac{1}{2}q$ is greater than $\sqrt{d^3 - \frac{1}{27}b^3 - \frac{1}{6}bp}$, one only affirmative Root will be exhibited.

In the fourth Place let the Equation be $z^3 - bz^2 - pz + q = 0$, which has two affirmative Roots and one negative, if the Center of the Circle is found in the indefinite Space between the Right Lines P Δ, P D, and the Curve of the Paraboloid Δ L; that is, ſuppoſing $d = \frac{1}{9}bb + \frac{1}{3}p$, if $\frac{1}{2}q$ is leſs than $\sqrt{d^3 + \frac{1}{27}b^3 + \frac{1}{6}bp}$; but if $\frac{1}{2}q$ is greater than this Quantity, there is only one negative Root.

Now the four remaining Equations, in which there is $+b$, as to the Limitation of the Number of Roots, do not differ from the foregoing, if the Sign of the laſt Term be changed, keeping the Sign of the third Term; but the Roots that in thoſe were affirmative will here be negative, and *vice verſa*. Thus in the Equation $z^3 - bz^2 + pz - q = 0$, either one or three are negative under the ſame Conditions, but none affirmative. So in

$z^3 + bz^2 + pz - q = 0$, two are negative and one affirmative, if p is leſs than $\frac{1}{3}bb$, and $\frac{1}{2}q$ leſs than $\sqrt{d^3} + \frac{1}{27}b^3 - \frac{1}{6}bp$, as in $z^3 - bz^2 + pz + q = 0$, two will be affirmative and one negative. But when p or q depart from the preſcribed Limits, here is only one affirmative Root which there was negative. In like manner in $z^3 + bz^2 - pz + q = 0$, either there are two affirmative and one negative Root, or one negative only. Laſtly for the ſame Reaſon in the Equation $z^3 + bz^2 - pz - q = 0$, there are two negative and one affirmative Root, or one affirmative only; as in the Equation $z^3 - bz^2 - pz + q = 0$, there are two affirmative and one negative, or only one Negative, according as $\frac{1}{2}q$ is greater or leſs than $\sqrt{d^3} + \frac{1}{27}b^3 + \frac{1}{6}bp$.

If the third Term is wanting, or $p = 0$, the Center R always falls in the Line IPEΔ; wherefore if it is $z^3 - bz^2 * - q = 0$, or $z^3 + bz^2 * + q = 0$, there can be only one Root, if 'tis $-b$ the Root is affirmative, if $+b$ negative. But if the Equation is $z^3 - bz^2 * + q = 0$, or $z^3 + bz^2 * - q = 0$, there may be two affirmative and one negative in the former, or one affirmative and two negative in the latter, the Center falling in the Line PΔ, between P and Δ, that is, if $\frac{1}{4}q$ is leſs than $\frac{1}{27}b^3$: But if it be greater, then can be only one Negative in the former, or one Affirmative in the latter.

Hitherto we have fully explained the Number of Roots in Cubic Equations; now we muſt add ſomething concerning the Quantity of thoſe Roots. Here it is firſt to be obſerved, that every Equation having three Roots may be reſolved expeditiouſly enough by means of the Table of Sines, that is, by the Triſection of an Angle. For by making $\sqrt{\frac{4}{9}bb - \frac{4}{3}p}$, or $\sqrt{4d}$ if it is $+p$ in the Equation, or $\sqrt{\frac{4}{9}bb + \frac{4}{3}p}$, if it is $-p$, the Radius of the Circle; and making that the Angle to be triſected, whoſe Sine is $\frac{\frac{1}{27}b^3 + \frac{1}{6}bp + \frac{1}{2}q}{\sqrt{d^3}}$ in the Table of Sines; when this Angle is found the Sine of a third Part of it, alſo the Sine of the third Part of its Complement to a Semicircle, and their Sum, will be known by the Table of Sines. But theſe Sines are to be drawn into the Radius $\sqrt{\frac{4}{9}bb + \frac{4}{3}p}$, and we ſhall have the Quantities (y &c, y &c, y &c, in the Figure) the Sum or Difference of which, and of $\frac{1}{3}b$, as the Caſe requires, will exhibit the true Roots of the Equation. All theſe Things are derived from what has been found by

Carteſius. Now that I may comprehend all the Caſes in as ſhort a Compaſs as poſſible. I ſay that in the firſt Form of Equations, if the Center R falls in the Space V G P, the two Sections Y, Y, fall between A and B, and therefore each of the leſſer Roots is leſs than $\frac{1}{3}b$, but the third and greater always exceeds $\frac{1}{3}b$, but is exceeded by b. Now if it falls in the Space G N V, two are greater than $\frac{1}{3}b$, but leſs than $\frac{2}{3}b$, and the third is b ſubſtracting the two others, and therefore leſs than $\frac{1}{3}b$. But admitting a Limitation of the Quantity p, the Roots are included within narrower Bounds. For the greateſt Root is leſs than $\sqrt{\frac{4}{9}bb-\frac{4}{3}p}+\frac{1}{3}b$, and greater than $\sqrt{\frac{1}{4}bb-p}+\frac{1}{2}b$. But when $\frac{1}{4}bb$ is leſs than p, that Limit becomes $\sqrt{\frac{1}{9}bb-\frac{1}{3}p}+\frac{1}{3}b$. The middle Root is always leſs than $\sqrt{\frac{1}{4}bb-p}+\frac{1}{2}b$, but greater than $\frac{1}{3}b-\sqrt{\frac{1}{9}bb-\frac{1}{3}p}$. The greateſt Root never exceeds this Limit, but vaniſhes with the Quantity q.

In the ſecond Form by the Rules preſcribed there are two affirmative and one negative Root, and the Center falling in the Space G P E, one of the Affirmatives is greater and the other leſs than $\frac{1}{3}b$. But the greater does not exceed b, and the Negative cannot be greater than $\sqrt{\frac{1}{3}bb}-\frac{1}{3}b$, and it is the Difference of b and of the Sum of the Affirmatives. But when the Center is placed in the Space E N G Δ, either of the Affirmatives is greater than $\frac{1}{3}b$, but leſs than $\sqrt{\frac{1}{3}bb}+\frac{1}{3}b$. But the Negative is always leſs than $\frac{1}{3}b$. But from p being given, the Limits become nearer, $\sqrt{\frac{1}{4}bb-p}+\frac{1}{2}b$ of the greateſt affirmative Root, than which it is always leſs, and greater than $\sqrt{\frac{1}{9}bb-\frac{1}{3}p}+\frac{1}{3}b$. Yet the other affirmative Root, which diminiſhes with the Quantity q, is leſs than this Limit. But the negative Root is always leſs than $\sqrt{\frac{4}{9}bb-\frac{4}{3}p}-\frac{1}{3}b$, and vaniſhes when the Quantity q is abſent.

In the third Form are two negative and one affirmative Root; in this, as alſo in the fourth, the Roots are not limited by the Quantity b. Now the Affirmative is always leſs than $\sqrt{\frac{4}{9}bb+\frac{4}{3}p}+\frac{1}{3}b$, but greater than $\sqrt{p+\frac{1}{4}bb}+\frac{1}{2}b$. But the greateſt of the Negatives is always greater than $\sqrt{\frac{1}{9}bb+\frac{1}{3}p}-\frac{1}{3}p$, and leſs than $\sqrt{p+\frac{1}{4}bb}-\frac{1}{2}b$. But the leſſer of the Negatives always is diminiſhed when the Quantity q is diminiſhed.

In the fourth Form the Center falls within the Space L Δ P D; if two Roots are affirmative and one negative, the greateſt of the Affirmatives cannot be greater than $\sqrt{p+\frac{1}{4}bb}+\frac{1}{2}b$, nor leſs than $\sqrt{\frac{1}{9}bb+\frac{1}{3}p}+\frac{1}{3}b$. But the leſſer Root is diminiſhed from this Limit, as the Quantity q is diminiſhed. The Negative is leſs than $\sqrt{\frac{4}{9}bb+\frac{4}{3}p}-\frac{1}{3}p$, but greater than $\sqrt{p+\frac{1}{4}bb}-\frac{1}{2}b$.

But here it muſt be noted, that the negative Roots are every where marked with the affirmative Sign, becauſe theſe are the affirmative Roots of thoſe four Equations, in which there is $+b$, and q is marked with a contrary Sign, as I obſerved before. The Demonſtration of all theſe follows from hence, that whenever the Center of the Circle R falls upon the Curve-Lines V P X or V Δ L, its Circumference touches the Parabola in a Point whoſe Diſtance from the Axis is $\sqrt{\frac{2}{3}}$ V H. But when the Center falls upon the Line D P D, one of the Roots becomes $=0$, and therefore the Cubic is reduced to a Quadratic, or to $z^2-bz+p=0$, whoſe Roots mark out the Limits where the Quantity q vaniſhes. And the leſs q is, ſo much the nearer the Roots approach to theſe Limits. It is alſo a Quadratic when the Center falls in the Axis; that is, when $\frac{1}{2}q=\frac{1}{6}bp-\frac{1}{27}b^3$, in the firſt Form, or $\frac{1}{2}q=\frac{1}{27}b^3+\frac{1}{6}bp$ in the ſecond. In the third it is impoſſible, but in the fourth when $\frac{1}{2}q=\frac{1}{27}b^3+\frac{1}{6}bp$. In which Caſe the leſſer of the affirmative Roots is $\frac{1}{3}b$, the greater $\sqrt{\frac{1}{3}b^2+p}+\frac{1}{3}b$. And the negative Root is $\sqrt{\frac{1}{3}bb+p}-\frac{1}{3}b$.

In the firſt Formula the Roots are $\frac{1}{3}b$ and $\frac{1}{3}b \pm \sqrt{\frac{1}{3}bb - p}$; in the ſecond $\frac{1}{3}b$ and $\sqrt{\frac{1}{3}bb - p} + \frac{1}{3}b$ are affirmative; but $\sqrt{\frac{1}{2}bb - p} - \frac{1}{3}b$ is negative.

And this ſeems to be ſufficient as to Cubics. But becauſe of the excellent Uſe of that Method, which finds the Roots of theſe Equations by means of the Table of Sines; I have thought proper to add an Example or two, to ſhew the Compendiouſneſs of this Praxis. Let there be propoſed this Equation $z^3 - 39z^2 + 479z - 1881 = 0$, of which the Roots z are required. We ſhall have $\sqrt{\frac{1}{9}bb - \frac{1}{3}p} = \sqrt{9\frac{1}{3}} = \sqrt{d}$, whoſe double $\sqrt{37\frac{1}{3}}$ is the Radius of the Circle; and $\frac{\frac{1}{27}b^3 + \frac{1}{2}q - \frac{1}{6}bp}{\sqrt{d^3}} = \frac{2197 + 940\frac{1}{2} - 3113\frac{1}{2}}{9\frac{1}{3}\sqrt{9\frac{1}{3}}} = \frac{24}{9\frac{1}{3}\sqrt{9\frac{1}{3}}}$ is the Tabular Sine of the Angle, that is, making the Diviſion by Help of the Logarithms, Log. 9. 9251560, to which anſwers the Angle 57° 19′ 11″ ½. A third Part of this is 19° 6′ 24″, and of the Complement 40° 53′ 36″. The Sines give Log. 9. 514933, and 9. 816011, which drawn into the Radius $\sqrt{37\frac{1}{3}}$, produce Y & and Y &, Log. 0. 301030, = 2, and Log. 0. 601059, = 4. But the third Y & is equal to their Sum, or 6. Therefore the Roots are $13 - 4 = 9$, $13 - 2 = 11$, and $13 + 6 = 19$; of all of which the foreſaid Equation is compoſed. Where it may be obſerved, that the two leſſer Roots do not exceed $\frac{1}{6}b$, or 13, becauſe in the Conſtruction the Center R falls to the Right Hand of the Axis; that is, $\frac{1}{6}bp$ is leſs than $\frac{1}{27}b^3 + \frac{1}{2}q$.

For another Example ſuppoſe $x^3 - 15x^2 - 229x - 525 = 0$, and let the Roots be ſought. We have $\sqrt{\frac{1}{9}bb + \frac{1}{3}p} = \sqrt{101\frac{1}{3}} = \sqrt{d}$, and the Radius of the Circle is $\sqrt{405\frac{1}{3}}$. Then $\frac{\frac{1}{27}b^3 + \frac{1}{6}bp + \frac{1}{2}q}{\sqrt{d^3}} = \frac{125 + 572\frac{1}{2} + 262\frac{1}{2}}{101\frac{1}{3}\sqrt{101\frac{1}{3}}} = \frac{960}{101\frac{1}{3}\sqrt{101\frac{1}{3}}} =$ Tabular Sine of an Arch, whoſe Log. 9. 9736426, and the Arch itſelf will be 70° 14′ 22″, whoſe third Part is 23° 24′ 47″ ½, and of its Complement 36° 35′ 12″ ½; whoſe Logarithmic Sines are 9. 599183, and 9. 775275; to which the Log. of $\sqrt{405\frac{1}{2}}$ being added, they become Log. 0. 903089. = 8, and Log. 1. 079181, = 12, and their Sum = 20. Hence it is concluded, that $20 \pm \frac{1}{3}b$, or 25, is equal to the affirmative Root, and $8 = \frac{1}{3}b$, or 3, and $12 - \frac{1}{3}b$, or 7, are equal to the negative Roots. Now if the Equation had been $x^3 + 15x^2 - 229x + 525 = 0$, the Roots 3 and 7 would have been affirmative, and 25 negative. But other Cubic Equations, which are explicable by one Root only, are to be reſolved by *Cardan's* Rules after the ſecond Term is taken away; nor can I perceive how it may be done by leſs Calculation. But if this Root were deſired, expreſſed by the Quantities b, p, q; I ſay in the firſt Form it would be $\frac{1}{3}b$, adding or ſubtracting the Sum or Difference of the Cubic Roots of $\sqrt{\frac{1}{4}qq - \frac{1}{108}p^2b^2 + \frac{1}{27}b^3q - \frac{1}{6}bpq + \frac{1}{27}p^3} \pm \frac{1}{27}b^3 + \frac{1}{2}q - \frac{1}{6}bp$; that is, it will be $+$ if $\frac{1}{27}b^3 + \frac{1}{2}q$ is greater than $\frac{1}{6}bp$, otherwiſe $-$. Alſo it will be the Sum as often as $\frac{1}{3}bb$ is greater than p, but if $\frac{1}{3}bb$ is leſs, then the Difference. And in the other Forms the Root is always made up of the ſame Elements, with Variations of the Signs $+$ and $-$, as any one will eaſily perceive who has a Mind to try.

By Help of the Logarithmic Table of Verſed Sines theſe Roots may be found readily enough; that is, if the Numeral Coefficients are Surds or Fractions, and the Roots are effable in Numbers, as it generally happens. Now this is the Rule. In the firſt and ſecond Formula, if $\frac{1}{3}bb$ is leſs than p, make $\frac{1}{3}p - \frac{1}{9}bb = d$, and taking for Radius the Difference between $\frac{1}{6}bp$ and $\frac{1}{27}b^3 + \frac{1}{2}q$, that is HR in the firſt, and between $\frac{1}{6}bp + \frac{1}{2}q$, and $\frac{1}{27}b^3$ in the ſecond; let the Angle be found whoſe Tangent is $d\sqrt{d}$. Then as the Coſine of this Angle to its Verſed Sine, ſo is the Difference taken for Radius to a fourth; whoſe Cubic Root by Triſection will be had for a Logarithm. Then dividing $\frac{1}{3}p - \frac{1}{9}bb$ by this Cubic Root, from the Quotient ſubtract the Diviſor, and the Remainder will be e Quantity Y &. The Sum of $\frac{1}{3}b$ and this Remainder, if the Center l on the Right Side the Axis, otherwiſe the Difference of the ſame, will the Root ſought. Now if $\frac{1}{3}bb$ is greater than p, taking HR for Ra- us, let $d\sqrt{d}$ or the Diſtance of the Paraboloid from the Axis, be the Sine of a certain Arch. Let the verſed Sine of this be multiplied by the Radius, or $\frac{1}{6}bp - \frac{1}{27}b^3 \pm \frac{1}{2}q$, and the Logarithm of the Product being triſected, its Cubic Root will be had, by which let $\frac{1}{9}bb - \frac{1}{3}p$ be divided. I ſay that the Sum of the Quotient and Diviſor in the ſame Manner added to or taken from $\frac{1}{3}b$, will exhibit the Root required. And in like manner in the third and fourth Forms, unleſs that $\frac{1}{27}b^3 + \frac{1}{6}bp \pm \frac{1}{2}q$ is to be aſſumed for the Radius, and $\frac{1}{9}bb + \frac{1}{3}p$ into $\sqrt{\frac{1}{9}bb + \frac{1}{3}b}$, or $d\sqrt{d}$ for the Sine. But theſe Precepts will be better underſtood by Examples.

Let the Cubic Equation be $z^3 - 17z^2 + 54z - 350 = 0$, and let the Root z be required. Here $\frac{1}{3}bb$ is greater than p, but q is greater than the Cube of $\frac{1}{3}b$; therefore the Equation is explicable by only one affirmative Root. Now $\frac{289}{9} - \frac{54}{3}$ is d, and $\frac{127}{9}\sqrt{\frac{127}{9}}$ is to be taken for the Sine to Radius $\frac{4913}{27} + 175 - 153$, that is $\frac{5507}{27}$. The Arch belonging to it is 15° : 3′ : 49″. The Logarithm of the verſed Sine of this is 8. 5362376, which added to the Logarithm of the Radius 2. 3095913, gives 0. 8457889, whoſe third Part 0. 2819276 is the Logarithm of the Cubic Root 1. 91394. This dividing $\frac{127}{9}$ or d, the Quotient is 7. 37281. Then the Sum of the Quotient and Diviſor adding $\frac{1}{3}b$ is the Root ſought, or 14, 9534, *&c.*

Having now done with Cubic Equations, we are to undertake Biquadratics. Theſe always have either none, or two, or four true Roots, the Determination of which depends partly on the Coefficients, and partly on the Sign and Magnitude of the given abſolute Number. In the Conſtruction of the Equation $z^4 - bz^3 + pz^2 - qz + r = 0$, make $BD = \frac{1}{4}b$, $AB = \frac{1}{16}bb$, $BK = \frac{1}{2}$, or half the *latus rectum*, $KC = 2AB = \frac{1}{8}bb$, $KE = \frac{1}{8}bb - \frac{1}{2}p$, $AE - \frac{1}{2} = \frac{3}{16}bb - \frac{1}{2}p$, $FE = \frac{1}{16}b^3 - \frac{1}{4}bp$, and $EG = \frac{1}{16}b^3 - \frac{1}{4}bp + \frac{1}{2}q$. Which being conſtructed a Fig. 49.

Circle with Center G and Radius $\sqrt{GDq - r}$ will interſect the Parabola, either in none, two, or four Points, which exhibit all the Values of z, or all the Roots, by Perpendiculars upon the Line H D. Now that there may be four, it is plain the Center of the Circle muſt be placed ſomewhere within the Space, from any Point of which three Perpendiculars can be let fall upon the Curve of the Parabola; and at the ſame Time the Radius muſt be leſs than the greateſt of thoſe Perpendiculars, and greater than the middlemoſt. Now if the Center be placed without this Space, ſo that only one Perpendicular can be let fall upon the Parabola, than which the Radius is greater; or if it be leſs than the middlemoſt of the three Perpendiculars, but greater than the leaſt of them, there can be only two Roots. But there will be none as often as the Radius $\sqrt{GDq - r}$ is leſs than the leaſt of the three, or than that one when there is but one. Now what Sort of Space this is, by what Limits it is circumſcribed, and upon what Conditions the Radius of the Circle is leſs or greater than the ſaid Perpendiculars, we muſt now inquire. And firſt it muſt be ſhewn, how a Perpendicular may be let fall upon the Parabola.

Let A B C be a Parabola, A E its Axis, A V its half *latus rectum*, and G the Point from whence the Perpendicular is to be drawn. Make G E perpendicular to the Axis, and let V E be biſected in F. And erecting the Perpendicular F H on the ſame Side of the Axis, make $FH = \frac{1}{4} GE$. I ſay, that a Circle deſcribed with Center H, and with Radius H A, will interſect the Parabola in three Points, or in one Z, to which the Right Lines G Z being drawn, will inſiſt perpendicularly upon the Curve of the Parabola. Fig. 53.

Now that there may be three ſuch Interſections, it is required that the Center of the Circle H ſhould be ſo placed, that it may be within the Space included by the Paraboloids. That is, that F H may be leſs than $\sqrt{\frac{8}{27} VFc}$, or FHq leſs than the Cube of $\frac{2}{3} VF$, and therefore $GE = 4FH$ will be leſs than $\sqrt{\frac{8}{27} VFc}$ or $4\sqrt{\frac{1}{27} VEc}$, that is, the Square of G E leſs than $\frac{16}{27} VEc$. Therefore theſe Limits coincide with the two Paraboids of the ſame Kind with thoſe which we made uſe of in Cubics, but whoſe atus *rectum* is as little again; that is, $\frac{27}{16}$ of the *latus rectum* of the Parabola, $\frac{27}{8}$ of A V. Therefore it is the very Curve by the Evolution of which he Parabola is generated, as Mr. *Huygens* has demonſtrated, and which the ine D F always touches, which is perpendicular to the Parabola in the oint D. But the Point P, in which the Right Line D F touches the Pa- Fig. 52.

raboloid; is the Center of the Circle, which being deſcribed with Radius D P coincides with the Parabola in the Point D, or is of the ſame Degree of Curvity; which is manifeſt of itſelf.

Therefore ſuch Paraboloids V X P, V N Δ, being deſcribed on each Side the Axis, it is plain unleſs the Center of the Circle is fixed within theſe Limits, it cannot cut the Parabola in more than two Points. Whence we may determine under what Conditions the Coefficients of the intermediate Terms are reſtrained in Biquadratic Equations, in order to have four Roots. And firſt it appears that p cannot be greater than $\frac{3}{8}\,b\,b$, in thoſe Forms in which it is $+\,p$, nor q greater than $\frac{1}{16}\,b^3$. But in general $\frac{1}{16}\,b^3 \mp \frac{1}{4}\,pb \mp \frac{1}{2}\,q$, that is, the Diſtance of the Center from the Axis E G, muſt be leſs than $EH = 4\sqrt{\frac{1}{27}}\,VEc$; that is, becauſe of $VE = \frac{3}{16}\,bb \mp \frac{1}{2}\,p$, leſs than $\frac{1}{4}\,bb \mp \frac{2}{3}\,p\sqrt{\frac{1}{16}\,bb \mp \frac{1}{6}\,p}$; the Signs $+$ and $-$ being left doubtful, that they may be varied according to the Exigence of any Equation; as we have ſhewn before in Cubics.

But the Limitation of the laſt Term r cannot ſo eaſily be found. And this becauſe to let fall a Perpendicular upon the Curve of a Parabola is a ſolid Problem, and which cannot be reſolved without the Solution of a Cubic Equation. Therefore firſt let the ſecond Term of the Equation be wanting, or if it be preſent let it be taken away, that the Equation may have this Form $z^4 : * : pz^2 : qz : r : = 0$. Now if it be $-\,r$, the Equation may always be explained by two or four Roots: But that there ſhould be four, the Center of the Circle muſt be had below the foreſaid Paraboloids, or that there may be $-\,p$, and qq leſs than $\frac{8}{27}\,p^3$, or than the Cube of $\frac{2}{3}\,p$. Then let there be found the Roots of this Equation, $y^3 : * : \frac{1}{2}\,py . \frac{1}{4}\,q = 0$, the ſame Signs being annexed to p and q as in the Biquadratic Equation. Now theſe Roots may eaſily be found by help of the Table of Sines. Thoſe three Values of y being found, (which are the Ordinates to the Axis of the Parabola, from the Points where the Perpendiculars fall upon the Curve, that is Z Y,) the Quantity $py^2 - 3y^4$ compoſed of the leſſer Root y will expreſs the greateſt Value of r, if it is $-\,r$; than which if r is leſs, the Equation will have four Roots, otherwiſe but two. But if it is $+\,r$, it muſt be leſs than $3y^4 - py^2$, compoſed of the middlemoſt y; for if it be greater, there can be but two Roots, at leaſt if r be leſs than $3y^4 - py^2$ compoſed of the greateſt y. But if it be greater than this, the Equation is explicable by no true Root at all. Now theſe ſame Limits may be otherwiſe marked out by the Quantity q. That is, $\frac{1}{2}qy - y^4$ in the firſt Caſe, $y^4 - \frac{1}{2}qy$ in the ſecond, and $y^4 + \frac{1}{2}qy$ in the third.

But it may happen that the two leſſer Quantities y are not far aſunder, ſo that each of the Perpendiculars may be greater than the Right Line G A, that is, when qq is greater than $\frac{4}{27}p^3$, but leſs than $\frac{8}{27}p^3$, the Center falling within the Space intercepted by the Paraboloids, Fig. 51, 52. In this Caſe if it is $+r$, there can be but two Roots, the Quantity $y^4 + \frac{1}{2}qy$ compoſed of the greateſt y being greater than r, otherwiſe none at all. But if $\frac{1}{2}qy - y^4$ compoſed of the leaſt y is greater than r having the Sign $-$, and r is greater than $\frac{1}{2}qy - y^4$ compoſed of the mean y, then there would be four Roots; and but two only if r ſhould be either greater than the former or leſs than the latter.

But in the Equation if it ſhould be $+p$, or if it is $-p$ and qq greater than $\frac{8}{27}p^3$, the Equation $y^3 * \frac{1}{2}py \cdot \frac{1}{4}q = 0$, is explicable only by one Root y, that is, one Perpendicular alone can be let fall from the Center of the Circle. Whence we may certainly conclude, that there can be had only two Roots from the given Equation, the Sum of which, if it is $-r$, increaſes with the Quantity r; but if it is $+r$, the Quantity y being found, that Quantity r muſt be leſs than $y^4 + \frac{1}{2}qr$. For if it be greater, the propoſed Equation will be abſurd and impoſſible.

It would be a tedious and unneceſſary Thing to run through all the Equations of this Sort, ſince it muſt be evident to any one that conſiders it, what Roots are negative and what affirmative; and that the Limits of theſe Roots may be had from the Quantities y being found. Yet for an Example, which may be imitated in others, let it be propoſed, to find the Limits or Conditions, under which in a Biquadratic Equation four affirmative Roots may be had. Now this will happen as often as the Center of the Circle G is placed
the Space V P K, and at the ſame Time there is $+r$, or the Radius of
Circle is leſs than G D. Whence it is plain that the Equation we are
ating of muſt be of this Form $z^4 - bz^3 + pz^2 - qz + r = 0$. But p
nnot be greater than $\frac{3}{8}bb$, nor in this Caſe $\frac{1}{4}pb$ greater than $\frac{1}{16}b^3 +$
q. Alſo it is neceſſary that $\frac{1}{4}bb - \frac{2}{3}p$ drawn into $\sqrt{\frac{1}{16}bb - \frac{1}{6}p}$
ſt be greater than $\frac{1}{6}b^3 + \frac{1}{2}q - \frac{1}{4}pb$; and from theſe Limitations

Fig. 52.

it will be certain, that the Center will be found within the Space V P K. Now that the Quantity r may be determined, we muſt firſt ſolve this Cubic Equation $y^3 : * : -\frac{3}{16}b^2 - \frac{1}{2}py = \frac{1}{32}b^3 + \frac{1}{4}q - \frac{1}{8}pb$, and the Points will be had, upon which the Perpendiculars from the Center will fall upon the Curve of the Parabola.

Now the three Values of this y being found, r muſt be leſs than $\frac{3}{256}b^4 + \frac{1}{4}bq - \frac{1}{16}b^2p + 3y^4 - \frac{3}{8}b^2y^2 + py^2$, compoſed of the middle Value of y, and greater than $\frac{3}{256}b^4 + \frac{1}{4}bq - \frac{1}{16}b^2p + 3y^4 - \frac{3}{8}b^2y^2 + py^2$, when compoſed of the leaſt y. But if r exceeds theſe Limits, there can be only two Roots. Laſtly if $\frac{3}{256}b^4 + \frac{1}{4}bq - \frac{4}{16}b^2p + 3y^4 - \frac{3}{8}b^2y^2 + py^2$, when compoſed of the greateſt y, ſhall be leſs than r, the Equation is impoſſible.

It may alſo happen that there are four affirmative Roots, when the Center G is placed in the little Space V T S, drawing R T S perpendicular upon the middle of the Line A D. This happens when p is greater than $\frac{5}{16}bb$, and $\frac{1}{4}bb - \frac{2}{3}p\sqrt{\frac{1}{16}bb - \frac{1}{6}p}$ is greater than $\frac{1}{8}pb - \frac{5}{128}b^3 - \frac{1}{2}q$. In which Caſe always two, and ſometimes three, of the Roots are greater than $\frac{1}{4}b$.

Fig. 52. But here it muſt be obſerved, that Limit produced from the leaſt y is ſometimes negative, or leſs than nothing; that is, whenever the greateſt of the three Perpendiculars is greater than G D. This may be diminiſhed to nothing, if it happens to be $+r$ from the preſcribed Limit compoſed of the middlemoſt y. And the Defect of the Limit from the leaſt y ſhews, how great $-r$ may be in the Equation, if there are three affirmative Roots in the Equation and one Negative: Which if it exceeds, there are only two Roots, one affirmative and one negative. All theſe Things are demonſtrated from hence, that the aforeſaid Limits of the Quantity r are the Differences of the Squares of the Line G D and of the Perpendiculars upon the Curve of the Parabola.

But becauſe of the intricate Reſtrictions which the Diverſity of Signs produces in theſe Equations, it is always beſt to take away the ſecond Term, and then by the Precepts already delivered to find the Number of the Roots and their Signs; eſpecially if thoſe Quantities y are not far remote from one another. Now of theſe four affirmative Roots, two are always leſs than $\frac{1}{4}b$, and two greater; that is, if D G is leſs than A G, or $\frac{1}{4}pb$ than $\frac{3}{64}b^3 + q$. But three of them are leſs than $\frac{1}{4}b$, as often as the middlemoſt Perpendicular, or that found from the middlemoſt y, is greater than A G, or $\frac{3}{8}bb$ greater than $3y^3 - py^2$ computed from the ſame middlemoſt y. But the fourth and greateſt Root is greater than the greateſt $y + \frac{1}{4}b$, and is equal to the Difference of b and the Sum of the other three Roots, and therefore is leſs than b. But I ſhall proceed no farther. Perhaps thoſe who have a greater Inſight into the Nature of the Parabola may be able to draw all theſe Concluſions more compendiouſly; but it may well be doubted, whether all theſe Quantities, b, p, q, and r, can be rightly determined, without the Solution of a Cubic Equation. For whatever can be done in this Matter by plane Equations, can only exhibit Approximations, and not the true Limits.

XX. Mr. *de Lagney* has lately given us two very compendious Rules for an Approximation to the Cubic Root; one of them rational, and the other irrational. That is, the Root of the Cube $a^3 + b$ is between $a + \frac{ab}{3a^3 + b}$ and $\sqrt{\frac{1}{4}a^2 + \frac{b}{3a}} + \frac{1}{2}a$. And the Root of the fifth Power $a^5 + b$ he thus expreſſes, $\frac{1}{2}a + \sqrt{\sqrt{\frac{1}{4}a^4 + \frac{b}{5a}} - \frac{1}{4}a^2}$. Now the aforeſaid Rules are demonſtrated from the Geneſis of the Cube and of the fifth Power. For ſuppoſing the Side of any Cube to be $a + e$, the Cube derived from thence will be $a^3 + 3a^2e + 3ae^2 + e^3$. So that if a^3 be ſuppoſed a Cubic Number which is the next leſs to a non-Cubic, e^3 will be leſs than Unity, and the Remainder or b will be equal to the other Members of the Cube $3a^2e + 3ae^2 + e^3$. And e^3 being rejected becauſe of its Smallneſs, it will be $b = a^2 e + 3ae^2$. And ſince a^2e is much greater than ae^2, $\frac{b}{3a^2}$ will not much ceed e; and putting $e = \frac{b}{3a^2}$, $\frac{b}{3a^2 + 3ae}$ (to which the Quantity e is arly equal) will be found equal to $\frac{b}{3a^2 + \frac{3ab}{3a^2}} = \frac{b}{3a^2 + \frac{b}{a}} = \frac{ab}{3a^3 + b}$ $= e$ *ferè*. So that the Side of the Cube $a^3 + b$ is equal to $a + e = a + \frac{ab}{}$

The Extraction of all Roots without any previous Reduction; by Mr. Ed. Halley. N. 210. p. 136. May, An. 1694.

$\frac{ab}{3a^3+b}$, nearly, which is the Rational Form of Mr. *de Lagney*. Now if a^3 be a Cubic Number next greater than the given Number, the Side of the Cube $a^3 - b$ by a like Way of reaſoning will be found $a - \frac{ab}{3a^3-b}$. And this Approximation to the Cubic Root, which is very eaſy and expeditious, makes but a very ſmall Error in defect, ſince e the Remainder of the Root found by this Means, will be but little leſs than the true Remainder. Now the irrational Form may likewiſe be eaſily derived after the ſame Manner. For becauſe $b = 3a^2e + 3ae^2$, or $\frac{b}{3a} = ae + ee$, and therefore $\sqrt{\frac{1}{4}a^2 - \frac{b}{3a}} = \frac{1}{2}a + e$, and $\sqrt{\frac{1}{4}a^2 - \frac{b}{3a}} + \frac{1}{2}a = a + e$ which is the Root required. And after the ſame Manner will be had $\frac{1}{2}a + \sqrt{\frac{1}{4}a^2 - \frac{b}{3a}}$ for the Side of the Cube $a^3 - b$. Now this Form approaches ſomething nearer to the true Root than the rational Form, erring in Exceſs as the other does in Defect, and ſeems to be more convenient for Practice, ſince the Reſtitution of the Calculation is nothing elſe but the continual Addition or Subtraction of the Quantity $\frac{e^3}{3a}$, as the little Supplement e becomes known. So that it may rather be wrote $\sqrt{\frac{1}{4}a^2 + \frac{b - e^3}{3a}} + \frac{1}{2}a$ in the former Caſe, and $\frac{1}{2}a + \sqrt{\frac{1}{4}a^2 + \frac{e^3 - b}{3a}}$ in the latter. Now by either Form the Figures already known in extracting the Root are at leaſt tripled, which I believe will be a very acceptable Compendium to Arithmeticians, and therefore I congratulate the Inventor upon it. But that the Uſefulneſs of theſe Rules may be the better perceived, I ſhall add an Example or two.

Example I. Let the Side of the double Cube be ſought, or make $a^3 + b = 2$. Hence $a = 1$, $b = 1$, and $\frac{b}{3a} = \frac{1}{3}$, and therefore $\frac{1}{2} + \sqrt{\frac{7}{12}}$, or 1, 26 will be found near the true Root. But the Cube of 1, 26 is 2, 000376, and therefore $0,63 + \sqrt{0,3969 - \frac{0,000376}{3,78}}$ or $0,63 + \sqrt{0,3968005291005291} = 1,259921049895$ —; which exhibits the Side of the double Cube as far as thirteen Figures with little Trouble, that is, by one Diviſion and Extraction of the Square-root; whereas had it been found by the common Way of Operation, every Arithmetician knows what Labour it muſt have coſt him. Now this Calculation may be continued at Pleaſure, by increaſing the Square by the Addition of $\frac{e^3}{3a}$: Which Correction in this Caſe brings only the Increaſe of an Unit to the fourteenth Figure.

Example II. Let the Side of a Cube be ſought, which is equal in Meaſure to a Gallon, containing 231 ſolid Inches. The next leſſer Cube is 216, whoſe Cubic Root is $6 = a$, and the Remainder $15 = b$. Therefore for the firſt Approximation we have $3 + \sqrt{9 + \frac{5}{6}} =$ to the Root. And becauſe $\sqrt{9,8333\ldots} = 3,1358\ldots$ it is plain that $6,1358 = a + e$. Now make $6,1358 = a$, and we ſhall have its Cube 231, 000853894712, and according to the Rule $3,0679 + \sqrt{9,41201041 - \frac{0,000853894712}{18,4074}}$ $= 6,135792439661958 97$, which is the Side of the given Cube very exactly, being true in the eighteenth Figure, and falling ſhort in the nineteenth; which Calculation I performed within an Hour's Time. Now this Form is deſervedly to be preferred before the rational one, which, becauſe of its large Diviſor, cannot be managed without a great deal of Trouble; whereas the Extraction of the Square-root proceeds much more eaſily, as manifold Experience has informed me.

Now the Rule for the Root of the pure Surſolid, or of the fifth Power, is of a little higher Inquiry, and yet performs the Matter much more perfectly. For it quintuples at leaſt the given Figures of the Root, nor does it require much or very operoſe Calculation. But the Author has no where given his Method of Inveſtigation or Demonſtration of it, tho' it ſeemed moſt to be wanting: Eſpecially as it is faultily printed in his Book, which may eaſily miſlead the Unſkilful. Now the fifth Power of the Side $a + e$ is made up of theſe Members following, $a^5 + 5a^4e + 10a^3e^2 + 10a^2e^3 + 5ae^4 + e^5 = a^5 + b$; whence $b = 5a^4e + 10a^3e^2 + 10a^2e^3 + 5ae^4$; (rejecting e^5 becauſe of its Smallneſs) wherefore $\frac{b}{5a} = a^3e + 2a^2e^2 + 2ae^3 + e^4$, and adding $\frac{1}{4}a^4$ on each Side, and extracting the Square-root, it will be $\sqrt{\frac{1}{4}a^4 + \frac{b}{5a}} = (\sqrt{\frac{1}{4}a^4 + a^3e + 2a^2e^2 + 2ae^3 + e^4} =) \frac{1}{2}$ $+ ae + e^2$. Then ſubtracting $\frac{1}{4}a^2$ from each Side, and extracting the quare-root again, $\sqrt{\sqrt{\frac{1}{4}a^4 + \frac{b}{5a}} - \frac{1}{4}a^2} = (\sqrt{\frac{1}{4}a^2 + ae + e^2} =)$ $a + e$, to which if you add $\frac{1}{2}a$, 'tis $a + e = \frac{1}{2}a + \sqrt{\sqrt{\frac{1}{4}a^4 + \frac{b}{5a}} - \frac{1}{4}a^2}$ Root of the Power $a^5 + b$. Now if it had been $a^5 - b$, (aſſuming a

greater than it ought to be) the Rule would have been thus, $\frac{1}{2}a + \sqrt{\sqrt{\frac{1}{4}a^4 - \frac{b}{5a}} - \frac{1}{4}aa} =$ Root of $a^5 - b$.

And this Rule approximates wonderfully, ſo that there can ſcarce be any Need of Reſtitution. And upon farther conſidering this Matter, I have fallen upon a certain general Method of Forms for any Power, which is elegant enough, and which I cannot prevail upon myſelf to conceal; ſince in the higher Powers they triplicate at leaſt the known Figures of the Root.

Now theſe Forms proceed after this Manner, as well the rational as the irrational.

$$\sqrt[2]{a^2 + b} = \tfrac{0}{1}a + \sqrt{\tfrac{1}{1}aa + \frac{b}{1a^0}}, \text{ or } a + \frac{ab}{2a^2 + \frac{1}{2}b},$$

$$\sqrt[3]{a^3 + b} = \tfrac{1}{2}a + \sqrt{\tfrac{1}{4}aa + \frac{b}{3a}}, \text{ or } a + \frac{ab}{3a^3 + \frac{2}{2}b},$$

$$\sqrt[4]{a^4 + b} = \tfrac{2}{3}a + \sqrt{\tfrac{1}{9}aa + \frac{b}{6a^2}}, \text{ or } a + \frac{ab}{4a^4 + \frac{3}{2}b},$$

$$\sqrt[5]{a^5 + b} = \tfrac{3}{4}a + \sqrt{\tfrac{1}{16}aa + \frac{b}{10a^3}}, \text{ or } a + \frac{ab}{5a^5 + \frac{4}{2}b},$$

$$\sqrt[6]{a^6 + b} = \tfrac{4}{5}a + \sqrt{\tfrac{1}{25}aa + \frac{b}{15a^4}}, \text{ or } a + \frac{ab}{6a^6 + \frac{5}{2}b}, \text{ \&c.}$$

And ſo in other higher Powers. Now if a ſhould be aſſumed greater than the Root ſought, (which would be done with Advantage as often as the Power to be reſolved is much nearer to the Power of the integer Number that is next greater, than it is to the next leſſer) changing what is to be changed, the ſame Expreſſions of the Roots ariſe.

$$\sqrt[2]{a^2 - b} = \tfrac{0}{1}a + \sqrt{\tfrac{1}{1}aa - \frac{b}{1a^0}}, \text{ or } - \frac{ab}{2a^2 - \frac{1}{2}b},$$

$$\sqrt[3]{a^2 - b} = \tfrac{1}{2}a + \sqrt{\tfrac{1}{4}aa - \frac{b}{3a}}, \text{ or } - \frac{ab}{3a^3 - \frac{2}{2}b},$$

$$\sqrt[4]{a^2 - b} = \tfrac{2}{3}a + \sqrt{\tfrac{1}{9}aa - \frac{b}{6a^2}}, \text{ or } - \frac{ab}{4a^4 - \frac{3}{2}b},$$

$$\sqrt[5]{a^2 - b} = \tfrac{3}{4}a + \sqrt{\tfrac{1}{16}aa - \frac{b}{10a^3}}, \text{ or } - \frac{ab}{5a^5 - \frac{4}{2}b},$$

$$\sqrt[6]{a^2 - b} = \tfrac{4}{5}a + \sqrt{\tfrac{1}{25}aa - \frac{b}{15a^4}}, \text{ or } - \frac{ab}{6a^6 - \frac{5}{2}b}, \text{ \&c.}$$

Now between theſe two Limits the true Root always conſiſts, being ſomething nearer the irrational than the rational. But e found according to the irrational Formula always errs in Exceſs, as the Quote reſulting from the rational always errs in Defect; therefore if it is $+ b$, the irrational gives the Root greater than the Truth, and the rational leſs. And juſt the contrary if it be $- b$. And this may ſuffice concerning the finding of the Roots of pure Powers, which for ordinary Uſes (and accurately enough) may be done more eaſily by Help of the Logarithms. But whenever the Root is to be extracted very accurately, and beyond the Reach of the Logarithms, we muſt neceſſarily have Recourſe to ſuch Methods as theſe. Beſides, as from the Diſcovery and Conſideration of theſe Forms, I have hit upon an univerſal Rule for affected Equations, which I believe will be made uſe of with good Advantage by all ſuch as are ſtudious of Geometry and Algebra; I was willing to lay open with all poſſible Perſpicuity the Foundation of this Diſcovery.

About the Year 1687 I made publick a Method then newly invented by me, which contained a neat and very eaſy general Conſtruction of affected Equations, not beyond Quadrato-quadratics; ſince which Time I have always had a very ſtrong Deſire of performing the ſame in Numbers. A little after that Mr. *Raphſon* ſeemed in a good Meaſure to have accompliſhed my Wiſhes; till Mr. *de Lagney* ſhewed me, by his little Book on this Subject, that the Thing might be ſtill performed more compendiouſly. Now my Method is this.

Let the Root of any Equation z be conceived to be compoſed of two Parts, $a +$ or $- e$, of which let a be aſſumed (by Suppoſition) as near as may be to the Quantity z, (which is convenient indeed, but not abſolutely neceſſary) and of the Quantity $a \pm e$ let there be formed all the Powers, to which z ariſes in the given Equation, to which let their Numeral Coefficients be affixed reſpectively. Then let the Power to be reſolved be ſubtracted from the Sum of the given Parts in the firſt Column, where e is not found, which is called the *Homogeneum Comparationis*, and let the Difference be $\pm b$. Then let there be found the Sum of all the Coefficients of the Side e in the ſecond Column, which let be s. Laſtly, in the third Column, let all the Coefficients be added of the Square ee, the Sum of which we may call t. Then the Root ſought z will be had in the rational Form $z = a \pm \frac{sb}{ss \pm tb}$, and in the irrational Form $z = a \pm \frac{\frac{1}{2}s \pm \sqrt{\frac{1}{4}ss \pm bt}}{t}$; which may be worth while to illuſtrate by Examples. But as a convenient Help may be proper to have at Hand a general Table, exhibiting the Geneſis all the Powers of $a \pm e$, which may be eaſily continued farther if neceſſary. ill carry it on to the ſeventh Power, becauſe few Problems go beyond t. This Table may juſtly be called a general Analytical Speculum. Now aforeſaid Powers, ariſing from the continual Multiplication of $z = a + e$, as follows with their Coefficients adjoined.

$$\begin{array}{llllllllll}
cz & = ca & +1ca^{0}e \\
dz^{2} & = da^{2} & +2da^{1}e & +1da^{0}e^{2} \\
fz^{3} & = fa^{3} & +3fa^{2}e & +3fa^{1}e^{2} & +1fa^{0}e^{3} \\
gz^{4} & = ga^{4} & +4ga^{3}e & +6ga^{2}e^{2} & +4ga^{1}e^{3} & +1ga^{0}e^{4} \\
hz^{5} & = ha^{5} & +5ha^{4}e & +10ha^{3}e^{2} & +10ha^{2}e^{3} & +5ha^{1}e^{4} & +1ha^{0}e^{5} \\
kz^{6} & = ka^{6} & +6ka^{5}e & +15ka^{4}e^{2} & +20ka^{3}e^{3} & +15ka^{2}e^{4} & +6ka^{1}e^{5} & +1ka^{0}e^{6} \\
lz^{7} & = la^{7} & +7la^{6}e & +21la^{5}e^{2} & +35la^{4}e^{3} & +35la^{3}e^{4} & +21la^{2}e^{5} & +7la^{1}e^{6} & +1la^{0}e^{7} \\
 & & s & t & u & w & x & y
\end{array}$$

Now if it ſhould be $a - e = z$, the Table would be compoſed of the ſame Parts, but only the odd Powers of e muſt be negative, as e, e^3, e^5, &c. and the even Powers e^2, e^4, e^6, &c. muſt be ſtill affirmative. Let the Sum of the Coëfficients of the Side e be denoted by s, of the Square e^2 by t, of the Cube e^3 by u, of the Biquadrate e^4 by w, of the Surſolid e^5 by x, of the Cubo-cube e^6 be y; and ſo on. And as e is ſuppoſed to be but a ſmall Part of the Root required, all the Powers of e become much leſs than the like Powers of a, and therefore for the firſt Poſition the higher Powers may be rejected, as has been ſhewn in pure Powers; and a new Equation being formed, we ſhall have, as ſaid before, $\pm b = \pm se \pm te^2$. For the better underſtanding of which take the following Examples:

Example I. Let the Equation propoſed be $z^4 - 3z^2 + 75z = 10000$. For the firſt Suppoſition make $a = 10$, and the following Equation will ariſe.

$$\begin{array}{rlllll}
z^{4} = & +a^{4} & \pm 4a^{3}e & +6a^{2}e^{2} & \pm 4ae^{3} & +e^{4} \\
-dz^{2} = & -da^{2} & \mp 2dae & -de^{2} \\
+cz = & +ca & \pm ce \\
 \\
= & +10000 & \pm 400e & +600e^{2} & \pm 40e^{3} & +e^{4} \\
 & -300 & \mp 60e & -3e^{2} \\
 & +750 & \pm 75e \\
 & -10000 \\
\hline
 & +450 & \pm 415e & +597e^{2} & \pm 40e^{3} & +e^{4} = 0, \\
 & & s & t & u
\end{array}$$

The Signs $\pm$ are left in Doubt in reſpect of e and e^3, till it is known whether e is negative or affirmative, which may admit of ſome Difficulty, ſince in Equations that have many Roots, the *Homogeneum Comparationis* (as it is called) may be increaſed by diminiſhing the Quantity a, and contrary-wiſe may be diminiſhed when it is increaſed. But the Sign of e is determined by the Sign of the Quantity b. For the Reſolvend being taken away from the *Homogeneum* formed by a, the Sign of se, and therefore of the Parts prevailing in its Compoſition, will always be contrary to the Sign of the Difference b. Whence it will appear whether it ſhould be $-e$ or $+e$, or whether a is greater or leſs than the true aſſumed Root. But e is always equal to $\frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - bt}}{t}$, whenever b and t have the ſame Sign. But when they are connected with different Signs, then the ſame e is $\frac{\sqrt{\frac{1}{4}ss + bt} - \frac{1}{4}s}{t}$. Now after it is found that it muſt be $-e$, in the affirmative Parts of the Equation let e, e^3, e^5, *&c.* be made negative, and in the negative let them be made affirmative, that is, let them all be wrote with a contrary Sign. But if it be $+e$, let their own Signs continue. Now in our Example we have 10450 inſtead of the Reſolvend 10000, or $b = +450$; whence it appears that a was taken above the true Root, and therefore it muſt be $-e$. Hence the Equation becomes $10450 - 4015e + 597e^2 - 40e^3 + e^4 = 10000$, that is, $450 - 4015e + 597e^2 = 0$. And therefore $450 = 4015e - 597e^2$, or $b = se - te^2$, whoſe Root e is $\frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - bt}}{t}$, or if you had rather $\frac{s}{2t} - \sqrt{\frac{ss}{4tt} - \frac{b}{t}}$; that is, in the preſent Caſe $e = \frac{2007\frac{1}{2} - \sqrt{3761406\frac{1}{4}}}{597}$, whence ariſes the Root ſought near the Truth $z = 9,886$. Then I ſubſtitute this for a ſecond Hypotheſis, and there ariſes $a + e = z = 9,8862603936495\ldots$ very accurately, hardly exceeding the Truth by 2 in the laſt Figure; that is, when $\frac{\sqrt{\frac{1}{4}ss + bt} - \frac{1}{2}s}{t} = e$. And even this, if there were Occaſion, might be verified much farther, by ſubtracting $\frac{\frac{1}{2}ue^3 + \frac{1}{4}e^4}{\sqrt{\frac{1}{4}ss + tb}}$ if it is $+e$, or adding $\frac{\frac{1}{2}ue^3 - \frac{1}{2}e^4}{\sqrt{\frac{1}{4}ss - tb}}$ if it be $-e$, from or to the Root before found: The Compendium of which is ſo much the more to be valued, that ſometimes from the firſt Suppoſition alone, but always from the ſecond, you may continue the Calculation as far as you pleaſe, by keeping to the ſame Coefficients. But the foregoing Equation has alſo a negative Root, which is $z = -10,26\ldots$ which any one that pleaſes may purſue.

Example II. Let $z^3 - 17z^2 + 54z = 350$, and make $a = 10$. Then according to the Rule.

$$\begin{array}{rcl} z^3 &=& a^3 + 3a^2e + 3ae^2 + e^3 \\ -dz^2 &=& da^2 - 2dae - de^2 \\ +cz &=& ca + ce \end{array}$$

$$\begin{array}{lrrrr} & b & s & t & \\ \text{That is,} & +1000 & +300e & +30e^2 & +e^3 \\ & -1700 & -340e & -17e^2 & \\ & +540 & +54e & & \\ & -350 & & & \\ \hline \text{Or,} & -510 & +14e & +13e^2 & +e^3 = 0. \end{array}$$

Now ſince there is -510, it is plain that a was aſſumed leſs than the Truth, and therefore e is affirmative. And from $510 = 14e + 13e^2$, there ariſes $\frac{\sqrt{bt + \frac{1}{4}ss} - \frac{1}{2}s}{t} = e = \frac{\sqrt{6679} - 7}{13}$; whence 'tis $z = 15{,}7\ldots$ which is too much, becauſe a was not taken near enough. Therefore, ſecondly, let us ſuppoſe $a = 15$, and by the ſame Way of arguing we ſhall have $e = \frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - tb}}{t} = \frac{109\frac{1}{2} - \sqrt{11710\frac{1}{4}}}{28}$; and therefore $z = 14{,}954068$. Now if we would renew the Calculation a third Time, we ſhould find the Root true to 25 Figures. But if we are contented with fewer, by writing $tb \pm te^3$ inſtead of tb, or by adding or ſubtracting $\frac{\frac{1}{2}e^3}{\sqrt{\frac{1}{4}ss + tb}}$ to or from the Root before found, we ſhall ſoon arrive at our Purpoſe. Now the propoſed Equation cannot be explained by any other Root, becauſe the Power to be reſolved 350 is greater than the Cube of $\frac{17}{3}$, or $\frac{1}{3}d$.

Example III. Let it be that Equation which the learned *Wallis*, in Chap. 62. of his Algebra, makes uſe of in the Reſolution of a moſt difficult Arithmetical Problem, in which he has attained the Root very accurately by the Method of *Vieta*. And the abovementioned Mr. *Raphſon*, Pag. 25, 26. brings the ſame Equation as an Example of his Method. The Equation is $z^4 - 80z^3 + 1998z^2 - 14937z + 5000 = 0$. Now this Equation is of ſuch a Form, as that it may have ſeveral affirmative Roots, and what increaſes the Difficulty, the Coefficients are very great in reſpect of the given Reſolvend. Now that it may be managed the better, let it be divided, and placed according to the known Rules of Punctuation, $-z^4 + 8z^3 - 20z^2 + 15z = 0{,}5$; where z is but one tenth Part of z in the Equation propoſed. Then for the firſt Suppoſition let us make $a = 1$. Wherefore $+2 - 5e - 2e^2 + 4e^3 - e^4 - 0{,}5 = 0$. That is, $1\frac{1}{2} = 5e +$

$2e^2$, and hence $\frac{\sqrt{\frac{1}{4}ss+bt}-\frac{1}{2}s}{t}=e=\frac{\sqrt{37-5}}{4}$, and therefore $z=1{,}27$. Hence it apppears, that 12,7 is the Root of the propoſed Equation pretty near the Truth. Now, in the ſecond Place, let it be ſuppoſed, that $z=12{,}7$, and according to the Table of Powers,

$$\begin{array}{llllll}
-\ 26014{,}4641 & -\ 8193{,}532\,e & -\ 967{,}74\,e^2 & -\ 50{,}8\,e^3 & -\ e^4 & \\
+\ 163870{,}640. & +\ 38709{,}60.\,e & +\ 3048{,}..\,e^2 & +\ 80{,}.\,e^3 & & \\
-\ 322257{,}42.. & -\ 50749{,}2..\,e & -\ 1998{,}..\,e^2 & & & \\
+\ 189699{,}9... & +\ 14937{,}...\,e & & & & \\
-\ 5000{,}.... & & & & & \\
\hline
+\ \underset{b}{298{,}6559} & -\ \underset{s}{5296{,}132}\,e & +\ \underset{t}{82{,}26}\,e^2 & +\ \underset{u}{29{,}2}\,e^3 & -\ e^4 & =0
\end{array}$$

Therefore $-298{,}6559=-5296{,}132\,e+82{,}26\,e^2$, whoſe Root e according to Rule is $\frac{\frac{1}{2}s-\sqrt{\frac{1}{4}ss-bt}}{t}=\frac{2648{,}066-\sqrt{6987686{,}106022}}{82{,}26}=0{,}05644080331\ldots=e$ leſs than the Truth. Now that it may be corrected, $\frac{\frac{1}{2}ue^3-e^4}{\sqrt{\frac{1}{4}ss-bt}}$ or $\frac{0{,}0026201\ldots}{2643{,}423\ldots}$ becomes 0,00000099117, and therefore e corrected will be 0,05644179448. Now if more Figures of the Root be wanted, of e corrected let there be formed $tue^3-te^4=0{,}43105602423\ldots$, then $\frac{\frac{1}{2}s-\sqrt{\frac{1}{4}ss-bt-tue^3+te^4}}{t}$, or $\frac{2648{,}066-\sqrt{6987685{,}67496597577\ldots}}{82{,}26}=0{,}0564417944807440\,2=e$. Whence $a+e=z$, or the Root very accurately comes out 12,75644179448074402, ſuch as was found by Dr. *Wallis* in the Place above quoted. Here it may be obſerved, that the renewing of the Calculus always triplicates the true Figures in the aſſumed a, which the firſt Correction, or $\frac{\frac{1}{2}ue^3-\frac{1}{2}e^4}{\sqrt{\frac{1}{4}ss-bt}}$, makes quintuple, which Operation is eaſily performed by the Logarithms. The other Correction after the firſt alſo adds a double Number of Figures, ſo that in the whole it makes the aſſumed Figures ſevenfold. But the firſt generally is abundantly ſufficient for all the Uſes of Arithmetiek. But what is here ſaid about the Number of Figures ghtly aſſumed in the Root, I would have ſo underſtood, that when a is ſtant from the true Root not above a tenth Part, the firſt Figure may be ghtly aſſumed; if within an hundredth Part, the two firſt Figures; if hin a thouſandth Part, the three firſt Figures may be true. Then when aged according to our Rule, the true Figures will immediately become ne.

It remains that I ſhould add a Word or two concerning our rational Form, $e = \frac{sb}{ss \pm tb}$, which will ſeem expeditious enough, and not much inferior to the former, ſince it is able to triplicate the given Figures. For an Equation being formed of $a \mp e = z$, as before, it will ſoon appear whether the aſſumed a be greater or leſs than the Truth, ſince se muſt always have a Sign contrary to the Sign of the Difference of the Reſolvend, and of its *Homogeneum* produced of a. Then ſuppoſing that $\pm b \mp se \pm te = 0$, the Diviſor becomes $ss - tb$ as often as b and t have the ſame Sign. But it becomes $ss + bt$ if their Signs are different. But it ſeems more accommodated to Practice if the Theorem were written $e = \frac{b}{s \pm \frac{tb}{s}}$; for then the Thing would be performed by one Multiplication and two Diviſions, which otherwiſe would require three Multiplications and one Diviſion. Let us alſo take an Example of this Method from the Root of the foregoing Equation 12, 7 ... in which $\underset{+b}{298,6559} \underset{-s}{-5296,132\,e} \underset{+t}{+82,26\,e^2} \underset{+u}{+29,2\,e^3} - e^4 = 0$, and therefore $\frac{b}{s - \frac{tb}{s}} = e$. That is, let it be s to t ſo b to $\frac{tb}{s} = 5296,132)$ 298,6559 into 82,26 (4,63875. Wherefore the Diviſor becomes $s = \frac{tb}{s} = 5291,49325 \ldots)$ 298,6559 (0,056441 ... $= e$, that is, five true Figures added to the aſſumed Root. But this Formula cannot be corrected as the foregoing irrational one; ſo that if more Figures of the Root are deſired, it is better to repeat the Calculation by making a new Aſſumption; and the new Quotient, by triplicating the known Figures in the Root, will abundantly ſatisfy even the moſt ſcrupulous Computer.

A Method of Raiſing an Infinite Multinomial to any given Power; by M. de Moivre. N. 230. *p.* 619. Jul. An. 1697.

XXI. *Theorem.*] $\overline{az + bz^2 + cz^3 + dz^4 + ez^5 + fz^6 + gz^7 + bz^8 + iz^9 \ \&c.}|^m = a^m z^m + \frac{m}{1} a^{m-1} b z^{m+1}$

$$+ \frac{m}{1} \times \frac{m-1}{2} a^{m-2} b^2 z^{m+2} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} a^{m-3} b^3 z^{m+3}$$

$$+ \frac{m}{1} a^{m-1} e \qquad\qquad + \frac{m}{1} \times \frac{m-1}{1} a^{m-2} bc$$

$$\times \frac{m}{1} a^{m-1} d$$

$+ \frac{m}{1}$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\,a^{m-4}\,b^{4}\,z^{m+4}$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}\,a^{m-3}\,b^{2}\,c$$

$$+\frac{m}{1}\times\frac{m-1}{1}\,a^{m-2}\,b\,d$$

$$+\frac{m}{1}\times\frac{m-1}{2}\,a^{m-2}\,c^{2}$$

$$+\frac{m}{1}\,a^{m-1}\,e$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{5}\,a^{m-5}\,b^{5}\,z^{m+1}$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{1}\,a^{m-4}\,b^{3}\,c$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}\,a^{m-3}\,b^{2}\,d$$

$$+\frac{m}{1}\times\frac{m-1}{1}\times\frac{m-2}{2}\,a^{m-3}\,b\,c^{2}$$

$$+\frac{m}{1}\times\frac{m-2}{1}\,a^{m-2}\,b\,c$$

$$+\frac{m}{1}\times\frac{m-1}{1}\,a^{m-2}\,c\,d$$

$$+\frac{m}{1}\,a^{m-1}\,f$$

$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{5}\times\frac{m-5}{6}a^{m-6}b^6z^{m+6}$$
$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{1}a^{m-5}b^4c$$
$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{1}a^{m-4}b^3d$$
$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}\times\frac{m-3}{2}a^{m-4}b^2c^2$$
$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}a^{m-3}b^2e$$
$$+\frac{m}{1}\times\frac{m-1}{1}\times\frac{m-2}{1}a^{m-3}bcd$$
$$+\frac{m-1}{1}\times\frac{m-1}{1}a^{m-2}bf$$
$$+\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}a^{m-3}c^3$$
$$+\frac{m}{1}\times\frac{m-1}{1}a^{m-2}ce$$
$$+\frac{m}{1}\times\frac{m-1}{2}a^{m-2}d^2$$
$$+\frac{m}{1}a^{m-1}g, \text{ \&c.}$$

I ſuppoſe that the Infinite Number Multinomial is $az+bzz+cz^3+dz^4+ez^5$, *&c.* m is the Index of the Power, to which this Multinomial ought to be raiſed; or, if you will, 'tis the Index of the Root which is to be extracted: I ſay, That this Power or Root of the Multinomial is ſuch a Series as I have expreſſed.

For the underſtanding of it, it is only neceſſary to conſider all the Terms by which the ſame Power of z is multiplied; in order thereto, I diſtinguiſh two things in each of theſe Terms; 1°. The Product of certain Powers of the Quantities, a, b, c, d, *&c.* 2°. The Unciæ (as *Oughtred* calls them) prefixed to theſe Products. To find all the Products belonging to the ſame Power of z, to that Product, for Inſtance, whoſe Index is $m+r$ (where r may denote any Integer Number) I divide theſe Products into ſeveral Claſſes; thoſe which immediately after ſome certain Power of a (by which all theſe Products begin) have b, I call Products of the firſt Claſſis; for Example, $a^{m-4}b^3e$ is the Product of the firſt Claſſis, becauſe b immediately follows a^{m-4}; thoſe which immediately after ſome Power of a have c, I call Products of the ſecond Claſſis, ſo $a^{m-3}ccd$, is a Product of the ſecond Claſſis; thoſe which immediately after ſome Power of a have d, I call Products of the third Claſſis; and ſo of the reſt.

This being done, I multiply all the Products belonging to z^{m+r-1} (which precedes immediately z^{m+r}) by b, and divide them all by a; 2°. I multiply by c, and divide by a, all the Products belonging to z^{m+r-2}, except thoſe of the firſt Claſſis; 3°. I multiply by d and divide by a, all the Products belonging to z^{m+r-3}, except thoſe of the firſt and ſecond Claſſis: 4°. I multiply by e and divide by a, all the Terms belonging to z^{m+r-4}, except thoſe of the firſt, ſecond, and third Claſſis, and ſo on, till I meet twice with the ſame Term. Laſtly, I add to all theſe Terms the Product of a^{m-1} into the Letter, whoſe Exponent is $r+1$.

Here I muſt take notice, that by the Exponent of a Letter, I mean the Number which expreſſes what Place the Letter has in the Alphabet: So 3 is the Exponent of the Letter c, becauſe the Letter c is the 3d in the Alphabet.

It is evident by this Rule, you may eaſily find all the Products belonging to the ſeveral Powers of z, if you have but the Product belonging to z^m, viz. a^m.

To find the Unciæ which ought to be prefixed to every Product, I conſider the Sum of Units contained in the Indices of the Letters which compoſe it (the Index of a excepted) I write as many Terms of the Series $m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3}$, &c. as there are Units in the Sum of theſe Indices; this Series is to be the Numerator of a Fraction whoſe Denominator in the Product of the ſeveral Series $1 \times 2 \times 3 \times 4 \times 5$, *&c.* $1 \times 2 \times 3 \times 4 \times 5$, *&c.* $1 \times 2 \times 3 \times 4 \times 5 \times 6$, *&c.* the firſt of which contains as many Terms as there are Units in the Index of b, the ſecond as many as there are Units in the Index of c, the third as many as there are Units in the Index of d, the fourth as many as there are Units in the Index of e, &c.

Demonſtration.] To raiſe the Series $az + bzz + cz^3 + dz^4$, &c. to any Power whatſoever, write ſo many Series equal to it, as there are Units in the Index of the Power demanded. Now it is evident that when theſe Series are alſo multiplied, there are ſeveral Products in which there is the ſame Power of z; thus if the Series $az + bzz + cz^3 + dz^4$, &c. is raiſed to its Cube, you have the Products $b^3 z^5$, $abcz$, $aadz^5$, in which you find the ſame Power z^5. Therefore let us conſider what is the Condition that can make ſome Products to contain the ſame Power of z; the firſt thing that will appear in relation to it, is, that in any Product whatſoever, the Index of z is the Sum of the particular Indices of z in the multiplying Terms (this follows from the Nature of the Indices): thus $b^3 z^6$ is the Product of bz^2, bz^2, bz^2, and the Sum of the Indices in the multiplying Terms, is $2 + 2 + 2 = 6$; $abcz^6$ is the Product of az, bzz, cz^3, and the Sum of the Indices of z in the multiplying Terms is $1 + 2 + 3 = 6$; $aadz^6$ is the Product of az, az, dz^4, and the Sum of the Indices of z in the multiplying Terms is $1 + 1 + 4 = 6$. The next thing that appears, is, that the Index of z in the multiplying Terms is the ſame with the Exponent of the Letter to which z is joined: From which two Conſiderations it follows, that, to have all the Products belonging to a certain Power of z, you muſt find all the Products where the Sum of the Exponents of the Letters which compoſe them, ſhall always be the ſame with the Index of that Power. Now this is the Method I uſe, to find eaſily all the Products belonging to the ſame Power of z, let $m + r$ be the Index of that Power, I conſider that the Sum of the Exponents of the Letters which compoſe the Products, muſt exceed by 1, thoſe which belong to z^{m+r-1}: Now becauſe the Exceſs of the Exponent of the Letter b, above the Exponent of the Letter a, is 1, it follows, that if each of the Products belonging to z^{m+r-1} is multiplied by b, and divided by a, you will have Products, the Sum of whoſe Exponents will be $m + r$; likewiſe the Sum of the Exponent of the Letters which compoſe the Products belonging to z^{m+r} exceeds by 2 the Sum of the Exponents of the Letters which compoſe the Products belonging to z^{m+r-1}: Now becauſe the Exponent of the Letter a is leſs by 2 than the Exponent of the Letter c, it follows, that if each Product belonging to z^{m+r-1} is multiplied by c, and divided by a, you will have other Products, the Sum of whoſe Exponents is ſtill $m + r$: Now if all the Products belonging to z^{m+r-2} were multiplied by c and divided by a, you would have ſome Products that would be the ſame as ſome of them found before; therefore you muſt except out of them thoſe that I have called Products of the firſt Claſſis. What I have ſaid ſhews why all the Products belonging to z^{m+r-3}, except thoſe of the firſt and ſecond Claſſis, muſt be multiplied by d, and divided by a. Laſtly, you ſee the Reaſon why to all theſe Products is added the Product of a^{m-1} by the Letter whoſe Exponent is $r + 1$; 'Tis becauſe the Sum of the Exponents is ſtill $m + r$.

As for what relates to the Unciæ; obſerve, that when you multiply $az +$, $bzz + cz^3 + dz^4$, &c. by $az + bzz + cz^3 + dz^4$, &c. each Letter, a, b, c, d, &c. of the ſecond Series, is multiplied by each of the Letters a; b, c, d, &c. of the firſt Series. Thus the Letter a of the ſecond Series is multiplied by the Letter b of the firſt Series, and the Letter b of the ſecond Series is multiplied by the Letter a of the firſt; therefore you may have the two Planes, ab, ab, or $2ab$; for the ſame Reaſon you have $2ac$, $2ad$, &c. Therefore you muſt prefix to each Plane of thoſe that compoſe the Square of the Infinite Series $az + bz + cz^3$, &c. the Number which expreſſes how many Ways the Letters of each Plane may be changed; likewiſe if you multiply the Product of the two preceding Series by $az + bzz + cz^3$, each Letter, a, b, c, d, of the third Series, is multiplied by each of the Planes formed by the Product of the firſt and ſecond Series: Thus the Letter a is multiplied by the Planes bc and cb; the Letter b is multiplied by ac and ca; the Letter c is multiplied by ab and ba; therefore you have the ſix Solids, abc, acb, bac, bca, cab, cba, or ſix abc: Therefore you muſt prefix to each Solid whereof the Cube of the Infinite Series is compoſed, the Number which expreſſes how many Ways the Letters of each Solid may be changed; and, generally, you muſt prefix to any Product, whereof any Power of the Infinite Series $az + bzz + cz^3$, &c. is compoſed, the Number which expreſſes how many ways the Letters of each Product may be changed.

Now to find how many Ways the Letters of any Product, for inſtance, $a^{m-n} h^h c^p d^r$, may be changed; this is the Rule which is commonly given: Write as many Terms of the Series $1 \times 2 \times 3 \times 4 \times 5$, *&c.* as there are Units in the Sum of the Indices, *viz.* $m - n + h + p + r$; let this Series be the Numerator of a Fraction, whoſe Denominator ſhall be the Product of the Series, $1 \times 2 \times 3 \times 4 \times 5$, *&c.* $1 \times 2 \times 3 \times 4 \times 5$, *&c.* $1 \times 2 \times 3 \times 4 \times 5 \times 6$, *&c.* $1 \times 2 \times 3 \times 4 \times 5$, *&c.* whereof the firſt is to contain as many Terms as there are Units in the firſt Index $m - n$; the ſecond as many as there are Units in the ſecond Index h; the third as many as there are Units in the third Index p; the fourth as many as there are Units in the fourth Index r. But the Numerator and Denominator of this Fraction have a common Diviſor, *viz.* the Series of $1 \times 2 \times 3 \times 4 \times 5$, *&c.* continued to ſo many Terms as there are Units in the firſt Index $m - n$; therefore let both this Numerator and Denominator be divided by this common Diviſor, then this new Numerator will begin with $m - n + 1$, whereas the other began with 1, and will contain ſo many Terms as there are Units in $h + p + r$, that is, ſo many as there are Units in the Sum of all the Indices excepting the firſt: As for the new Denominator, it will be the Product of three Series only; that is, of ſo many as there are Indices, excepting the firſt. But if it happens withal, that n be equal to $h + p + r$, as it always happens in our *Theorem*, then the Numerator beginning by $m - n + 1$, and being continued to ſo many Terms as there are Units in $h + p + r$ or n, the laſt Term will be m neceſſarily: So if you invert the Series, and make that the firſt Term which was the laſt, the Numerator will be $m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3}$, *&c.* continued to ſo many Terms as there are Units in the *Sum* of the Indices of each Product, excepting the firſt Index. And whatever is here ſaid of Powers whoſe Index is an Integer, may be adapted to Roots or Powers whoſe Index is a Fraction.

XXII. *Theorem.*] If $az + hzz + cz^3 + dz^4 + ez^5 + fz^5$, *&c.* $= gy + hyy + iy^3 + ky^4 + ly^5 + my^6$, *&c.* then will z be $= \frac{g}{a}y + \frac{h - bAA}{a}y^2$, $+ \frac{i - 2bAB - cA^3}{a}y^3 + \frac{k - bBB - 2bAC - 3cAAB - dA^4}{a}y^4 + \frac{l - 2bBC - 2bAD - 3cABB - 3cAAC - dA^3B - eA^5}{a}y^5 + \frac{m - 2bBD - bCC - 2bAE - cB^3 - ABC - 3cAAD - 6dAABB - 4dAC - 5eA^4B - fA^6}{a}y^5$, &c.

The Extraction of a Root of an Infinite Equation; by Mr. Abr. de Moivre. N. 240. *p.* 190. May, An. 1698.

For the underſtanding of this Series, and in order to continue it as far as we pleaſe, it is to be obſerved, 1. That every Capital Letter is equal to the Coefficient of each preceding Term; thus the Letter B is equal to the Coefficient $\frac{h - bAA}{a}$. 2. That the Denominator of each Coefficient is always a. 3. That the firſt Member of each Numerator is always a Coefficient of the Series $gy + hyy + iy^3$, &c. *viz.* the firſt Numerator begins with the firſt Coefficient g, the ſecond Numerator with the ſecond Coefficient h, and ſo on. 4. That in every Member after the firſt, the Sum of the Exponents of the Capital Letters is always equal to the Index of the Power to which this Member belongs: Thus, conſidering the Coefficient $\frac{k - bBB - 2bAC - 3cAAB - dA^4}{a}$, which belongs to the Power^4, we ſhall ſee that in every Member bBB, $2bAC$, $3cAAB$, dA^4, the Sum of the Exponents of the Capital Letters is 4, (where I muſt take notice, that by the Exponent of a Letter, I mean the Number which expreſſes what place it has in the Alphabet; thus 4 is the Exponent of the Letter D) hence I derive this Rule for finding the Capital Letters of all the Members that belong to any Power; Combine the Capital Letters as often as you can make the Sum of their Exponents equal to the Index of the Power to which they belong. 5. That the Exponents of the ſmall Letters, which are written before the Capitals, expreſs how many Capitals there are in each Member. 6. That the Numerical Figures or Unciæ that occur in theſe Members, expreſs the Number of Permutations which the Capital Letters of every Member are capable of.

For the *Demonſtration* of this, Suppoſe $z = Ay + Byy + Cy^3 + Dy^4$, &c. Subſtitute this Series in the room of z, and the Powers of this Series in the room of the Powers of z, there will ariſe a new Series: Then take the Coefficients which belong to the ſeveral Powers of y in this new Series, and make them equal to the correſponding Coefficients of the Series $gy + hyy + iy^3$, &c. and the Coefficients A, B, C, D, &c. will be found ſuch as I have determined them.

But if any one deſires to be ſatisfied, that the Law by which the Coefficients are formed will always hold, I'll deſire them to have recourſe to the Sect. XXI. *Theorem* I have given for *Raiſing an Infinite Series to any Power*, or extracting any Root of the ſame; for if they make uſe of it, for taking ſucceſſively the Powers of $Ay + Byy + Cy^3$, &c. they will ſee that it muſt of neceſſity be ſo. I might have made the *Theorem* I give here much more general than it is; for I might have ſuppoſed, $az^m + bz^{m+1} + cz^{m+2} + dz^{m+3}$, &c. $=$ $gy^m + hy^{m+1} + iy^{m+2}$, &c. then all the Powers of the Series $Ay + Byy + Cy^3$, &c. deſigned by the univerſal Indices, muſt have been taken ſucceſſively; but thoſe who will pleaſe to try this, may eaſily do it, by means of the *Theorem* for raiſing an Infinite Series to any Power, &c.

This *Theorem* may be applied to what is called the Reverſion of Series; ſuch as finding the Number from its Logarithm given; the Sign from the Arch; the Ordinate of an Ellipſe from an Area given to be cut from any Point in the Axis: But to make a particular Application of it, I will ſuppoſe we have this Problem to ſolve; *viz.* *The Chord of an Arc being given, to find the*

the Chord of another Arc, that ſhall be to the firſt as n *to* 1. Let y be the Chord given, z the Chord required; now the Arc belonging to the Chord y is $y + \frac{y^3}{6dd} + \frac{3y^5}{40d^4} + \frac{5y^7}{112d^6}$, &c. And the Arc belonging to the Chord z is $z + \frac{z^3}{6dd} + \frac{3z^5}{40d^4} + \frac{5z^7}{112d^6}$, &c. The firſt of theſe Arcs is to the ſecond as 1 to n; therefore multiplying the Extremes and Means together, we ſhall have this Equation: $z + \frac{z^3}{6dd} + \frac{3z^5}{40d^4} + \frac{5z^7}{112d^6}$, &c. $= ny + \frac{ny^3}{6dd} + \frac{3ny^5}{40d^4} + \frac{5ny^7}{112d^6}$, &c.

Compare theſe two Series with the two Series of the Theorem, and you will find $a = 1$, $b = 0$, $c = \frac{1}{6dd}$, $d = 0$, $e = \frac{3}{40d^4}$, $f = 0$, &c. $g = n$, $h = 0$, $i = \frac{n}{6dd}$, $k = 0$, $l = \frac{3n}{40d^4}$, $m = 0$, &c. Hence z will be $= ny + \frac{n - n^3}{6dd} y^3$, &c. or $ny + \frac{1 - nn}{2 \times 3dd} yy A$, &c. ſuppoſing A to denote the whole preceding Term; which will be the ſame Series as Mr. *Newton* has firſt found.

By the ſame Method this general *Problem* may be ſolved: *The Abſciſs correſponding to a certain Area in any Curve being given, to find the Abſciſs whoſe correſponding Area ſhall be to the firſt in a given Ratio.*

The Logarithmic Series might alſo be found without borrowing any other Idea, than that Logarithms are the Indices of Powers: Let the Number, whoſe Logarithm we enquire, be $1 + z$; ſuppoſe its Log. to be $az + bzz + cz^3$, &c. Let there be another Number $1 + y$; therefore its Logarithm will be $ay + byy + cy^3$, &c. Now if $1 + z = \overline{1+y}^{n}$, it follows that $az + bzz + cz^3$, &c. : $ay + byy + cy^3$, &c. :: n : 1; that is, $az + bzz + cz^3$, &c. $= nay + nbyy + ncy^3$, &c. therefore we may find a Value of z expreſſed by the Powers of y. Again, ſince $1 + z = \overline{1+y}^{n}$, therefore $z = \overline{1+y}^{n} - 1$; that is, $z = ny + \frac{n}{1} \times \frac{n-1}{2} yy + \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} y^3$, &c. Therefo doubly expreſſed by the Powers of y. Compare theſe two Values to ther and the Coefficients a, b, c, &c. will be determined, except the fir ich may be taken at pleaſure, and gives accordingly all the diffe re Species of Logarithms.

II. You need be no longer in Concern, how the infiniteſimal Diffe- re of Infiniteſimals are to be explained. For as you will eaſily grant, that any

Doctrine of Exhauſtions; by Dr. Wallis. N. 255. *p.* 281. Aug. An. 1699.

any Multiple of Nothing is ſtill Nothing; I ſhall allow with the ſame Eaſe, that you may ſafely neglect infiniteſimal Differences drawn into Infiniteſimals. But though this may be done ſafely, yet it ought to be done cautiouſly. For in any kind of Quantities, thoſe Things which differ by leſs than a given or aſſignable Difference, are to be conſidered as equal. Upon this the whole Doctrine of Exhauſtions is founded, which is neceſſary both to the Antients and Moderns.

The Approximation of the Ancients in the Extracting of Roots, improv'd by Dr. Wallis. N. 215. *p.* 2. Jan. An. 1695.

XXIV. It is agreed by all, that if a Number propoſed be not a true Square, it is in vain to hope for a juſt Quadratic Root thereof, explicable by rational Numbers, Integers or Fracted. And therefore in ſuch Caſes we muſt content ourſelves with Approximations (ſomewhat near the Truth) without pretending to Accuracy.

And ſo, for the Cubic Root of what is not a perfect Cube. And the like for ſuperior Powers.

Now the Ancients (being aware of this) had their Methods of Approximation, which though ſcarce applied by them beyond the Quadratic, or perhaps the Cubic Root, yet are equally practicable (by due Adjuſtments) to the ſuperior Powers alſo.

I ſhall begin with the Square Root: For which the ancient Method is to this Purpoſe.

From the propoſed Non-quadrate (ſuppoſe N) ſubtract (in the uſual manner) the greateſt Square in Integers therein contained (ſuppoſe Aq.) The Remainder (ſuppoſe $B = 2AE + Eq$) is to be the Numerator of a Fraction, for deſigning the near Value of E (the remaining Part of the Root ſought $A + E = \sqrt{N}$) whoſe Denominator or Diviſor is to be $2A$ (the double Root of the ſubtracted Square) or $2A + 1$ (that double Root increaſed by 1) the true Value falling between theſe two; ſometime the one, ſometime the other, being neareſt to the true Value. But (for avoiding of Negative Numbers) the latter is commonly directed.

This Method Monſieur *De Lagny* affirms to be more than 200 Years old. And it is ſo; for I find it in *Lucas Pacciolus* (otherwiſe called *Lucas de Burgo*, or *de Burgo Sancti Sepulchri*) printed at *Venice* in the Year 1494, (if not even ſooner than ſo, for I find there have been ſeveral Editions of it.) And how much older than ſo, I cannot tell: For he doth not deliver it as a new Invention of his own, but as a received Practice, and derived from the *Moors* or *Arabs*, from whom they had their *Algoriſm*, or Practice of Arithmetick by the ten numeral Figures now in Uſe.

And it is continued down hitherto in Books of practical Arithmetick in all Languages, which teach the Extraction of the Square Root, and (therein) this Method of Approximation, in caſe of a Non-quadrate.

The true Ground of the Rule is this: Aq being (by Conſtruction) the greateſt Integer Square contained in N, 'tis evident that E muſt be leſs than 1; (otherwiſe not Aq, but the Square of $A + 1$, or ſome greater than ſo, would be the greateſt Integer Square contained in N.) Now if the Remainder $B = 2AE + Eq$ be divided by $2A$, the Reſult will be too great for E, (the Diviſor being too little; for it ſhould be $2\ A + E$, to make the Quotient E.) But if (to rectify this) we diminiſh the Quotient by increaſing the Diviſor, adding 1 to it, it then becomes too little; becauſe the Diviſor is now too big. For (E being leſs than 1) $2\ A + 1$ is more than $2\ A + E$; and therefore too big.

As for Inſtance: If the Non-quadrate propoſed by $N = 5$, the greateſt Integer Square therein contained is $A\,q = 4$ (the Square of $A = 2$:) which being ſubtracted, leaves $N - A\,q = 5 - 4 = 1 = B = 2\ AE + E\,q$. Which divided by $2\ A = 4$ gives $\frac{1}{4}$; but divided by $2\ A + 1 = 4 + 1 = 5$, gives $\frac{1}{5}$. That too great, and this too little, for E. And therefore the true Root $(A + E = \surd\ N)$ is leſs than $2\,\frac{1}{4} = 2.\ 25$, but greater than $2\,\frac{1}{5} = 2.\ 2$: And this was anciently thought an Approach near enough.

If this Approach be not now thought near enough, the ſame Proceſs may be again repeated; and that as oft as is thought neceſſary.

Take now for A, $2\,\frac{1}{5} = 2.\ 2$, whoſe Square is $4.\ 84 = A\,q$ (now conſidered as an Integer in the ſecond Place of Decimal Parts.) This ſubtracted from 5. 00, (or, which is the ſame, 0. 84, the Exceſs of this Square above the former, from 1, which was then the Remainder) leaves a new Remainder $B + 0.\ 16$: which divided by $2\ A = 4.\ 4$, gives $\frac{0.\ 16}{4.\ 40} = \frac{2}{55} = 0.\ 03636+$, too much. But divided by $2\ A + 1 = 4.\ 5$, it gives $\frac{0.\ 16}{4.\ 50} = \frac{8}{225} = 0.\ 03555+$ too little. The true Value (between theſe two) being 2. 236 *proxime*, whoſe Square is 4. 999696.

If this be not thought near enough, ſubtract the Square from 5.000000: the Remainder $B = 0.\ 000304$, divided by $2\ A = 4.\ 472$, or by $2\ A + 1 = 4.\ 473$, gives (either way) $0.\ 000068-$; which added to $A = 2.\ 236$, makes $2.\ 236068-$, ſomewhat too big; but $2.\ 236067+$ would be much more too little.

Which gives us the Square Root of 5, adjuſted to the ſixth Place of Decimal Parts, at three Steps. And by the ſame Method, if it be thought needful, we may proceed further.

For the Cubic Root the Rule is this:

From the Non-Cubic propoſed, (ſuppoſe N) ſubtract the greateſt Cube in Integers therein contained, (ſuppoſe $A\,c$:) the Remainder (ſuppoſe $B = 3A\,q\,E + 3\ A\ E\,q + E\,c$,) is to be the Numerator of a Fraction for deſigning the Value of E, (the remaining Part of the Root ſought $A + E = \surd\,c\ N$.) To this Numerator, if (for the Denominator or Diviſor) we ſubjoin $3\ A\,q$, the Reſult will certainly be too great for E, becauſe the Diviſor is too little: (For it ſhould be $3\ A\,q + 3\ A\,E + E\,q$, to give the true Value of E.) If for the Diviſor we take $3\ A\,q + 3\ A + 1$, it will certainly be too little, becauſe the Diviſor is too great. (For E by Conſtruction is leſs than 1.) It muſt therefore (between theſe Limits) be more than this latter. And therefore this latter Reſult being added to A, will give a Root whoſe Cube may be ſubtracted from the Non-Cubic propoſed, in order to another Step.

But if, for the Diviſor, we take $3\,Aq + 3\,A$, (or even leſs than ſo) the Reſult may be too great; or (in caſe B be ſmall) it may be too little, and oft is ſo. Which comes to paſs from hence; becauſe E (by Conſtruction) is leſs than 1; and therefore $3\,AE$ leſs than $3\,A$; and perhaps ſo much as that the Addition of Eq will not redreſs it. And when it ſo happens $3\,Aq + 3\,A$ is a better Diviſor than $3\,Aq + 3\,A + 1$ (or even ſomewhat leſs than either.) But becauſe it doth not always ſo happen (though for the moſt part it doth) the Rule doth rather direct the other; as which doth certainly give a Root leſs than the true Value, whoſe Cube may always be ſubtracted from the Non-Cubic propoſed: the Deſign being to have ſuch a Cube as (being ſubtracted) may leave another B, to be ordered in like manner for a new Approach.

But, for the moſt part, $3\,Aq$ may be ſafely taken for the Diviſor. For tho' the Reſult will then be ſomewhat too big, yet the Exceſs may be ſo ſmall as to be neglected; or, at leaſt we may thence eaſily judge what Number (ſomewhat leſs than it) may be ſafely taken. And if we chance to take it ſomewhat too big, the Inconvenience will be but this, that B for the next Step will be a Negative. Of which Caſe we ſhall ſpeak anon.

Thus for Inſtance: If the Non-Cube propoſed be $9 = N$: The greateſt Integer Cube therein contained is $8 = Ac$, (whoſe Cubic Root is $A = 2$.) Which Cube ſubtracted, leaves $9 - 8 = 1 = B = 3\,AqE + 3\,AEq + Ec$. This divided by $3\,Aq = 12$, gives $\frac{1}{12} = 0.08333 +$, too big for E. But the ſame divided by $3\,Aq + 3\,A + 1 = 12 + 6 + 1 = 19$, gives $\frac{1}{19} = 0.05263 +$, too little. Or if but by $3\,Aq + 3\,A = 12 + 6 = 18$, it gives $\frac{1}{18} = \frac{5}{90} = 0.05555 +$, yet too little. For the Cube of $A + 0.06, = 2.06$, is but $8.742 -$, which is ſhort of 9; and ſo much ſhort of it, that we may ſafely take 2. 07 as not too big: Or perhaps 2. 08, which upon Trial will be found not too big; for the Cube of 2. 08, is but 8. 998912.

If this firſt Step be not near enough, this Cube ſubtracted from 9.000000, leaves a new $B = 0.001088$, which divided by $3\,Aq = 12.9796$, gives $0.000084 -$, which will be ſomewhat too big, but not much. (For E is now ſo ſmall, as that $3\,AE$ may be ſafely neglected, and Eq much more.) So that if to 2. 08 we add $0.3000084 -$, the Reſult 2.080084 will be too big; but 2. 080083 will be more too little, as will appear if we take the Cube of each.) So that either of them, at the ſecond Step, gives the true Root within an Unite in the ſixth Place of decimal Parts. But when I ſay, taking the Cube of each (which I do that the Thing may be more clearly apprehended) it is not neceſſary that we trouble ourſelves with the whole Cube. For Ac being already ſubtracted, for finding $B = 3\,AqE + 3\,AEq + Ec$, we have no more to try, but whether $3\,AqE + 3\,AEq + Ec$, be greater or leſs than B, according as we take 0.000084, or 0.000083, for E.

Which may conveniently be done in this manner: Take $3\,A + E$, and multiply this by E (or E by it) ſo have we $3\,AE + Eq$. To this add $3\,Aq$, and multiply the whole by E (ſo have we $3\,AqE + 3\,AEq + Ec$) to ſee whether this be greater or leſs than B.

That is, in the preſent Caſe, if we take $E = 0.000084$, and add to this $3\,A = 6.24$, then is $6.240084 = 3\,A + E$. This multiplied by $E =$

0. 000084,

0.000084, is $3\,AE+Eq=0.000524+$. To which if we add $3\,Aq=12.9792$, it is $3\,Aq+3\,AE+Eq=12.979724$. Which multiplied again by $E=0.000084$, is $0.0010902+=3\,AqE+3\,AEq+Ec$, which is more than $B=0.001088$.

But if we take $E=0.000083$, and proceed as before, we ſhall have $3\,AqE+3\,AEq+Ec=0.001077+$, which is leſs than $B=0.001088$. And therefore (if we ſubtract that from this) the Remainder, 0.000011, will be another B for the next Step, if we pleaſe to proceed farther.

Hitherto I have purſued the Method moſt affected by the Ancients, in ſeeking a Square or Cube (and the like of other Powers) always leſs than the juſt Value, that it might be ſubtracted from the Number propoſed, leaving B a poſitive Remainder; thereby avoiding Negative Numbers.

But ſince the Arithmetic of Negatives is now ſo well underſtood, it may in this (and other Operations of like nature) be adviſeable to take the neareſt, whether it be greater or leſs than the juſt Value.

According to this Notion, for the Square Root of 5, I would ſay it is (2+) ſomewhat more than 2; and enquire how much more. But for the Square Root of 8, I would ſay, it is (3—) ſomewhat leſs than 3; and enquire how much leſs: taking (in both Caſes) that which is neareſt to the juſt Value.

Thus in the Cubic Root before us; I would take for E (in the laſt Enquiry) 0.000084— (where for the next Step we have $B=-0.000002$) rather than 0.000083+ (where for the next Step we ſhould have $B=+0.000011$.) In the latter Caſe, we are to divide $B=+0.000011$, by $3\,Aq=12.980236-$, to find (by the Quotient) how much is to be added to 0.000083. In the other Caſe, we are to divide $B=+0.000002$, by $3\,Aq=12.98028$, to find (by the Quotient) what is to be abated of 0.000084; ſo have we $\frac{0.000011}{12.980236}=0.00000085+$ to be added to 6.240083: Or $\frac{0.000002}{12.980248}=0.00000015+$ to be abated of 6.240084; (Or it may ſuffice in either to divide by 12.98+, or even by 13—, without being incumbered with a long Diviſor) either of which gives us, for the Root ſought, 2.08008385 *proxime*; true (at the third Step) to the eighth Place of decimal Parts. And if this be not near enough, the Cube of this, compared with the Number propoſed, will give us another B for the next Step: And ſo onwards as far as we pleaſe.

Now what is ſaid of the Cube, is eaſily applicable to the higher Powers.

I ſhall omit that of the Biquadrate; becauſe here perhaps it may be thought moſt adviſeable to extract the Square Root of the Number propoſed, and then the Square Root of that Root. But if we would do it at once, we are from N (the Number propoſed, being not a Biquadrate) to ſubtract Aqq (the greateſt Biquadrate contained in it) to find the Remainder $B=4\,AcE+6\,AqEq+4\,AEc+Eqq$. Which Remainder, if we divide by $4\,Ac$, the Quotient will certainly be too big for E, (though perhaps not much:) If by $4\,Ac+6\,Aq+4\,A+1$, it will certainly be too little (for Reaſons before-mentioned.) And we are to uſe our Diſcretion in taking ſome intermediate Number. And if we chance not to hit on the neareſt, the Inconvenience will be but this, that our Leap will not be ſo great as otherwiſe it might be. Which will be rectified by another B at the next Step.

For the Surſolid (of five Dimenſions) we are, from N (the Number propoſed, being not a perfect Surſolid) to ſubtract $A\,qc$ (the greateſt Surſolid therein contained) to find the Remainder $B = 5\,Aqq\,E + 10\,Ac\,Eq + 10\,Aq\,Ec + 5\,A\,E\,qq + E\,qc$. Which (as before) if we divide by $5\,A\,qq$, the Reſult will be ſomewhat too big, (becauſe the Diviſor is too little:) If by $5\,Aqq + 10\,Ac + 10\,Aq + 5\,A + 1$, the Reſult will certainly be leſs than the true E. The juſt Value of E being ſomewhat between theſe two, where we are to uſe our Diſcretion what intermediate Number to take. Which according as it proves too great or too little, is to be rectified at the next Step.

But for the moſt part it will be ſafe enough (and leaſt trouble) to divide by $5\,A\,qq$, which gives a Quotient ſomewhat too big; which we may either rectify at Diſcretion, by taking a Number ſomewhat leſs, or proceed to another B, (affirmative or negative, as the Caſe ſhall require) and ſo onward to what Exactneſs we pleaſe. Which is, for Subſtance, in a manner co-incident with Mr. *Raphſon*'s Method, even for affected Equations.

Thus, in the preſent Caſe: If the Number propoſed be $N = 33$, then is $A\,qc = 32$, and $B = 33 - 32 = 1 = 5\,Aqq\,E + 10\,A\,c\,Eq + 10\,Aq\,Ec + 5\,A\,E\,qq + E\,qc$. Which if we divide by $5\,A\,qq = 5 \times 16 = 80$, the Reſult $\frac{1}{80} = 0.0125$, is ſomewhat too big for E, but not much. And if we examine it, by taking the Surſolid of 2.0125, or of $2\frac{1}{80}$, we ſhall find a Negative B (for the next Step), but not very conſiderable. Or if we think it conſiderable, we may proceed farther to another Step, or more than ſo.

The like Method may be applied (and with more Advantage) in the higher Powers, according as the Compoſition of each Power requires.

And the ſame Method may be of Uſe (with good Advantage) in long Numbers (if duly applied) even before we come to the Place of Units; for the ſame will equally hold there alſo. Which the Reader may eaſily apprehend, without a long Diſcourſe upon it.

The Proportion of infinite Quantities; by Mr. E. Halley. n. 195. *p.* 556.

XXV. The very Idea of Magnitudes infinitely great, or ſuch as exceed any aſſignable Quantity, does include a Negation of Limits: yet if we nearly examine this Notion, we ſhall find that ſuch Magnitudes are not equal amongſt themſelves, but that there are really beſides infinite Length, and infinite Area, three ſeveral Sorts of infinite Solidity: all of which are *Quantitates ſui generis*; and that thoſe of each Species are in given Proportions.

Infinite Length, or a Line infinitely long, is to be conſidered either as beginning at a Point, and ſo infinitely extended one Way, or elſe both Ways from the ſame Point; in which Caſe the one, which is a beginning Infinity, is the one half of the whole, which is the Sum of the beginning and ceaſing Infinity; or, as I may ſay, of Infinity *à parte ante* and *à parte poſt*, which is analogous to Eternity in Time or Duration, in which there is always as much to follow as is paſt, from any Point or Moment of Time: Nor doth the Addition or Subduction of finite Length, or Space of Time, alter the Caſe either in Infinity or Eternity, ſince both the one or the other cannot be any Part of the Whole.

As to Infinite Surface or Area, any Right Line infinitely extended both Ways on an infinite Plane, does divide that infinite Plane into equal Parts, the one to the right, and the other to the left of the ſaid Line; but if from any Point in ſuch a Plane, two right Lines be infinitely extended, ſo as to make an Angle, the infinite Area, intercepted between thoſe infinite right Lines, is to the whole infinite Plane, as the Arch of a Circle on the Point of Concourſe on thoſe Lines as a Center, intercepted between the ſaid Lines, is to the Circumference of the Circle; or as the Degrees of the Angle to the 360 Degrees of a Circle. For Example: Two right Lines meeting at a right Angle do include, on an infinite Plane, a quarter Part of the whole infinite Area of ſuch a Plane.

But if ſo be two parallel infinite Lines be ſuppoſed drawn on ſuch an infinite Plane, the Area intercepted between them will be likewiſe infinite; but at the ſame time will be infinitely leſs than that Space, which is intercepted between two infinite Lines that are inclined, tho' with never ſo ſmall an Angle; for that in the one Caſe, the given finite Diſtance of the parallel Lines diminiſhes the Infinity in one Degree of Dimenſion; whereas in a Sector, there is Infinity in both Dimenſions: and conſequently the Quantities are the one infinitely greater than the other, and there is no Proportion between them.

From the ſame Conſideration ariſe the three ſeveral Species of infinite Space or Solidity; for a Parallelopipid, or a Cylinder infinitely long, is greater than any finite Magnitude, how great ſoever; and all ſuch Solids ſuppoſed to be formed on given Baſes, are as thoſe Baſes in proportion to one another. But if two of theſe three Dimenſions are wanting, as in the Space intercepted between two parallel Planes infinitely extended, and at a finite Diſtance; or with infinite Length and Breadth, with a finite Thickneſs, all ſuch Solids ſhall be as the given finite Diſtances one to another; but theſe Quantities, tho' infinitely greater than the other, are yet infinitely leſs than any of thoſe wherein all the three Dimenſions are infinite. Such are the Spaces intercepted between two inclined Planes infinitely extended; the Space intercepted by the Surface of a Cone, or the Sides of a Pyramid, likewiſe infinitely continued, *&c.* of all which, notwithſtanding, the Proportions one to another, and to the τὸ πᾶν or vaſt Abyſs of infinite Space (wherein is the *Locus* of all Things that are or can be; or to the Solid of infinite Length, Breadth, and Thickneſs taken all manner of ways) are eaſily aſſignable. For the Space between two Planes is to the whole, as the Angle of thoſe Planes to the 360 Degrees of the Circle. As for Cones and Pyramids, they are as the ſpherical Surface intercepted by them is to the Surface of the Sphere, and therefore Cones are as the verſed Sines of half their Angles to the Diameter of the Circle: Theſe three Sorts of infinite Quantity are analogous to a Line, Surface, and Solid, and after the ſame manner cannot be compared, or have no Proportion the one to the other.

Infinitely-infinite Fractions; by D. R. Wood. Ph. Col. n. 3. p. 45.

XXVI. Infinitely-infinite Fractions, or all the Powers of the Fractions whoſe Numerator is 1, are all of them together equal to (1) an Unit.

$$\begin{array}{ccccccc}
 & & & & \text{P.} & & \\
 & \text{A.} & \text{R.} & \text{q.} & \text{c.} & \text{qq.} & \\
1 = & \frac{1}{1} = & \frac{1}{2} : + & \frac{1}{4} + & \frac{1}{8} + & \frac{1}{16} & \text{\&c.} \\
 & \frac{1}{2} = & \frac{1}{3} : + & \frac{1}{9} + & \frac{1}{27} + & \frac{1}{18} & \text{\&c.} \\
 & \frac{1}{3} = & \frac{1}{4} : + & \frac{1}{16} + & \frac{1}{64} + & \frac{1}{256} & \text{\&c.} \\
 & \frac{1}{4} = & \frac{1}{5} : + & \frac{1}{25} + & \frac{1}{125} + & \frac{1}{625} & \text{\&c.} \\
 & \frac{1}{5} = & \frac{1}{6} : + & \frac{1}{36} + & \frac{1}{216} + & \frac{1}{1296} & \text{\&c.} \\
 & \text{\&c.} & \text{\&c.} & & & &
\end{array}$$

Or thus:

$$\begin{array}{ccc}
 & \text{1 anor.+} & \\
\frac{1}{1\times1} = & \frac{1}{2} + & \frac{1}{1\times2} = \\
\frac{1}{2\times1} = & \frac{1}{3} + & \frac{1}{2\times3} = \\
\frac{1}{3\times1} = & \frac{1}{4} + & \frac{1}{3\times4} = \\
\frac{1}{4\times1} = & \frac{1}{5} + & \frac{1}{4\times5} = \\
\frac{1}{5\times1} = & \frac{1}{6} + & \frac{1}{5\times6} =
\end{array}$$

$$\begin{array}{ccccc}
\text{2 anor.+} & & \text{3 anor.+} & & \text{4 anor.+} \\
\frac{1}{2^2} + & \frac{1}{1\times2^2} = & \frac{1}{2^3} + & \frac{1}{1\times2^3} = & \frac{1}{2^4} + \text{\&c.} \\
\frac{1}{3^2} + & \frac{1}{2\times3^2} = & \frac{1}{3^3} + & \frac{1}{2\times3^3} = & \frac{1}{3^4} + \text{\&c.} \\
\frac{1}{4^2} + & \frac{1}{3\times4^2} = & \frac{1}{4^3} + & \frac{1}{3\times4^3} = & \frac{1}{4^4} + \text{\&c.} \\
\frac{1}{5^2} + & \frac{1}{4\times5^2} = & \frac{1}{5^3} + & \frac{1}{4\times5^3} = & \frac{1}{5^4} + \text{\&c.} \\
\frac{1}{6^2} + & \frac{1}{5\times6^2} = & \frac{1}{6^3} + & \frac{1}{5\times6^3} = & \frac{1}{6^4} + \text{\&c.}
\end{array}$$

A. Is a File or Row of abſolute Numbers, or rather of all the Fractions, whoſe Numerator is 1; which Row is ſuppoſed to be continued *in infinitum* (downwards.)

R. Is another File or Row of all the Roots (whoſe Numerator is 1) of all the Powers of ſuch Fractions, ſuppoſed likewiſe to be continued *in infinitum* (downwards).

P. Are all the reſpective Powers of ſuch Fractions, (*viz.* Squares, Cubes, *&c.*) or ſo many Ranks of Geometrical Proportionals, ſuppoſed to be continued *in infinitum*, both to the Right-hand, and alſo downwards.

Lemma.] *Each of the ſaid Ranks of Powers, together with their reſpective Roots, is equal to each of the ſeveral Numbers under* A *reſpectively.*

Demonſtration.] If from the Line *a b* you take (for inſtance) $\frac{1}{4}$ part towards *a*, ſuppoſe *a c*; and alſo from the other End of the ſame Line *a b*, you take two ſuch Parts (or $\frac{1}{4}$ Part) towards *b*, ſuppoſe *b d*, (*viz.* a Number of Parts leſs by two than the whole Line *a b* was firſt ſuppoſed to be divided into) there will remain the Line $c\,d = a\,c = \frac{1}{4}$ of *a b*. Then again, if from *c d* you take $\frac{1}{4}$ Part thereof towards *a*, ſuppoſe *c e*, and from the other End $\frac{1}{4}$ Part of the ſame *c d*, ſuppoſe *d f*, there will remain only $e\,f = c\,e = \frac{1}{4}$ of *c d*. And if you ſtill go on without ceaſing, to take on the Side towards *a*, $\frac{1}{4}$ Part of what was taken laſt before, and twice as much on the other Side towards *b*, there ſhall be found between the two Lines laſt taken always remaining $\frac{1}{4}$ Part of the Line from which they were taken. From which $\frac{1}{4}$ Part there may ſtill after the ſame manner be ſuppoſed to be taken two other ſuch Lines. But if this be ſuppoſed to be done infinite Times actually, then there will nothing more remain (between), and ſo the continued Diviſion on either Side will come exactly to the Point *g*, ſuppoſing *a g* to be $\frac{1}{3}$ of *a b*, and $b\,g = 2\,a\,g$: For, becauſe that which was taken away towards *b*, was always twice as much as that which was taken away towards *a*, the total Sum of all the Lines taken away towards *b*, (which all together do make up the Line *b g*) muſt be twice as much as the Line *a g*, (which is the total Sum of all the Lines taken away towards *a*) viz. $b\,g = 2\,a\,g$; and conſequently $b\,g + a\,g$ (or the whole Line *a b*) is equal to $3\,a\,g$; and therefore $a\,g = \frac{1}{3}$ of *a b*. *Q. E. D.* Fig. 54.

The like *Conſtruction* and *Demonſtration* (*mutatis mutandis*) may be made uſe of in taking away any other Part of any Quantity, and the like Part again of the firſt mentioned Part, and ſo *in infinitum*. The total Sum of all the Parts ſo taken, or ſuppoſed to be taken, ſhall be equal to any certain Quantity, or Part, or Fraction, whoſe Denominator ſhall be leſs by an Unit than the Denominator of the firſt mentioned Part; as $\frac{1}{6} = \frac{1}{7} + \frac{1}{49} + \frac{1}{343}$ &c. $\frac{1}{9} = \frac{1}{10} + \frac{1}{100} + \frac{1}{1000}$ &c. And ſo, that which the incomparable *Archimedes* (in his Squaring the *Parabola*) has only demonſtrated in one particular Caſe, *viz.* $\frac{1}{3} = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \frac{1}{1024}$ &c. and that too, not without an huge Apparatus of Preliminary Propoſitions, amounting to a whole Book, is here univerſally demonſtrated in all Caſes (which are infinite) and by a very ſimple and eaſy Method (in *Des Cartes*'s Way) and on one ſingle Page.

Now if each of the ſaid Ranks of Powers, together with their reſpective Roots, be equal to the ſeveral Numbers of Fractions under A; (as is demonſtrated by the *Lemma*) then is A the Sum of them all, or equal to them all; that is to ſay, $R + P = A = R + 1$; for R is the ſame with A wanting $\frac{1}{1}$, or 1, (as appears upon View) or but $(\frac{1}{\infty})$ one infinite Part bigger. Wherefore $P = 1$, *Q. E. D. viz.* Infinitely-infinite Fractions are equal to Unity, that is, to the leaſt Root that is an Integer.

Corollaries. Hence it is plain,

1. That a Progreſs *ad infinitum* muſt be allowed.
2. And not only to one Infinite, but to ſeveral; or rather to infinite Infinites, or infinitely Infinites.
3. And that this may be done, that is, this Calculation may be performed, by a very finite or bounded Capacity.
4. And that this whole Progreſs, or ſuch infinite Progreſſes, may be caſt up, or collected into one Sum.
5. And into a Sum that is not only not infinite, but ſo ſmall that it is leſs then any Number.

It appears farther,

That of Infinites ſome are equal, and others unequal.

And that one Infinite may be equal to two, three, or ſeveral, either Finites or Infinites.

For 1. The infinite Powers of the firſt Rank are $= \frac{1}{2} = \frac{1}{1 \times 2}$, and alſo equal to all the infinitely-infinite Powers of all the other Ranks.

The infinite Powers of the ſecond Rank are $= \frac{1}{6} = \frac{1}{2 \times 3}$

Thoſe of the third Rank are $= \frac{1}{12} = \frac{1}{3 \times 4}$

Thoſe of the fourth Rank are $= \frac{1}{20} = \frac{1}{4 \times 5}$

Thoſe of the fifth Rank are $= \frac{1}{30} = \frac{1}{5 \times 6}$

&c. *in infinitum.*

viz. equal to the reſpective mean proportional Number between the Square Numbers reſpectively. *e. g.*

4, 6, 9
9, 12, 16
16, 20, 25
25, 30, 36
&c. *in infinitum.*

2. The infinite Powers of the two firſt Ranks are $= \frac{2}{3}$

Thoſe of the three firſt are $= \frac{3}{4}$

Thoſe of the four firſt are $= \frac{4}{5}$

Thoſe of the five firſt are $= \frac{5}{6}$

&c. *in infinitum.*

3. All the Powers of all the infinite Ranks, except the firſt are $= \frac{1}{2}$

All, except the two firſt, are $= \frac{1}{3}$

All, except the three firſt, are $= \frac{1}{4}$

All, except the four firſt, are $= \frac{1}{5}$

&c. *in infinitum.*

The latter *Corollaries* may all appear by ſimple Addition and Subduction; and ſo may many more.

The Antiquity of the Numeral Figures; by Dr. J. Wallis. N. 154. *p.* 399. Dec. An. 1683.

XXVII. 1. That the Numeral Figures now in uſe, with the manner of Computation by them (and the Names of Algoriſm, appropriated to that Way of Computation) came to us from the *Arabs* (but ſomewhat altered, as to the Shape of the Figures, in ſucceeding Ages) and to them from the *Indians*, is generally agreed. But it is not ſo generally agreed, of what Antiquity the Uſe of them, in our *European* Parts, hath been.

Jo. Gerard Voſſius (De Scientiis Mathematicis) thinks they came not in Uſe here till about the Year of our Lord 1300; or at the fartheſt, later than the Year 1250. And *P. Mabillon* (*De Re Diplomatica*) tells us, that he hath not found them any where uſed ſooner than the 14th Century. But I think their Uſe in theſe Parts was as old at leaſt as the Times of *Hermannus Contractus*, who lived about the Year of our Lord 1050 (that is, about the middle of the 11th Century,) if not ſo frequently in ordinary Affairs; yet at leaſt in Mathematical Things, and eſpecially in Aſtronomical Tables.

Fig. 55.

But I do not remember that I have any where ſeen any Monument of them ore antient than the Mantle-tree of the Parlour Chimney at the Dwelling-Iouſe of Mr. *Will. Richards*, the Rector of *Helmdon* in *Northamptonſhire*.

The Sides of the Chimney, by which the Mantle-tree is ſupported, are of one; but the Mantle-tree itſelf is of Wood. It is all over as black as k, having by Age and Smoak contracted that Colour. It may yet contie many hundreds of Years; for I did not diſcern in it any Thing either of orm, or Rottenneſs, or any Tendency to it. The Length of it is five Foot ne Inches; its Breadth or Depth at the Ends, (A B) is 11 $\frac{1}{2}$ Inches, but at the Middle, as C D, ſomewhat leſs. It is alſo carved from End to End, and the lower Part of it is abated, as in the Mouldings of other Chimneys. On the Front of the upper Part there is (beginning at the Middle) on three Squares parted from each other by a deep Furrow or Channel, the Date (I ſuppoſe, when it was firſt made) deſcribed partly in Numeral Figures, A° Doⁱ M° 133; on a fourth a Flower, and on a fifth the two Letters W. R. with an Eſcutcheon, repreſenting (I ſuppoſe) the Name of him to whom it did then belong. Both the Letters and the Figures are of an Antique Form; agreeing well enough with that Age. They are not engraved or cut in, but prominent on their ſeveral Squares (by way of *Bas-relief*) the Wood being abated round about them. The o over the A is a round o, but that over the M is a ſquare o.

Hence it appears, that the Uſe of ſuch Figures here in *England*, even on ordinary Occaſions, is at leaſt as ancient as the Year 1133. And I judge it to have been yet ſomewhat ancienter, becauſe the Shape of the Figures, though not come juſt to the Shape which we now uſe, was even then conſiderably varied from the Shape of the *Arabick* Figures; which argues they had then been for ſome Time in uſe; ſuch Change of Shape in Figures and Letters coming on gradually with Time.

I need not move any Scruple at all, that Part of the Numbers is expreſſed by the Numeral Letter M (or the Word *Milleſimo*, of which M° is but a Contraction) while the reſt is expreſſed in Numeral Figures: For the like doth oft occur in old *Manuſcripts*; and ſometimes even at this Day. And it doth rather favour the Simplicity of that Age, (not very nice in ſuch Things, eſpecially amongſt Mechanics) than any Deſign of Impoſture.

By Mr. Tho. Luffkin. N. 255. p. 287. N. 266. p. 677. Aug. An. 1690.

2. Over-againſt the *Market-place* in *Colcheſter* ſtands the Houſe of Mr. *Furley*, a Linnen-draper; ſome of the backermoſt Part of which is an ancient *Roman* Building, but the Front is of leſſer ſtanding, and timbered. Upon the bottom Cell (which is almoſt in the Form of a triangular Priſm) of one of the Windows of the Front, between two carved Lions, ſtands an Eſcutcheon, containing only theſe Figures 1090. The Periphery of the Cyphers, and Nine, are rather fracted than flected, prominent, large, and very fair; but to make them the more perſpicuous, they are gilded by the Proprietor. The Window looks directly North, the Date being thereby preſerved from the ſcorching Heat of the Sun; and by its Inclination (falling from the *Vertex* or *Perpendicular* by an Angle of about 60 Degrees) from Rain, Snow, &c. If it be objected, that the ſecond and fourth Figures may repreſent that among the *Arabians*, which is with us as five; I anſwer, that the o is not uſed with all the *Arabs* for 5, but with ſome for a Cypher, and ſo it was uſed by the *Moors* in *Spain*, who firſt brought theſe Figures into our Parts; nor is the ſquare o a *Arabick* Letter, but an *Engliſh* Letter of that Age. And the Form of the Figures ſoon degenerated from that of the *Arabs*, into ſuch as we now uſe, not at the firſt Reception from the *Arabs* [or *Moors*], certainly long befo 1595, as this *Conſtruction* would make it.

Fig. 56.

The Conſtruction of Logarithms; by Mr. Edm. Halley. N. 216. *p.* 58. Mar. An. 1695.

XXVIII. The old Definition of *Logarithms*, that they are the equally-differing Aſſociates of Proportional Numbers, is too ſcanty to define them fully

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For they may much more properly be ſaid to be *Numeri Rationum Exponentes*; wherein *Ratio* is conſidered as a *Quantitas ſui generis*, beginning from the Ratio of Equality, or 1 to 1 = 0; being Affirmative when the Ratio is increaſing, as of Unity to a greater Number, but Negative when decreaſing; and theſe *Rationes* we ſuppoſe to be meaſured by the Number of the *Ratiunculæ* contained in each. Now theſe Ratiunculæ are to be ſo underſtood as in a continued Scale of Proportionals, infinite in Number between the two Terms of the Ratio; which infinite Number of mean Proportionals is to that infinite Number of the like and equal Ratiunculæ between any other two Terms, as the Logarithm of the one Ratio is to the Logarithm of the other. Thus if there be ſuppoſed between 1 and 10 an infinite Scale of mean Proportionals, whoſe Number is 100000, &c. *in infinitum*; between 1 and 2 there ſhall be 30102, &c. of ſuch Proportionals, and between 1 and 3 there will be 47712, &c. of them; which Numbers therefore are the Logarithms of the Rationes of 1 to 10, 1 to 2, and 1 to 3; and not ſo properly to be called the Logarithms of 10, 2 and 3.

This being laid down, it is obvious that if between Unity and any Number propoſed, there be taken any Infinity of mean Proportionals, the infinitely little Augment or Decrement of the firſt of thoſe Means from Unity, will be a Ratiuncula, that is, the Momentum or Fluxion of the Ratio of Unity to the ſaid Number: And ſeeing that in theſe continual Proportionals all the Ratiunculæ are equal, their Sum, or the whole Ratio will be as the ſaid Momentum is directly; that is, the Logarithm of each Ratio will be as the Fluxion thereof. Wherefore if the Root of any infinite Power be extracted out of any Number, the Differentiola of the ſaid Root from Unity, ſhall be as the Logarithm of that Number. So that Logarithms thus produced, may be of as many Forms as you pleaſe to aſſume infinite Indices of the Power whoſe Root you ſeek: as if the Index be ſuppoſed 100000, &c. infinitely, the Roots ſhall be the Logarithms invented by the Lord *Napier*; but if the ſaid Index were 2302585, &c. Mr. *Briggs's* Logarithms would immediately be produced. And if you pleaſe to ſtop at any Number of Figures, and not to continue them on, it will ſuffice to aſſume an Index of a Figure or two more than your intended Logarithm is to have, as Mr. *Briggs* did, who to have his Logarithms true to 14 Places, by continual Extraction of the Square Root, at laſt came to have the Root of the 140737488355328th Power; but how operoſe that Extraction was, will be eaſily judged by whoſo ſhall undertake to examine his *Calculus*.

Now, tho' the Notion of an infinite Power may ſeem very ſtrange, and to thoſe
at know the difficulty of the Extraction of the Roots of high Powers, perhaps
practicable; yet by the help of that admirable Invention of Mr. *Newton*,
hereby he determines the Unciæ or Numbers prefix'd to the Members compo-
g Powers, (on which chiefly depends the *Doctrine of Series*) the Infinity of
e Index contributes to render the Expreſſion much more eaſy: For if the
inite Power to be reſolved be put (after Mr. *Newton*'s Method) $\overline{p+pq}$

$\overline{p+pq}|^{\frac{1}{m}}$, or $\overline{1+q}|^{\frac{1}{m}}$, inſtead of $1+\frac{1}{m}q+\frac{1-m}{2mm}qq+\frac{1-3m+2mm}{6m^3}q^3+\frac{1-6m+11mm-6m^3}{24m^4}q^4$, &c. (which is the Root when m is finite), becomes $1+\frac{1}{m}q-\frac{1}{2m}qq+\frac{1}{3m}q^3+\frac{1}{4m}q^4+\frac{1}{5m}q^5$, &c. mm being *infinitè* infinite; and conſequently whatever is divided thereby vaniſhing. Hence it follows that $\frac{1}{m}$ multiplied into $q-\frac{1}{2}qq+\frac{1}{3}q^3-\frac{1}{4}q^4+\frac{1}{5}q^5$, &c. is the Augment of the firſt of our mean Proportionals between Unity and $1+q$, and is therefore the Logarithm of the Ratio of 1 to $1+q$; and whereas the Infinite Index m may be taken at pleaſure, the ſeveral Scales of Logarithms to ſuch Indices will be as $\frac{1}{m}$, or reciprocally as the Indices. And if the Index be taken 10000, *&c.* as in the caſe of *Napier*'s Logarithms, they will be ſimply $q-\frac{1}{2}qq+\frac{1}{3}q^3-\frac{1}{4}q^4+\frac{1}{5}q^5-\frac{1}{6}q^5$, &c.

Again; If the Logarithm of a decreaſing Ratio be ſought, the infinite Root of $1-q$, or $\overline{1-q}|^{\frac{1}{m}}$ is $1-\frac{1}{m}q-\frac{1}{2m}q^2-\frac{1}{3m}q^3-\frac{1}{4m}q^4-\frac{1}{5m}q^5-\frac{1}{6m}q^6$, &c. whence the Decrement of the firſt of our Infinite Number of Proportionals will be $\frac{1}{m}$ into $q+\frac{1}{2}qq+\frac{1}{3}q^3+\frac{1}{4}q^4+\frac{1}{5}q^5+\frac{1}{6}q^6$, &c. which therefore will be as the Logarithm of the Ratio of Unity to $1-q$: But if m be put 10000, *&c.* then the ſaid Logarithm will be $q+\frac{1}{2}qq+\frac{1}{3}q^3+\frac{1}{4}q^4+\frac{1}{5}q^5+\frac{1}{6}q^6$, *&c.*

Hence the Terms of any Ratio being a and b, q becomes $\frac{b-a}{a}$ or the Difference divided by the leſſer Term, when 'tis an increaſing Ratio, or $\frac{b-a}{b}$ when 'tis decreaſing, or as b to a; whence the Logarithm of the ſame Ratio may be doubly expreſſed; for, putting x for the Difference of the Terms a and b, it will be either

$$\frac{1}{m}\text{ into }\frac{x}{b}+\frac{x^2}{2bb}+\frac{x^3}{3b^3}+\frac{x^4}{4b^4}+\frac{x^5}{5b^5}+\frac{x^6}{6b^6}\text{ \&c.}$$

$$\text{or }\frac{1}{m}\text{ into }\frac{x}{a}-\frac{x^2}{2aa}+\frac{x^3}{3b^3}-\frac{x^4}{4a^4}+\frac{x^5}{5a^5}-\frac{x^5}{6a^5}\text{ \&c.}$$

But if the Ratio of *a* to *b* be ſuppoſed to be divided into two Parts, *viz.* into the Ratio of *a* to the *Arithmetical Mean* between the Terms, and the Ratio of the ſaid *Arithmetical Mean* to the other Term *b*; then will the Sum of the Logarithms of thoſe two Rationes be the Logarithm of the Ratio of *a* to *b*; and, ſubſtituting $\frac{1}{2}z$ inſtead of $\frac{1}{2}a+\frac{1}{2}b$, the ſaid *Arithmetical Mean*, the Logarithms of thoſe Rationes will be, by the foregoing Rule,

$$\frac{1}{m}\text{ into }\frac{x}{z}+\frac{xx}{2zz}+\frac{x^3}{3z^3}+\frac{x^4}{4z^4}+\frac{x^5}{5z^5}+\frac{x^6}{6z^6}\text{ \&c.}$$

$$\text{and }\frac{1}{m}\text{ into }\frac{x}{z}-\frac{xx}{2zz}+\frac{z^3}{3z^3}-\frac{x^4}{4z^4}+\frac{x^5}{5z^5}-\frac{x^5}{6z^6}\text{ \&c.}$$

the Sum whereof $\frac{1}{m}$ into $\frac{2x}{z}*+\frac{2x^3}{3z^3}*+\frac{2x^5}{5z^5}*+\frac{2x^7}{7z^7}$ &c. will be the Logarithm of the Ratio of *a* to *b*, whoſe Difference is *x*, and Sum *z*. And this Series converges twice as ſwift as the former, and therefore is more proper for the Practice of making Logarithms; which it performs with that Expedition, that where *x* the Difference is but the hundredth Part of the Sum, the firſt Step, $\frac{2x}{z}$, ſuffices to ſeven Places of the Logarithm, and the ſecond Step to twelve; but if *Briggs*'s firſt twenty *Chiliads* of Logarithms be ſuppoſed made, as he very carefully computed them, to fourteen Places, the firſt Step alone is capable to give the Logarithm of any intermediate Number, true to all the Places of thoſe Tables.

After the ſame Manner may the Difference of the ſaid two Logarithms be very aptly applied to find the Logarithm of prime Numbers, having the Logarithms of the two next Numbers above and below them: For the Difference of the Ratio of *a* to $\frac{1}{2}z$, and of $\frac{1}{2}z$ to *b*, is the Ratio of *ab* to $\frac{1}{4}zz$, and the Half of that Ratio is that of $\sqrt{ab}$ to $\frac{1}{2}z$, or of the Geometrical Mean to the Arithmetical. And conſequently the Logarithm thereof will be the Half-difference of the Logarithms of thoſe Rationes, *viz.*

$$\frac{1}{m}\text{ into }\frac{xx}{2zz}+\frac{x^4}{4z^4}+\frac{x^5}{6z^5}+\frac{x}{8z^3},\text{ \&c.}$$

Which is a *Theorem* of good Diſpatch to find the Logarithm of $\frac{1}{2}z$. But the ſame is yet much more advantageouſly performed by a Rule derived from the foregoing, and beyond which, in my Opinion, nothing better can be hoped. For the Ratio of *ab* to $\frac{1}{4}zz$, or $\frac{1}{4}aa+\frac{1}{2}ab\frac{1}{2}+\frac{1}{4}bb$, has the Difference of its Terms, $\frac{1}{4}aa-\frac{1}{2}ab+\frac{1}{4}bb$, or the Square of $\frac{1}{2}a-\frac{1}{2}b=\frac{1}{4}xx$, which, in the preſent Caſe of finding the Logarithms of Prime Numbers, is always Unity; and calling the Sum of the Terms $\frac{1}{4}zz+ab=yy$, the Logarithm of the Ratio of $\surd ab$ to $\frac{1}{2}a+\frac{1}{2}b$, or $\frac{1}{2}z$ will be found,

$$\frac{1}{m} \text{ into } \frac{1}{yy}+\frac{1}{3y^6}+\frac{1}{5y^{10}}+\frac{1}{7y^{14}}+\frac{1}{9y^{18}}, \text{ \&c.}$$

which converges very much faſter than any *Theorem* hitherto publiſhed for this Purpoſe.

Here note, that $\frac{1}{m}$ is all along applied to adapt theſe Rules to all Sorts of Logarithms. If m be 10000, *&c.* it may be neglected, and you will have *Napier*'s Logarithms, as was hinted before; but if you deſire *Briggs*'s Logarithms, which are now generally received, you muſt divide your Series by 2, 302585092994045684017991454684364207610101488628779760333 28, or multiply it by the Reciprocal thereof, *viz.* 0, 43429448190325182765112891891660508229439700580366656611 4454.

But to ſave ſo operoſe a Multiplication (which is more than all the reſt of the Work) it is expedient to divide this Multiplicator by the Powers of z or y continually, according to the Direction of the *Theorem*, eſpecially where x is ſmall and Integer, reſerving the proper Quotes to be added together, when you have produced your Logarithm to as many Figures as you deſire; of which Method I will give a Specimen, in the Logarithms of the firſt prime Numbers under 20 to ſixty Places, computed by Mr. *Abraham Sharp*, as they were communicated to me by our common Friend, Mr. *Euclid Speidal.*

Num.	Logarithms,
2.	0, 301029995663981195213738894724493026768189881462108541310427
3.	0, 477121254719662437295027903255115309200128864190695864829866
7.	0, 845098040014256830712216258592636193483572396323965406503835
11.	1, 041392685158225040750199971243024241706702190466453094596539
13.	1, 113943352306837769206541895026246254561189005053673288598083
17.	1, 230448921378273028540169894328337030007567378425046397380368
19.	1, 278753600952828961536333475756929317951192337394497598906819

The next Prime Number is 23, which I will take for an Example of the foregoing Doctrine; and by the firſt Rules, the Logarithm of the Ratio of 22 to 23 will be found to be either

$$\frac{1}{22}-\frac{1}{968}+\frac{1}{31944}-\frac{1}{937024}+\frac{1}{25768160}, \text{ \&c.}$$

$$\text{or } \frac{1}{23}+\frac{1}{1058}+\frac{1}{36531}+\frac{1}{1119364}+\frac{1}{32181715}, \text{ \&c.}$$

As likewiſe that of the Ratio of 23 to 24 by a like Proceſs.

$$\frac{1}{23} - \frac{1}{1058} + \frac{1}{36501} - \frac{1}{1119364} + \frac{1}{32181715}, \text{ \&c. of}$$

$$\frac{1}{24} + \frac{1}{1152} + \frac{1}{41471} + \frac{1}{1327104} + \frac{1}{39813120}, \text{ \&c.}$$

And this is the Reſult of the Doctrine of *Mercator*, as improved by the learned Dr. *Wallis*. But by the ſecond *Theorem*, viz. $\frac{2x}{z} + \frac{2x^3}{3z^3} + \frac{2x^5}{5z^5}$, *&c.* the ſame Logarithms are obtained by fewer Steps; to wit,

$$\frac{2}{45} + \frac{2}{273375} + \frac{2}{922640625} + \frac{2}{2615686171875}, \text{ \&c. And}$$

$$\frac{2}{47} + \frac{2}{311469} + \frac{2}{1146725035} + \frac{2}{3546361843241}, \text{ \&c.}$$

which was invented and demonſtrated in the Hyperbolical Spaces analogous to the Logarithms, by the excellent Mr. *James Gregory*, in his *Exercitationes Geometricæ*, and ſince further proſecuted by the aforeſaid Mr. *Speidall*, in a late Treatiſe in *Engliſh* by him publiſhed on this Subject. But the Demonſtration, as I conceive, was never till now perfected without the Conſideration of the Hyperbola, which in a Matter purely Arithmetical, as this is, cannot ſo properly be applied. But what follows, I think, I may more juſtly claim as my own, *viz. That the Logarithm of the Ratio of the Geometrical Mean to the Arithmetical*, between 22 and 24, or of $\surd 528$ to 23, will be found to be either,

$$\frac{1}{1058} + \frac{1}{1119364} + \frac{1}{888215334} + \frac{1}{626487882248}, \text{ \&c. or}$$

$$\frac{1}{1057} + \frac{1}{3542796579} + \frac{1}{6596765584852 85}, \text{ \&c.}$$

All theſe Series being to be multiplied into 0,4342944819, *&c.* if you deſign to make the *Logarithm* of *Briggs*. But with great Advantage in reſpect of the Work, the ſaid 0,4342944819, *&c.* is divided by 1057, and the Quotient thereof again divided by three times the Square of 1057, and that Quotient again by $\frac{5}{3}$ of that Square, and that Quotient by $\frac{7}{5}$ thereof, and ſo forth, till you have as many Figures of your Logarithm as you deſire. As for Example, the Logarithm of the Geometrical Mean between 22 and 24 is found by the Logarithms of 2, 3, and 11, to be

$$\begin{array}{rrl}
 & & 1,36131696126690612945009172669805\\
1057)\ 43429\ \&c. & (& ---\ 41087462810146814347315886368\\
3\ in\ 1117249)\ 41087\ \&c. & (& -------\ 1225852154418182946007 4\\
\frac{5}{3}\ in\ 1117249)\ 12258\ \&c. & (& -------------\ 6583235184376175\\
\frac{7}{5}\ in\ 1117249)\ 65832\ \&c. & (& ------------------\ 4208829765\\
\frac{9}{7}\ in\ 1117249)\ 42088\ \&c. & (& ----------------------\ 2930\\
\text{Summa} & & 1,36172783601757287886777711225119
\end{array}$$

Which is the Logarithm of 23 to thirty-two Places, and obtained by five Diviſions with very ſmall Diviſors; all which is much leſs Work than ſimply multiplying the Series into the ~~ſaid~~ Multiplicator 0,43429, &c.

From the Logarithm given to find what Ratio it expreſſes, is a *Problem* that has not been ſo much conſidered as the former, but which is ſolved with the like Eaſe, and demonſtrated by a like Proceſs, from the ſame general *Theorem* of Mr. *Newton*; for as the Logarithm of the Ratio of 1 to $1+q$ was proved to be $\overline{1+q}^{\frac{1}{m}} - 1$, and that of the Ratio of 1 to $1-q$ to be $1 - \overline{1-q}^{\frac{1}{m}}$: ſo the Logarithm, which we will from henceforth call L, being given, $1+L$ will be equal to $\overline{1+q}^{\frac{1}{m}}$ in the one Caſe, and $1-L$ will be equal to $\overline{1-q}^{\frac{1}{m}}$ in the other: Conſequently $\overline{1+L}^{m}$ will be equal to $1+q$, and $\overline{1-L}^{m}$ to $1-q$; that is, according to Mr. *Newton*'s ſaid Rule, $1 + mL + \frac{1}{2}m^2L^2 + \frac{1}{6}m^3L^3 + \frac{1}{24}m^4L^4 + \frac{1}{120}m^5L^5$, &c. will be $=$ to $1+q$; and $1 - mL + \frac{1}{2}m^2L^2 - \frac{1}{6}m^3L^3 + \frac{1}{24}m^4L^4 - \frac{1}{120}m^5L^5$, &c. will be equal to $1-q$; m being any infinite Index whatſoever: which is a full and general *Propoſition* from the Logarithm given to find the Number, be the Species of Logarithms what it will. But if *Napier*'s Logarithm be given, the Multiplication by m is ſaved, (which Multiplication is indeed no other than the reducing the other Species to his) and the Series will be more ſimple, *viz.* $1 + L + \frac{1}{2}L^2 + \frac{1}{6}L^3 + \frac{1}{24}L^4 + \frac{1}{120}L^5$, &c. or $1 - L + \frac{1}{2}L^2 - \frac{1}{6}L^3 + \frac{1}{24}L^4 - \frac{1}{120}L^5$, &c. This Series, eſpecially in great Numbers, converges ſo ſlowly, that it were to be wiſhed it could be contracted.

If one Term of the Ratio, whereof L is the Logarithm, be given, the other Term will be had eaſily by the ſame Rule: For if L were *Napier*'s Logarithm of the Ratio of a the leſſer to b the greater Term, b would be the Product of a into $1+\mathrm{L}+\frac{1}{2}\mathrm{L}^2+\frac{1}{6}\mathrm{L}^3$, &c. $=a+a\mathrm{L}+\frac{1}{2}a\mathrm{L}^2+\frac{1}{6}a\mathrm{L}^3$, &c. But if b were given, a would be $=b-b\mathrm{L}+\frac{1}{2}b\mathrm{L}^2-\frac{1}{6}b\mathrm{L}^3$, &c. Whence, by the help of the *Chiliads*, the Number appertaining to any Logarithm will be exactly had to the utmoſt Extent of the Tables. If you ſeek the neareſt next Logarithm, whether greater or leſſer, and call its Number a if leſſer, or b if greater; then the given L, and the Difference thereof from the ſaid neareſt Logarithm you call l; it will follow that the Number anſwering to the Logarithm L will be either a into $1+l+\frac{1}{2}l^2+\frac{1}{6}l^3+\frac{1}{24}l^4+\frac{1}{120}l^5$, &c. or elſe b into $1-l+\frac{1}{2}l^2-\frac{1}{6}l^3+\frac{1}{24}l^4-\frac{1}{120}l^5$, &c. wherein as l is leſs, the Series will converge the ſwifter. And if the firſt 20000 Logarithms be given to 14 Places, there is rarely occaſion for the three firſt Steps of this Series to find the Number to as many Places. But for *Ulacq*'s great *Canon* of 100000 Logarithms, which is made but to ten Places, there is ſcarce ever need for more than the firſt Step $a+al$, or $a+mal$ in one Caſe, or elſe $b-bl$, or $b-mbl$ in the other, to have the Number true to as many Figures as thoſe Logarithms conſiſt of.

There is another Series which is not indeed ſo ſimple and uniform, yet the firſt Step thereof is moſt commodious for Practice, and exact enough for Tables not exceeding 14 Places: It is thus; $a+\frac{a\,l}{1-\frac{1}{2}}l$ or $b-\frac{b\,l}{1+\frac{1}{2}l}$ will be the Number anſwering to the Logarithm given, differing from the Truth but by one half of the third Step from the former Series. But that which renders it yet more eligible is, that with equal Facility it ſerves for *Briggs*'s or any other ſort of Logarithms, with the only Variation of writing $\frac{1}{m}$ inſtead of 1, that is $a+\frac{a\,l}{\frac{1}{m}-\frac{1}{2}l}$, and $b-\frac{b\,l}{\frac{1}{m}+\frac{1}{2}l}$ or $\frac{\frac{1}{m}a+\frac{1}{2}la}{\frac{1}{m}-\frac{1}{2}l}$ and $\frac{\frac{1}{m}b-\frac{1}{2}lb}{\frac{1}{m}+\frac{1}{2}l}$ which easily reſolved into Analogies, *viz.*

$$\left.\begin{array}{l}\text{As } 42429 \text{ \&c.} -\frac{1}{2}l : \text{to } 43429+\frac{1}{2}l :: \text{ſo is } a : \\ \text{As } 43429 \text{ \&c.} +\frac{1}{2}l : \text{to } 43429-\frac{1}{2}l :: \text{ſo is } b :\end{array}\right\}\text{ to the Number ſought.}$$

If more of this Series be deſired, it will be found as follows, $a + \frac{:al}{2 - \frac{1}{3}l} - \frac{\frac{1}{12}al^3}{1 - l} + \frac{\frac{1}{12}al^5}{1 - 2l}$, &c. as may eaſily be demonſtrated by working out the Diviſions in each Step, and collecting the Quotes, whoſe Sum will be found to agree with our former Series; which is no other than an eaſy *Corollary* to Mr. *Newton*'s general *Theorem* for forming *Roots* and *Powers*.

XXIX. *Papers of leſs General Uſe omitted.*

Tangents to Curves. N. 81. *p.* 4010. Mar. An. 1672.

1. A Breviat of Dr. *Wallis*'s two Methods of drawing Tangents; Extracted by him from his *Con. Sect.* and other Parts of his *Mathematical Works*.

Rectification of Curves. N. 98. *p.* 6146, 6149. Nov. An. 1673.

2. Mr. *Huygens* in his *Hor. Oſcill.* having given M. *Huraet* the Honour of inventing a Curve equal to a Straight Line in the Year 1659; Dr. *Wallis* here aſſerts this Invention to Mr. *William Neile* (Son of Sir *Paul Neile*), who diſcovered and demonſtrated the Equality of a *Paraboloid* to a Straight Line two Years before. The ſame was ſoon after otherwiſe demonſtrated by my Lord *Brounker*, and Sir *Chriſtopher Wren*, in *June* and *July* 1657; and the Demonſtrations inſerted by Dr. *Wallis*, in his Tract *de Cycloide* 1659, with a fair Relation of the whole Matter. Beſides, Sir *Chriſtopher Wren* found a ſtraight Line equal to that of a *Cycloid* in the Year 1658: Yet he freely confeſſes Mr. *Neile*'s Invention of a Curve capable of *Rectification* the Year before.

Ib. p. 6150.

Transformation of Curves. N. 214. *p.* 233. Nov. An. 1694.

3. The Abbot *Galloys*, having, in the Year 1693, aſſerted that Mr. *James Gregory* and Dr. *Barrow* ſtole their General Propoſitions concerning the *Transformation of Curves* from Mr. *Robervall*; Dr. *David Gregory* here fully refutes that Aſſertion. For Mr. *Gregory* publiſhed his Book at *Padua* 1668, and Dr. *Barrow* his *Lectiones Geometricæ* 1674, which Mr. *Robervall* doubtleſs had a Sight of before he died (which was not till *October* 1675), yet he never complained of any ſuch Injury done him.

Cycloidal Spaces perfectly Quadrable. N. 217. *p.* 111. Oct. An. 1695.

4. Beſides that Segment of the *Semicycloidal Figure*, firſt obſerved by Sir *Chriſtopher Wren*, and after him by Mr. *Huygens*, and a Trilinear Part of it, which are capable of being Geometrically Squared; Dr. *Wallis* here produces from his Tracts *de Cycloide*, and *de Motu*, ſome other Portions thereof equally capable of Quadrature.

The Cycloid conſider'd long ago. N. 229. *p.* 561. June An. 1697.

5. Dr. *Wallis* finds among the Mathematical Works of *Bovillus*, publiſhed at ſeveral times between the Years 1501 and 1510, that the Curve (which is now called the *Cycloid*,) was then conſidered. But he alſo finds that *Bovill* was not the firſt who conſidered it: For Cardinal *Cuſanus*, as appears by a ancient Manuſcript of his Works (tranſcribed by *J. Scoblant* in the Year 145 had conſidered it ſome time before. The Figure indeed (thro' the Unſkilfulneſs of the Tranſcriber) both in the MS. and the *Baſil* Edition, A. 1565, very ill drawn; but being corrected according to the true Meaning of th *Cardinal*'s own Words, it evidently repreſents the modern *Cycloid*. Fro hence 'tis manifeſt, that this Curve was not firſt taken into Conſiderati

either by *Merſennus* or *Galileo*, but ſome Ages before, tho' never well underſtood till this preſent Age.

6. Some Papers ſent by Mr. *Jo. Collins* to Dr. *Wallis*, giving his Thoughts about ſome Defects in Algebra; which he did not live to finiſh. *Defects in Algebra.* N. 159. *p.* 575. May An. 1684.

XXX. *Accounts of Books, with Editions, Emendations,* &c. *omitted.*

1. EUclidis *Elementa Geometrica, novo ordine ac methodo demonſtrata.* Lond. 1666. N. 15. *p.* 261.

2. Archimedis *Opera*; Apollonii Perg: *Conic. Libri* iv; Theodoſii *Sphærica, Methodo nova illuſtrata, & ſuccincte demonſtrata: ab* Iſa. Barrow, *R. S. S.* Lond. 1675. *in 4to.* N. 114. *p.* 314.

3. Ἀρχιμήδους τοῦ Συρακουσίου Ψαμμίτης, καὶ Κύκλου μέτρησις: Εὐτοκίου Ἀσκαλωνίτου εἰς αὐτὴν ὑπόμνημα, &c. *Cum Verſione & Notis* Jo. Wallis, S. S. Th. D. *Oxon.* 1676. N. 123. *p.* 567.

4. *Theon Smyrnæus*, publiſhed at *Paris* by *Iſmael Bulialdus* in Greek and Latin. N. 80. *p.* 3095.

5. Diophanti Alexandrini *Arithmeticorum Libri ſex, & de Numeris Multangulis Liber unus*; *cum Commentariis* C. G. Bacheti, *& Obſervationibus* D. P. de Fermat, *Senatoris* Tholoſani: *cui acceſſit Doctrinæ Analyticæ Inventum Novum.* Toloſæ 1670. *in Folio.* N. 72. *p.* 2185.

6. The Works of Monſieur *de Fermat.* N. 1. *p.* 15.

7. Franciſci du Laurens *Specimina Mathematica, duobus Libris comprehenſa. Horum prior Syntheticus agit de Genuinis Matheſeos Principiis in genere*; *in ſpecie autem de veris Geometriæ Elementis hucuſque nondum traditis. Poſterior Analyticus de Methodo Compoſitionis atque Reſolutionis fuſe differit, & multa nova complectitur, quæ ſubtiliſſimam Analyſeos Artem mirum in modum promovent.* This Book is here cenſured, ſome Miſtakes in it corrected, and the Cenſure vindicated, by Dr. *Wallis.* N. 30. 580. N. 34. p. 654. N. 38. *p.* 744. N. 39. *p.* 775. N. 41. *p.* 825.

8. R. P. Andreæ Taquet, *è S. J. Opera Mathematica.* Antwerp. 1669. in *Folio.* N. 43. *p.* 869.

9. A Mathematical Compendium, collected out of the Notes and Papers of Sir *Jonas Moore*, by *Nicholas Stevenſon. Lond.* 1674. in 12*mo.* N. 104. *p.* 83.

10. R. P. Claudii Franc. Milliet de Chales, è S. J. *Curſus ſeu Mundus Mathematicus, univerſam Matheſin tribus Tomis complectens.* Lugd. 1674. in *Folio.* N. 110. *p.* 229.

11. The Mathematical Works of Dr. *Jo. Wallis, Savilian Profeſſor of Geometry* in the Univerſity of *Oxford*, & F. R. S. in three Volumes in *Folio.* N. 216. *p.* 73. N. 154. *p.* 259.

7.

2. An Introduction to Algebra, tranſlated out of *High Dutch* into *Eng-* by *Tho. Branker*, M. A. much altered and augmented by Dr. *J. Pell.* ſo a Table of ſuch odd Numbers as are leſs than One Hundred Thouſand, ewing thoſe that are *Incompoſit*, and reſolving the reſt into their *Factors*, or *efficients. Lond.* in 4*to.* N. 35. *p.* 688.

13. *Labyrinthus Algebræ. Auth.* Joh. Jac. Ferguſon. 1667. in 4*to.* N. 49. *p.* 996.

N. 90. p. 5152. N. 95. p. 6073. 14. The Elements of that Mathematical Science called *Algebra*; by *Jo.*
N. 108. p. 192. *Kerſey*. Lond. 1673. in *Folio*.
N. 143. p. 21. N. 173. p. 1095. 15. A Treatiſe of *Algebra*, both Hiſtorical and Practical; by *Jo. Wallis*,
N. 233. p. 730. D. D. In the 109th Chapter there are ſome Numbers miſtaken, which are
here rectified by the Author.

N. 14. p. 253. 16. *De Principiis & Ratiocinatione Geometrarum*; *contra Faſtum Profeſſo-*
N. 16. p. 289. *rum Geometriæ. Authore* Tho. Hobbes. This Book is here animadverted on,
and anſwered, by Dr. *Wallis*.

N. 48. p. 971. 17. Thomæ Hobbes *Quadratura Circuli, Cubatio Sphæræ, Duplicatio Cubi,*
confutata. Auth. Jo. Wallis. S. T. D. Oxon. 1669. in *Quarto*.

N. 55. p. 1121. 18. Thomæ Hobbes *Quadratura Circuli, Cubatio Sphæræ, Duplicatio Cubi*
(*ſecundo edita*) *denuo refutata. Auth.* Jo. Wallis. S. T. D. Oxon. 1669.

N. 72. p. 2185. 19. *Roſetum Geometricum, cum Cenſura brevi* Doctrinæ Walliſianæ *de Motu.*
Auth. Tho. Hobbes *Malmeſburienſi*. Lond. 1671. in *Quarto*. This Book is
N. 73. p. 2202. here anſwered by Dr. *Wallis*.

N. 75. p. 2241. 20. Four Papers of Mr. *Hobbs*'s, publiſhed in the Months of *Auguſt* and
September 1671. which are here anſwered by —— —

N. 86. p. 5047. 21. *Lux Mathematica, Colliſionibus* Johannis Walliſii, S. T. D. & Thomæ
Hobbeſii Malmſburienſis, *excuſſa, multis & fulgentiſſimis aucta radiis. Auth.*
R. R. *Adjuncta Cenſura Doctrinæ* Walliſianæ *de Libra, una cum* Roſeto Hobbe-
N. 87. p. 5067. ſii. Lond. 1672. in *Quarto*. This Book is here anſwered by Dr. *Wallis*.

N. 97. p. 6131. 22. *Principia & Problemata aliquot Geometrica, ante deſperata, nunc brevi-*
ter explicata & demonſtrata. Auth. T. H. Malmsburienſi. Lond. 1673. in
Quarto.

N. 185. p. 245. 23. *Le Grand & Fameux Probleme de la Quadrature du Cercle reſolu Geome-*
triquement par le Cercle & la Ligne droit, par M. Mallement de Meſſange.
à Paris. 1686. in *Twelves*. This Book is here refuted by M. D. *Cluverius*.
R. S. S.

N. 32. p. 625. 24. *Nouveaux Elemens de Geometrie*: Or a Mathematical Treatiſe, entituled
New Elements of Geometry. Paris 1667. in *Quarto*.

N. 79. p. 3064. 25. *Elemens de Geometrie*; *par le* P. Ignace Gaſton Pardies, *de la Comp. de*
J. à Paris 1671. in *Twelves*.

N. 33. p. 640. 26. 1. *Vera Circuli & Hyperbolæ Quadratura, in propria ſua Proportionis*
Specie inventa & Demonſtrata, à Jac. Gregorio Scoto. Patavii. in *Quarto*.
Ib. p. 641. This Subject is here further conſidered, and the Area of an Hyperbole ex-
plained; by Mr. *J. Collins*.

N. 37. p. 732. 2. M. *Huygens* having publiſhed Animadverſions upon this Book, in the
Journal de Sçavans, 1668. Mr. *Gregory* here anſwers them. To this M. *Huy-*
N. 44. p. 882. *gens* replied in a following Journal of that Year; and Mr. *Gregory*, further to
elucidate the Controverſy, here returns a ſecond Anſwer.

N. 216. p. 65. 3. In the 48th Page of this Book, Mr. *Halley* has diſcovered and correct-
ed a ſmall Miſtake in the Logarithm of 10.

N. 35. p. 685. 27. *Geometriæ pars Univerſalis, Quantitatum Curvarum Tranſmutationi &*
Menſuræ inſerviens. Auth. Jac. Gregorio *Scoto*. Patavii 1668. in *Quarto*.

N. 37. p. 378. 28. *De Infinitis Spiralibus inverſis, Infinitiſque Hyperbolis, aliiſque Geome*
tricis. Auth. F. Stephano de Angelis *Veneto*. Patavii. in *Quarto*.

29. *Michaelis Angeli Ricci Exercitatio Geometrica.* Romæ. in 4*to*. Reprinted N. 37. *p.* 738.
at *London*, and annexed to *Mercator*'s *Logarithmotechnia*.

30. Renati Franc. Sluſii *Meſolabum. Cui acceſſit pars altera de Analyſi, &* N. 45. *p.* 903.
Miſcellanea. Leodii Eburonum 1668. in 4*to*.

31. *Elementa Geometriæ Planæ. Authore* Ægidio Franciſco de Gottignies N. 67. *p.* 2054.
Bruxellenſi. S. J. Romæ. 1669. in 12*mo*.

32. *Synopſis Geometrica; cum tribus Opuſculis, de Linea Sinuum & Cycloide;* N. 67. *p.* 2054.
de Maximis & Minimis, Centuria; & Synopſis Geometriæ Planæ. Auth. Honor.
Fabry. *S. J.* Lugduni Galliarum 1669. in 12*mo*.

33. *Lectiones* xiii. *Geometricæ; in quibus (præſertim) Generalia Linearum* N. 75. *p.* 2260.
Curvarum Symptomata declarantur, ab Iſaaco Barrow. Lond. 1669. in 4*to*. To
theſe Lectures the Author here adds ſeveral *Corollaries* and *Theorems*.

34. Eraſmi Bartholini *Selecta Geometrica.* Hauniæ. 1674. in 4*to*. N. 106. *p.* 137.

35. *Elemens des Mathematiques, ou Principes Generaux de toutes les Sciences* N. 126. *p.* 638.
qui ont les Grandeurs pour Object. Par J. P. à Paris 1675. in 4*to*.

36. *Nouvelle Methode en Geometrie pour les Sections des Superficies Coniques* N. 129. *p.* 745.
& Cylindriques; qui ont pour Baſe des Circles, ou des Paraboles, des Ellipſes,
& des Hyperboliques; par Ph. de la Hire. à Paris 1673. in 4*to*.

37. *De Cycloide & Sectionibus Conicis.* Ph. de la Hire. *Ibid. p.* 746.

38. The Geometrical Key, or Conſtruction of all Equations, Linear, N. 157. *p.* 349.
Quadratic, Cubic, and Biquadratic, by a Circle and one only Parabola;
by Mr. *Tho. Baker*.

39. *Exercitatio Geometrica de Dimenſione Figurarum. Auth.* Davide Gregorio. N. 163. *p.* 730.
Edinb. 1684. in 4*to*.

40. *Methodus Figurarum Lineis Rectis & Curvis comprehenſarum Quadraturas* N. 183. *p.* 185.
determinandi, Auth. J. Craig. Lond. 1685. in 4*to*. To this Tract the *Ibid. p.* 186.
Author here makes an Addition; and takes notice of ſome Remarks made on N. 235. *p.* 786.
it in the *Act. Lipſ.* by M. *Leibnitz*, and M. *J. Bernoulli*.

41. *Tractatus Mathematicus de Figurarum Curvilinearum Quadraturis &* N. 209. *p.* 113.
Locis Geometricis. Auth. J. Craig. Lond. 1693. in 4*to*.

42. *Tractatus de Principiis Calculi Exponentialis. Auth.* D. Bernoullio; N. 245. *p.* 374.
wherein a Miſtake is here diſcovered and corrected, by Mr. *Craig*.

43. *Analyſis Geometrica, ſive nova & vera Methodus Reſolvendi, tam Problemata* N. 257. *p.* 351.
Geometrica, quam Arithmeticas Quæſtiones. Pars prima, de Planis.
Auth. D. Antonio Hugone de Omerique *Sanlucarenſe*.

44. Stereometrical Propoſitions, variouſly applicable, but particularly intended N. 39. *p.* 785.
for *Gauging*; by *Rob. Anderſon. Lond.* 1668. in 8*vo*.

45. *Gauging* promoted; being an Appendix to Stereometrical Propoſitions; N. 47. *p.* 960.
Rob. Anderſon. Lond. 1669. in 8*vo*.

6. *Gauging* Epitomized; by *Mich. Dary. Lond.* 1699. upon one Folio N. 52. *p.* 1054.
ge.

. *Tabula Numerorum Quadratorum decies Millium, una cum ipſorum Late-* N. 82. *p.* 4050.
s *ab Unitate incipientibus, & Ordine Naturali uſque ad* 10000 *progredien-*
us. Lond. 1672.

8. The Deſcription and Uſe of two Arithmetick Inſtruments, *&c.* by N. 94. *p.* 6048.
r *Sam. Moreland. Lond.* 1673.

N. 139. p. 980. 49. Johannis Walliſii S. T. D. *Exercitationes tres: 1. De Cometarum Diſtantiis inveſtigandis. 2. De Rationum & Fractionum Reductione. 3. De Periodo Juliano.* Lond. 1678.

N. 38. p. 753. Ibid. p. 756. Ibid. p. 759. 50. *Logarithmotechnia* Nicholai Mercatoris. Lond. 1668. in *Quarto*. This Author's Method of ſquaring the Hyperbola, and of finding the Sum of the Logarithms is here improved by Dr. *Wallis*; and further explicated by the Author himſelf.

CHAP. II.

Trigonometry, Surveying.

A Chronographical Problem propoſed by Mr. Rich. Townley; *ſolved by Mr.* John Collins. N. 69. p. 2093. Mar. An. 1674.

I. Prob. *THE Diſtances of three Objects in the ſame Plane being given, as A, B, C; the Angles made at a fourth Place in the ſame Plane, as at S, are obſerved: the Diſtances from the Place of Obſervation to the reſpective Objects are required.*

The *Problem* hath ſix *Caſes*.

Fig. 57. *CASE* 1. If the Station be taken without the Triangle made by the Objects, but in one of the Sides thereof produced, as at S: find the Angle A C B; then in the Triangle A C S, all the Angles and the Side A C are known; whence either or both the Diſtances S A, or S C, may be found.

Fig. 58. *Caſe* 2. If the Station be in one of the Sides of the Triangle, as at S: then having the three Sides, A C, C B, B A, given, find the Angle C A B; then again in the Triangle S A B all the Angles, and the Side A B are known; whence may be found either A S, or S B, Geometrically; if you make the Angle C A D equal to the obſerved Angle C S B, and draw B S parallel to D A, you determine the Point of Station S.

Fig. 59. *Caſe* 3. If the three Objects lie in a right Line as A C D (ſuppoſe it done), and that a Circle paſſeth through the Station S, and the two Exterior Objects A, B: then is the Angle A B D equal to the obſerved Angle A S C (by 21. 3. *E.*) as inſiſting on the ſame Arch A D; and the Angle B A D in like manner equal to the obſerved Angle C S B: By this means the Point D is determined. Join D C, and produce the ſame, then a Circle paſſing through the Points A, B, D, interſects D C produced, at S, the Place of Station.

Calculation.] In the Triangle A B D, all the Angles and the Side A B, are known, whence may be found the Side A D.

Then in the Triangle C A D, the two Sides C A, and A D, are known, and their contained Angle C A D is known; whence may be found the Angles C D A, and A C D, the Complement whereof to a Semicircle is the Angle S C A: in which Triangle the Angles are now all known, and the Side A C: whence may be found either of the Diſtances S C, or S A.

Caſe 4: If the Station be without the Triangle made by the Objects, the Sum of the Angles obſerved is leſs than four Right Angles. The *Conſtruction* is the ſame as in the laſt *Caſe*, and the *Calculation* likewiſe; ſaving that you muſt make one Operation more; having the three Sides A C, C B, B A, thereby find the Angle C A B, which add to the Angle E A D, then you have the two Sides, *viz.* A C, being one of the Diſtances, and A D, (found as in the former *Caſe*) with their contained Angle C A D, given, to find the Angles C D A, and A C D, the Complement whereof to a Semicircle is the Angle S C A: Now in the Triangle S C A, the Angle at C being found, and at S obſerved, and given by Suppoſition, the other at A is likewiſe known, as being the Complement of the two former to a Semicircle, and the Side A C given; hence the Diſtances C S, or A S, may be found. Fig. 60.

Caſe 5. If the Place of Station be at ſome Point within the Plane of the Triangle, made by the three Objects, the *Conſtruction* and *Calculation* are the ſame as in the laſt, ſaving only, that inſtead of the obſerved Angle A S C, the Angle A B D is equal to the Complement thereof to a Semicircle, to wit, it is equal to the Angle A S D; both of them inſiſting on the ſame Arch A D: And in like manner the Angle B A D is equal to the Angle D S B, which is the Complement of the obſerved C S B; and in this *Caſe* the Sum of the three Angles obſerved, is equal to four Right Angles. Fig. 61.

In theſe three latter *Caſes* no uſe is made of the Angle obſerved between the two Objects, as A and B, that are made the Baſe-line of the *Conſtruction*; yet the ſame is of ready uſe for finding the third Diſtance or laſt Side ſought; as in the Triangle S A B, there is given the Diſtance A B, its oppoſite Angle equal to the Sum of the two obſerved Angles, and the Angle S A B attained, as in the fourth *Caſe*: Hence the third Side, or laſt Diſtance S B, may be found. Fig. 60.

And here it may be noted, that the three Angles C A S, A S B, S B C, are together equal to the Angle A C B; for the two Angles C S B and C B S are equal to E C B, as being the Complement of S C B to two Right Angles; and the like in the Triangle on the other Side. *Ergo*, &c.

Caſe 6. If the three Objects be A, B, C, and the Station at S, as before, it may happen, according to the former *Conſtructions*, that the Points C and D may fall cloſe together, and ſo a right Line joining them, ſhall be produced with Uncertainty; in ſuch *Caſe* the Circle may be conceived to paſs through the Place of Station at S, and any two of the Objects, as through B and C; wherein making the Angle D B C equal to the obſerved Angle A S C, and B C D equal to the Complement to 180 deg. of both the obſer- l Angles in D S B; thereby the Point D is determined, through which, d the Points C, B, the Circle is to be deſcribed; and joining D A, (pro- ed when Need requireth) where it interſects the Circle, as at S, is the ce of Station ſought. Fig. 61.

This *Problem* may be of good uſe for the due Situation of Sands and Rocks, t are within ſight of three Places upon Land, whoſe Diſtances are well own; or for *Chorographical Uſes*, &c. eſpecially now there is a Method of ſerving Angles nicely accurate by the Aid of a Teleſcope.

II. The

Three Chorographic Problems ſolved by a Member of the Philoſophical Society at Oxford. N. 177. p. 1231. *Dec.* An. 1685.

II. The three following *Problems* may occur at Sea, in finding the Diſtance and Poſition of Rocks, Sands, *&c.* from the Shore; or in Surveying the Sea-Coaſt; when only two Objects, whoſe Diſtance from each other is known, can be ſeen at one Station: But eſpecially they may be uſeful to one, that would make a Map of a Country by a Series of Triangles derived from one or more meaſured Baſes; which is the moſt exact Way of finding the Bearing and Diſtance of Places from each other, and thence their true Longitude and Latitude; and may conſequently occur to one that would in that Matter meaſure a Degree on the Earth.

Fig. 63. Prob. 1.] *There are two Objects B and C, whoſe Diſtance B C is known; and there are two Stations at A and E, where the Objects B, C, being viſible, and the Stations one from another, the Angles B A C, B A E, A E B, A E C, are known by Obſervation,* (which may be made with an ordinary *Surveying Semicircle,* or *Croſs-Staff*; or, if the Objects are beyond the View of the naked Eye, with a *Teleſcopic Quadrant*;) *To find the Diſtances or Lines A B, A C, A E, E C.*

Fig. 64. *Conſtruction.*] In each of the Triangles, B A E, C A E, two Angles at A, E, being known, the third is alſo known; then take any Line $\alpha\varepsilon$ at pleaſure, on which conſtitute the Triangles, $\beta\alpha\varepsilon$, $\alpha\varepsilon\gamma$, reſpectively equiangular to the Triangles B A E, A E C; join $\beta\gamma$: Then upon B C conſtitute the Triangles B C A, B C E, equiangular to the correſpondent Triangles $\beta\gamma\alpha$, $\beta\gamma\varepsilon$, join A E, and the Thing is manifeſtly done.

The Calculation.] Aſſuming $\alpha\varepsilon$, of any Number of Parts, in the Triangles $\alpha\beta\varepsilon$, $\alpha\gamma\varepsilon$, the Angles being given, the Sides $\alpha\beta$, $\alpha\gamma$, $\varepsilon\beta$, $\varepsilon\gamma$, may be found by Trigonometry: Then in the Triangle $\beta\alpha\gamma$, having the Angle $\beta\alpha\gamma$, and the Legs $\alpha\beta$, $\alpha\gamma$, we may find $\beta\gamma$. Then $\beta\gamma : BC :: \beta\alpha : BA :: \beta\varepsilon : BE :: \gamma\alpha : CA :: \gamma\varepsilon : CE$.

Fig. 65. Prob. 2.] *Three Objects B, C, D, are given, or (which is the ſame) the Sides and conſequent Angles of the Triangle B C D are given; alſo there are two Points or Stations A, E, ſuch that at A may be ſeen the three Points B, C, E, but not D, and at the Station E, may be ſeen A, C, D, but not B; that is, the Angles B A C, B A E, A E C, A E D, (and conſequently E A C, A E C) are known by Obſervation: To find the Lines A B, A C, A E, E C, E D.*

Fig. 66. *Conſtruction.*] Take any Line $\alpha\varepsilon$ at pleaſure, and at its Extremities make the Angles $\varepsilon\alpha\gamma$, $\varepsilon\alpha\beta$, $\alpha\varepsilon\gamma$, $\alpha\varepsilon\delta$, equal to the correſpondent obſerved Angles E A C, E A B, A E C, A E D. Produce $\beta\alpha$, $\delta\varepsilon$, till they meet in φ; join $\varphi\gamma$: then upon C B deſcribe (according to 33. 3. *E.*) a Segment of a Circle, that may contain an Angle $= \gamma\varphi\beta$; and upon C D deſcribe a Segment of a Circle capable of an Angle $= \gamma\varphi\delta$: Suppoſe F the common Section of theſe 2 Circles; join F B, F C, F D; then from the Point C, draw for the Lines C A, C E, ſo that the Angle F C A may be $= \varphi\gamma\alpha$, and F C E $= \varphi\gamma\varepsilon$: ſo A, E, the common Sections of C A, C E, with F B, F D, will be the Points required, from whence the reſt is eaſily deduced.

Calculation.] Aſſuming $\alpha\epsilon$ of any Number, in the Triangles $\alpha\gamma$, $\alpha\epsilon$, $\phi\epsilon$, all the Angles being given, with the Side $\alpha\epsilon$ aſſumed, the Sides $\alpha\gamma$, $\epsilon\gamma$, $\alpha\phi$, $\epsilon\phi$, will be known; then in the Triangle $\gamma\alpha\phi$, the Angle $\gamma\alpha\phi$, with the Legs $\alpha\gamma$, $\alpha\phi$, being known, the Angles $\alpha\phi\gamma$, $\alpha\gamma\phi$, with the Side $\phi\gamma$, will be known: Then as for the reſt of the Work, the Triangle B C D having all its Sides and Angles known, and the Angles B F C, B F D, being equal to the found $\beta\phi\gamma$, $\beta\phi\delta$; how to find F B, F C, F D, by *Calculation* (and alſo *Protraction*) has been already ſhewn above by Mr. *Collins*, as to all its *Caſes*. Sect. 1.

But it muſt here be noted, that if the Sum of the obſerved Angles B A E, A E D, is 180 deg. then A B, and E D, cannot meet, becauſe they are parallel, and conſequently the given Solution cannot take Place; for which Reaſon I here ſubjoin another.

Another Solution.] Upon B C deſcribe a Segment B A C, of a Circle, ſo that the Angle of the Segment may be equal to the obſerved Angle $\beta\alpha\gamma$, (which is ſhewn 33. 3. *E.*) and upon C D deſcribe a Segment C E D, of a Circle, capable of an Angle equal to the obſerved C E D; from C draw the Diameters of theſe Circles C G, C H; then upon C G deſcribe a Segment of a Circle G F C, capable of an Angle equal to the obſerved Angle A B C; likewiſe upon C H, deſcribe a Circle's Segment C F A, capable of an Angle equal to the obſerved Angle C A E: ſuppoſe F the common Section of the two laſt Circles H F C, G F C; join F H, cutting the Circle H E C in E; join alſo F G, cutting the Circle G A C in A: I ſay, that A, E, are the Points required. Fig. 67.

Demonſtration.] For the Angle B A C is $= \beta\alpha\gamma$, by *Conſtruction* of the Segment; alſo the Angles C E H, C A G, are right, becauſe each exiſts in a Semicircle: Therefore a Circle being deſcribed upon C F, as a Diameter, will paſs thro' E, A; therefore the Angle C A E $=$ C F E $=$ C E H $=$ (by *Conſtruction*) to the obſerved Angle $\gamma\alpha\epsilon$. In like manner the Angle C E A $=$ C F A $=$ C F G $=$ obſerv'd Angle $\gamma\epsilon\alpha$.

If the Stations A, E, fall in a right Line with the Point C; the Lines G A, H E, being parallel, cannot meet; but in this *Caſe* the Problem is indeterminate, and capable of infinite Solutions. For, as before, upon C G, deſcribe a Segment of a Circle capable of the obſerved Angle $\gamma\epsilon\alpha$, and upon C H, deſcribe a Segment capable of the obſerved Angle $\gamma\alpha\epsilon$: then through C draw a Line any way, cutting the Circles in A, E, theſe Points will anſwer the Queſtion.

Problem 3.] *Four Points, B, C, D, F, or the four Sides of a Quadrilateral,* Fig. 68.
h the Angles comprehended, are given; alſo there are two Stations A and E,
h, that at A, only B, C, E, are viſible, and at E, only A, D, F; that is, the
gles B A C, B A E, A E D, D E F, are given: To find the Places of the two
nts A, E; and conſequently the Lengths of the Lines AB, AC, A E, E D, E F.

onſtruction.] Upon B C (by 33. 3. *E.*) deſcribe a Segment of a Circle, that
y contain an Angle equal to the obſerved Angle B A C; then from C draw
Chord C M, or a Line cutting the Circle in M, ſo that the Angle B C M,
y be equal to the Supplement of the obſerved Angle B A E, *i. e.* its Reſidue

to 180 deg. In like manner on D F deſcribe a Segment of a Circle, capable of an Angle equal to the obſerved D E F; and from D draw the Chord D N, ſo that the Angle F D N may be equal to the Supplement of the obſerved Angle A E F; join M N, cutting the two Circles in A, E: I ſay, A, E, are the two Points required.

Demonſtration.] Join A B, A C, E D, E F; then is the Angle M A B = B C M (by 21. 3. *E.*) = Supplement of the obſerved Angle B A E, by *Conſtruction*; therefore the conſtructed Angle B A E, is equal to that which was obſerv'd. Alſo the Angle B A C, of the Segment, is, by Conſtruction of the Segment, equal to the obſerved Angle B A C. In like manner the conſtructed Angles A E F, D E F, are equal to the correſpondent obſerved Angles A E F, and D E F; therefore A, E, are the Points required.

Calculation.] In the Triangle B C M, the Angle B C M, (= Supplement of B A E) and Angle B M C, (= B A C) are given, with the Side B C; thence M C may be found: in like manner D N, in the Triangle D N F, may be found. But the Angle M C D (= B C D — B C M) is known, with its Legs M C, C D; therefore its Baſe M D, and Angle M D C, may be known. Therefore the Angle M D N (= C D F — C D M — E D N) is known, with its Legs M D, D N; thence M N, with the Angles D M N, D N M, will be known. Then the Angle C M A (= D M C + D M N) is known, with the Angle M A C (= M A B + B A C) and M C, before found; therefore M A, and A C, will be known. In like manner in the Triangle E D N, the Angles E, N, with the Side D N, being known, the Sides E N, E D, will be known; therefore A E (= M N — M A — E N) is known. Alſo in the Triangle A B C, the Angle A, with its Sides B C, C A, being known, the Side A B will be known, with the Angle B C A; ſo in the Triangle E F D, the Angle E, with the Sides E D, D F, being known, E F will be found, with the Angle E D F. Laſtly, in the Triangle A C D, the Angle A C D (= B C D — B C A) with its Legs A C, C D, being known, the Side A D will be known; and in like manner E C, in the Triangle E D C.

Note, That in this *Problem*, as alſo in the firſt and ſecond, if the two Stations fall in a right Line with either of the given Objects, the *Locus* of A or E being a Circle, the particular Point of A or E cannot be determined from the Things given.

As to the other *Caſes* of this third *Problem*, wherein A and E may ſhift Places, *i. e.* only D, F, E, may be viſible at A, and only A, B, C, at E; or wherein B, D, E, may be viſible at A, and only C, F, A, at E; or wherein A may be on one ſide of the Quadrilateral, and E on the other; or one of the Stations within the Quadrilateral, and the other without it; I preſume that the *Surveyor* will eaſily direct himſelf, by what has been already ſaid.

The *Solution* of this third *Problem* is general, and ſerves alſo for both the precedent. For ſuppoſe C, D, the ſame Point in the laſt Figure, and it gives the *Solution* of the ſecond *Problem*; but if B, C, be ſuppoſed the ſame Points with D, F, by proceeding as in the laſt, you may directly ſolve the firſt *Problem*.

III. The

An Error of Common Surveyors, in comparing Surveys taken at long Intervals of Time with the magnetic Needle, demonſtrated; by Mr. William Molyneux. N. 230. *p.* 625. July An. 1697.

III. The Variation of the magnetic Needle is ſo commonly known, that I need not inſiſt much on the Explication thereof; 'tis certain that the true ſolar Meridian, and the Meridian ſhewn by a Needle, agree but in very few Places of the World; and this too, but for a little Time (if a Moment) together; the Difference between the true Meridian and magnetic Meridian perpetually varying and changing in all Places, and at all Times; ſometimes to the Eaſtward, and ſometimes to the Weſtward:

On which Account 'tis impoſſible to compare two *Surveys* of the ſame Place, taken at diſtant Times, by magnetic Inſtruments (ſuch as the Circumferentor, by which the *Down Survey*, or Sir *William Petty*'s *Survey* of *Ireland* was taken) without due Allowance be made for this Variation. To which purpoſe, we ought to know the Difference between the magnetic Meridian and true Meridian, at that time of the *Down Survey*, and the ſaid Difference at the Time when we make a *New Survey* to compare with the *Down Survey*.

But here I would not be underſtood, as if I propoſed hereby to ſhew, that a Map of the ſame Place, taken by magnetic Inſtruments at never ſo diſtant Times, ſhould not at one Time give the ſame Figure and Contents as at another Time. This certainly it will do moſt exactly, the Variation of the Needle having nothing to do either in the Shape or Contents of the *Survey*. All that is affected thereby, is the Bearings of the Lines run by the Chain, and the Boundaries between Neighbours. And how this may cauſe a conſiderable Error (unleſs due Allowance be made for it) is what I ſhall prove moſt fully.

In order to which, let us ſuppoſe that about the Year 1657 (at which time the *Down Survey* was taken) the magnetic Meridian and true Meridian did agree at *Dublin*, or pretty nigh all over *Ireland*; that is to ſay, that there was no Variation. And indeed by Experiment it was at that time found, as I am well aſſured, that at *Dublin* it was hardly half a Degree.

Let us ſuppoſe, that in the Year 1695 the Variation was 7 Degrees from the North to the Weſtward: That it was really ſo, I believe, I am pretty well aſſured, from an Experiment made by *myſelf* with all Diligence. But this is not material; let us now only ſuppoſe it.

Let A, B, repreſent the Survey of two Town Lands, one in the Poſſeſſion of A, and the other in Poſſeſſion of B, taken by the *Down Survey*, *Anno* 1657, when there was no Variation. *Fig.* 69.

Let the Line N S, running through the Point P, be the true Meridian, and conſequently the magnetic Meridian alſo at that Time, becauſe of the ſuppoſed no Variation; and let this Line N S, be alſo the Boundary between e two Town Lands A, and B.

In the Year 1695, when the Variation is 7 Degrees from the North to the eſtward, B having a Map of the *Down Survey*, and being ſuſpicious that his eighbour A had encroached on him by a Ditch P Q, employs a Surveyor to quire into the Matter: The Surveyor finds by his Map, that the Boundary tween B and his Neighbour A, run from the Point P, through a Meadow, ectly according to the magnetic Meridian S P N; but obſerving the Ditch Q caſt up much to the Eaſtward of the preſent magnetic Meridian, he ncludes that A has encroached upon B, and that the Ditch ought to have been caſt up along the Line Pq, the Angle QPq being an Angle of 7 deg. that is, the preſent Variation of the Needle, and the Line Pq, the preſent magnetic Meridian; for which Variation not making any Allowance, he poſitively determines that B has all the Land in the Triangle QPq, more than he ought to have; and that his Ditch ought to run along the Line Pq.

'Tis true indeed, if the *Surveyor* go the whole Surround of the Land A, and B, he will find their Figure and Contents exactly agreeable to the Map here expreſſed. But then the Bearings of the Lines are all 7 deg. different from the Bearings in the Map, and they will run in and out upon the adjacent neighbouring Lands, and cauſe endleſs Differences between their Poſſeſſors; as is manifeſt from the Figure: Wherein the prick'd Lines repreſent the Diſagreement in the Bearings of the Lines, protracted from the Point P; and we ſee A encroaching upon his Neighbours on the Weſtward, as he encroaches upon B, and B's Eaſtward Neighbours encroaching on him, and ſo forward and clear round. Whereas by a due Allowance for the Variation of the Needle, all this Confuſion and Diſagreement is avoided, and every thing hits right.

Thus, for Inſtance, in the Caſe before us, knowing that the magnetic Variation has cauſed the preſent magnetic Meridian to fall in the Line $nqPs$, 7 deg. from the North to the Weſtward; to reduce this to the magnetic Meridian at the Time of the *Down Survey*, I muſt make the Meridian of my Map to fall 7 deg. to the Eaſtward of my magnetic Meridian, as we ſee the Line PQ falls 7 deg. to the Eaſtward of the Line Pq.

What is here ſaid on Suppoſition that the Magnet had no Variation at the Time of the firſt Survey taken, and that it had 7 deg. Variation Weſtward at the Time of the ſecond Survey, may eaſily be accommodated to the Suppoſal of any other Variations at the firſt and ſecond Surveys, *mutatis mutandis*; for knowing the Variations we know their Difference; and if we know their Difference, this gives us the Angle QPq, by which we reduce them to each other. The beſt Way therefore to make Maps invariable, conſtant, and everlaſting, were for the Surveyors, who uſe magnetic Inſtruments, to make always Allowance for the magnetic Variation, and to protract and lay down Plats by the true Meridian.

Perhaps it may be objected, That Surveys may be taken without magnetic Inſtruments, and that therefore this Error ariſing from the magnetic Variation, and Change of the Bearing of Lines, may be avoided. To which I anſwer, Firſt, That granting a Survey may be taken without magnetic Inſtruments, this is nothing againſt what we have laid down, relating to Surveys that are taken with magnetic Inſtruments, as the *Down Survey* actually was, and moſt *Surveys* at preſent actually are taken therewith. Secondly, Tho' a Survey may be taken truly without magnetic Inſtruments, ſo as to ſhew the exact Angles and Lines of the Plat, and conſequently the true Contents; yet this will not give the true Bearings of the Lines, or ſhew my Poſition in relation to my Neighbours, or other Parts of the Country. This muſt be ſupplied by the Magnet, or ſomething equivalent thereto, as finding a true meridian Line on your Land by celeſtial Obſervation. And I doubt not but the ancient *Ægyptians*, before the Diſcovery of the Magnet, were forced to ſome ſuch Expedient in their

Surveys and Applotments of Lands, between Neighbour and Neighbour, after the *Inundations* of the *Nile*; which, we are told, gave the firſt Original to *Geometry* and *Surveying:* Abſolute Neceſſity and Uſe having introduced theſe, as Delight and Diverſion introduced *Aſtronomy* amongſt the *Chaldeans*.

And this brings me to another Objection, which may be made againſt the Inſtance before laid down: It may be ſaid, That certainly the Surveyor which B employed was very ignorant, who would chuſe to judge of the Line PQ rather by its Bearing, than by determining the Point Q by meaſuring from H and G. To this I anſwer, What if both the Points H and G were vaniſhed ſince the *Down Survey* was taken? What if the whole Face of the Country were changed, ſave only the Point P, and the Line PQ? How ſhall the Surveyor then judge of the Line PQ, but by its Bearing? That this is no extravagant Suppoſition, we have an Example in *Ægypt* abovementioned, where the *Nile* lays all flat before it, and ſo uniformly covers all with Mud, that there is no Diſtinction. In ſuch a Caſe your Bearing muſt certainly help you out; there is no other Way.

But I anſwer, ſecondly, To ſay that the Surveyor might have determined the Point Q by Admeaſurement from G and H, or any other adjoining noted Points, as from F, K, I, *&c.* 'tis very true; but then 'tis againſt our *Suppoſition*. I am upon ſhewing an Error that ariſes from judging of the Line PQ by *magnetic Bearing*; and to tell me that this might be avoided by another Way, is to ſay nothing. I myſelf ſhew how it may be avoided, by allowing for the Variation; but ſtill it is an Error till it be avoided.

But, thirdly, If B's Surveyor do not allow for the Variation of the Needle, he will never exactly determine even the Points G, F, H, K, *&c.* or any other Points in the Plat; but inſtead thereof, will fall on the Points *g*, *h*, *f*, *k*.

From what has been laid down, we may ſee the abſolute Neceſſity of allowing for the Variation of the Magnet, in comparing old Surveys with new ones; for want of which, great Diſputes may ariſe between neighbouring Proprietors of Lands: And it were to be wiſhed, that our honourable and learned Judges would take this Matter into their Conſideration, whenever any Buſineſs of this Kind comes before them.

IV. I have invented a *Level* with a Tube, with Glaſſes and a Thread, hanging between four Points, with a Weight in a Box ſo contrived, that as ſoon as the Inſtrument is ſet down, you have your *Point* of Horizon with a great deal of Exactneſs. I am making another, which playeth on one Steel Point, ſtanding on a Diamond. *A new Level; by Mr.* Butterfield. N. 141. *p.* 1026. Sept. An. 1678.

V. *An Account of a Book omitted; viz.* The Art of Levelling; by *M.* *Mariotte.* N. 74. *p.* 2237.

CHAP. III.

OPTICS.

A new Theory about Light and Colours; by Mr. Iſaac Newton. N. 80. *p.* 3075. Feb. An. 1672.

I. IN the Year 1666 (at which time I applied myſelf to the Grinding of Optic Glaſſes of other Figures than Spherical) I procured me a Triangular Glaſs-Priſm, to try therewith the celebrated Phænomena of Colours. And in order thereto, having darkened my Chamber, and made a ſmall Hole in my Window-ſhuts, to let in a convenient Quantity of the Sun's Light, I placed my Priſm at its Entrance, that it might be thereby refracted to the oppoſite Wall. It was at firſt a very pleaſing Divertiſement, to view the vivid and intenſe Colours produced thereby; but after a while applying myſelf to conſider them more circumſpectly, I became ſurpriſed to ſee them in an oblong Form; which, according to the received Laws of Refractions, I expected ſhould have been circular. They were terminated at the Sides with ſtreight Lines, but at the Ends the Decay of Light was ſo gradual, that it was difficult to determine juſtly what was their Figure, yet they ſeemed Semicircular.

Comparing the Length of this coloured *Spectrum* with its Breadth, I found it about five Times greater; a Diſproportion ſo extravagant, that it excited me to a more than ordinary Curioſity of examining from whence it might proceed. I could ſcarce think, that the various Thickneſs of the Glaſs, or the Termination with Shadow or Darkneſs, could have any Influence on Light to produce ſuch an Effect; yet I thought it not amiſs, firſt to examine thoſe Circumſtances, and ſo tried what would happen by tranſmitting Light through Parts of the Glaſs of divers Thickneſſes, or through Holes in the Window of divers Bigneſſes, or by ſetting the Priſm without, ſo that the Light might paſs through it, and be refracted, before it was terminated by the Hole: But I found none of thoſe Circumſtances material. The Faſhion of the Colours was in all theſe Caſes the ſame.

Then I ſuſpected, whether by any Unevenneſs in the Glaſs, or other contingent Irregularity, theſe Colours might be thus dilated. And to try this, I took another Priſm like the former, and ſo placed it, that the Light paſſing thro' them both, might be refracted contrary ways, and ſo by the latter returned into that Courſe from which the former had diverted it: For by this means I thought the regular Effects of the firſt Priſm would be deſtroyed by the ſecond Priſm, but the irregular Ones more augmented, by the Multiplicity of Refractions. The Event was, that the Light, which by the firſt Priſm was diffuſed into an oblong Form, was by the ſecond reduced into an orbicular One, with as much Regularity as when it did not at all paſs through them.

N. 83. *p.* 4061. *Fig.* 70.

That this Experiment may be better apprehended; let E G deſign the Window; F, the Hole in it, thro' which the Light arrives at the Priſms; A B C, the firſt Priſm, which refracts the Light towards P T, painting there the Colour in an Oblong, and $\alpha\beta\gamma$, the ſecond Priſm, which refracts back again the Rays to Q, where the long Image P T is contracted into a round one. I ſuppoſe the Plane $\alpha\gamma$ parallel to B C, and $\beta\gamma$ to A C, that the Rays may be equally refracted contrary ways in both Priſms. The Priſms alſo muſt be placed very near to one another; for if their Diſtance be ſo great, that the Colours begin to appear in the Light, before its Incidence on the ſecond Priſm, thoſe Colours will not be deſtroyed by the contrary Refractions of that Priſm. And if a *Lens* be placed in the Hole F, or immediately after the Priſms, ſo that its *Focus* be at the Image Q, or P T, the Perimeter of the Image Q, and the ſtraight Sides of the Image P T, will become much better defined than otherwiſe. So that, whatever was the Cauſe of that Length, 'twas not any contingent Irregularity. N. 80. p. 3076. Feb. An. 1672.

I then proceeded to examine more critically, what might be effected by the Difference of the Incidence of Rays coming from divers Parts of the Sun; and to that end, meaſured the ſeveral Lines and Angles belonging to the Image. Its Diſtance from the Hole or Priſm was 22 Feet; its utmoſt Length 13 ¼ Inches; its Breadth 2 ⅝; the Diameter of the Hole ¼ of an Inch. The Angle which the Rays, tending towards the middle of the Image, made with thoſe Lines, in which they would have proceeded without Refraction, was 44 deg. 56 min. and the Vertical Angle of the Priſm, 63 deg. 12 min. Alſo the Refractions on both Sides the Priſm, that is, of the Incident and Emergent Rays, were, as near as I could make them, equal, and conſequently about 54 deg. 4 min. And the Rays fell perpendicularly upon the Wall. Now ſubducting the Diameter of the Hole from the Length and Breadth of the Image, there remains 13 Inches in the Length, and 2 ⅜ the Breadth, comprehended by thoſe Rays, which paſſed through the Center of the ſaid Hole; and conſequently the Angle of the Hole, which that Breadth ſubtended, was about 31 min. anſwerable to the Sun's Diameter; but the Angle which its Length ſubtended, was more than five ſuch Diameters, namely 2 deg. 49 min.

Having made theſe Obſervations, I firſt computed from them the refractive Power of that Glaſs, and found it meaſured by the Ratio of the Sines, 20 to 31; and then by that Ratio I computed the Refractions of two Rays flowing from oppoſite Parts of the Sun's *Diſcus*, ſo as to differ 31 min. in their Obliquity of Incidence, and found that the emergent Rays ſhould have comprehended an Angle of about 31 min. as they did before they were incident.

But becauſe this Computation was founded on the Hypotheſis of the Protionality of the Sines of Incidence and Refraction, which though by my n Experience I could not imagine to be ſo erroneous, as to make that An- but 31 min. which in reality was 2 deg. 49 min. yet my Curioſity cauſed again to take my Priſm: And having placed it at my Window, as before, bſerved, that by turning it a little about its Axis to and fro, ſo as to vary Obliquity to the Light, more than an Angle of 4 or 5 Degrees, the Co- rs were not thereby ſenſibly tranſlated from their Place on the Wall; and ſequently by that Variation of Incidence, the Quantity of Refraction was ſenſibly varied. By this Experiment therefore, as well as by the former

Computation, it was evident, that the Difference of the Incidence of Rays, flowing from divers Parts of the Sun, could not make them after Decuſſation diverge at a ſenſibly greater Angle, than that at which they before converged; which being, at moſt, but about 31 or 32 min. there ſtill remained ſome other Cauſe to be found out, from whence it could be 2 deg. 49 min.

Then I began to ſuſpect, whether the Rays, after their Trajection through the Priſm, did not move in curve Lines, and according to their more or leſs Curvity tend to divers Parts of the Wall. And it increaſed my Suſpicion, when I remember'd that I had often ſeen a Tennis-Ball ſtruck with an oblique Racket, deſcribe ſuch a curve Line. For, a circular as well as a progreſſive Motion being communicated to it by that Stroke, its Parts on that Side, where the Motions conſpire, muſt preſs and beat the contiguous Air more violently than on the other, and there excite a Reluctancy and Re-action of the Air proportionably greater. And for the ſame Reaſon, if the Rays of Light ſhould poſſibly be globular Bodies, and by their oblique Paſſage out of one Medium into another acquire a circulating Motion, they ought to feel the greater Reſiſtance from the ambient Æther on that Side where the Motions conſpire, and thence be continually bowed to the other. But notwithſtanding this plauſible Ground of Suſpicion, when I came to examine it, I could obſerve no ſuch Curvity in them. And beſides (which was enough for my purpoſe) I obſerved, that the Difference betwixt the Length of the Image, and the Diameter of the Hole through which the Light was tranſmitted, was proportionable to their Diſtance.

The gradual Removal of theſe Suſpicions at length led me to the *Experimentum Crucis*, which was this; I took two Boards, and placed one of them cloſe *behind the Priſm at the Window*, ſo that the Light might paſs through a ſmall Hole, made in it for the purpoſe, and fall on the other Board, which I placed at about 12 Feet diſtance, having firſt made a ſmall Hole in it alſo for ſome of that incident Light to paſs through. Then I placed another Priſm behind this ſecond Board, ſo that the Light trajected through both the Boards might paſs through that alſo, and be again refracted before it arrived at the Wall. This done, I took the firſt Priſm in my Hand, and turned it to and fro ſlowly about its Axis, ſo much as to make the ſeveral Parts of the Image, caſt on the ſecond Board, ſucceſſively paſs through the Hole in it, that I might obſerve to what Places on the Wall the ſecond Priſm would refract them. And I ſaw by the Variation of thoſe Places, that the Light, tending to that End of the Image towards which the Refraction of the firſt Priſ— was made, did in the ſecond Priſm ſuffer a Refraction conſiderably greater tha the Light tending to the other End. And ſo the true Cauſe of the Lengt Ibid. p. 3081. of that Image was detected to be no other, than that *Light* is not ſimilar c homogeneal, but conſiſts of *difform Rays, ſome of which are more refrangib than others*; ſo that without any Difference in their Incidence on the ſam Medium, ſome ſhall be more *refracted* than others; and therefore that, according to their *particular Degrees of Refrangibility*, they were tranſmitte through the Priſm to divers Parts of the oppoſite Wall.

I ſhall now proceed to acquaint you with another more notable *Difformity* in its Rays, wherein the Origin of Colours is unfolded: Concerning which I ſhall lay down the Doctrine firſt; and then, for its Examination, give you an Inſtance or two of the Experiments, as a Specimen of the reſt.

The Doctrine you will find comprehended and illuſtrated in the following Propoſitions.

1. As the Rays of Light differ in Degrees of Refrangibility, ſo they alſo differ in their Diſpoſition to exhibit this or that particular Colour. Colours are not Qualifications of Light, derived from Refractions, or Reflections of natural Bodies (as 'tis generally believed) but original and connate Properties, which in divers Rays are divers. Some Rays are diſpoſed to exhibit a Red Colour, and no other; ſome a Yellow, and no other; ſome a Green, and no other; and ſo of the reſt. Nor are there only Rays proper and particular to the more eminent Colours, but even to all their intermediate Gradations.

2. To the ſame Degree of Refrangibility ever belongs the ſame Colour, and to the ſame Colour ever belongs the ſame Degree of Refrangibility. The leaſt refrangible Rays are all diſpoſed to exhibit a Red Colour; and contrarily, thoſe Rays which are diſpoſed to exhibit a Red Colour, are all the leaſt refrangible: So the moſt refrangible Rays are all diſpoſed to exhibit a deep Violet Colour; and contrarily, thoſe which are apt to exhibit ſuch a Violet Colour, are all the moſt refrangible: And ſo to all the intermediate Colours in a continued Series belong intermediate Degrees of Refrangibility. And this Analogy betwixt Colours and Refrangibility is very preciſe and ſtrict; the Rays always either exactly agreeing in both, or proportionally diſagreeing in both.

3. The Species of Colour, and Degree of Refrangibility proper to any particular Sort of Rays, is not mutable by Refraction, nor by Reflection from natural Bodies, nor by any other Cauſe that I could yet obſerve. When any one Sort of Rays hath been well parted from thoſe of other Kinds, it hath afterwards obſtinately retained its Colour, notwithſtanding my utmoſt Endeavours to change it. I have refracted it with Priſms, and reflected it with Bodies, which in Day-light were of other Colours; I have intercepted it with the coloured Film of Air, interceding two compreſſed Plates of Glaſs, tranſmitted it through coloured Mediums, and through Mediums irradiated with other Sorts of Rays, and diverſly terminated it; and yet could never produce any new Colour out of it. It would by contracting or dilating become more briſk, or faint, and, by the loſs of many Rays, in ſome Caſes very obſcure and dark; but I could never ſee it changed in Specie.

4. Yet ſeeming Tranſmutations of Colours may be made, where there is any Mixture of divers ſorts of Rays: For in ſuch Mixtures, the component Colours appear not; but by their mutual allaying each other, conſtitute a middling Colour. And therefore, if by Refraction, or any other of the aforeſaid Cauſes, the difform Rays, latent in ſuch a Mixture, be ſeparated, there ſhall emerge Colours different from the Colour of the Compoſition. Which Colours are not new generated, but only made apparent by being parted; for

if they be again intirely mixed and blended together, they will again compoſe that Colour, which they did before Separation. And for the ſame Reaſon, Tranſmutations made by the convening of divers Colours are not real; for when the difform Rays are again ſevered, they will exhibit the very ſame Colours which they did before they entered the Compoſition; as you ſee Blue and Yellow Powders, when finely mixed, appear to the naked Eye, Green; and yet the Colours of the component Corpuſcles are not thereby really tranſmuted, but only blended. For when viewed with a good Microſcope, they ſtill appear Blue and Yellow interſperſedly.

5. There are therefore two ſorts of Colours, the one Original and Simple, the other compounded of theſe. The original or primary Colours are, Red, Yellow, Green, Blue, and a Violet-Purple, together with Orange, Indico, and an indefinite Variety of intermediate Gradations.

6. The ſame Colours in Specie with theſe primary Ones, may be alſo produced by Compoſition. For a Mixture of Yellow and Blue makes Green; of Red and Yellow makes Orange; of Orange and Yellowiſh Green makes Yellow. And in general, if any two Colours be mixed, which in the Series of thoſe generated by the Priſm are not too far diſtant one from another, they by their mutual Alloy compound that Colour, which in the ſaid Series appeareth in the Midway between them. But thoſe which are ſituated at too great a Diſtance, do not ſo. Orange and Indico produce not the intermediate Green, nor Scarlet and Green the intermediate Yellow.

7. But the moſt ſurprizing and wonderful Compoſition was that of Whiteneſs. There is no one ſort of Rays which alone can exhibit this. 'Tis ever compounded; and to its Compoſition are requiſite all the aforeſaid primary Colours, mixed in a due Proportion. I have often with Admiration beheld, that all the Colours of the Priſm being made to converge, and thereby to be again mixed, as they were in the Light before it was incident upon the Priſm, reproduced Light, entirely and perfectly White, and not at all ſenſibly differing from a direct Light of the Sun, unleſs when the Glaſſes I uſed were not ſufficiently clear; for then they would a little incline it to their Colour.

8. Hence therefore it comes to paſs, that Whiteneſs is the uſual Colour of Light; for Light is a confuſed Aggregate of Rays indued with all ſorts of Colours, as they were promiſcuouſly darted from the various Parts of luminous Bodies. And of ſuch a confuſed Aggregate, as I ſaid, is generated Whiteneſs, if there be a due Proportion of the Ingredients; but if any one predominate, the Light muſt incline to that Colour; as it happens in the blue Flame of Brimſtone; the yellow Flame of a Candle; and the various Colours of the Fixed Stars.

9. Theſe Things conſidered, the Manner how Colours are produced by the Priſm is evident. For, of the Rays, conſtituting the incident Light, ſince thoſe which differ in Colour proportionally differ in Refrangibility, they by their unequal Refractions muſt be ſevered and diſperſed into an oblong Form in an orderly Succeſſion, from the leaſt refracted Scarlet, to the moſt refracted Violet. And for the ſame Reaſon it is, that Objects when looked upon through a Priſm, appear coloured. For the difform Rays, by their unequal

Refractions, are made to diverge towards ſeveral Parts of the *Retina*, and there expreſs the Images of things coloured, as in the former Caſe they did the Sun's Image upon a Wall. And by this Inequality of Refractions, they become not only coloured, but alſo very confuſed and indiſtinct.

10. Why the Colours of the Rainbow appear in falling Drops of Rain, is alſo from hence evident. For thoſe Drops which refract the Rays, diſpoſed to appear Purple, in greateſt Quantity to the Spectator's Eye, refract the Rays of other ſorts ſo much leſs, as to make them paſs beſide it; and ſuch are the Drops on the Inſide of the primary Bow, and on the Outſide of the ſecondary or exterior One. So thoſe Drops, which refract in greateſt Plenty the Rays, apt to appear Red, toward the Spectator's Eye, refract thoſe of other ſorts ſo much more, as to make them paſs beſide it; and ſuch are the Drops on the exterior Part of the primary, and interior Part of the ſecondary Bow.

11. The odd Phænomena of an Infuſion of *Lignum Nephriticum*, Leaf-Gold, Fragments of coloured Glaſs, and ſome other tranſparently coloured Bodies, appearing in one Poſition of one Colour, and of another in another, are on theſe Grounds no longer Riddles. For thoſe are Subſtances apt to reflect one ſort of Light, and tranſmit another; as may be ſeen in a dark Room, by illuminating them with ſimilar or uncompounded Light. For then they appear of that Colour only, with which they are illuminated; but yet in one Poſition more vivid and luminous than in another, accordingly as they are diſpoſed more or leſs to reflect or tranſmit the incident Colour.

12. From hence alſo is manifeſt the Reaſon of an unexpected Experiment, which Mr. *Hook*, ſomewhere in his *Micrography*, relates to have made with two Wedge-like tranſparent Veſſels, filled the one with a Red, the other with a Blue Liquor: namely, that though they were ſeverally tranſparent enough, yet both together became opake; for if one tranſmitted only Red, and the other only Blue, no Rays could paſs through both.

13. I might add more Inſtances of this Nature, but I ſhall conclude with this general one: That the Colours of all natural Bodies have no other Origin than this, that they are variouſly qualified to reflect one ſort of Light in greater Plenty than another. And this I have experimented in a dark Room, by illuminating thoſe Bodies with uncompounded Light of divers Colours. For by that means any Body may be made to appear of any Colour. They have there no appropriate Colour, but ever appear of the Colour of the Light caſt upon them; but yet with this Difference, that they are moſt briſk and vivid in the Light of their own Day-light Colour. *Minium* appeareth there of any Colour indifferently, with which it is illuſtrated, but yet moſt luminous in Red; and ſo Biſe appeareth indifferently of any Colour, with which it is illuſtrated, but yet moſt luminous in Blue: and therefore *Minium* reflecteth Rays of any Colour, but moſt copiouſly thoſe endowed with Red, and conſequently when illuſtrated with Day-light, that is, with all ſorts of Rays promiſcuouſly blended, thoſe qualified with Red ſhall abound moſt in the reflected Light, and by their Prevalence cauſe it to appear of that Colour. And for the ſame Reaſon Biſe, reflecting Blue moſt copiouſly, ſhall appear Blue by the Exceſs of thoſe Rays in its reflected Light; and the like of other Bodies. And that this is the en-

tire and adequate Cauſe of their Colours, is manifeſt, becauſe they have no Power to change or alter the Colours of any ſort of Rays incident apart, but put on all Colours indifferently, with which they are enlighten'd.

Theſe things being ſo, it can be no longer diſputed, whether there be Colours in the Dark, or whether they be the Qualities of the Objects we ſee; no, nor perhaps, whether Light be a Body. For, ſince Colours are the Qualities of Light, having its Rays for their intire and immediate Subject, how can we think thoſe Rays Qualities alſo, unleſs one Quality may be the Subject of, and ſuſtain another? which, in effect, is to call it Subſtance. We ſhould not know Bodies for Subſtances, were it not for their ſenſible Qualities; and the Principal of thoſe being now found due to ſomething elſe, we have as good Reaſon to believe that to be a Subſtance alſo.

Beſides, Who ever thought any Quality to be a heterogeneous Aggregate, ſuch as Light is diſcovered to be? But to determine more abſolutely what Light is, after what Manner refracted, and by what Modes or Actions it produceth in our Minds the Phantaſms of Colours, is not ſo eaſy: And I ſhall not mingle Conjectures with Certainties.

Reviewing what I have written, I ſee the Diſcourſe itſelf will lead to divers Experiments ſufficient for its Examination: And therefore I ſhall not trouble you further than to deſcribe one of thoſe which I have already inſinuated.

In a darkened Room make a Hole in the Shut of a Window, whoſe Diameter may conveniently be about a third Part of an Inch, to admit a convenient Quantity of the Sun's Light: And there place a clear and colourleſs Priſm, to refract the entring Light towards the further part of the Room; which, as I ſaid, will thereby be diffuſed into an oblong coloured Image. Then place a Lens of about 3 Foot Radius (ſuppoſe a broad Object-Glaſs of a three Foot Teleſcope) at the Diſtance of about 4 or 5 Feet from thence, through which all thoſe Colours may at once be tranſmitted, and made by its Refraction to convene at a further Diſtance of about 10 or 12 Feet. If at that Diſtance you intercept this Light with a Sheet of white Paper, you will ſee the Colours converted into Whiteneſs again by being mingled. But it is requiſite that the Priſm and Lens be placed ſteddy, and that the Paper, on which the Colours are caſt, be moved to and fro; for by ſuch Motion you will not only find at what Diſtance the Whiteneſs is moſt perfect, but alſo ſee how the Colours gradually convene and vaniſh into Whiteneſs; and afterwards, having croſſed one another in that Place where they compound Whiteneſs, are again diſſipated and ſevered, and in an inverted Order retain the ſame Colours which they had before they entred the Compoſition. You may alſo ſee, that if any of the Colours at the Lens be intercepted, the Whiteneſs will be changed into the other Colours. And therefore, that the Compoſition of Whiteneſs be perfect, Care muſt be taken that none of the Colours fall beſide the Lens.

Fig. 71. Thus in the Deſign of this Experiment, A B C expreſſeth the Priſm ſet endwiſe to ſight, cloſe by the Hole F, of the Window E G. Its vertical Angle A C B may conveniently be about 63 deg. M N deſigneth the Lens. Its Breath 2 ½, or 3 Inches. S F, one of the ſtraight Lines, in which *difform Rays* may be conceived to flow ſucceſſively from the Sun. F P, and F R, two

of thoſe Rays unequally refracted, which the Lens makes to converge towards Q, and after Decuſſation to diverge again. And H I, the Paper, at divers Diſtances, on which the Colours are projected; which in Q conſtitute Whiteneſs, but are Red and Yellow in R, r, and ϱ, and Blue and Purple in P, p, and ϖ.

If you proceed further to try the Impoſſibility of changing any uncompounded Colour (which I have aſſerted in the third and thirteenth Propoſitions) 'tis requiſite that the Room be made very dark, leſt any ſcattering Light, mixing with the Colour, diſturb and allay it, and render it compound, contrary to the Deſign of the Experiment. 'Tis alſo requiſite, that there be a perfecter Separation of the Colours, than, after the Manner above deſcribed, can be made by the Refraction of one ſingle Priſm; and how to make ſuch further Separations, will ſcarce be difficult to them that conſider the diſcovered Laws of Refractions. But if Trial ſhall be made with Colours not thoroughly ſeparated, there muſt be allowed Changes proportionable to the Mixture. Thus, if compound Yellow Light fall upon the Blue Biſe, the Biſe will not appear perfectly Yellow, but rather Green; becauſe there are in the yellow Mixture many Rays endued with Green, and Green being leſs remote from the uſual Blue Colour of Biſe than Yellow, is the more copiouſly reflected by it.

In like manner, if any one of the priſmatic Colours, ſuppoſe Red, be intercepted, on Deſign to try the aſſerted Impoſſibility of reproducing that Colour out of the others which are pretermitted; 'tis neceſſary, either that the Colours be very well parted before the Red be intercepted, or that together with the Red, the neighbouring Colours, into which any Red is ſecretly diſperſed (that is, the Yellow, and perhaps Green too) be intercepted; or elſe, that Allowance be made for the emerging of ſo much Red out of the Yellow-Green, as may poſſibly have been diffuſed, and ſcatteringly blended in thoſe Colours. And if theſe Things be obſerved, the new Production of Red, or any intercepted Colour, will be found impoſſible.

Some Experiments propoſed in relation to this Theory. N. 83. *p.* 4059. May, An. 1672.

II. 1. To contract the Beams of the Sun without the Hole of the Window, and to place the Priſm between the Focus of the Lens and the Hole.

2. To cover over both Ends of the Priſm with Paper at ſeveral Diſtances from the Middle; or with moveable Rings, to ſee how that will vary or divide the Length of the Figure.

3. To move the Priſm ſo, as the End may turn about, the Middle being ſteddy.

4. To move the Priſm by ſhoving it, till firſt the one Side, then the Middle, then the other Side paſs over the Hole, obſerving the ſame Paralleliſm.

Obſervations on this Propoſal by Mr Newton. N. 83. *p.* 4060. May, An. 1672.

2. I ſuppoſe the Deſign of the *Propoſer* of theſe Experiments is, to have their Events expreſſed, with ſuch Obſervations as may occur concerning them. Touching the *firſt*, I have obſerved, that the ſolar Image falling on a Paper placed at the Focus of the Lens, was by the interpoſed Priſm drawn out in Length proportional to the Priſm's Refraction or Diſtance from that Focus. And the chief Obſervable here, which I remember, was, that the ſtreight Edges of the oblong Image were diſtincter than they would have been without the Lens.

Conſidering that the Rays coming from the Planet *Venus*, are much leſs inclined one to another, than thoſe which come from the oppoſite Parts of the Sun's Diſk; I once tried an Experiment or two with her Light. And to make it ſufficiently ſtrong, I found it neceſſary to collect it firſt by a broad Lens; and then interpoſing a Priſm between the Lens and its Focus, at ſuch Diſtance that all the Light might paſs through the Priſm, I found the Focus, which before appeared like a lucid Point, to be drawn out into a long ſplendid Line by the Priſm's Refraction.

Concerning the *ſecond* Experiment, I have occaſionally obſerved, that by covering both Ends of the Priſm with Paper at ſeveral Diſtances from the Middle, the Breadth of the ſolar Image will be increaſed or diminiſhed as much, as is the Aperture of the Priſm, without any Variation of the Length: Or, if the Aperture be augmented on all ſides, the Image on all ſides will be ſo much and no more augmented.

Of the *third* Experiment I have occaſion to ſpeak in my Anſwer to another Perſon; where you will find the Effects of two Priſms, in all croſs Poſitions of one to another, deſcribed. But if one Priſm alone be turned about, the coloured Image will only be tranſlated from Place to Place, deſcribing a Circle, or ſome other conic Section on the Wall, on which it is projected, without ſuffering any Alteration in its Shape, unleſs ſuch as may ariſe from the Obliquity of the Wall, or caſual Change of the Priſm's Obliquity to the Sun's Rays.

The Effect of the *fourth* Experiment I have already inſinuated, telling you that Light paſſing through Parts of the Priſm of divers Thickneſſes, did ſtill exhibit the ſame Phænomena.

The Genuine Method of examining this Theory; by Mr. Newton; N. 85. *p.* 5004. July, An. 1672.

III. I cannot think it effectual for determining Truth, to examine the ſeveral Ways by which Phænomena may be explained, unleſs where there can be a perfect Enumeration of all thoſe Ways. You know, the proper Method for enquiring after the Properties of Things, is to deduce them from Experiments. And I told you that the *Theory* which I propounded, was evinced to me, not by inferring *'tis thus, becauſe 'tis not otherwiſe*; that is, not by deducing it only from a Confutation of contrary Suppoſitions, but by deriving it from Experiments concluding poſitively and directly. The Way therefore to examine it, is, by conſidering, Whether the Experiments which I propound do prove thoſe Parts of the *Theory* to which they are applied, or by proſecuting other Experiments which the *Theory* may ſuggeſt, for its Examination. And this I would have done in a due Method; the Laws of Refraction being thoroughly enquired into and determined, before the Nature of Colours be taken into Conſideration. It may not be amiſs to proceed according to the Series of theſe Queries; which I could wiſh were determined by the Event of proper Experiments, declared by thoſe that may have the Curioſity to examine them.

1. Whether *Rays*, that are alike incident on the ſame Medium, have *unequal Refractions?* And how great are the *Inequalities* of their Refractions at any Incidence?

2. What is *the Law* according to which each Ray is *more or leſs refracted*; whether it be that the ſame Ray is ever refracted according to the *ſame Ratio* of the Sines of Incidence and Refraction; and divers Rays, according to divers Ratio's; or that the Refraction of each Ray is greater or leſs without any certain Rule? That is, whether each Ray have a certain Degree of Refrangibility, according to which its Refraction is performed; or is refracted without that Regularity?

3. Whether *Rays*, which are endued with particular Degrees of *Refrangibility*, when they are by any Means ſeparated, have particular *Colours conſtantly belonging to them*; *viz.* the leaſt Refrangible, Scarlet; the moſt Refrangible, deep Violet; the middle, Sea-Green; and others, other Colours? And on the contrary?

4. Whether *the Colour* of any ſort of *Rays apart* may be changed by *Refraction*?

5. Whether *Colours* by coaleſcing do really *change* one another to produce a *New Colour*, or produce it by *mixing* only?

6. Whether *a due Mixture of Rays*, endued with all variety of Colours, produces *Light* perfectly *like that of the Sun*, and which hath all the ſame Properties, and exhibits the ſame Phænomena?

7. Whether the *component Colours* of each Mixture be really *changed*; or be only *ſeparated*, when from that Mixture various Colours are produced again by Refraction?

8. Whether there be any *other Colours* produced by *Refraction*, than ſuch as ought to reſult from the *Colours* belonging to the *diverſly Refrangible Rays*, by their being ſeparated or mixed by *that Refraction*?

To determine by Experiments theſe and ſuch like Queries, which involve the propounded Theory, ſeems the moſt proper and direct Way to a Concluſion. And therefore I could wiſh all Objections were ſuſpended, taken from Hypotheſes, or any other Heads than theſe two; of ſhewing the Inſufficiency of Experiments to determine theſe Queries, or prove any other Parts of my Theory, by aſſigning the Flaws and Defects in my Concluſions drawn from them; or of producing other Experiments which directly contradict me, if any ſuch may ſeem to occur. For if the Experiments, which I urge, be defective, it cannot be difficult to ſhew the Defects; but if valid, then by proving the Theory they muſt render all Objections invalid.

Animadverſions upon this Theory by R. P. Ign. Gaſton Pardies. N. 84. *p.* 4087. June, An. 1672.

IV. 1. This ſo extraordinary an Hypotheſis, which overturns the very Foundations of Dioptrics, and makes uſeleſs all the Practice that has hitherto obtained, is intirely built upon that Experiment of the Glaſs Priſm, in which the Rays of Light entering at the Hole of a Window into a dark Room, and then ſtriking againſt the Wall, or received upon a Paper, are not gathered into a round Spot, as Mr. *Newton* ſeemed to expect from the received Laws of Refractions, but appeared to be extended into an oblong Figure. Whence he concluded, that this oblong Figure proceeded from this Cauſe, that ſome of the Rays were refracted more and ſome leſs.

But it ſeems to me, that according to the common and received Laws of Dioptrics, that Figure ought not to be round but oblong. For ſince the Rays proceeding from the oppoſite Parts of the ſolar Diſk have a different Inclination in paſſing through the Priſm, they ought to be differently refracted; and ſince the Inclination of ſome is greater than the Inclination of others by at leaſt 30 Minutes, the Refraction alſo muſt become greater.

Therefore the oppoſite Rays, emerging from the other Superficies of the Priſm, diverge and divaricate more than if all had proceeded without Refraction, or at leaſt had been equally refracted. But that Refraction of the Rays is performed only towards thoſe Parts which may be conceived to be in Plains perpendicular to the Axis of the Priſm: For there is no Inequality of Refraction towards thoſe Parts which are underſtood to be in Plains parallel to the Axis; as may be eaſily demonſtrated. For the two Superficies of the Priſm may be judged as parallel to one another, in reſpect to the Inclination of the Axis, ſince each is parallel to the Axis. But the Refraction through two parallel plain Superfices is reckoned as none, becauſe as much as the Ray is deflected one Way by the firſt Superficies, ſo much it is turned the other Way by the other Superficies. Therefore ſince the Rays of the Sun paſſing from the Hole through the Priſm are not refracted at the Sides, they proceed farther, as if no Superficies of the Priſm had intervened (reſpect only being had, as I ſaid before, to that lateral Divarication;) but ſince the ſame Rays are refracted towards the upper or lower Parts, ſome indeed more and ſome leſs, as being unequally inclined; it is neceſſary that they muſt divaricate more, and therefore muſt be extended into a longer Figure.

Now if a Calculation be rightly made; as the lateral Rays are found by the learned *Newton* in ſuch a Breadth as ſubtends an Arch of 31 Minutes, which Arch anſwers to the Sun's Diameter; ſo I don't at all doubt, but that Height alſo of the Image being found, which ſubtends an Arch of 2°. 49′. is that itſelf which in that Caſe anſwers to the ſame Diameter of the Sun after the unequal Refractions.

Fig. 72. And in reality, ſuppoſing the Priſm to be A B C whoſe Angle A is 60 Degrees, the Ray to be D E, which makes an Angle of 30 Degrees with the Perpendicular E H; I find it at emerging through F G to make with the Perpendicular F I an Angle of 76°. 22′. But ſuppoſing another Ray *d* E, which with the Perpendicular E H makes an Angle of 29°. 30′. I find this, as it emerges through *f g*, to make with the Perpendicular *f i* an Angle of 78°. 45′. Whence thoſe two Rays D E, *d* E, which are ſuppoſed to proceed from oppoſite Parts of the Solar Diſk, make an Angle with each other of 30 Minutes; but the ſame at emerging in the Lines F G, *f g*, ſo diverge, as to conſtitute an Angle of 2°. 23′. with each other. Now if two other Rays were aſſumed, that approach more to the Perpendicular E H (which make, for inſtance, with the ſame Perpendicular, one an Angle of 29°. 30′, the other an Angle of 29°. 0′,) then the ſame Rays at their emerging would diverge ſtill more, and would make a greater Angle, even ſometimes more than three Degrees. And moreover that Interval of the refracted Rays is farther increaſed from hence, that the two Rays D E, *d* E, meeting in E, immediately begin to divaricate, and hit upon two ſeparate Points of the other Superficies, that is, in F and *f*. Therefore it is not ſufficient for duly performing the Calculation, from the Length of the Image projected upon the Paper to ſubtract the Magnitude of the Hole of the Window; ſince even ſuppoſing the Hole E to be indiviſible, there would be as it were another broad Hole in the other Superficies, which is F *f*.

As to what he calls *Experimentum Crucis*, to me it ſeems to fall in with the commonly received Rules of Refraction. For, as I have now ſhewn, the Solar Rays, which approaching and converging make an Angle of 30 Degrees, after they have paſſed through the indiviſible Foramen, diverge into an Angle of two or three Degrees. Therefore it is no wonder if the ſame Rays, ſingly impinging upon another Priſm which is alſo opened with a ſmall Hole, ſhould be unequally refracted, ſince their Inclination is unequal. Nor is it to the purpoſe that thoſe Rays are raiſed or depreſſed by the Rotation of the firſt Priſm, the ſecond Priſm remaining without Motion, (which yet cannot be done in every Caſe;) or that, the firſt remaining immoveable, the ſecond ſhould be moved, that it may ſucceſſively receive the coloured Rays of the whole Image, and tranſmit them through its own Hole. For in either Way it is neceſſary, that thoſe extreme Rays, that is the Red and Violet, ſhould fall upon the ſecond Priſm at an unequal Angle, and thereby that their Refraction ſhould be unequal, ſo that That of the Violet Rays ſhould be the greater.

Since therefore a manifeſt Cauſe appears, why the Figure of the Rays ſhould be oblong, and that Cauſe ariſes from the very Nature of Refraction; there ſeems no Occaſion to have Recourſe to any other Hypotheſis, or to admit any different Frangibility of the Rays.

What afterwards he has conceived concerning Colours, that indeed commodiouſly follows from the foregoing Hypotheſis; yet even that is obnoxious to ſome Difficulties. For as he affirms, that no Colour at all, but rather Whiteneſs, will appear, when the Rays of all Colours are mixed promiſcuouſly, that does not ſeem conformable to all the Phænomena. Surely the Varieties which are obſerved in the mingling of different Bodies, which are imbued with different Colours, the very ſame are perceived in the mingling of different Rays that are alſo imbued with different Colours. And he well obſerves, that as from a Yellow and Blue Body a Green Colour ariſes, ſo from a Yellow and Blue Ray a Green Colour will alſo ariſe. So that if all the Rays of all Colours were confounded together, it is neceſſary in that Hypotheſis, that the ſame Colour ſhould appear which really appears in a Mixture of all ints. But now if thoſe Colours, that is, Red and Yellow, together with ue and Purple, and all others, if there be more, were beaten together and ngled; not White would appear, but a dark ſad Colour. Therefore a e Colour would appear in the ordinary Rays of Light, if it conſiſted of a ixture of all ſorts of Colours.

Now at firſt Aſpect nothing ſeems more ingenious or more proper, than hat he ſays concerning the Experiment of the moſt acute Mr. *Hook*, in hich two different Liquors, the one Red the other Blue, and both ſeparately

rately tranſparent, but when mixed together become opaque. This, the learned *Newton* ſays, ariſes from hence, that one of the Liquors can only tranſmit the Red Rays, the other only the Blue; whence being mingled they can tranſmit none. This, I ſay, at firſt ſight ſeems very oppoſite; nevertheleſs it ſhould follow from hence, that a like Opacity would ariſe from the Mixture of any Liquors of different Colours, which yet is not true.

Anſwer'd by Mr. Newton. N. 84. *p.* 4091. June, An. 1672. *Fig.* 73.

2. The Reverend Father *Pardies* makes the Refractions on the different Sides of the Priſm as unequal as he can, whereas I make them equal, both in my Experiments, and in the Calculation founded upon thoſe Experiments. Let A B C be a Section of the Priſm perpendicular to its Axis, F L, K G, two Rays croſſing one another in *x* the middle of the Hole, and falling upon that Priſm at G and L, and let their refracted Rays be G H, L *m*, and again H I and *m n*. And whereas I have ſuppoſed that their Refractions at the Side A C are nearly equal to thoſe at the Side B C; if A C and B C are put equal, the Rays G H and L *m* will have a like Inclination to the Baſe of the Priſm A B; and therefore the Angle C L *m* = Ang. C H G, and Ang. C *m* L = Ang. C G H. Wherefore alſo the Refractions in G and *m* will be equal, as likewiſe in L and H, and ſo Ang. K G A = Ang. *n m* B, and Ang. F L A = Ang. B H I: And therefore the refracted Rays H I and *m n* will have the ſame Inclination to one another as the incident Rays F L and K G. Therefore if the Angle F *x* K be 30 Minutes, or equal to the Sun's Diameter, then alſo the Angle comprehended by H I and *m n* will be 30 Minutes, if the Rays F L and K G are ſuppoſed equally refrangible. But to me, who have tried it, that Angle comes out about 2°. 49′, which the Ray H I exhibiting the utmoſt Violet Colour, contains with the Ray *m n* of a Blue Colour; and therefore it muſt be neceſſarily granted, that thoſe Rays are differently refrangible, or that the Refractions are performed according to an unequal Ratio of the Sines of Incidence and Refraction.

The Reverend Father adds beſides, that for rightly performing the Calculation, it is not ſufficient to ſubtract the Bigneſs of the Hole from the Length of the Image projected on the Paper; becauſe even ſuppoſing the Hole to be indiviſible, there would ſtill be as it were another wide Hole in the poſterior Superficies of the Priſm. Yet this notwithſtanding it ſeems to me, that the Refractions of the Rays, croſſing at the anterior as well as poſterior Surface of the Priſm, may be rightly computed from the Principles laid down. But if the Matter were otherwiſe, the Breadth of the Chaſm in the latter Superficies, which is as a Hole, could hardly make an Error of two Seconds; and in practical Matters I think it not worth while to take Notice of ſuch minu Differences.

The Reverend Father does not contradict that which I called *Experimentu Crucis*, while he contends, that the unequal Refractions of the Rays, endue with different Colours, are produced by the unequal Incidences. For when the Rays paſs through two very ſmall, diſtant, and immoveable Holes, tho Incidences (as I made the Experiment) were plainly equal, and yet theſe Refractions were very unequal. But if he has any Doubt about my Experiments, I beg that he would try himſelf, and meaſure the Refractions of the Rays en-

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dued with different Colours, from equal Incidences, and he will find them to be unequal. If he does not like the Way I made uſe of to try this, (than which I think none can be more ſatisfactory) it is eaſy to contrive others; as I myſelf have experienced not a few, and with good Succeſs.

It is objected to the Theory about Colours, that Powders of different Colours being mixed, they exhibit not a White, but a dark Brown Colour. Now to me White, Black, and all intermediate Brown Colours, which can be compoſed of White and Black mixed, ſeem to differ not ſo much in their Species of Colour as in the Quantity of Light. And in the Mixture of Paints, ſince the ſeveral Corpuſcles reflect only their own Colour, and therefore the greateſt Part of the incident Light is ſuppreſſed and ſtifled; the reflected Light muſt become obſcure, and mixed as it were with Darkneſs, ſo that it cannot exhibit intenſe Whiteneſs, but ſuch as would be cauſed by a Mixture of Black, that is, a dark Brown.

It is then objected, that from any Liquors of different Colours mixed in the ſame Veſſel, as well as being contained in different Veſſels, Opacity ſhould be produced, which yet (he ſays) is not true. But I do not ſee the Conſequence. For very many Liquors act upon one another, and ſecretly induce a new Configuration of Parts upon one another; whence they may become opaque, tranſparent, or endued with various Colours, ſuch as could by no means ariſe from a Mixture of Colours. And for this I always thought, that Experiments of this kind were leſs fit, from whence Concluſions could be drawn. Yet here I will obſerve, that for this Experiment Liquors are required, which are endued with ſtrong and intenſe Colours, which tranſmit but few Rays except thoſe of their own Colour. Such are rarely to be met with, as may be ſeen by illuminating Liquors in a dark Room with the different Colours of the Priſm. For few are found which appear very tranſparent in their own Colours, and opaque in others. It is alſo convenient, that the Colours made uſe of ſhould be oppoſite to one another, ſuch as I think Red and Blue to be, or Yellow and Violet, or alſo Green and that Purple which approaches to Scarlet. And perhaps ſome of theſe Liquors, whoſe tinging Particles will not come together, when mixed may become more opaque. But I am nothing ſolicitous about the Event, as well becauſe the Experiment will be more perſpicuous in Liquors when ſeparate, as becauſe I propoſed that Experiment (as likewiſe that of the Rainbow, of the Tincture of *Lignum Nephriticum*, and the Phænomena of other natural Bodies) not ſo much to prove the Doctrine as to illuſtrate it.

Now as to the Reverend Father's calling my Theory *an Hypotheſis*, I take not amiſs, becauſe to him it may not yet appear. But I propoſed it with nother Intent, and it ſeems to contain nothing elſe but certain Properties of ght, which being now found I think it not difficult to prove; and which I did not know to be true, I ſhould rather chooſe to reject as vain and mpty Speculation, than to own them as my Hypotheſis.

3. In that Hypotheſis which our *Grimaldus* explains at large, in which it s ſuppoſed that Light is a certain Subſtance moving with a moſt rapid Moion, ſome Diffuſion of Light might be made after paſſing through the Hole,

Some farther Objections, by R. P. Pardies. N. 85. p. 5012. Jul. An. 1672.

and the Decuſſation of the Rays. Alſo in that Hypotheſis, in which Light is ſuppoſed to proceed through certain Undulations of ſubtile Matter, as the very ingenious *Hook* explains; Colours might be accounted for by ſome kind of Diffuſion and Expanſion of Undulations, which may be made at the Sides of the Rays beyond the Hole, by Contact itſelf, and by the Continuation of the Matter. And indeed I have recourſe to ſuch an Hypotheſis, in the Diſſertation concerning the Motion of Undulation, which is the ſixth Part of my Mechanics. Where I ſuppoſe, that thoſe appearing Colours are produced by that Communication of Motion alone, which is propagated at the Sides *Fig.* 74. by the Undulations going directly on. As if Rays entering at the Hole *a* go on towards *b*, the direct Undulations (having regard to ſtrait and natural Motion) ought to be terminated at the Right Line *ab*; yet becauſe of the Continuity of Matter, there will be ſome Communication of Motion towards the Sides *ec*, where a certain tremulous and ſubſultory Succuſſion will be excited. Now if Colours may be ſuppoſed to conſiſt in that lateral ſubſultory Motion, I imagine all the Phænomena of Colours might be explained, as I have endeavoured to do more at large in the Diſſertation now mentioned. From which being ſuppoſed it will alſo appear, why the Breadth of the Colours muſt be expanded farther than the Divarication of the Rays themſelves would require.

As to the *Experimentum Crucis*, I do not at all doubt, but that in his Experiment he makes uſe of ſuch a Situation, as that there may be an equal Inclination of the incident Rays; ſince he expreſly affirms it was ſo done by him. But I was not able to gueſs that from what I had read before, where are ſuppoſed two ſmall and very diſtant Holes, and one Priſm near the firſt Hole which is in the Window; through which Priſm the coloured Rays proceeding, fall upon the other diſtant Hole. It was alſo added, that in order that all the Rays might fall ſucceſſively on that ſecond Hole, the firſt Priſm was turned about its Axis. But by this means it muſt neceſſarily be, that the Inclination of the Rays falling upon the ſecond Hole muſt be changed. I alſo ſuggeſted, that the Thing would be the ſame, whether the firſt Priſm remaining immoveable, the ſecond Hole was raiſed or depreſſed, that it might receive ſucceſſively all the Rays of the painted Solar Image; or, that ſecond Hole remaining immoveable, the firſt Priſm was turned about, ſo that the ſame Image might change its Situation, and all the Parts might ſucceſſively impinge upon the ſecond Hole. But without doubt the moſt ſagacious *Newton* made uſe of other Cautions.

As to what I had objected about the Colours, I think it very well ſolved. And that I called his Theory an Hypotheſis, I did it without any Deſign, but made uſe of the Word that firſt occurred. Therefore I deſire him not to think, that I uſed that Word in any kind of Contempt.

Anſwer'd by Mr. Newton. *Ibid. p.* 5014. July An. 1672. 4. The Reverend Father ſays, that it is poſſible to explain the Length of the Colours, without a different Refrangibility of the different Rays; ſuppoſe from the Hypotheſis of P. *Grimaldus*, by the Diffuſion of Light, which is ſuppoſed to be a certain Subſtance moved with the greateſt Rapidity; or by the Hypotheſis of our *Hook*, by the Diffuſion or Expanſion of Undulations,

tions, which he pretends are excited in the Æther by lucid Bodies, and to be propagated every Way. To which I add, that by the Hypotheſis of *Carteſius* may alſo be imagined a like Diffuſion or Endeavour of the Preſſure of the Globules, in like Manner as is ſuppoſed in the Explication of the Tail of a Comet. And the ſame Diffuſion or Expanſion may be feigned according to any other Hypotheſis, in which Light is ſuppoſed to be a Force, Action, Quality, or Subſtance of any kind, emitted on all Sides from luminous Bodies.

That I *may* anſwer this, it is to be obſerved, that the Doctrine I have explained concerning Refractions and Colours, conſiſts only in certain Properties of Light, neglecting all Hypotheſes by which thoſe Properties may be explained. Wherefore I thought proper here wholly to refrain from the Conſideration of Hypotheſes, as being no fit Place of Argumentation; and I will abſtract the Force of the Objection, that it may receive a fuller and more general Anſwer.

Therefore by Light I underſtand any Being or Power of a Being, (whether it be a Subſtance, or any Power, Action, or Quality of it) which proceeding directly from a lucid Body is adapted to excite Viſion. And by Rays of Light I underſtand the leaſt Parts of it, or any indefinitely little Parts of it, which do not depend on one another; ſuch as are all thoſe Rays which luminous Bodies emit according to Right Lines, either at the ſame Time or ſucceſſively. For thoſe as well collateral as ſucceſſive Parts of Light are independent, ſince ſome without others may be intercepted, and may be ſeparately reflected or refracted any Way. And this being allowed, all the Force of the Objection conſiſts in this, that the Colours may be ſtretched out in Length by ſome Diffuſion of the Light beyond the Hole, which does not ariſe from the unequal Refrangibility of different Rays, or of the independent Parts of Light.

But I have ſhewn above, that they are not lengthened by any other Means; and that I might confirm the whole in the ſtricteſt Manner, I have added that Experiment which now is well known by the Name of *Experimentum Crucis*; of the Conditions of which ſince the Reverend Father has had ſome Doubt, I have thought fit to repreſent it by a Scheme. Let B C be the *Fig.* 75. former little Board to which the Priſm A is immediately prefixed, and let D E be the latter Board, at the Diſtance of about 12 Feet from the former, behind which the other Priſm F is fix'd. But let the Boards be ſo perforated at *x* and *y*, that a little of the Light refracted by the former Priſm may be tranſmitted through each of the Holes to the ſecond Priſm, and there again refracted. Now let the former Priſm be converted about its Axis by a ciprocal Motion, and the Colours falling upon the latter Board D E, will raiſed and depreſſed by Turns, by which Means another and another Co- ur at Pleaſure may be ſucceſſively made to paſs through its Hole *y* to the tter Priſm, while the other Colours will fall upon the Board. Then you ill ſee, that the Rays which are endued with different Colours will ſuffer a fferent Refraction in that latter Priſm, becauſe they will arrive to different laces of the oppoſite Wall, or of any Obſtacle G H removed at the Diſtance of ſome Feet farther. The Violet Rays will come ſuppoſe to H, the Red to G, and intermediate ones to intermediate Places. And yet becauſe of the determinate Poſition of the Holes, the Incidence of the Rays of each Colour paſſing through each Priſm muſt needs be alike. And thus it appears by meaſuring, that the Rays affected with different Colours will have different Laws of Refractions.

But I ſuſpect what it was that led the Reverend Father into a State of Doubt; he ſeems to have placed his firſt Priſm A behind the Board BC, and thus by converting it about its Axis, it is probable that the Inclination of the Rays, which come between the two Holes, may have ſuffered ſome Change becauſe of the intermediate Refraction. But by the Deſcription before explained, that Board ought to be placed after the Priſm, that the Rays may fall directly between the Holes, as may appear from my Words; *I took two Boards, and placed one of them cloſe behind the Priſm at the Window.* And the Deſign of the Experiment requires the ſame Thing.

But farther I ſhall obſerve, that in this Experiment, becauſe of the Refraction of the ſecond Priſm, the coloured Light is far leſs diffuſed, and divaricates leſs, than when it is quite White, ſo that the Image at G or H is almoſt Circular; eſpecially if the Priſms are made parallel, and in a contrary Situation of their Angles, as is repreſented in the Scheme. And farther, if the Diameter of the Hole *y* is equal to the Breadth of the Colours, there will be no Extenſion of the ſame coloured Light in Length; but the Image which is formed by any Colour at G or H will be really Circular; ſuppoſing the Holes to be Circular, and the Refraction of the latter Priſm not to be greater than of the former, the Rays being nearly perpendicular to the Obſtacle. This ſhews the Diffuſion mentioned before to proceed from a certain Law of the Refractions of every kind of Rays, and not from the Contact or Continuity of Matter undulating or moving very ſwiftly, or any the like Cauſe. But why that Image ſhould be Circular in one Caſe, and in others ſomething ſtretched out in Length, and how the Extenſion of the Light into Length may be leſſened at Pleaſure in any Caſe, I ſhall leave to be determined by Geometricians, and then to be compared with Experiment.

After the Properties of Light have been ſufficiently examined by theſe and the like Experiments, by conſidering the Rays as Parts of it either collateral or ſucceſſive, of which we have found by their Independence that they are diſtinct from one another; Hypotheſes are thence to be judged of, and thoſe are to be rejected which cannot be reconciled with the Phænomena. But it is an eaſy Matter to accommodate Hypotheſes to this Doctrine. For if any one has a Mind to defend the *Carteſian* Hypotheſis, he may ſay that the Globules are unequal, or that the Preſſures of the Globules are ſome ſtronger than others, and thence they become differently refrangible, and proper to excite a Senſation of different Colours. And ſo according to Mr. *Hook's* Hypotheſis, it may be ſaid, that the Undulations of the Æther are ſome greater or denſer than others. And ſo of the reſt. For this ſeems to be the moſt neceſſary Law and Condition of Hypotheſes, in which natural Bodies are ſuppoſed to conſiſt of a Multitude of Corpuſcles cohering together, that from the different Particles of lucid Bodies, or from the different Parts of the ſame Corpuſcle, (according as they differ in Motion, Figure, Bulk, or any other Qualities) unequal Preſſures, Motions, or moved Corpuſcles, may be propagated every Way through the Æther, from which being confuſedly mixed together, Light may be ſuppoſed to be conſtituted. And nothing can be more difficult in theſe Hypotheſes than the contrary Suppoſition.

From the Aperture or Dilatation of the Light in the latter Face of the Priſm, which the Reverend Father ſays is a Hole as it were, it is ſufficient that no ſenſible Error can ariſe, if there is any at all. And if a Calculation is made exactly according to the Obſervations, the Error will be none. For the Diameter of the Hole being ſubtracted from the Length of the Image, a Length will remain which the Image would have, if the Hole before the Priſm were an indiviſible Point, and that notwithſtanding the aforeſaid Dilatation of the Light in the latter Face of the Priſm; as may eaſily be ſhewn. Then from that given Length of the Image, and its Diſtance from the indiviſible Hole, as alſo from the Poſition and Form of the Priſm, and from the Inclination of the incident Rays to it, and from the Angle which the refracted Rays tending to the Middle of the Image make with the incident Rays from the Center of the Sun, all the other Things may be determined. And what determines the Refractions and Poſitions of the Rays are ſufficient to make a true Calculation of thoſe Refractions. But this Matter is not of ſuch Conſequence as to make any Difficulty.

As to the Reverend Father's calling our Doctrine an Hypotheſis, I believe it proceeded from nothing elſe, but that he uſed the Word which firſt occurred to him, for a Cuſtom has prevailed, that whatever is explained in Philoſophy is called an Hypotheſis. And I had no other Deſign in ſhewing my Diſlike to that Word, than to obviate an Appellation, which may miſlead thoſe who deſire to philoſophize the right Way.

5. Mr. *Newton*'s laſt Anſwer to my Objections has intirely ſatisfied me. The laſt Scruple that I had, about the *Experimentum Crucis*, is fully removed. And now I clearly underſtand by his Figure what I did not underſtand before. When the Experiment was performed after his Manner, every Thing ſucceeded, and I had nothing farther to deſire.

To the Satiſfaction of P. Ig. Gaſt. Pardies. N. 85. *p.* 5018. July, An. 1672.

V. The Conſideration on my *Theories* conſiſts in aſcribing an Hypotheſis to me, which is not mine; in aſſerting an Hpotheſis, which, as to the principal Parts, is not againſt me; in granting the greateſt Part of my Diſcourſe, if explicated by that Hypotheſis; and in denying ſome Things, the Truth of which would have appeared by an Experimental Examination.

Some Conſiderations upon this Theory; by ---- Anſwer'd by Mr. Newton. N. 88. *p.* 5086. Nov. An. 1672.

Of theſe Particulars I ſhall diſcourſe in Order. And *firſt* of the Hypothe- s, which is aſcribed to me in theſe Words: *But grant this firſt Suppoſition, hat Light is a Body, and that as many Colours or Degrees as there may be, ſo many Bodies there may be; all which compounded together would make White,* &c. This, it ſeems, is taken for my Hypotheſis. 'Tis true, that from my *Theory* I argue the Corporeity of Light; but I do it without any abſolute Poſitiveneſs, as the Word *perhaps* intimates; and make it at moſt but a very plauſible Conſequence of the Doctrine, and not a fundamental Suppoſition, nor ſo much as any Part of it; which was wholly comprehended in the precedent Propoſitions. And I ſomewhat wonder, how the Objector could imagine, that, when I had aſſerted the *Theory* with the greateſt Rigour, I ſhould be ſo forgetful as afterwards to aſſert the fundamental Suppoſition itſelf with no more than a *perhaps*. Had I intended any ſuch Hypotheſis, I ſhould ſomewhere have explained it: But I knew, that the Properties, which I declared of Light, were in ſome meaſure capable of being explicated, not only by that, but by many other mechanical Hypotheſes; and therefore I choſe to decline them all, and to ſpeak of Light in general Terms, conſidering it abſtractly, as ſomething or other propagated every Way in ſtreight Lines from luminous Bodies, without determining what that Thing is; whether a confuſed Mixture of difform Powers, or Beings whatſoever. And for the ſame Reaſon I choſe to ſpeak of Colours according to the Information of our Senſes, as if they were Qualities of Light without us. Whereas by that Hypotheſis, I muſt have conſidered them rather as Modes of Senſation, excited in the Mind by various Motions, Figures, or Sizes of the Corpuſcles of Light, making various mechanical Impreſſions on the Organ of Senſe: as I expreſſed it in that Place, where I ſpake of the Corporeity of Light.

But ſuppoſing I had propounded that Hypotheſis, I underſtand not why the Objector ſhould ſo much endeavour to oppoſe it: For certainly it has a much greater Affinity with his own Hypotheſis, than he ſeems to be aware of; the Vibrations of the Æther being as uſeful and neceſſary in this, as in his; for aſſuming the Rays of Light to be ſmall Bodies, emitted every Way from ſhining Subſtances, thoſe, when they impinge on any refracting or reflecting Superficies, muſt as neceſſarily excite Vibrations in the Æther, as Stones do in Water when thrown into it. And ſuppoſing theſe Vibrations to be of ſeveral Depths or Thickneſſes, accordingly as they are excited by the ſaid corpuſcular Rays of various Sizes and Velocities; of what Uſe they will be for explicating the Manner of Reflection or Refraction, the Production of Heat by the Sun-Beams, the Emiſſion of Light from Burning, Putrifying, or other Subſtances, whoſe Parts are vehemently agitated, the Phænomena of thin tranſparent Plates and Bubbles, and of all natural Bodies, the Manner of Viſion, and the Difference of Colours, as alſo their Harmony and Diſcord; I ſhall leave to their Conſideration, who may think it worth their Endeavour, to apply this Hypotheſis to the Solution of Phænomena.

In the *ſecond* Place, I told you, That the Objector's Hypotheſis, as to the fundamental Part of it, is not againſt me. That fundamental Suppoſition is; *That the Parts of Bodies, when briſkly agitated, do excite Vibrations in the Æther, which are propagated every Way from thoſe Bodies in ſtreight Lines, and cauſe a Senſation of Light by beating and daſhing againſt the Bottom of the Eye, ſomething after the Manner that Vibrations in the Air cauſe a Senſation of Sound by beating againſt the Organs of Hearing.* Now, the moſt free and natural Application of this Hypotheſis to the Solution of Phænomena, I take to be this: That the agitated Parts of Bodies, according to their ſeveral Sizes, Figures, and Motions, do excite Vibrations in the Æther of various Depths or

Bigneſſes, which being promiſcuouſly propagated through that Medium to our Eyes, effect in us a Senſation of Light of a white Colour; but if by any means thoſe of unequal Bigneſſes be ſeparated from one another, the largeſt beget a Senſation of a red Colour; the leaſt or ſhorteſt, of a deep Violet; and the intermediate ones, of intermediate Colours; much after the manner that Bodies, according to their ſeveral Sizes, Shapes, and Motions, excite Vibrations in the Air of various Bigneſſes, which, according to thoſe Bigneſſes, make ſeveral Tones in Sound: That the largeſt Vibrations are beſt able to overcome the Reſiſtance of a refracting Superficies, and ſo break through it with leaſt Refraction; whence the Vibrations of ſeveral Bigneſſes, that is, the Rays of ſeveral Colours, which are blended together in Light, muſt be parted from one another by Refraction, and ſo cauſe the Phænomena of Priſms, and other refracting Subſtances: And that it depends upon the Thickneſs of a thin tranſparent Plate or Bubble, whether a Vibration ſhall be reflected at its further Superficies, or tranſmitted; ſo that, according to the Number of Vibrations interceding the two Superficies, they may be reflected or tranſmitted for many ſucceſſive Thickneſſes. And ſince the Vibrations which make Blue and Violet, are ſuppoſed ſhorter than thoſe which make Red and Yellow, they muſt be reflected at a leſs Thickneſs of the Plate: Which is ſufficient to explicate all the ordinary Phænomena of thoſe Plates or Bubbles, and alſo of all natural Bodies, whoſe Parts are like ſo many Fragments of ſuch Plates.

Theſe ſeem to be the moſt plain, genuine, and neceſſary Conditions of this Hypotheſis. And they agree ſo juſtly with my Theory, that if the Animadverſor think fit to apply them, he need not, on that Account, apprehend a Divorce from it. But yet how he will defend it from other Difficulties, I know not; for to me the fundamental Suppoſition itſelf ſeems impoſſible, namely, that the Waves or Vibrations of any Fluid, can, like the Rays of Light, be propagated in ſtreight Lines, without a continual and very extravagant ſpreading and bending every Way into the quieſcent Medium, where they are terminated by it. I miſtake, if there be not both Experiment and Demonſtration to the contrary. And as to the other two or three Hypotheſes, which he mentions, I had rather believe them ſubject to the like Difficulties, than ſuſpect the Animadverſor ſhould ſelect the worſt for his own.

What I have ſaid of this, may be eaſily applied to all other mechanical Hypotheſes, in which Light is ſuppoſed to be cauſed by any Preſſion or Motion whatſoever, excited in the Æther by the agitated Parts of luminous Bodies; for it ſeems impoſſible that any of thoſe Motions or Preſſions can propagated in ſtreight Lines, without the like ſpreading every Way into ſhadowed Medium on which they border. But yet, if any Man can nk it poſſible, he muſt at leaſt allow, that thoſe Motions, or Endeavours Motion, cauſed in the Æther by the ſeveral Parts of any lucid Body, that ffer in Size, Figure, and Agitation, muſt neceſſarily be unequal: Which is ough to denominate Light an Aggregate of *Difform Rays*, according to y of thoſe Hypotheſes. And if thoſe original Inequalities may ſuffice to ference the Rays in Colour and Refrangibility, I ſee no reaſon, why they, that adhere to any of thoſe Hypotheſes, ſhould ſeek for other Cauſes of theſe Effects, unleſs (to uſe the Objector's Argument) *they will multiply Entities without Neceſſity.*

The *third* Thing to be conſidered is, the Condition of the Animadverſor's Conceſſions, which is, That I would explicate my Theories by his Hypotheſis: And if I could comply with him in that Point, there would be little or no Difference between us: For he grants, That, without any reſpect to a different Incidence of Rays, there are different Refractions; but he would have it explicated, not by the different Refrangibility of ſeveral Rays, but by the ſplitting and rarefying Æthereal Pulſes. He grants my third, fourth, and ſixth Propoſitions; the Senſe of which is, That uncompounded Colours are unchangeable, and that compounded Ones are changeable only by reſolving them into the Colours of which they are compounded: and that all the Changes which can be wrought in Colours, are effected only by variouſly mixing or parting them: But he grants them on Condition that I will explicate Colours by the two Sides of a ſplit Pulſe, and ſo make but two Species of them, accounting all other Colours in the World to be but various Degrees and Dilutings of thoſe two. And he further grants, that Whiteneſs is produced by the Convention of all Colours; but then I muſt allow it to be not only by Mixture of thoſe Colours, but by a farther uniting of the Parts of the Ray ſuppoſed to be formerly ſplit.

If I would proceed to examine theſe his Explications, I think it would be no difficult Matter to ſhew, that they are not only inſufficient, but in ſome reſpects (to me at leaſt) unintelligible: For though it be eaſy to conceive, how Motion may be dilated and ſpread, or how parallel Motions may become diverging; yet I underſtand not, by what Artifice any linear Motion can by a refracting Superficies be infinitely dilated and rarefied, ſo as to become ſuperficial: Or, if that be ſuppoſed, yet I underſtand as little, why it ſhould be ſplit at ſo ſmall an Angle only, and not rather ſpread and diſperſe through the whole Angle of Refraction. And further, though I can eaſily imagine, how unlike Motions may croſs one another; yet I cannot well conceive, how they ſhould coaleſce into one uniform Motion, and then part again, and recover the former Unlikeneſs; notwithſtanding that I conjecture the Ways by which the Animadverſor may endeavour to explain it. So, that the direct, uniform, and undiſturb'd Pulſes ſhould be ſplit and diſturbed by Refraction; and yet the oblique and diſturbed Pulſes perſiſt without ſplitting, or further Diſturbance, by following Refractions, is (to me) as unintelligible: and there is as great a Difficulty in the Number of Colours; as you will ſee hereafter.

But whatever be the Advantages or Diſadvantages of this Hypotheſis, I hope I may be excuſed from taking it up, ſince I do not think it needful to explicate my Doctrine by any Hypotheſis at all: For if Light be conſidered abſtractedly without reſpect to any Hypotheſis, I can as eaſily conceive, that the ſeveral Parts of a ſhining Body may emit Rays of different Colours and other Qualities, of all which Light is conſtituted, as that the ſeveral Parts of a falſe or uneven String, or of unevenly agitated Water in a Brook or Cataract, or the ſeveral Pipes of an Organ inſpir'd all at once, or all the Variety of ſounding Bodies in the World together, ſhould produce Sounds of ſeveral Tones, and propagate them thro' the Air confuſedly intermixed. And if there were any natural Bodies that could reflect Sounds of one Tone, and ſtifle or tranſmit thoſe of another; then, as the Echo of a confuſed Aggregate of all Tones would be that particular Tone, which the echoing Body is diſpoſed to reflect; ſo, ſince (even by the Animadverſor's Conceſſions) there are Bodies apt to reflect Rays of one Colour, and ſtifle or tranſmit thoſe of another; I can as eaſily conceive, that thoſe Bodies, when illuminated by a Mixture of all Colours, muſt appear of that Colour only which they reflect.

But when the Objector would inſinuate a Difficulty in theſe Things, by alluding to Sounds in the String of a muſical Inſtrument before Percuſſion, or in the Air of an Organ-Bellows before its Arrival at the Pipes; I muſt confeſs, I underſtand it as little, as if one had ſpoken of Light in a Piece of Wood before it be ſet on Fire, or in the Oil of a Lamp before it aſcend up the Match to feed the Flame.

You ſee therefore how much it is beſide the Buſineſs in Hand to diſpute about Hypotheſes. For which Reaſon I ſhall now, in the laſt Place, proceed to abſtract the Difficulties in the Animadverſor's Diſcourſe, and without having regard to any Hypotheſis, conſider them in general Terms. And they may be reduced to theſe three Queries:

1. *Whether the unequal Refractions, made without reſpect to any Inequality of Incidence, be cauſed by the different Refrangibility of ſeveral Rays; or by the ſplitting, breaking, or diſſipating the ſame Ray into diverging Parts?*
2. *Whether there be more than two Sorts of Colours?*
3. *Whether Whiteneſs be a Mixture of all Colours?*

The *firſt* of theſe *Queries* you may find determined above, by an Experiment: The Deſign of which was to ſhew, that the Length of the colour'd Image proceeded not from any Unevenneſs in the Glaſs, or any other contingent Irregularity in the Refractions. Amongſt other Irregularities, I know not what is more obvious to ſuſpect, than a fortuitous Dilating and Spreading of Light, after ſome ſuch manner, as *Des Cartes* has deſcribed in his *Æthereal Refractions*, for explicating the Tail of a Comet; or as the Animadverſor now ſuppoſes to be effected by the ſplitting and rarefying of his æthereal Pulſes. And to prevent the Suſpicion of any ſuch Irregularities, I told you, that I refracted the Light contrary ways, with two Priſms ſucceſſively, to deſtroy ereby the regular Effects of the firſt Priſm by the ſecond, and to diſcover e irregular Effects by augmenting them with iterated Refractions. Now, nongſt other Irregularities, if the firſt Priſm had ſpread and diſſipated every ay into an indefinite Number of diverging Parts, the ſecond ſhould in like nanner have ſpread and diſſipated every one of thoſe Parts into a further inefinite Number, whereby the Image would have been ſtill more dilated, conary to the Event. And this ought to have happened, becauſe thoſe linear iverging Parts depend not on one another for the manner of their Refraction, but are every one of them as truly and compleatly Rays, as the whole was before its Incidence; as may appear by intercepting them ſeverally.

The Reaſonableneſs of this Proceeding will, perhaps, better appear by acquainting you with this further Circumſtance. I ſometimes placed the ſecond Priſm in a Poſition tranſverſe to the firſt, on Deſign to try if it would make the long Image become four-ſquare, by Refractions croſſing thoſe that had drawn the round Image into a long one. For if, amongſt other Irregularities, the Refraction of the firſt Priſm did, by ſplitting, dilate a linear Ray into a ſuperficial; the croſs Refractions of that ſecond Priſm ought, by further ſplitting, to dilate and draw that ſuperficial Ray into a pyramidical Solid. But, upon Trial, I found it otherwiſe; the Image being as regularly oblong as before, and inclined to both the Priſms at an Angle of 45 Degrees.

I tried alſo all other Poſitions of the ſecond Priſm, by turning the Ends about its middle Part; and in no caſe could obſerve any ſuch Irregularity. The Image was ever alike inclined to both Priſms, its Breadth anſwering to the Sun's Diameter, and its Length being greater or leſs, according as the Refractions more or leſs agreed or contradicted one another.

And by theſe Obſervations, ſince the Breadth of the Image was not augmented by the croſs Refraction of the ſecond Priſm, that Refraction muſt have been performed without any ſplitting or dilating of the Ray; and therefore at leaſt the Light incident on that Priſm muſt be granted an Aggregate of Rays *unequally Refrangible* in my Senſe. And ſince the Image was equally inclined to both Priſms, and conſequently the Refractions alike in both, it argues that they were performed according to ſome conſtant Law, without any Irregularity.

To determine the *ſecond Query*, The Animadverſor refers to an Experiment made with two Wedge-like Boxes, recited in the *Micrography* of the ingenious Mr. *Hook*, *Obſerv.* 10. *Pag.* 73. The Deſign of which was to produce all Colours out of a Mixture of two. But there is, I conceive, a double Defect in this Inſtance: For it appears not, that by this Experiment all Colours can be produced out of two; and if they could, yet the Inference would not follow.

That all Colours cannot by that Experiment be produced out of two, will appear by conſidering, that the Tincture of Aloes, which afford one of thoſe Colours, was not all over of one uniform Colour, but appeared Yellow near the Edge of the Box, and Red at other Places where it was thicker; affording all Variety of Colours, from a pale Yellow to a deep Red or Scarlet, according to the various Thickneſs of the Liquor. And ſo the Solution of Copper, which afforded the other Colour, was of various Blues and Indicoes. So that inſtead of two Colours, here is a great Variety made uſe of for the Production of all others. Thus, for Inſtance, to produce all Sorts of Greens, the ſeveral Degrees of Yellow and pale-Blue muſt be mix'd; but to compound Purples, the Scarlet and deep Blue are to be the Ingredients.

Now if the Animadverſor contend, that all the Reds and Yellows of the one Liquor, or Blues and Indicoes of the other, are only various Degrees and Dilutings of the ſame Colour, and not divers Colours; that is a begging of the Queſtion: And I ſhould as ſoon grant, that the Two Thirds or Sixths in Muſick are but ſeveral Degrees of the ſame Sound, and not divers Sounds. Certainly it is much better to believe our Senſes, informing us, that Red and Yellow are divers Colours; and to make it a philoſophical Query, Why the ſame Liquor doth, according to its various Thickneſs, appear of thoſe divers Colours; than to ſuppoſe them to be the ſame Colour, becauſe exhibited by the ſame Liquors. For if that were a ſufficient Reaſon, then Blue and Yellow muſt alſo be the ſame Colour, ſince they are both exhibited by the ſame Tincture of Nephritic Wood. But that they are divers Colours, you will more fully underſtand by the Reaſon, which in my Judgment is this: The Tincture of Aloes is qualified to tranſmit moſt eaſily the Rays endued with Red, moſt difficultly the Rays endued with Violet, and with intermediate Degrees of Facility, the Rays endued with intermediate Colours. So that where the Liquor is very thin, it may ſuffice to intercept moſt of the Violet, and yet tranſmit moſt of the other Colours; all which together muſt compound a middle Colour, that is, a faint Yellow. And where it is ſo much thicker, as alſo to intercept moſt of the Blue and Green, the remaining Green, Yellow, and Red, muſt compound an Orange. And where the Thickneſs is ſo great, that ſcarce any Rays can paſs through it beſides thoſe endued with Red, it muſt appear of that Colour, and that ſo much the deeper and obſcurer, by how much the Liquor is thicker. And the ſame may be underſtood of the various Degrees of Blue, exhibited by the Solution of Copper, by reaſon of its Diſpoſition to intercept Red moſt eaſily, and tranſmit a deep Blue or Indico Colour moſt freely.

But ſuppoſing that all Colours might, according to this Experiment, be produced out of two by Mixture; yet it follows not, that thoſe two are the only original Colours; and that for a double Reaſon: Firſt, Becauſe thoſe two are not themſelves original Colours, but compounded of others; there being no Liquor, nor any other Body in Nature, whoſe Colour in Day-light is wholly uncompounded. And then, becauſe though thoſe two were original, and all others might be compounded of them, yet it follows not that they cannot be otherwiſe produced. For I ſaid that they had a double Origin, the ſame Colours to Senſe being in ſome Caſes compounded, and in others uncompounded; and ſufficiently declared in my *third* and *fourth Propoſitions*, and in the *Concluſion*, by what Properties the one might be known and diſtinguiſh'd from the other. But becauſe I ſuſpect by ſome Circumſtances, that the Diſtinction might not be rightly apprehended, I ſhall once e declare it, and further explain it by Examples.

at Colour is Primary or Original, which cannot by any Art be changed, whoſe Rays are alike refrangible: And that Compounded, which is geable into other Colours, and whoſe Rays are not alike refrangible. For ance: To know whether the Colour of any green Object be compounded not, view it through a Priſm; and if it appear confuſed, and the Edges d with Blue, Yellow, or any Variety of other Colours, then is that n compounded of ſuch Colours as at its Edges emerge out of it: But appear diſtinct, and well defined, and entirely Green to the very Edges, without any other Colours emerging, it is of an original and uncompounded Green. In like manner, if a refracted Beam of Light, being caſt on a white Wall, exhibit a green Colour, to know whether that be compounded, refract the Beam with an interpoſed Priſm; and if you find any Difformity in the Refractions, and the Green be transformed into Blue, Yellow, or any Variety of other Colours, you may conclude that it was compounded of thoſe that emerge: But if the Refractions be uniform, and the Green perſiſt without any Change of Colour, then is it original and uncompounded. And the Reaſon why I call it ſo is, becauſe a Green endued with ſuch Properties cannot be produced by any mixing of other Colours.

Now if two green Objects may to the naked Eye appear of the ſame Colour, and yet one of them through a Priſm ſeem confuſed and variegated with other Colours at the Edges, and the other diſtinct and entirely green; or if there may be two Beams of Light, which falling on a white Wall, do to the naked Eye exhibit the ſame green Colour, and yet one of them, when tranſmitted through a Priſm, be uniformly and regularly refracted, and retain its Colour unchanged, and the other be irregularly refracted, and made to divaricate into a Multitude of other Colours: I ſuppoſe theſe two Greens will in both Caſes be granted of a different Origin and Conſtitution. And if by mixing Colours, a Green cannot be compounded with the Properties of the unchangeable Green, I think I may call that an uncompounded Colour, eſpecially ſince its Rays are alike refrangible and uniform in all Reſpects.

The ſame Rule is to be obſerved in examining whether Red, Orange, Yellow, Blue, or any other Colour, be compounded or not. And, by the way, ſince all white Objects through the Priſm appear confuſed and terminated with Colours, Whiteneſs muſt, according to this Diſtinction, be ever compounded, and that the moſt of all Colours, becauſe it is the moſt confuſed and changed by Refractions.

There now remains the *third* Query to be conſidered, which is, Whether Whiteneſs can be an uniform Colour, or a diſſimilar Mixture of all Colours? The Experiment which I brought to decide it, the Animadverſor thinks, may be otherwiſe explained; and ſo concludes nothing. But he might eaſily have ſatisfied himſelf by trying what would be the Reſult of a Mixture of all Colours. And that very Experiment might have ſatisfied him, if he had pleaſed to examine it by the various Circumſtances. One Circumſtance I there declared, of which I ſee no Notice taken; and it is, That if any Colour at the Lens be intercepted, the Whiteneſs will be changed into other Colours: If all the Colours but Red be intercepted, that Red alone in the Concourſe or croſſing of the Rays, will not conſtitute Whiteneſs, but continues as much Red as before; and ſo of the other Colours. So that the Buſineſs is not only to ſhew how Rays, which before the Concourſe exhibit Colours, do in the Concourſe exhibit White; but to ſhew, how, in the ſame Place, where the ſeveral Sorts of Rays apart exhibit ſeveral Colours, a Confuſion of all together makes White. For Inſtance, If Red alone be firſt tranſmitted to the Paper at the Place of Concourſe, and then the other Colours be let fall on that Red, the Queſtion will be, Whether they convert it into White by mixing with it only, as Blue falling on a Yellow Light is ſuppoſed to compound Green? Or whether there be ſome further Change wrought in the Colours by their mutual acting on one another, until, like contrary *peripatetic Qualities*, they become aſſimilated? And he that ſhall explicate this laſt Caſe mechanically, muſt conquer a double Impoſſibility. He muſt firſt ſhew, that many unlike Motions in a Fluid can by claſhing ſo act on one another, and change each other, as to become one uniform Motion; and then, that an uniform Motion can of itſelf, without any new unequal Impreſſions, depart into a great Variety of Motions regularly unequal. And after this he muſt further tell me, why all Objects appear not of the ſame Colour; that is, why their Colours in the Air, where the Rays that convey them every Way are confuſedly mix'd, do not aſſimilate one another, and become uniform before they arrive at the Spectator's Eye?

But if there be yet any doubting, 'tis better to put the Event on further Circumſtances of the Experiment, than to acquieſce in the Poſſibility of any hypothetical Explications: As for Inſtance, by trying what will be the Apparition of theſe Colours in a very quick Conſecution of one another. And this may be eaſily performed by the rapid Gyration of a Wheel with many Spokes or Coggs in its Perimeter, whoſe Interſtices and Thickneſſes may be equal, and of ſuch a Largeneſs, that, if the Wheel be interpoſed between the Priſm and the white Concourſe of the Colours, one half of the Colours may be intercepted by a Spoke or Cogg, and the other half paſs through an Interſtice. The Wheel being in this Poſture, you may firſt turn it ſlowly about, to ſee all the Colours fall ſucceſſively on the ſame Place of the Paper, held at their aforeſaid Concourſe; and if you then accelerate its Gyration, until the Conſecution of *thoſe* Colours be ſo quick, that you cannot diſtinguiſh them ſeverally, the reſulting Colour will be a Whiteneſs perfectly like that, which an unrefracted Beam of Light exhibits, when in like manner ſucceſſively interrupted by the Spokes or Coggs of that circulating Wheel. And that this Whiteneſs is produced by a ſucceſſive Intermixture of the Colours, without their being aſſimilated, or reduced to any Uniformity, is certainly beyond all Doubt, unleſs Things that exiſt not at the ſame Time, may notwithſtanding act on one another.

There are yet other Circumſtances, by which the Truth might have been decided; as by viewing the white Concourſe of the Colours through another Priſm placed cloſe to the Eye, by whoſe Refraction that Whiteneſs may appear again transformed into Colours: And then, to examine their Origin, if Aſſiſtant intercept any of the Colours at the Lens before their Arrival at the hiteneſs, the ſame Colours will vaniſh from amongſt thoſe, into which that hiteneſs is converted by the ſecond Priſm. Now, if the Rays which diſap- r be the ſame with thoſe that are intercepted, then it muſt be acknowledg- that the ſecond Priſm makes no new Colours in any Rays, which were t in them before their Concourſe at the Paper. Which is a plain Indication, at the Rays of ſeveral Colours remain diſtinct from one another in the White- ſs, and that from their previous Diſpoſitions are derived the Colours of the ſecond Priſm. And, by the Way, what is ſaid of their Colours may be applied to their Refrangibility.

The aforeſaid Wheel may alſo here be made uſe of; and, if its Gyration be neither too quick nor too ſlow, the Succeſſion of the Colours may be diſcern'd thro' the Priſm, whilſt to the naked Eye of a By-ſtander they exhibit Whiteneſs.

There is ſomething ſtill remaining to be ſaid of this Experiment. But this, I conceive, is enough to enforce it, and ſo to decide the Controverſy. However, I ſhall now proceed to ſhew ſome other Ways of producing Whiteneſs by Mixtures, ſince I perſuade myſelf, that this Aſſertion above the reſt appears paradoxical, and is with moſt Difficulty admitted. And becauſe the Animadverſor deſires an Inſtance of it in Bodies of divers Colours, I ſhall begin with that. But in order thereto it muſt be conſider'd, that ſuch colour'd Bodies reflect but ſome Part of the Light incident on them; as is evident by the *13th Propoſition*: And therefore the Light reflected from an Aggregate of them will be much weaken'd by the Loſs of many Rays, Whence a perfect and intenſe Whiteneſs is not to be expected, but rather a Colour between thoſe of Light and Shadow, or ſuch a Grey or dirty Colour as may be made by mixing White and Black together.

And that ſuch a Colour will reſult, may be collected from the Colour of Duſt found in every Corner of an Houſe, which hath been obſerv'd to conſiſt of many colour'd Particles. There may alſo be produced the like dirty Colour, by mixing ſeveral Painters Colours together. And the ſame may be effected by painting a Top (ſuch as Boys play with) of divers Colours. For when it is made to circulate by whipping it, it will appear of ſuch a dirty Colour.

Now the compounding of theſe Colours is proper to my Purpoſe, becauſe they differ not from Whiteneſs in the Species of Colour, but only in a Degree of Luminouſneſs: Which (did not the Animadverſor concede it) I might thus evince. A Beam of the Sun's Light being tranſmitted into a darkened Room, if you illuminate a Sheet of white Paper by that Light, reflected from a Body of any Colour, the Paper will always appear of the Colour of that Body, by whoſe reflected Light it is illuminated. If it be a Red Body, the Paper will be Red; if a Green Body, it will be Green; and ſo of the other Colours. The Reaſon is, that the Fibres or Threads, of which the Paper conſiſts, are all tranſparent and ſpecular; and ſuch Subſtances are known to reflect Colours without changing them. To know, therefore, to what Species of Colour a Grey belongs, place any Grey Body (ſuppoſe a Mixture of Painters Colours) in the ſaid Light, and the Paper being illuminated by its Reflection, ſhall appear White. And the ſame Thing will happen, if it be illuminated by Reflection from a Black Subſtance.

Theſe therefore are all of one Species; but yet they ſeem diſtinguiſh'd not only by Degrees of Luminouſneſs, but alſo by ſome other Inequalities, whereby they become more harſh or pleaſant. And the Diſtinction ſeems to be, that Greys, and perhaps Blacks, are made by an uneven Defect of Light, conſiſting as it were of many little Veins or Streams, which differ either in Luminouſneſs, or in the unequal Diſtribution of diverſly colour'd Rays; ſuch as ought to be cauſed by Reflection from a Mixture of White and Black, or of diverſly coloured Corpuſcles. But when ſuch imperfectly mix'd Light is by a ſecond Reflection from the Paper more evenly and uniformly blended, it becomes more pleaſant, and exhibits a faint or ſhadowed Whiteneſs. And that ſuch little Irregularities as theſe may cauſe theſe Differences, is not improbable, if we conſider, how much Variety may be cauſed in Sounds of the ſame Tone, by irregular and uneven Jarrings. And beſides, thoſe Differences are ſo little, that I have ſometimes doubted, whether they be any at all, when I have conſidered, that a Black and White Body being placed together, the one in a ſtrong Light, and the other in a very faint Light, ſo proportioned that they might appear equally luminous, it has been difficult to diſtinguiſh them, when viewed at Diſtance, unleſs when the Black ſeemed more Bluiſh, and the White Body in a Light ſtill fainter, hath, in Compariſon of the Black Body, itſelf appeared Black.

This leads me to another Way of compounding Whiteneſs; which is, that, if four or five Bodies of the more eminent Colours, or a Paper painted all over in ſeveral Parts of it with thoſe ſeveral Colours in a due Proportion, be placed in the ſaid Beam of Light, the Light reflected from thoſe Colours to another White Paper, held at a convenient Diſtance, ſhall make that Paper appear White. If it be held too near the Colours, its Parts will ſeem of thoſe Colours that are neareſt them; but by removing it further, that all its Parts may be equally illuminated by all the Colours, they will be more and more diluted, until they become perfectly White. And you may further obſerve, that if any of the Colours be intercepted, the Paper will no longer appear White, but of the other Colours which are not intercepted. Now that this Whiteneſs is a Mixture of the ſeverally coloured Rays, falling confuſedly on the Paper, I ſee no reaſon to doubt; becauſe, if the Light became uniform and ſimilar before it fell confuſedly on the Paper, it muſt much more be uniform, when at greater Diſtance it falls on the Spectator's Eye; and ſo the Rays, which come from ſeveral Colours, would in no Qualities differ from one another, but all of them exhibit the ſame Colour to the Spectator, contrary to what he ſees.

Not much unlike this Inſtance it is, that if a poliſh'd Piece of Metal be ſo placed, that the Colours appear in it as in a Looking-Glaſs, and then the Metal be made rough, that by a confuſed Reflection thoſe apparent Colours may be blended together, they ſhall diſappear, and by their Mixture cauſe the Metal to look White.

But further to enforce this Experiment: If inſtead of the Paper, any White Froth, conſiſting of ſmall Bubbles, be illuminated by Reflection from the aforeſaid Colours, it ſhall to the naked Eye ſeem White, and yet through a good Microſcope the ſeveral Colours will appear diſtinct on the Bubbles, as if ſeen by Reflection from ſo many ſpherical Surfaces. With my naked Eye, being very near, I have alſo diſcerned the ſeveral Colours on each Bubble; and yet at a greater Diſtance, where I could not diſtinguiſh them apart, the Froth hath appeared entirely White. And at the ſame Diſtance, when I look'd intently, I have ſeen the Colours diſtinctly on each Bubble; and yet by ſtraining my Eyes, as if I would look at ſomething afar off beyond them, thereby to render the Viſion confuſed, the Froth has appeared without any other Colour than Whiteneſs. And what is here ſaid of Froths, may eaſily be underſtood of the Paper, or Metal, in the foregoing Experiments. For their Parts are ſpecular Bodies, like theſe Bubbles; and perhaps with an excellent Microſcope the Colours may alſo be ſeen intermixedly reflected from them.

In proportioning the ſeveral colour'd Bodies to produce theſe Effects, there may be ſome Niceneſs; and it will be more convenient to make uſe of the Colours of the Priſm, caſt on a Wall, by whoſe Reflection, the Paper, Metal, Froth, and other white Subſtances may be illuminated. And I uſually made my Trials this Way, becauſe I could better exclude any ſcattering Light from mixing with the Colours to dilute them.

To this Way of compounding Whiteneſs may be referred that other, by mixing Light after it hath been trajected through tranſparently colour'd Subſtances. For Inſtance, if no Light be admitted into a Room, but only through colour'd Glaſs, whoſe ſeveral Parts are of ſeveral Colours in a pretty equal Proportion; all white Things in the Room ſhall appear White, if they be not held too near the Glaſs: And yet this Light, with which they are illuminated, cannot poſſibly be uniform; becauſe, if the Rays, which at their Entrance are of divers Colours, do in their Progreſs through the Room ſuffer any Alteration to be reduced to an Uniformity, the Glaſs would not in the remoteſt Parts of the Room appear of the very ſame Colour, which it doth when the Spectator's Eye is very near it: Nor would the Rays, when tranſmitted into another dark Room through a little Hole in an oppoſite Door or Partition-Wall, project on a Paper the Species or Repreſentation of the Glaſs in its proper Colours.

And, by the by, this ſeems a very fit and cogent Inſtance of ſome other Parts of my *Theory*, and particularly of the 13th *Propoſition*. For in this Room all natural Bodies whatever appear in their proper Colours. And all the Phænomena of Colours in Nature, made either by Refraction or without it, are here the ſame as in the open Air. Now the Light in this Room being ſuch a diſſimilar Mixture, as I have deſcribed in my *Theory*, the Cauſes of all theſe Phænomena muſt be the ſame that I have here aſſigned. And I ſee no reaſon to ſuſpect, that the ſame Phænomena ſhould have other Cauſes in the open Air.

The Succeſs of this Experiment may be eaſily conjectured by the Appearances of Things in a Church or Chapel, whoſe Windows are of colour'd Glaſs; or in the open Air, when it is illuſtrated with Clouds of various Colours.

There are yet other Ways by which I have produced Whiteneſs; as by caſting ſeveral Colours from two or more Priſms upon the ſame Place; by refracting a Beam of Light with two or three Priſms ſucceſſively, to make the diverging Colours converge again; by reflecting one Colour to another; and by looking through a Priſm on an Object of many Colours; and (which is equivalent to the above-mentioned Way of mixing Colours by concave Wedges filled with coloured Liquors) I have obſerved the Shadows of a painted Glaſs-

Window to become White, where thoſe of many Colours have at a great Diſtance interfered. But yet for further Satisfaction, the Animadverſor may try, if he pleaſe, the Effects of four or five ſuch Wedges fill'd with Liquors of as many ſeveral Colours.

Beſides all theſe, the Colours of Water-bubbles, and other thin pellucid Subſtances, afford ſeveral Inſtances of Whiteneſs produced by their Mixture; with one of which I ſhall conclude this Particular. Let ſome Water, in which a convenient Quantity of Soap or Waſhball is diſſolv'd, be agitated into Froth, and after that Froth has ſtood a while without further Agitation, 'till you ſee the Bubbles, of which it conſiſts, begin to break, there will appear a great Variety of Colours all over the Top of every Bubble, if you view them near at hand; but if you view them at ſo great a Diſtance that you cannot diſtinguiſh the Colours one from another, the Froth will appear perfectly White.

Thus much concerning the Deſign and Subſtance of the Animadverſor's Conſiderations. There are yet ſome Particulars to be taken notice of, before I conclude; as the Denial of the *Experimentum Crucis.* On this I choſe to lay the whole Streſs of my Diſcourſe; which therefore was the principal Thing to have been objected againſt. But I cannot be convinced of its Inſufficiency by a bare Denial, without aſſigning a Reaſon for it. I am apt to believe it has been miſunderſtood; for otherwiſe it would have prevented the Diſcourſes about rarefying and ſplitting of Rays; becauſe the Deſign of it is to ſhew, that Rays of divers Colours conſidered apart, do at equal Incidences ſuffer unequal Refractions, without being ſplit, rarefied, or any ways dilated.

Some Conſiderations upon this Doctrine of Colours; from Paris, *by* ------ N. 96. *p.* 6086. July, An. 1673.

VI. 1. Methinks that the moſt important Objection which is made againſt Mr. *Newton* by way of Query, is that, Whether there be more than two ſorts of Colours? For my part, I believe, that an Hypotheſis, that ſhould explain Mechanically, and by the Nature of Motion, the Colours Yellow, Green, and Blue, would be ſufficient for all the reſt, in regard thoſe others, being only more deeply charged, (as appears by the Priſms of Mr. *Hook)* do produce the dark or deep Red and Blue; and that of theſe four all the other Colours may be compounded. Neither do I ſee why Mr. *Newton* doth not content himſelf with the two Colours, Yellow and Blue; for it will be much more eaſy to find any Hypotheſis by Motion, that may explicate theſe two Differences, than for ſo many Diverſities as there are of other Colours. And 'till he hath found this Hypotheſis, he hath not taught us, what it is wherein conſiſts the Nature and Difference of Colours, but only this Accident (which cer-
inly is very conſiderable) of their *different Refrangibility.*

As for the Compoſition of White made by all the Colours together, it may
oſſibly be, that Yellow and Blue might alſo be ſufficient for that: Which is
orth while to try; and it may be done by the Experiment which Mr. *Newton*
opoſeth, by receiving againſt a Wall of a darkened Room the Colours of
e Priſm, and to caſt their reflected Light upon white Paper. Here you
ſt hinder the Colours of the Extremities, *viz.* the Red and Purple, from
iking againſt the Wall, and leave only the intermediate Colours, Yellow,
reen, and Blue, to ſee whether the Light of theſe alone would not make the

Paper appear White, as well as when they all gave Light. I even doubt, whether the lighteſt Place of the Yellow Colour may not all alone produce that Effect, and I mean to try it at the firſt Conveniency; for this Thought never came into my Mind but juſt now. Mean time you may ſee, that if theſe Experiments do ſucceed, it can no more be ſaid, that all the Colours are neceſſary to compound White, and that 'tis very probable, that all the reſt are nothing but Degrees of Yellow and Blue, more or leſs charged.

Anſwered by Mr. Newton. N. 97. *p.* 6108. Aug. An. 1673.

2. It ſeems to me, that *N.* takes an improper Way of examining the Nature of Colours, whilſt he proceeds upon compounding thoſe that are already compounded. Perhaps he would ſooner ſatisfy himſelf by reſolving Light into Colours, as far as may be done by Art, and then by examining the Properties of thoſe Colours apart, and afterwards by trying the Effects of re-conjoining two or more, or all of thoſe; and laſtly, by ſeparating them again, to examine what Changes that Re-conjunction had wrought in them. I have formerly ſhewn, That all Colours cannot practically be deriv'd out of the Yellow and Blue, and conſequently that thoſe Hypotheſes are groundleſs, which imply they may. If you aſk, what Colours cannot be derived out of Yellow and Blue? I anſwer, None of thoſe which I defined to be Original; and if he can ſhew by Experiment how they may, I will acknowledge myſelf in an Error. Nor is it eaſier to frame an Hypotheſis by aſſuming only two original Colours, rather than an indefinite Variety; unleſs it be eaſier to ſuppoſe that there are but two Figures, Sizes, and Degrees of Velocity or Force of the Æthereal Corpuſcles or Pulſes, rather than an indefinite Variety; which certainly would be a harſh Suppoſition. No Man wonders at the indefinite Variety of Waves of the Sea, or of Sands on the Shore; but, were they all but two Sizes, it would be a very puzzling Phænomenon. And I ſhould think it as unaccountable, if the ſeveral Parts or Corpuſcles, of which a ſhining Body conſiſts, which muſt be ſuppoſed of various Figures, Sizes, and Motions, ſhould impreſs but two Sorts of Motion on the adjacent Æthereal Medium, or any other Way beget but two Sorts of Rays. But to examine how Colours may be explain'd hypothetically, is beſide my Purpoſe. I never intended to ſhew wherein conſiſts the Nature and Difference of Colours, but only to ſhew, that *de facto* they are original and immutable Qualities of the Rays which exhibit them; and to leave it to others to explicate by mechanical Hypotheſes, the Nature and Difference of theſe Qualities; which I take to be no difficult Matter. But I would not be underſtood, as if their Difference conſiſted in the *different Refrangibility* of thoſe Rays; for that *different Refrangibility* conduces to their Production no otherwiſe, than by ſeparating the Rays whoſe Qualities they are. Whence it is, That the ſame Rays exhibit the ſame Colours when ſeparated by any other Means; as by their *different Reflexibility*, a Quality not yet diſcourſed of.

In the next Particular, where *N.* would ſhew, that it is not neceſſary to mix all Colours for the Production of White; the Mixture of Yellow, Green and Blue, without Red and Violet, which he propounds for that End, will not produce White, but Green; and the brighteſt Part of the Yellow will afford no other Colour but Yellow, if the Experiment be made in a

Room well darkened, as it ought; becauſe the coloured Light is much weakened by the Reflection, and ſo apt to be diluted by the mixing of any other ſcattering Light. But yet there is an Experiment or two formerly mentioned, by which I have produced White out of two Colours alone, and that variouſly; as out of Orange and a full Blue, and out of Red and a pale Blue, and out of Yellow and Violet, as alſo out of other Pairs of intermediate Colours. The moſt convenient Experiment for performing this, was that of caſting the Colours of one Priſm upon thoſe of another, after a due Manner. But what *N.* can deduce from hence, I ſee not; for the two Colours were compounded of all others; and ſo the reſulting White (to ſpeak properly) was compounded of them all, and only decompounded of thoſe two. For Inſtance, the Orange was compounded of Red, Orange, Yellow, and ſome Green; and the Blue, of Violet, full Blue, light Blue, and ſome Green, with all their intermediate Degrees; and conſequently the Orange and Blue together made an Aggregate of all Colours to conſtitute the White. Thus if one mix Red, Orange, and Yellow Powders to make an Orange; and Green, Blue and Violet Colours to make a Blue; and laſtly, the two Mixtures to make a Grey; that Grey, though decompounded of no more than two Mixtures, is yet compounded of all the ſix Powders, as truly as if the Powders had been all mixed at once.

This is ſo plain, that I conceive there can be no further Scruple; eſpecially to them who know how to examine, whether a Colour be ſimple or compounded, and of what Colours it is compounded; which having explain'd in another Place, I need not now repeat. If therefore *N.* would conclude any Thing, he muſt ſhew how White may be produced out of two uncompounded Colours: Which when he hath done, I will farther tell him, why he can conclude nothing from that. But I believe there cannot be found an Experiment of that Kind; becauſe, as I remember, I once try'd, by gradual Succeſſion, the Mixture of all Pairs of uncompounded Colours; and though ſome of them were paler, and nearer to White than others, yet none could be truly called White. But it being ſome Years ſince this Trial was made, I remember not well the Circumſtances, and therefore recommend it to others to be tried again.

3. Seeing that Mr. *Newton* maintains his Opinion with ſo much Concern, I liſt not to diſpute. But what means it, I pray, that he ſaith, *Though I ſhould ſhew him, that the White could be produced only of two uncompounded Colours, yet I could conclude nothing from that?* And yet he hath affirm'd, that t ompoſe the White, all primitive Colours are neceſſary. *A Reply, by Monſieur N. N. 97. p. 6112.*

. In my ſaying, that when Monſieur *N.* hath ſhewn *how White may be p duced out of two uncompounded Colours, I will tell him, why he can conclude* n *hing from that*; my Meaning was, That ſuch a White (were there any ſuch) w uld have different Properties from the White which I had reſpect to, when I ſcribed my *Theory*, that is, from the White of the Sun's immediate Light, o he ordinary Objects of our Senſes, and of all white Phænomena that have h erto fallen under my Obſervation. And thoſe different Properties would e ıce it to be of a different Conſtitution: Inſomuch, that ſuch a Production of *Anſwered, by Mr. Newton. N. 96. p. 6087. July, An. 1673.*

of White would be ſo far from contradicting, that it would rather illuſtrate and confirm my Theory; becauſe by the Difference of that from other Whites, it would appear, that other Whites are not compounded of only two Colours like that. And therefore if Monſieur *N.* would prove any Thing, it is requiſite that he do not only produce out of two primitive Colours a White, which to the naked Eye ſhall appear like other Whites, but alſo ſhall agree with them in all other Properties.

But to let you underſtand wherein ſuch a White would differ from other Whites, and why from thence it would follow that other Whites are otherwiſe compounded, I ſhall lay down this Poſition.

That a compounded Colour can be reſolved into no more ſimple Colours than thoſe of which it is compounded.

This ſeems to be ſelf-evident, and I have alſo tried it ſeveral Ways, and Fig. 76. particularly by this which follows: Let *α* repreſent an oblong Piece of White Paper about $\frac{1}{3}$ or $\frac{1}{4}$ of an Inch broad, and illuminated in a dark Room, with a Mixture of two Colours caſt upon it from two Priſms, ſuppoſe a deep Blue and Scarlet, which muſt ſeverally be as uncompounded as they can conveniently be made; then, at a convenient Diſtance, ſuppoſe of ſix or eight Yards, view it through a clear Triangular Glaſs or Cryſtal Priſm held parallel to the Paper, and you ſhall ſee the two Colours parted from one another in the Faſhion of two Images of the Paper, as they are repreſented at *β* and *γ*; where ſuppoſe *β* the Scarlet, and *γ* the Blue, without Green, or any other Colour between them.

Now from the aforeſaid *Poſition* I deduce theſe two *Concluſions*; 1. That if there were found out a Way to compound White of two ſimple Colours only, that White would be again reſolvable into no more than two. 2. That if other Whites, as that of the Sun's Light, *&c.* be reſolvable into more than two ſimple Colours (as I find by Experiment that they are) then they muſt be compounded of more than two.

To make this plainer, ſuppoſe that A repreſents a white Body, illuminated by a direct Beam of the Sun tranſmitted through a ſmall Hole into a dark Room, and *α* ſuch another Body, illuminated by a Mixture of two ſimple Colours; which, if poſſible, may make it alſo appear of a white Colour exactly like A: Then, at a convenient Diſtance, view theſe two Whites through a Priſm, and A will be changed into a Series of all Colours, Red, Yellow, Green, Blue, Purple, with their intermediate Degrees ſucceeding in Order from B to C. But *α*, according to the aforeſaid Experiment, will only yield thoſe two Colours of which it was compounded, and thoſe not conterminate like the Colours at B C, but ſeparate from one another, as at *β* and *γ*, by means of the different Refrangibility of the Rays to which they belong. And thus by comparing theſe two Whites, they would appear to be of a different Conſtitution, and A to conſiſt of more Colours than *α*. So that what Monſieur *N.* contends for, would rather advance my Theory by the Acceſs of a new Kind of White, than conclude againſt it. But I ſee no Hopes of compounding ſuch a White.

As for Monſieur *N.* his Expreſſion, *That I maintain my Doctrine with ſome Concern*, I confeſs it was a little ungrateful to me, to meet with Objections which had been anſwered before, without having the leaſt Reaſon given me why thoſe Anſwers were inſufficient. Thoſe Anſwers were to ſhew, that there are other ſimple Colours beſides Blue and Yellow; I inſtanced in a ſimple or homogeneal Green, ſuch as cannot be made by mixing Blue and Yellow, or any other Colours. And I alſo ſhew'd why, ſuppoſing that all Colours might be produced out of two, yet it would not follow that thoſe two are the only original Colours. The Reaſons I deſire you would compare with what hath now been ſaid of White. And ſo the Neceſſity of all Colours to produce White, might have appear'd by that Experiment, where I ſay, That if any Colour at the Lens be intercepted, the Whiteneſs (which is compounded of them all) will be changed into (the Reſult of) the other Colours. *Vid.* Suppl. *p.* 149, & ſeq.

However, ſince there ſeems to have happened ſome Miſunderſtanding between us, I ſhall endeavour to explain myſelf a little further in theſe Things, according to the following Method.

Definitions.] 1. I call that Light homogeneal, ſimilar, or uniform, whoſe Rays are equally refrangible.

2. And that heterogeneal, whoſe Rays are unequally refrangible.

Note, There are but three Affections of Light, in which I have obſerved its Rays to differ, *viz. Refrangibility*, *Reflexibility*, and *Colour*; and thoſe Rays which agree in Refrangibility, agree alſo in the other two, and therefore may well be defined homogeneal, eſpecially ſince Men uſually call thoſe Things homogeneal, which are ſo in all Qualities that come under their Knowledge, tho' in other Qualities, that their Knowledge extends not to, there may poſſibly be ſome Heterogeneity.

3. Thoſe Colours I call ſimple, or homogeneal, which are exhibited by homogeneal Light.

4. And thoſe compounded or heterogeneal, which are exhibited by heterogeneal Light.

5. Different Colours, I call not only the more eminent Species, Red, Yellow, Green, Blue, Purple, but all other the minuteſt Gradations; much after the ſame manner that not only the more eminent Degrees in Muſick, but all the leaſt Gradations are eſteemed different Sounds.

Propoſitions.] 1. The Sun's Light conſiſts of Rays differing by indefinite Degrees of Refrangibility.

Rays which differ in Refrangibility, when parted from one another, do p ortionally differ in the Colours which they exhibit. Theſe two *Propoſi ns* are Matter of Fact.

. There are as many ſimple or homogeneal Colours, as Degrees of Refr gibility: For to every Degree of Refrangibility belongs a different Colc r, by *Prop.* 2. and that Colour is ſimple, by *Def.* 1, and 3.

. Whiteneſs, in all Reſpects like that of the Sun's immediate Light, and o ll the uſual Objects of our Senſes, cannot be compounded of two ſimple

Colours alone: For ſuch a Compoſition muſt be made by Rays that have only two Degrees of Refrangibility, by *Def.* 1, and 3; and therefore it cannot be like that of the Sun's Light, by *Prop.* 1; nor, for the ſame Reaſon, like that of ordinary white Objects.

5. Whiteneſs, in all Reſpects like that of the Sun's immediate Light, cannot be compounded of Simple Colours, without an indefinite Variety of them: For to ſuch a Compoſition there are requiſite Rays endued with all the indefinite Degrees of Refrangibility, by *Prop.* 1. And thoſe infer as many Simple Colours, by *Def.* 1, and 3. and *Prop.* 2, and 3.

To make theſe a little plainer, I have added alſo the *Propoſitions* that follow.

6. The Rays of Light do not act on one another in paſſing through the ſame *Medium*. This appears by ſeveral former Paſſages, and is capable of further Proof.

7. The Rays of Light ſuffer not any Change of their Qualities from Refraction.

8. Nor afterwards from the adjacent quiet Medium. Theſe two Propoſitions are manifeſt *de facto* in homogeneal Light, whoſe Colour and Refrangibility is not at all changeable, either by Refraction, or by the Contermination of a quiet Medium. And as for heterogeneal Light, it is but an Aggregate of ſeveral Sorts of homogeneal Light; no one Sort of which ſuffers any more Alteration than if it were alone, becauſe the Rays act not on one another, by *Prop.* 6. and therefore the Aggregate can ſuffer none. Theſe two Propoſitions alſo might be further proved apart by Experiments, too long to be here deſcribed.

9. There can no homogeneal Colours be educed out of Light by Refraction, which were not commixt in it before; becauſe by *Prop.* 7. and 8. Refraction changeth not the Qualities of the Rays, but only ſeparates thoſe which have diverſe Qualities, by means of the different Refrangibility.

10. The Sun's Light is an Aggregate of an indefinite Variety of homogeneal Colours; by *Prop.* 1, 3, and 9. And hence it is, that I call homogeneal Colours alſo primitive or original.

Animadverſions on this Theory of Light and Colours; by Mr. Fr. Linus. N. 110. *p.* 217. Jan. An. 1675.

VII. 1. I doubt not of what Mr. *Newton* affirms; and have myſelf ſometimes in like Circumſtances obſerved the like Difference between the Length and Breadth of the coloured *Spectrum*; but never found it ſo when the Sky was clear and free from Clouds, near the Sun: But then only appeared this Difference of Length and Breadth, when the Sun either ſhined thro' a white Cloud, or enlightened ſome ſuch Clouds near unto it. And then indeed it was no marvel, the ſaid *Spectrum* ſhould be longer than broad; ſince the Cloud or Clouds ſo enlightened, were in order to thoſe Colours like to a great Sun, making a far greater Angle of Interſection in the Hole, than the true Rays of the Sun do make; and therefore are able to enlighten the whole Length of the Priſm, and not only ſome ſmall Part thereof, as we ſee enlightened by the true Sun-Beams coming thro' the ſame little Hole. And this we behold alſo in the true Sun-Beams, when they enlighten the whole Priſm; for altho' in a clear Heaven, the Rays of the Sun paſſing thro' the ſaid Hole, never make a

Spectrum longer than broad, becauſe they then can occupy but a ſmall Part of the Priſm; yet if the Hole be ſo much bigger as to enlighten the whole Priſm, you ſhall preſently ſee the Length of the Spectrum much exceed its Breadth; which Exceſs will always be ſo much the greater, as the Length of the Priſm exceeds its Breadth. From whence I conclude, That the Spectrum, this *Learned Author* ſaw, much longer than broad, was not effected by the true Sun-Beams, but by Rays proceeding from ſome bright Cloud, as is ſaid; and by Conſequence, that the *Theory of Light* grounded upon that *Experiment* cannot ſubſiſt.

What I have here ſaid, needs no other Confirmation than meer Experience, which any one may quickly try: neither have I only tried the ſame upon this Occaſion, but near 30 Years ago ſhew'd the ſame, together with divers other Experiments of Light, to that worthy Promoter of Experimental Philoſophy, Sir *Kenelm Digby*, who coming into theſe Parts to take the *Spaw*-Waters, reſorted oftentimes to my darkned Chamber *, to ſee the various Phænomena of Light, made by divers Refractions and Reflections, and took Notes upon them; which Induſtry if they alſo had us'd, who endeavour to explicate the aforeſaid Difference between the Length and Breadth of this colour'd *Spectrum*, by the received Laws of Refraction, would never have taken ſo impoſſible a Task in Hand.

2. Theſe Animadverſions ſeem to need no other Anſwer but this, that you would be pleaſed to conſider the Scheme in Mr. *Newton's* ſecond Anſwer to *P. Parides*, and reſt aſſured, that the Experiment, as 'tis repreſented, was tried in clear Days, and the Priſm placed cloſe to the Hole in the Window, ſo that the Light had no room to diverge, and the coloured Image made not parallel (as in that Conjecture) but tranſverſe to the Axis of the Priſm.

Anſwered by · · · · · · No. 110. p. 219. Jan. An. 1675.

3. *If theſe Aſſertions be admitted*, they do indeed directly cut off what I ſaid of Mr. *Newton*'s being deceived by a *bright Cloud*. But if we compare them with Mr. *Newton*'s firſt Relation of the Experiment, it will evidently appear, they cannot be admitted, as being directly contrary to what is there delivered. For there he tells us, *The Ends of the coloured Image, he ſaw on the oppoſite Wall near five times as long as broad, ſeem to be ſemicircular. Now* theſe *ſemicircular Ends* are never ſeen in a clear Day, as Experience ſhews. From whence follows againſt the *firſt* Aſſertion, that the Experiment was not made in a clear Day. Neither are thoſe ſemicircular Ends ever ſeen when the Priſm is placed cloſe to the Hole; which contradicts the *ſecond* Aſſertion. Neither are they ever ſeen when the Image is tranſverſe to the Length or Axis he Priſm; which directly oppoſes the *third* Aſſertion. But if in any of e three Caſes, the Image be made ſo much longer than broad (as eaſily ay, by turning the Priſm a little about its Axis, near *five times* as long as ad) then the one End thereof will run out into a ſharp Cone or Pyramis the Flame of a Candle, and the other into a Cone ſomewhat more nt; both which are far from ſeeming ſemicircular: Whereas, if the age be made not in a clear Day but with a bright Cloud, and the Priſm not ced cloſe to the Hole but in a competent Diſtance from the ſame, then theſe icircular Ends always appear, with the Sides thereof, ſtreight Lines, juſt as

A Reply by Mr. Fr. Linus. N. 121. p. 499. Jan. An. 16-6.

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* *At* Leige.

as Mr. *Newton* deſcribes them. Neither is the Length of the Image tranſverſe, but parallel to the Length of the Priſm. Out of all which it evidently follows, that the Experiment was not made in a clear Day; nor with the Priſm cloſe to the Hole; nor yet with the Image tranſverſe, but by a bright Cloud and a parallel Image (as I conjectured); and I hope you will alſo now ſay, I had good Reaſon ſo to conjecture, ſince it ſo well agrees with the Relation. And Experience will alſo ſhew you, if you pleaſe to make Trial, as it was made in a dark Chamber, and obſerve the Difference between ſuch an Image made by a bright Cloud, and another made by the immediate Rays of the Sun: For the former you ſhall always find parallel, with the Ends ſemicircular; but the latter you ſhall find tranſverſe, with the Ends pyramidical, as aforeſaid, whenſoever it appears ſo much longer than broad.

More might be ſaid out of the ſame Relation, to ſhew that the Image was not tranſverſe. For if it had been tranſverſe, Mr. *Newton*, ſo well skill'd in *Optics*, could not have been ſurprized (as he ſays he was) to ſee the Length thereof ſo much to exceed the Breadth; it being a Thing ſo obvious and eaſy to be explicated by the ordinary Rules of Refraction. That other Place alſo (where he ſays the *Incident* Refractions were made in the Experiment *equal to the Emergent*) proves again, that the ſaid oblong Image was not tranſverſe, but parallel. For it is impoſſible the tranſverſe Image ſhould be ſo much longer than broad, unleſs thoſe two Refractions be made very unequal; as both the Computation according to the common Rules of Refraction and Experience teſtify.

Anſwer'd by Mr. Newton. N. 121. *p.* 501. N. 123. *p.* 556. Mar. An. 1676.

4. What it is that impoſes upon Mr. *Line* I cannot imagine; but I ſuſpect he has not try'd the Experiment ſince he acquainted himſelf with my Theory, but depends upon his old Notions, taken up before he had any Hint given to obſerve the Figure of the coloured Image. I ſhall deſire him therefore, before he returns any Anſwer, to try it once more for his Satisfaction, and according to this Manner.

Let him take any Priſm, and hold it ſo that its Axis may be perpendicular to the Sun's Rays, and in this Poſture let it be placed as cloſe as may be to the Hole through which the Sun ſhines into a dark Room, which Hole may be about the Bigneſs of a Pea. Then let him turn the Priſm ſlowly about its Axis, and he ſhall ſee the Colours move upon the oppoſite Wall, firſt towards that Place to which the Sun's direct Line would paſs, if the Priſm were taken away, and then back again. When they are in the Middle of theſe two contrary Motions, that is, when they are neareſt that Place to which the Sun's direct Ray tends, there let him ſtop; for then are the Rays equally refracted on both Sides the Priſm. In this Poſture of the Priſm let him obſerve the Figure of the Colours, and he ſhall find it not round, as he contends, but oblong, and ſo much the more oblong as the Angle of the Priſm, comprehended by the refracting Planes, is bigger, and the Wall, on which the Colours are caſt, more diſtant from the Priſm; the Colours Red, Yellow, Green, Blue, Purple, ſucceeding in Order, not from one Side of the Figure to the other, as in Mr. *Line*'s Conjecture, but from one End to the other; and the Length of the Figure being not parallel but tranſverſe to the

Axis of the Priſm. After this manner I uſed to try the Experiment; and it will not ſucceed well if the Day be not clear, and the Priſm placed cloſe to the Hole, or ſo near at leaſt, that all the Sun's Light that comes from the Hole may paſs through the Priſm alſo, ſo as to appear in a round Form, if intercepted by a Paper immediately after it has paſſed the Priſm.

When Mr. *Line* has try'd this, I could wiſh he would proceed a little further, to try that which I call'd the *Experimentum Crucis.* For when he has try'd them (which by his denying them, I know he has not done yet as they ſhould be try'd) I preſume he will reſt ſatisfied. It may be try'd (tho' not ſo perfectly) even without darkning a Room, or the Expence of any more Time than half a Quarter of an Hour.

5. Mr. *Linus* (now deceaſed) *try'd the Experiment* again and again, and called divers on purpoſe to ſee it, nor ever made Difficulty to ſhew it to any one, who either by Chance came into his Chamber as he was doing it, or ſhewed the leaſt Deſire to ſee the ſame, ſo that for Point of Experience, Mr. *Newton* cannot be more confident on his Side, than we are here on the other; who are fully perſuaded, that, unleſs the Diverſity of placing the Priſm, or the Bigneſs of the Hole, or ſome other ſuch Circumſtance, be the Cauſe of the Difference betwixt them, Mr. *Newton*'s Experiment will hardly ſtand.

The Experiment of Mr. Line *affirmed by Mr.* Gaſcoigne, N. 121. *p.* 503. *marg.* Jan. An. 1676.

6. By Mr. *Gaſcoigne*'s Letter one might ſuſpect, that Mr. *Linus* try'd the Experiment ſome other Way than I did; and therefore I ſhall expect, till his Friends have try'd it according to my late Directions. In which Trial it may poſſibly be a further Guidance to them, to acquaint them that the Priſm caſts from it ſeveral Images. One is, that oblong One of Colours which I mean; and this is made by two Refractions only. Another there is made by two Refractions and an intervening Reflexion; and this is round and colourleſs, if the Angles of the Priſm be exactly equal; but if the Angles at the reflecting Baſe be not equal, it will be colour'd, and that ſo much the more, by how much unequaller the Angles are, but yet much more unround, unleſs the Angles be very unequal. A third Image there is, made by one ſingle Reflection, and this is always round and colourleſs. The only Danger is, in miſtaking the *Second* for the *Firſt*. But they are diſtinguiſhable not only by the Length and lively Colours of the *Firſt*, but by its different Motion too: For, whilſt the Priſm is turned continually the ſame Way about its Axis, the *Second* and *Third* move ſwiftly, and go always on the ſame Way, till they diſappear; but the *Firſt* moves ſlow, and grows continually ſlower, till it be Stationary, and then turns back again, and goes back faſter and faſter, till it vaniſh in the Place where it began to appear.

Anſwered by Mr. Newton, *Ibid.* N. 123. *p.* 556. Mar. An. 1676.

f, without darkning their Room, they hold the Priſm at their Window he Sun's open Light, in ſuch a Poſture that its Axis be perpendicular to Sun-Beams, and then turn it about its Axis, they cannot miſs of ſeeing firſt Image; which having found, they may double up a Paper once or ce, and make a round Hole in the middle of it, about $\frac{1}{2}$ or $\frac{3}{4}$ of an Inch ad, and hold the Paper immediately before the Priſm, that the Sun may ne on the Priſm through that Hole; and the Priſm being ſtayed, and d ſteddy in that Poſture which makes the Image ſtationary, if the Image then fall directly on an oppoſite Wall, or on a Sheet of Paper placed at the Wall, ſuppoſe 15 or 20 Feet from the Priſm, or further off; they will ſee the Image in ſuch an oblong Figure as I have deſcribed, with the Red at one End, the Violet at the other, and the bluiſh Green in the Middle: And if they obſcure their Room as much as they can, by drawing Curtains, or otherwiſe, it will make the Colours the more conſpicuous.

This Direction I have ſet down, that no Body, into whoſe Hands a Priſm ſhall happen, may find Difficulty and Trouble in trying it. But when Mr. *Linus*'s Friends have try'd it thus, they may proceed to repeat it in a dark Room with a leſs Hole made in their Window-ſhut. And then I ſhall deſire that they will ſend a full and clear Deſcription how they try'd it. I ſhould be glad too, if they will favour me with a Deſcription of the Experiment as it hath been hitherto try'd by Mr. *Linus*; that I may have an Opportunity to conſider what there is in that which makes againſt me.

Exceptions by Mr. Lucas. N. 128. *p.* 692. Aug. An. 1676.

7. Mr. *Gaſcoigne* wanting Convenience to make the Experiment, according to the freſh Directions from Mr. *Newton*, requeſted me to ſupply this Want. The vertical Angle of my Priſm was 60 Degrees; the Diſtance of the Wall, whereon the coloured *Spectrum* appeared from the Window, about 18 Feet; the Diameter of the Hole in the Window-ſhuts about ½ Inch, which, upon Occaſions, I contracted to half the ſaid Diameter, but ſtill with equal Succeſs as to the main of the Experiment. The Refractions on both Sides the Priſm were, as near as I could make them, equal; and conſequently about 48 Deg. 40 Min. the refractive Power of Glaſs being computed according to the Ratio of the Sines 2 to 3. The Diſtance of the Priſm from the Hole in the Shuts was about 2 Inches; the Room darkned to that Degree as to equal the darkeſt Night, while the Hole in the Shuts was covered.

Now as to the Iſſue of my Trials; I conſtantly found the Length of the coloured Image (tranſverſe to the Axis of the Priſm) conſiderably greater than its Breadth, as often as the Experiment was made on a clear Day; but if a bright Cloud were near the Sun, I found it ſometimes exactly as Mr. *Line* wrote to you, namely, broader than long, eſpecially while the Priſm was placed at a great Diſtance from the Hole: Which Experiment will not, I conceive, be queſtion'd by Mr. *Newton*, it being ſo agreeable to the received Laws of Refractions. And indeed the Obſervations of theſe two learned Perſons, as to this Particular, are eaſily reconcileable to each other, and both to Truth; Mr. *Newton* contending only for the Length of the Image (tranſverſe to the Axis of the Priſm) in a very clear Day; whereas Mr. *Line* only maintained the Exceſs of Breadth, parallel to the ſame Axis, while the Sun is in a bright Cloud. Tho' as to what is further deliver'd by Mr. *Newton*, and oppoſed by Mr. *Line*, namely that the Length of the coloured Image was *five times* the Diameter of its Breadth; I never yet have found the Exceſs above thrice the Diameter, or at moſt 3 ½, while the Refractions on both Sides the Priſm were equal. So much as to the Matter of Fact.

Now as to Mr. *Newton*'s *Theory of Light* and *Colours*, I confeſs his neat Sett of very Ingenious and Natural Inferences was to me, upon the firſt Peruſal, a ſtrong Conjecture in Favour of his new Doctrine; I having formerly obſerved the like Chain of Inferences upon Search into natural Truths. But ſince ſeveral Experiments of Refractions remain ſtill untouched by him, I conceived a further Search into them would be very proper, in order to a further Diſcovery of the Truth of his Aſſertion. For accordingly as they are found, either agreeing with, or diſagreeing from his new Theory, they muſt needs much ſtrengthen, or wholly overthrow the ſame. The Experiments I pitched upon for this Purpoſe are as follow:

1. Having frequently obſerved, that the Form of Objects viewed in the Microſcope (or rather of the Microſcope itſelf) conſiſts almoſt in an indiviſible Point, I concluded two very ſmall Pieces of Silk, the one Scarlet, the other Violet Colour, placed near together, ſhould, according to Mr. *Newton*'s Theory, appear in the Microſcope in a very different Degree of Clarity, in regard their unequal Refrangibility muſt cauſe the Scarlet Rays, or Species, to over-reach the *Retina*, while placed in the due Focus of the Violet ones, and conſequently muſt occaſion a ſenſible Confuſion in the Viſion of the former, one and the ſame Point of the Scarlet Object affecting ſeveral Nerves in the *Retina*. Yet upon frequent Trials I have not been able to perceive any Inequality in this Point.

2. The ſecond Experiment I made in Water. I took a Braſs Ruler, and faſtning thereunto ſeveral Pieces of Silk, Red, Yellow, Green, Blue, and Violet, I placed it at the bottom of a Square Veſſel of Water: Then I retired from the Veſſel ſo far, as not to be able to ſee the aforeſaid Ruler and coloured Silks, otherwiſe than by the Help of the refracted Ray. Now, did Mr. *Newton*'s Doctrine hold, I conceived I ſhould not ſee all the mentioned Colours in a ſtreight Line with the Ruler, in regard the unequal Refrangibility of different Rays muſt needs diſplace ſome more than others. Yet in Effect, upon many Trials, I conſtantly found them in as ſtreight a Line, as the bare Ruler had appeared in.

3. To advance this Experiment, I adjoined a ſecond Refraction to the former of Water, by placing my Priſm ſo, as to receive perpendicularly the refracted Species of the Silk and Ruler; whereby only the emergent Species ſuffered a ſecond Refraction: but ſtill with equal Succeſs, as to their appearing in a ſtreight Line to the Eye placed behind the Priſm.

4. To theſe two Refractions I further added a third, by receiving the coloured Species obliquely upon the Priſm; whereby both incident and emergent Species ſuffered their reſpective Refractions: But ſtill with the ſame Succeſs as formerly, as to the ſtreight Line they appeared in.

For further Aſſurance in this Experiment, leſt Prepoſſeſſion, occaſion'd previous Knowledge of the Silk's Situation in a ſtreight Line, might bly prejudice the Judgment of the Eye (as ſometimes I have obſerv'd happen to the Judgment the Eye paſſeth upon the Diſtance of Objects) ll'd into the Room ſome unconcern'd Perſons, wholly ignorant of what Experiment aim'd at; and demanding whether they ſaw not the coloured s and Ruler in a crooked Line, they anſwer'd in the Negative.

The next Experiment I made in uncompounded Colours (as Mr. *Newton* us them, *Prop.* 5. and 13.) as follows. Having caſt two coloured Images upon the Wall, ſo as the Scarlet Colour of the one did fall in a ſtreight Line (parallel to the Horizon) with the Violet of the other; I then looked upon both through another Priſm, and found them ſtill appear in a ſtreight Line parallel to the Horizon, as they had formerly done to the naked Eye. Now according to Mr. *Newton*'s Aſſertion of different Refrangibility in different Rays, I conceive the Violet Rays ſhould ſuffer a greater Refraction in the Priſm at the Eye, than the Scarlet ones; and conſequently both Colours ſhould not appear in a ſtreight Line parallel to the Horizon.

6. Another Experiment I made, in order to ſome further Diſcovery of that ſurprizing Phænomenon of the coloured Image, which occaſioned Mr. *Newton*'s ingenious Theory of Light and Colours, as alſo of his excellent Invention of the reflecting Teleſcope and Microſcope. Having then ſometimes ſuſpected, that not only the direct Sun-Beams, but alſo other extraneous Light, might poſſibly influence the coloured *Spectrum*, I hoped to diſcover the Truth of this Suſpicion by means of the Sun-Spots, made to appear in the coloured Image, by placing a Teleſcope behind the Priſm. But my Endeavours proving ineffectual herein, by reaſon of ſome intervening Difficulties, I thought at length of a more feaſible Method, in order to the deſigned Diſcovery; as in the following Experiment.

Fig. 77. I faſtned a very white Paper-Circle (about an Inch in Diameter) upon my Window ſhuts; and beholding it thro' my Priſm, I found a coloured Image painted thereby upon my *Retina*, anſwerable in almoſt all reſpects to the former of the Sun-Beams upon the Wall, eſpecially when the Paper-Circle was indifferently well illuminated. This Image indeed appeared contrary to the former, as to the Situation of Colours, that is, the Scarlet appearing above, the Violet below, tho' but faint. But this I was not ſurprized at, having obſerved upon diſſecting the Eye, that Objects are painted on the Retina after a contrary Poſture to what they appear to ſight. Having thus rendered the coloured Image much more tractable than formerly it was, I conceived good Hopes of ſome further Diſcovery in the Point mentioned.

In Purſuance then of my former Suſpicion, having fixed my Priſm in a ſteady Poſture, I cauſed the Paper C to be applied cloſe up to the Paper-Circle *a b d*; whereupon the former Violet *d*, and the Scarlet Colour of C, vaniſhed into Whiteneſs. Next I removed the mentioned Circle from the Shuts, and placed it in the open Window, ſupported only by the Edge *d*: whereupon, to my Aſtoniſhment, all the former Colours exchanged Poſtures in the *Retina*; the Scarlet now appearing below, the Violet above, the intermediate Colours ſcarce diſcernible. And here, by the by, 'tis very remarkable, that during this Obſervation I clearly perceived both Blue and Scarlet Light to be tranſparent, I being able to diſcern ſeveral Objects thro' both, namely, Steeples oppoſite to my Window: Whence it follows, that theſe Colours do in great Part ariſe from the neighbouring Light. Laſtly, I placed the Paper-Circle anew, ſo as the one half *b* was faſtned to the Shuts, the other Semicircle *a* being expoſed to the open Air. Whereupon the Semicircle *a* became bordered with Violet above, Scarlet below; but the other Semicircle *b*, quite contrary. Hence I make the following Inferences.

Firſt, That not only the Light reflected from the Paper-Circle, but alſo from the ambient Air, hath great Influence upon the coloured Image, eſpecially as to the Violet and Scarlet Colours. Whence, perchance, it will not hereafter ſeem ſtrange that the coloured Spectrum on the Walls is ſo long, but only that the Breadth is not greater. Secondly, Were there a more luminous Body behind the Sun, we ſhould in all likelihood have the Colours of the Spectrum in a contrary Situation to what they appear in at preſent: Whence, Thirdly, it ſeems to follow, that the preſent Situation and Order of Colours ariſeth not from any intrinſical Property of *Refrangibility*, (as maintained by Mr. *Newton*) but from contingent and extrinſical Circumſtances of neighbouring Objects: For accordingly as the Body behind the Paper-Circle was more or leſs illuminated than the Circle itſelf, all the ſeveral Colours changed their Situation.

8. The next Experiment was made in order to Mr. *Newton*'s Doctrine of primary Colours, as *Prop.* 5. Having covered the Hole in the Window-ſhuts with a thin Slice of Ivory, the tranſmitted Light appeared Yellow; but upon adding three, four, or more Slices, it became Red. Whence it ſeems to follow, that Yellowneſs of Light is not a primary Colour, but a Compound of Red, *&c.*

9. The laſt Experiment was made in reference to Mr. *Newton*'s 12. *Prop.* where from his own Principles he renders a very plauſible Reaſon of a ſurprizing Phænomenon, related by Mr. *Hooke*; namely, of two Liquors, the one Blue, and the other Red, both ſeverally tranſparent; yet both, if placed together, became opake. The Reaſon whereof, ſaith Mr. *Newton*, is, becauſe if one Liquor tranſmitted only Red, the other only Blue, no Rays could paſs thro' both.

In reference then to this Point, I filled two ſmall Glaſſes with flat poliſhed Bottoms, the one with *Aqua fortis* deeply died Blue, the other with Oil of Turpentine died Red, both to that Degree, as to repreſent all Objects thro' them reſpectively Blue or Red: Then placing the one upon the other, I was able to diſcern ſeveral Bodies thro' both. Whereas, according to Mr. *Newton*'s Theory, no Object ſhould appear through both Liquors; becauſe if one tranſmit only Red, the other only Blue, no Rays can paſs thro' both.

P. S. *Juſt upon the cloſe of the adjoined Letter, I received from Mr.* Gaſcoigne *yours of* May *the 4th; wherein you are pleaſed to favour us with an exact Account of the famous Experiment of the coloured Spectrum, lately exhibited before the* Royal Society. *I was much rejoiced to ſee the Trials of that Illuſtrious Company agree ſo exactly with ours here; tho' in ſomewhat ours diſagree from Mr.* Newton.

o. The Things oppoſed by Mr. *Line* being upon Trials found true and inted me, I begin with the new Queſtion about the Proportion of the Length the Image to its Breadth: And it is no wonder that Mr. *Lucas* found the nage ſhorter than I did, ſeeing he tried the Experiment with a leſs ngle. *Anſwered by Mr.* Newton. *Ibid. p.* 658.

The Angle indeed which I uſed was but about 63 Degrees, 12 Min. and his is ſet to 60 Degrees; the Difference of which from mine being but 3 Deg. 12 Min. is too little to reconcile us; but yet it will bring us conſiderably nearer together. And if this Angle was not exactly meaſured, but the round Number of 60 Degrees ſet down by gueſs, or by a leſs accurate Meaſure (as I ſuſpect by the conjectural Meaſure of the Refraction of his Priſm, by the Ratio of the Sines as 2 to 3, ſet down at the ſame Time, inſtead of an Experimental one) then might it be two or three Degrees leſs than 60 Deg. if not ſtill leſs: And all this, if it ſhould be ſo, would take away the greateſt Part of the Difference between us.

But however it be, I am well aſſured my own Obſervation was exact enough. For I have repeated it divers times ſince the Receipt of Mr. *Lucas*'s Letter, and that without any conſiderable Difference of my Obſervations, either from one another, or from what I wrote before: And that it might appear experimentally, how the Increaſe of the Angle increaſes the Length of the Image, and alſo that no body, who has a mind to try the Experiment exactly, might be troubled to procure a Priſm which has an Angle juſt of the Bigneſs aſſigned by me, I tried the Experiment with divers Angles, and have ſet down my Trials in the following Table; where the firſt Column expreſſes the Angles of two Priſms which I uſed, which are meaſured as exactly as I could, by applying them to the Angle of a Sector; and the ſecond Column expreſſes, in Inches, the Length of the Image made by each of thoſe Angles; its Breadth being two Inches, its Diſtance from the Priſm 18 Feet and 4 Inches, and the Breadth of the Hole in the Window-ſhut ¼ of an Inch.

	Angles	*Lengths*		*Angles*	*Lengths*
The firſt Priſm	56° 10′	7¼	The ſecond Priſm	54° 0′	7⅕
	60 24	9½		62 12	10⅓
	63 26	10⅓		63 48	10¾

You may perceive, that the Length of the Images, in reſpect of the Angles that made them, are ſomething greater in the ſecond Priſm than in the firſt; but that was becauſe the Glaſs, of which the ſecond Priſm was made, had the greater refractive Power.

The Days in which I made theſe Trials were pretty clear, but not ſo clear as I deſired; and therefore, afterwards meeting with a Day as clear as I deſired, I repeated the Experiment with the ſecond Priſm, and found the Lengths of the Image made by its ſeveral Angles, to be about ¼ of an Inch greater than before; the Meaſures being thoſe ſet down in the Table.

	Angles	*Lengths*
The ſecond Priſm	54° 0′	7⅓
	62 12	10½
	63 48	11

The Reaſon of this Difference, I apprehend, was, that in the cleareſt Days the Light of the White Skies, which dilutes and renders inviſible the fainteſt Colours at the Ends of the Image, is a little diminiſhed in a clear Day, and ſo gives leave to the Colours to appear to a great Length; the Sun's Light at the ſame time becoming briſker, and ſo ſtrengthening the Colours, and making the faint ones at the two Ends more conſpicuous: For I have obſerved, that in Days ſomething cloudy, whilſt the Priſm has ſtood unmoved at the Window, the Image would grow a little longer or a little ſhorter, accordingly as the Sun was more or leſs obſcured by thin Clouds which paſſed over it; the Image being ſhorteſt while the Cloud was brighteſt, and the Sun's Light fainteſt. Whence it is eaſy to apprehend, that if the Light of the Clouds could be quite taken away, ſo that the Sun might appear ſurrounded with Darkneſs, or if the Sun's Light were much ſtronger than it is, the Colours would ſtill appear to a greater Length.

In all theſe Obſervations the Breadth of the Image was juſt two Inches. But obſerving that the Sides of the two Priſms I uſed were not exactly plain, but a little Convex, (the Convexity being about ſo much as that of a double Convex-Glaſs of a ſixteen or eighteen Foot Teleſcope) I took a third Priſm, whoſe Sides were as much Concave as thoſe of the other were Convex; and this made the Breadth of the Image to be two Inches and a third Part of an Inch; the Angles of this Priſm, and the Lengths of the Image made by each of thoſe Angles, being thoſe expreſſed in this Table.

Angles	*Lengths*
58°	8½
59½	9
62½	10⅓

In this Caſe you ſee the Concave Figure of the Sides of the Priſm, by making the Rays diverge a little, cauſes the Breadth of the Image to be greater in proportion to its Length than it would be otherwiſe. And this I thought fit to give you notice of, that Mr. *Lucas* may examine, whether his Priſm hath not this Fault. If a Priſm may be had with Sides exactly plain, it may do well to try the Experiment with that; but 'tis better if the Sides be about ſo much Convex as thoſe of mine are, becauſe the Image will thereby become much better defined: For this Convexity of the Sides does the ſame Effect, as if you ſhould uſe a Priſm with Sides exactly plain, and between it and the Hole in the Window-ſhut, place an Object-Glaſs of an 18 Foot Teleſcope, make the round Image of the Sun appear diſtinctly defined on the Wall n the Priſm is taken away, and conſequently the long Image made by the ſm to be much more diſtinctly defined (eſpecially at its ſtreight Sides) than would be otherwiſe.

One Thing more I ſhall add: That the utmoſt Length of the Image, n the fainteſt Red at one End, to the fainteſt Blue at the other, muſt be aſured. For in my firſt Letter about Colours, where I ſet down the Length be five Times the Breadth, I called that Length the utmoſt Length of the Image; and I meaſured the utmoſt Length, becauſe I account all that Length to be cauſed by the immediate Light of the Sun, ſeeing the Colours (as I noted above) become viſible to the greateſt Length in the cleareſt Days, that is, when the Light of the Sun tranſcends moſt the Light of the Clouds. Sometimes there will happen to ſhoot out from both Ends of the Image a glaring Light a good way beyond theſe Colours; but this is not to be regarded, as not appertaining to the Image. If the Meaſures be taken right, the whole Length will exceed the Length of the ſtreight Sides by about the Breadth of the Image.

By theſe Things ſet down thus circumſtantially, I preſume Mr. *Lucas* will be enabled to accord his Trials of the Experiment with mine; ſo nearly at leaſt, that there ſhall not remain any very conſiderable Difference between us. For if ſome little Difference ſhould ſtill remain, that need not trouble us any further, ſeeing there may be many various Circumſtances which may conduce to it; ſuch as are not only the different Figures of Priſms, but alſo the different refractive Power of Glaſſes, the different Diameters of the Sun at divers Times of the Year, and the little Errors that may happen in meaſuring Lines and Angles, or in placing the Priſm at the Window; though, for my part, I took Care to do theſe Things as exactly as I could. However, Mr. *Lucas* may make ſure to find the Image as long or longer than I have ſet down, if he take a Priſm whoſe Sides are not hollow ground, but plain, or (which is better) a very little convex, and whoſe refracting Angle is as much greater than that I uſed, as that he hath hitherto tried it with, is leſs; that is, whoſe Angle is about 66 or 67 Degrees, or (if he will) a little greater.

Concerning Mr. *Lucas*'s other Experiments, I am much obliged to him that he would take theſe Things ſo far into Conſideration, and be at ſo much Pains for examining them; and I thank him ſo much the more, becauſe he is the firſt that hath ſent me an experimental Examination of them. But yet it will conduce to his more ſpeedy and full Satisfaction, if he a little change the Method which he has propounded, and, inſtead of a Multitude of Things, try only the *Experimentum Crucis*: For it is not Number of Experiments, but Weight to be regarded; and where one will do, what need many?

The main Thing he goes about to examine is, the *Different Refrangibility of Light*; and this I demonſtrated by the *Experimentum Crucis*. Now if this Demonſtration be good, there needs no further Examination of the Thing; if not good, the Fault of it is to be ſhewn: For the only way to examine a demonſtrated Propoſition is to examine the Demonſtration. Let that Experiment therefore be examined in the firſt Place, and that which it proves be acknowledged; and then, if Mr. *Lucas* wants my Aſſiſtance to unfold the Difficulties which he fancies to be in the Experiments he has propounded, he ſhall freely have it. At preſent I ſhall ſay nothing in Anſwer to his Experimental Diſcourſe, but this in general, That it has proceeded partly from ſome Miſunderſtanding of what he writes againſt, and partly for want of due Caution in trying Experiments; and that amongſt his Experiments, there is one, which, when duly tried, is, next to the *Experimentum Crucis*, the moſt conſpicuous Experiment, I know, for proving the different Refrangibility of Light, which he brings it to prove againſt.

By the Poſtſcript of Mr. *Lucas*'s Letter, one not acquainted with what has paſſed, might think that he quotes the Obſervation of the *Royal Society* againſt me; whereas the Relation of their Obſervation, which you ſent to *Liege*, contained nothing at all about the juſt Proportion of the Length of the Image to its Breadth according to the Angle of the Priſm, nor any Thing more (ſo far as I can perceive by your laſt) than what was pertinent to the Things then in Diſpute, *viz.* that they found them ſucceed as I had affirmed. And therefore, ſince Mr. *Lucas* has found the ſame Succeſs, I ſuppoſe, that when he expreſſed, That *he much rejoiced to ſee the Trials of the* R. Society *agree ſo exactly with his*, he meant only ſo far as his agreed with mine.

P. S. *I had like to have forgotten to adviſe, that the* Experimentum Crucis, *and ſuch others as ſhall be made for knowing the Nature of Colours, be made with Priſms that refract ſo much, as to make the Length of the Images five Times its Breadth, and rather more than leſs; for otherwiſe, Experiments will not ſucceed ſo plainly with others, as they have done with me.*

VIII. I took a ſtiff Piece of brown Paper, and pricking a ſmall Hole therein, I held it at a little Diſtance from me; then applying a Needle to my Eye, I was ſurprized to ſee the Point of it inverted. The nearer the Needle was to the Hole, it was ſo much the more magnified, but leſs diſtinct; and if it were ſo held, as that its Image was near to the Edge of the Hole, its Point ſeemed crooked. So that, it ſeems, theſe ſmall Holes, or ſomewhat in them, perform the Effects of a Concave Speculum; and ſo I take leave to call them *Aerial Speculums.*

An Optical Experiment; by Mr. Steph. Gray. N. 221. *p.* 286. June, An. 1696.

IX. 1. Having this Opportunity I ſend you a Conſtruction of the Problem of *Albazen*, which my Colleges here approve of very much. The Problem is this: *A Concave or Convex Speculum being given, alſo the Eye and a Point of the Object, to find the Point of Reflexion.*

A Problem of Albazen, *ſolved by Mr.* Chr. Huygens. N. 97. *p.* 6119. Oct. An. 1673. *Fig.* 78.

Let the Speculum be a Part of a Sphere whoſe Center is the Point A, let the Eye be at B, and the viſible Point at C, and let the Plain drawn through A, B, C, make a Circle D*d* in the Sphere, in which the Points of Reflexion are to be found. Through the three Points A, B, C, let the Circumference of a Circle be deſcribed, whoſe Center is Z: Let A E produced meet it in R being perpendicular to B C, and to the two Lines R A, O A, let N A be a hird Proportional, and N M parallel to B C will be one of the Aſymptotes. A ain, let theſe be Proportionals E A, $\frac{1}{2}$ A O, A I, and the Sum I Y being ec al to I N, let Y M be drawn parallel to A Z; which will be the other A ymptote. Laſtly taking I X, I S, each of which is equal in Power to half th Square A O, together with the Square A I; the Points *x* and *s* will be in th Hyperbola, or the oppoſite Sections D *d* to be deſcribed to the Aſymptoto s now found, whoſe Interſections with the Circumference D O will ſhew th Points of Reflexion required. This Conſtruction takes Place in every

Caſe in which the Problem is ſolid, except in one, wherein a Parabola and not an Hyperbola is to be deſcribed. That is, when the Circumference through the Points A, B, C, touches the Right Line A E.

By Mr. Sluſius. *ibid.* 2. When I reduced the Conſtruction of the ingenious Mr. *Huygens* to Calculation, I found he had follow'd the ſame Analyſis as myſelf. But ſince two Effections may be derived from it, each of them by the Hyperbola about its Aſymptotes, he made Choice of one, and I of the other as being the eaſier. Now it is plain, that nothing elſe is required in this Problem, if we reduce it to mere Geometrical Terms, unleſs that in a given Circle, whoſe Center is A and Radius AP, ſome Point as P ſhould be found, from whence drawing Right Lines PE, PB, to the given Points E, B, at an unequal Diſtance from the Center A, the Right Line AP being produced may biſect the Angle E P B. Now this admits of a Variety of Caſes. Either the Perpendicular from A upon the Right Line E B, that is A O, falls between E and B, or beyond B. If beyond, the Rectangle E O B is either equal to the Square of A O, or is greater or leſs. Concerning the Caſe of Equality we ſhall ſee afterwards; now we ſhall comprehend the other three Caſes nearly *Fig.* 79, 80, 81. in the ſame Conſtruction. Let a Circle paſs through the three Points A, E, B, to the Circumference of which let A O be produced to D. And if the Point O falls between E and B, the Right Line A O is to be produced towards O; but if it falls beyond B, and the Rectangle E O B be greater than the Square of A O, it muſt be produced towards A; but if that Rectangle be leſs than the Square, the Circle will cut the Right Line A O in the Point D. Then drawing A X parallel to E B, cutting the given Circle in N, let it be made, as the Rectangle D A O is to the Square of A N, ſo ½ A X to A H, which muſt be taken towards X if O falls between E and B, or the Rectangle E O B be leſs than the Square of O A, or on the contrary Side if it be greater. Now let us ſuppoſe O Q to be equal to A H, (directly to E B in the firſt and ſecond Caſe, but towards E in the third) then let theſe be made Proportionals X A, N A, H K, to be taken in all Caſes towards X; and A O being divided in V, that K A to A V may have the ſame Ratio as A D to A X; let K V be joined, and produced till it meet the Right Line Q M, parallel to O A, indefinitely produced in the Point L. Then in every Caſe K L and Q L will be the Aſymptotes of the Hyperbola, which being deſcribed through the Point O will anſwer the Purpoſe: Yet with this Difference, that in the firſt and ſecond Caſe an Hyperbola through O will ſolve the Problem in the Convex Speculum, but the Section oppoſite to it in the Concave. But in the third Caſe on the contrary, the Hyperbola through O will ſerve for the Concave, and the oppoſite Hyperbola for the Convex. And thus it will be when the Point V falls between A and O; for if it ſhould fall beyond O, one Hyperbola alone deſcribed between the ſame Q L, K L, would ſuffice both for the Concave and Convex. But if V ſhould fall upon the Point O, then the Problem would become plane, and the Right Lines L Q, L K, would perform it. Whence it appears, that there are infinite Caſes of this Problem, which may be ſolved by what is called *Locus planus*: So that they ſeem to deſerve to be forgiven, who have thought it may be ſolved univerſally by the ſame *Locus*; becauſe thus ſometimes the Calculation has been ſucceſsful. For no Poſition can be given of the three Points A, E, B, (as to the Caſe of Equality between the Rectangle E O B and the Square O A, we ſhall ſee preſently) which does not admit that ſome Circle may be deſcribed from the Center O [illegible]t whoſe Circumference the Problem may be ſolved by a plane Place. [illegible]ow the Radius of this Circle, if it be worth while, may thus be found. In the firſt and ſecond Caſe of the Conſtruction above, let it be made, as the Square of A X together with the double of the Rectangle O A D, to the double Square of A D, ſo the Square of A O to the Square of A N; A N will be the Radius required. But in the third Caſe it muſt be made, as the Square of A X ſubtracting the double Rectangle O A D, to the double Square of A D, ſo is the Square of A O to the Square of A N.

Now there remains another Caſe to be conſtructed, that is, when the Rectangle E O B is equal to the Square of A O, or in which the Circle deſcribed through the Points A, E, B, touches the Right Line A O. For Mr. *Huygens* has rightly admoniſhed, that in this Caſe a Parabola muſt be deſcribed. Which yet is not ſo to be underſtood, as though it could not be ſolved by an Hyperbola, ſince it admits of either an Hyperbola, or an Ellipſis, nay an infinite Number of them, if any one ſhall proceed by our Method. However it admits of a Solution by a Parabola, which the other Caſes refuſe. For the ſame Reaſon that muſt be limited when he ſays, that his Conſtruction takes Place in every Caſe where the Problem is ſolid; for he means, that by a ſmall Alteration an Hyperbola may always be found, which will ſerve the Purpoſe; which will appear to any one that ſhall compare the Caſes above conſtructed with his Conſtruction. Now that I may return to the Caſe of Equality, and that I may not ſeem to have made a raſh Aſſertion, here you have not one but two Parabola's, and oppoſite Hyperbola's beſides, that will anſwer the Purpoſe. Let the given Points be E, B, as *Fig.* 82. before; let a Circle be deſcribed with Center A, and another through the three Points A, E, B, whoſe Tangent is A O, and Center D. Drawing the Diameter N A D X, let there be three Proportionals X A, N A, Z A, the half of which is A L. Again let there be three Proportionals 2 O A, N A, I A, whoſe half is K A, and let the Rectangle L A O V be compleated; and L V being produced to S, till V S be a third Proportional to A I, O V; with Axis S L and *latus rectum* A I, and Vertex S, let a Parabola be deſcribed; for this will cut the Circle in the Points required, P, P. Another will do the ſame Thing; thus the Rectangle D A K C being compleated, and K C being produced to T, ſo that C T may be a third Proportional to A Z, D C, it may be deſcribed about the Axis T K, with the Vertex T, and he *latus rectum* Z A; for it will meet the Circle in the ſame Points P, P. The Conſtruction is ſtill eaſier by the oppoſite Sections; for making as before the three Proportionals X A, N A, Z A, let fall the Perpendicular Z I, *Fig.* 83. being a third Proportional to the double of A O, and A N. Therefore will Z I be greater than Z A, ſince the double of A O is leſs than X A. Then in the Point I let the Right Lines I Q, I M, be inclined to the Right Line I Z on both Sides to half a Right Angle, and be produced indefinitely both

Ways. Then about them as Aſymptotes let an Hyperbola be deſcribed through A, and another oppoſite to it; for this will ſatisfy the Problem in the Convex Speculum, and the other in the Concave. But ſince Z I is always greater than Z A, as we have ſhewn, the Right Line I M will never paſs through A. Therefore there will be no Caſe in which by this Conſtruction (as in the former) the Problem can be ſolved by th[illegible]ſymptotes themſelves. And yet this ſometimes may admit a *Locus Planus*, when it happens, that the Right Line X O, drawn to the Center D, may touch the Circle N P P; for then the Point of Contact itſelf will ſatisfy the Queſtion. And ſo much concerning a Problem, which has exerciſed the Wits of many, and whoſe Solution I compleated ſome Years ago.

Otherwiſe by Mr. Sluſius. *ibid.* p. 6123. *Fig.* 84.

I ſend you here my ſecond Thoughts about the Problem of *Albazen*.

Let a Circle be given whoſe Center is A; and D and d are Points given. Let that which is inquired be ſuppoſed to be done, and let the incident Ray be D E; the reflected Ray E d, and from the Point of Reflexion E let the Perpendicular E I fall upon the joined Line D A; and upon the ſame the Perpendicular d N from d, and let the Tangent E C and the Ray d E meet the ſame produced in B. Now make D A $= z$, A I $= a$, N A $= n$, E I $= e$, d N $= b$, B A $= y$, A E $= q$, and C A $= x$. Therefore, ſince the Angles D E C and C E B are equal, and C E A is a Right Angle, by the Hypotheſis the three Lines D A, C A, B A, will be Harmonically Proportionals, which is eaſily ſhewn. Therefore it will be as D A to B A ſo is D C to C B; or in Analytical Terms; $z . y :: z - x . x - y$, and $2zy - xy = zx$, or $\frac{2zy}{z+y} = x$. Now ſince the Rectangle C A I, or xa, is equal to the Square of A E, or qq, it will be $x = \frac{qq}{a}$, and conſequently $\frac{2zy}{z+y} = \frac{qq}{a}$, or $\frac{zqq}{2za - qq} = y$. Again, it is as d N to E I, ſo is N B to I B; or $b . e :: y - n . y - a$. Therefore $ye - ne = by - ba$, and $y = \frac{ba - ne}{b - e}$. Therefore $\frac{zqq}{2za - qq} = \frac{ba - ne}{b - e}$, or $2zbaa - 2znae - qqba + qqne = bzqq - zqqe$. Which is an Equation to the Hyperbola about its Aſymptotes, the Conſtruction of which with a given Circle ſatisfies the Problem. But becauſe of the Circle, ſince it is $qq = aa + ee$, if inſtead of $2bzaa$ its Value $2bzqq - 2bzee$ is ſubſtituted, we ſhall have $bzqq - 2bzee - 2znae - qqba + qqne = - zqqe$, which is another Equation to the Hyperbola about the Aſymptotes. And by this Method, or by that which we have explained in our Treatiſe of Analyſis, infinite Equations will come out to Hyperbolas and Ellipſes, which with a given Circle will perform the Problem; except that the Effections will generally become ſo intricate, as it may not be worth while to attempt them. Yet they may be conſtructed after that Manner which we have made uſe of in the Ellipſis.

We have reduced, as you will perceive, the Sum of our Calculation to the Line D A; but you may obſerve, that with the ſame eaſe it might have been referred to d A, which is alſo given, by drawing thoſe Lines which in the Scheme are ſhadowed out by Points. But there is no need of the Labour of a new Calculation. For if you apply to the Right Line d A and to its Parts the ſame Analytical Terms as before, that is, if you make $d\,A = z$, $Dn = b$, $n\,A = n$, $AI = a$, $iE = e$, &c. the ſame Equation will come out as before, and you will obtain infinite other Hyperbola's and Ellipſes, which with the given Circle will ſatisfy the Porblem. I ſhould be tireſome if I was to purſue all the Caſes, ſince their Equations differ only by the Signs + and —. I except only one, which is when d A D is a right Angle; for its Equation will be had only by expunging out of the former thoſe Terms which are effected by n, which then vaniſhes into nothing. Which Equation will be this, $2zbaa - qqba = bzqq - zqqe$; or this, $zbqq - qqba = 2zbee - zqqe$, inſtead of $2zbaa$ writing its Value.

Yet it is to be obſerved, that although by referring the Analyſis to the right Line D A, two Hyperbola's in the Equation preſently offer themſelves; and others as many different from the former, when the Calculation is referred to the Right Line d A; yet the very ſame Parabola's come forth, when the Analyſis is referred to either of the Right Lines d A or D A. The Reaſon of which you will perceive by a little Conſideration.

Now give me Leave, learned Sir, to apply the foregoing Analyſis to all the Problems which are uſed to be propoſed about the Reflection of Spherical *Specula*, and that by a new Scheme. Therefore let there be a Circle as before, whoſe Center is A, D a point given, and from that an incident Ray *Fig.* 85. DE, whoſe reflected Ray is E Q. Let D A be joined, and to it be drawn the Tangent E C, and perpendicular E I. Let the Right Line Q E B be produced to the ſame, and the Parts be denominated as before; that is, $DA = z$, $CA = x$, $AE = q$, $BA = y$, $AI = a$, $IE = e$. Now becauſe of the three Lines harmonically Proportionals D A, C A, B A, and the three Geometrically Proportionals C A, A E, A I, we ſhall always have the Equation $y = \frac{zqq}{2za - qq}$, upon whatever point of the Circle the Ray D E may fall. Therefore if the Point E be required, in which if the Ray D E falls, it may be reflected parallel to the Diameter L A V perpendicular to D A, the reflected Ray Q E produced will paſs through I, as is plain; and I and B will coincide. Therefore $a = y = \frac{zqq}{2za - qq}$, or $aa - \frac{1}{2} \times \frac{qqa}{z} = \frac{1}{2}qq$, and the Problem will be ſolved by Plains.

If the Point be required, from whence a Ray may be reflected parallel to any other Line, as A K drawn from the Center A; from the point L draw a Tangent to it $KL = d$; it is plain the Triangles A K L, E I B, will be similar, ſince all the Sides of one are parallel to the Sides of the other. Therefore A L to L K, as E I to I B, or $q.d::e.a - y$; and $\frac{qa - de}{q} =$

$y = \frac{zqq}{2za - qq}$; and $zq^3 = 2qzaa - 2zdae - q^3a + qqde$, or for aa putting $qq - ee$, $zq^3 = 2zq^3 - 2zqee - 2zdae - q^3a + q^2de$. But either of the Equations is to an Hyperbola about the Aſymptotes, which with a given Circle ſolves the Problem.

Let it now be propoſed to cauſe, that the reflected Ray ſhall paſs through a given Point N, as in the Problem of *Albazen*, or that being produced towards the Point of Reflection E, it may meet the given Point N. From N let NO $= n$ fall perpendicularly upon AL, and make AO $= b$. It is plain it will be, as AO to the difference of ON and AB, ſo is EI to IB; that is, $b . n - y :: e . a - y$: Or $b . y - n :: e . y - a$. Therefore $\frac{ba - ne}{b - e} = y = \frac{zqq}{2za - qq}$. Whence $2zbaa - 2znae - qqba + qqne = bzqq - zqqe$; which is the very Equation of *Albazen's* Problem, which we deduced above. Or in the ſecond caſe $\frac{ba + ne}{b + e} = y = \frac{zqq}{2za - qq}$, or $2zbaa + 2znae - qqba - qqne = zbqq + zqqe$.

And theſe are the Problems commonly propoſed about the Point of Reflection, in which hitherto we have ſuppoſed the Diſtance of the point D to be finite. But the Analyſis will be eaſier if we ſuppoſe it infinite. For CA being divided equally in G, it is plain from the Property of the three harmonical Proportionals DA, CA, BA, that the three Lines DG, CG, BG, will be Geometrical Proportionals, whatever the Diſtance of the point D is ſuppoſed to be. Therefore if it is ſuppoſed infinite, BG will become nothing, and the Points B, G, will coincide. Therefore AB will always be equal to BC, and CA $= 2y$, and the Rectangle CAI being equal to the Square of AE, will give in Analytical Terms $2ay = qq$, or $y = \frac{qq}{2a}$. And ſince the Diſtance of the Point D is ſuppoſed infinite, ED will be parallel to AC. Therefore if the reflected Ray parallel to AL is required, becauſe in this Caſe a and y coincide, it will be $a = y = \frac{qq}{2a}$, or $aa = \frac{1}{2}qq$; if it is deſired to be parallel to AK, it will be again $q . d :: e . a - y$, and $\frac{qa - de}{q} = y = \frac{qq}{2a}$, or $2qaa - dae = q^3$. If it is required to paſs through N, it will be as above $\frac{ba \pm ne}{b \pm e} = y = \frac{qq}{2a}$, and therefore $2baa \pm 2nae = bqq \pm qqe$. Which Equations are alſo to Hyperbola's about their Aſymptotes, unleſs when the Point N is ſuppoſed to be in AL; for whereas then n becomes nothing, taking away thoſe Terms from the Equation in which n is found, the remaining Terms will give an Equation to a Parabola, as we have taken Notice before.

You cannot expect, learned Sir, that as hitherto I have given Examples only in concave Speculum's, ſo now I ſhould proceed to convex. For you know the Analyſis is the ſame in both, and their Equations differ only in varying the Signs + and —. You know the Parabola or Ellipſis that ſatiſfies one, will ſatisfy the other alſo; and if the Hyperbola ſolves the Problem in the Convex, the oppoſite Hyperbola will do the ſame in the Concave. Therefore omitting theſe I ſhall only add, that by the ſame Analyſis in concave Speculum's we may find their Focus's, and the Spaces taken up by the Rays in the Axis, at any diſtance of the lucid Point: But with great Facility when the Rays come parallel, which yet I have ſeen demonſtrated by ſome in a round-about way. For in the Concave Speculum E E, whoſe Center is A, if the extreme Ray is ſuppoſed to be reflected to the Axis A R in B, *Fig.* 86. drawing the Tangent E C it will be $CB = BA$. Let the Semiaxis A R be biſected in Q; therefore Q will be the Focus; and Q B will be the Space required. But Q B is half C R, becauſe of the Equals A Q, Q R, A B, B C, that is, half of the Exceſs of the Arch E R above the whole Sine. Therefore if the Arch E R, for Example, be Nine Degrees, A C will be 101246, and $BQ = \frac{623}{100000}$ of A R.

4. This is the Compendium that I found at the ſame time, about the firſt Conſtruction communicated to you at firſt. Drawing the Line A T, parallel to C B, and that being biſected in V, this is that point through which one of the oppoſite Hyperbola's ought to paſs, whoſe Aſymptotes are found to be Y M, M N. *Otherwiſe by Mr.* Huygens, Nº 98. *p.* 6140. D. An. 1673. *Fig.* 87, 88.

But here is that genuine Conſtruction which is ſufficient in all caſes. Let the given Circle be E D whoſe Center is A, and the points given B and C. Drawing the Lines A B, A C; let theſe be Proportionals, B A, the Radius of the Circle, and F A; and likewiſe C A, the Radius of the Circle, and G A. Then let F G be joined, and let it be biſected in H. Through this Point let the Lines L H K, M H N, be drawn, interſecting one another at Right Angles, of which let L H K be parallel to the Line which biſects the Angle B A C. Theſe are the two Aſymptotes of the Hyperbola's to be deſcribed through the Points F and G, one of which will paſs alſo through the Center A; whoſe Interſections with the Periphery of the Circle will mark out the Points of Reflection required.

5. Here the great *Huygens* has well obſerved, how the Equilateral Hyperbola may be accommodated to all the caſes, which, as I inſinuated in my forier, immediately offered itſelf in the caſe of a Right Angle. Alſo of thoſe ifinite Ellipſes which might be uſed, one may be choſen of no difficult Conruction. But it is tedious to dwell ſo long upon one Problem. But one iing ſtill remains of no diſagreeable Speculation: That is, ſince the Sections hich with the given Circle are made uſe of for the Solution of the Problem, ut it in four Points, of which only two can ſerve for the Reflexion; it may e inquired, what Problem is ſolved by the other two, and how is the Propoion to be expreſſed ſo as to include all thoſe four Caſes? And again, do *Farther conſidered by Mr.* Sluſius. *ib. p.* 6141.

not thoſe four Caſes occur, when thoſe Points are equally diſtant from the Center?

Again ib. The learned *Huygens* makes uſe of no other than my Analyſis, which admits of a Parabola only in one Caſe. That this may appear more evidently to you, I will here produce the Equation which he has conſtructed. Recollect (if you pleaſe) what I wrote to you, when I ſent you my ſecond Thoughts, and you will find, that I aſſign'd two Equations proper for ſolving the Problem by an Hyperbola about its Aſymptotes. They were theſe following.

$$2zbaa - 2znae - qqba + qqne = bzqq - zqqe,$$

And $$bzqq - 2znae - qqba + qqne = 2zbee - zqqe;$$

Then I added, that by a ſmall Alteration (for inſtance, by ſubſtituting for qq its Value $aa + ee$,) infinite Hyperbola's and Ellipſes might be found, which with the given Circle would ſolve the Problem. Now in the former of theſe Equations for $bzqq$ let its Value be ſubſtituted; then

$$zbaa - 2znae - qqba + qqne = bzee - zqqe;$$

Or, $$aa - \frac{qqa}{z} = ee - \frac{qqe}{b} + \frac{2nae}{b} - \frac{qqne}{zb}.$$

And this is the Equation which that very learned Gentleman has conſtructed, with great Ingenuity and equal Facility.

Again ib. Fig. 89. I happened lately upon the following Conſtruction, which I could not forbear ſubmitting to your Judgment and Cenſure, believing that a ſhorter can hardly be given. Let the given Points be E, B, the Circle with Center A. Joining EA, BA, cutting the Circle in F and C; let EA, FA, VA, be three Proportionals, and three others BA, CA, XA. Then VX being joined and produced at Pleaſure, with Vertex X, *latus tranſverſum* VX, and *latus rectum* equal to it, let the Hyperbola XP be deſcribed, whoſe Ordinates to the Diameter VXG are parallel to the Right Line AB. For this ſatisfies the Propoſition in the Caſe of a Convex Speculum, and its oppoſite in the Caſe of a Concave. If you deſire to have the Aſymptotes they will eaſily be found, by producing VX till it meets EB produced in L. Then biſecting VX in I, and taking LD equal to LI; for DI being joined will be one of the Aſymptotes, upon which the other falls perpendicularly at I.

But perhaps it will not be unacceptable to you to know how I arrived at

Fig. 90. this Conſtruction. Know then that I deduced it thus from my former Analyſis. The ſame things being given as before, let fall the Perpendicular AO upon EB, and let the Point deſired be P, from which let PR fall perpendicularly upon AO. Make AO $= b$, EO $= z$, OB $= d$, AP $= q$, PR $= e$, AR $= a$. Then this Equation is eaſily derived.

$$\frac{2zdae + 2bbae - 2bqqe}{zb - bd} + ee = aa - \frac{qqa}{b},$$ which may be changed into theſe,

$$\frac{zdae + bbae - bqqe}{zb - bd} = aa - \tfrac{1}{2}qq - \frac{\frac{1}{2}qqa}{b},$$

And $$\frac{zdae + bbae - bqqe}{zb - bd} + ee = \tfrac{1}{2}qq - \frac{\frac{1}{2}qa}{b}.$$

The Conſtruction of this laſt I have ſent to you already, and Mr. *Huygens* has ſent you the Conſtruction of the other. But as to the firſt, tho' it preſently came into my View, yet I almoſt neglected it, becauſe its Conſtruction ſeemed to be difficult. But I find myſelf to have been deceived by a needleſs Fear, ſince I have lately found it to bring me to this Conſtruction which I now ſend you. For the ſake of abbreviating the Calculation, make $z - d = k$, $zd + bb = bm$; it will be $ee + \frac{2mae - 2qqe}{k} = aa - \frac{qqa}{b}$. And adding $\frac{q^4 + m^2a^2 - 2q^2ma}{kk}$ on both Sides, it will be $ee + \frac{mae - 2qqe}{k} + \frac{q^4 + mmaa - 2qqma}{kk}$ (that is, the Square of $e - \frac{qq + ma}{k}$) $= aa - \frac{qqa}{b} + \frac{q^4 + m^2a^2 - 2q^2ma}{kk}$. Therefore there will be this Analogy, $kk . kk + mm :: aa - \frac{kkqqa}{bkk + bmm}, - \frac{2qqma + q^4}{kk + mm}$. And the Square of $e - \frac{qq + ma}{k}$. Which may be reduced to an eaſier Equation, if making $kk + mm = pp$, it ſhall be $\frac{ky}{p} = a$. At length it is, the Square of $e - \frac{qq}{k} + \frac{my}{p} = yy - \frac{qqky}{bp} - \frac{2qqmy}{kp} + \frac{q^4}{kk}$, which Equation you will find to anſwer the former Conſtruction, if you undertake the Calculation. And at the ſame time you will obſerve, to which ever of the Lines E A, A B, B E, the Analyſis is refer'd, the ſame Sections will always ariſe, though by a longer Proceſs and very different Equations.

From this Equation by Analogy we may deduce an Effection of the other Problem, that is, when a Point is ſought from whence the Reflected Ray ſhall be parallel to any given Line. Thus if the luminous Point B being given, and the Circle with Center A, the reflected Ray parallel to the Right Line A E were Fig. 91.
ſought. For it is the ſame thing as if in the other Problem the Diſtance of the Points A and E were ſuppoſed infinite. In which Caſe the third Proportional to E A and F A would vaniſh into nothing, and the Points A and V would coincide. Therefore V X would be equal to A X, and A E parallel to P E. Therefore apply the foregoing Conſtruction and you will ſolve the Problem. That is, with Vertex X, and *latus tranſverſum* V X or AX, and *latus rectum* equal to it, deſcribe the Hyperbola X E, whoſe Ordinates to the Diameter A X are parallel to the Right Line A E.

6. It is true, and likewiſe wonderful, that the Conſtruction I ſent you for- *By Mr.* Huygens.
merly, may alſo be found by Mr. *Sluſius's* Calculation, after the Change of *ibid. p.* 6143.
q into $aa + ee$. But this ſeems to be done by chance, nor does the Sim- *Fig.* 92.
plicity of the Conſtruction appear there, till after we have apply'd ourſelves to it.

The Problem of Alhazen] *A Circle being given whoſe Center is A, Radius A D, and two Points B, C; to find a Point H in the Circumference of the given*

Circle, whence drawing the Lines HB, HC, they ſhall make equal Angles at the Circumference.

Suppoſe it found, and drawing the Right Line AM which ſhall biſect the Angle BAC, draw HF perpendicular to it, as alſo BM, CL. Then join AH, to which let HE be perpendicular, and let HM meet the Lines BH, HC, in the Points K, G.

Now let it be $AM = a$, $MB = b$, $AL = c$, $LC = n$, Radius $AD = d$, $AF = x$, and $FH = y$. Now becauſe the Angles KHE and CHZ are equal, as alſo EHG; and EHA is a Right Angle; it will be as KE to EG, ſo KA to AG. And becauſe BM to MD, ſo HF to FK, it will be as BM + HF to HE, ſo is MF to FK. That is, $b + y . y :: a - x . \frac{ay - xy}{b + y}$. Add $FA = x$, then $KA = \frac{ay + bx}{b + y}$.

Again, becauſe CL to LG, as HF to FG, it will be *permutando & dividendo*, CL — HF to HF, ſo is LF to FG. That is, $n - y . y :: c - x . \frac{cy - xy}{n - y}$. Which being taken away from $AF = x$, tis $GA = \frac{nx - cy}{n - y}$.

But it is $EA = \frac{dd}{x}$, becauſe FA, AH, AE, are Proportionals. Therefore $EA - GA = EG = \frac{dd}{x} - \frac{nx + cy}{n - y}$. And $KA - EA = KE = \frac{ay + bx}{b + y} - \frac{dd}{x}$.

But we have ſaid it is KE to EG, ſo is KA to AG. That is, $\frac{ay + bx}{b + y} - \frac{dd}{x} . \frac{dd}{x} - \frac{nx + cy}{n - y} :: \frac{ay + bx}{b + y} . \frac{nx - cy}{n - y}$. Whence is found

$$2anxxy + 2bnx^3 - ddbnx - ddnxy = naddy + nbddx - 2acxyy - 2bcxxy + ddbcy + ddcyy - addyy - bddxy.$$

And becauſe $n = \frac{bc}{a}$, it becomes $\frac{2bbc}{a}x^3 - \frac{bbddcx}{a} - \frac{2bbcyyx}{a}$, becauſe $xx = dd - yy$. But it is $\frac{2bbc}{a}x^3 = \frac{2bbcddx}{a} - \frac{2bbcyyx}{a}$, becauſe $xx = dd - yy$. Therefore $-\frac{2bbcxyy}{a} - \frac{ddbcxy}{a} - 2acxyy + ddcyy = -addyy - bddxy$. And dividing all by y, and multiplying by a,

$$-2bbcxy - ddbcx - 2aacxy + ddcay = -aaddy - bddax$$

$$abddx - cbddx + acddy + aaddy = 2aacxy + 2bbcxy$$

$$\frac{abddx - cbddx + acddy + aaddy}{2aac + 2bbc} = xy,$$

which is an Equation to an Hyperbola.

Or becauſe $bc = na$, $\frac{abddx - anddx + acddy + aaddy}{2aac + 2bbc} = xy$.

Let $\frac{add}{aa + bb} = p$; therefore $\frac{pbx - pnx + pcy + pay}{2c} = xy$.

Fig. 93.

Now from hence the following Conſtruction was not difficult to find. Let BA, AC, be joined, and the Square of the Radius AD being ſeparately applied to each, let AP and AQ be produced by that Application. Then joining PQ let it be biſected in R, and through the Point R let RD, RN, be drawn cutting each other at Right Angles, of which let RD be parallel to AD, which biſects the Angle BAC. Now RD, RN, will be the Aſymptotes of the oppoſite Hyperbolas, one of which muſt paſs through the Center A, and which will cut the Circumference in H the Points required. Alſo the Hyperbolas will paſs through the Points P, Q.

The Reaſon of the Conſtruction appears, when Pγ, Qζ, are drawn, perpendicular to AM. For it is A$\gamma = \frac{add}{aa + bb} = p$, and A$\zeta = \frac{ap}{c}$. Alſo P$\gamma = \frac{pn}{c}$ and Q$\zeta = \frac{pb}{c}$. Therefore AO $= \frac{pc + pa}{2c}$, and OR $= \frac{pb - pn}{2c}$. Whence the reſt will be eaſy.

By Mr. Sluſius. ib. p. 6143. Fig. 94.

7. You will ceaſe to wonder, learned Sir, that in the Problem of *Albazen* the ſame Conſtruction ſhould be derived from different Equations, when you conſider that all we have hitherto made uſe of, are contained in one and the ſame general Analyſis. Now to ſhew this, let a Circle be given whoſe Center is A, the Points H and I; and let the Point required be K, to which from the Points I and H let be drawn the Right Lines HK, IK, and the Tangent KD. Then from A let any Line AG be drawn, meeting HK in E, IK in B, the Tangent KD in D; the Lines if needful being produced. Theſe Things ſuppoſed it is plain, becauſe of equal Angles EKD and DKB, and the Right Angle AKD, that the three Lines AE, BE, DE, will be Harmonically Proportional. Therefore drawing KC, IF, HG, perpendicular to AE, and calling AK $= q$, AC $= a$, CK $= e$, HG $= b$, AG $= d$, FA $= z$, FI $= n$, by the Method I formerly uſed in my ſecond Analyſis of this Problem, we ſhall have this general Equation, $ndaa - bzaa - nqqa + bqqa = ndee - zbee + 2bnae + 2zdae - dqqe - zqqe$.

Now ſuppoſe AG to be perpendicular to HI; there will be no Variety in the Equation, except that AF and AG (that is d and z) will be equal. Writing therefore d for z, the Equation will become $ndaa - bdaa - nqqa + bqqa = ndee - dbee + 2bnae + 2ddae - 2dqqe$. Or applying all to $nd - db$, the Equation is $aa - \frac{qqa}{d} = ee + \frac{2bnae + 2ddae - 2dqqe}{nd - db}$.

Which is the ſame that I deduced from my firſt Analyſis, though another Way, and which I lately ſent you conſtructed after an eaſy Manner.

Then ſuppoſe A G to coincide with A H; therefore H G or b vaniſhes into nothing. Therefore the Terms being expunged in which b is found, there will remain $ndaa - nqqa = ndee + 2zdae - dqqe - qqze$. This as you may remember I gave you as my ſecond Thoughts, and another like to it, in the Caſe wherein the Right Line A G paſſes through I.

Then let us ſuppoſe the Right Line A G to biſect the Angle H A I; then becauſe of the Similitude of the Triangles H A G, I A F, it will be as H G to G A, ſo is I F to F A; or $b . d :: n . z$, or $nd = bz$. Therefore taking away Equals, 'tis $bqqa - nqqa = 2bnae + 2zdae - dqqe - qqze$. The ſame which, as I underſtand by your Letters, Mr. *Huygens* has conſtructed.

Laſtly, let it be ſuppoſed that the Right Line H G biſects the Right Line H I; therefore H G = I G, or $b = n$. Then it will be by taking away Equals, $bdaa - bzaa = bdee - bzee + 2bbae + 2zdae - dqqe - qqze$. This though no difficult one, none of us have yet conſtructed. But theſe, and the general Equation itſelf, may be divided into two others, by ſubſtituting (as you know) for aa or ee their Values $qq - ee$ or $qq - aa$.

You ſee therefore that whatever has been done yet may be reſolved into the ſame Analyſis, which comprehends alſo infinite other Conſtructions by the given Circle and an Hyperbola. But to inveſtigate them is of no great Conſequence, ſince in this Problem, though formerly Solutions were wanting, yet now we have Plenty enough. I will only add a Conſtruction by the Parabola, and that two Ways; which though it may ſeem more operoſe than thoſe by the Hyperbola, yet it makes amends by the Simplicity of the Curve, in which the Parabola has the Advantage of the other Conic Sections.

Fig. 95. The ſame Things then being given, let A I be joined and produced to S, 'till A S become equal to A H, and H S be joined, biſect I S in M, and through M let R M Q be drawn perpendicular to H S, upon which let fall the Perpendicular A Q, parallel to M Q let the Ray A C be drawn. Then making three Proportionals I A, A C, A E, let it be as S A to A E, ſo is M Q to A D, and R S to A P, in the Right Line A Q towards Q. And in the ſame on the other Side take D O equal to D C. Then biſect P D in X, and let the Right Line V X L be inclined through X in half a Right Angle to A X, meeting the Perpendicular erected at D, in the Point V, and upon which from O let fall the Perpendicular O B. I ſay, if it be V X to X B, ſo is X B to B L, the Point L will be the Vertex, L V the Axis, and X V the *latus rectum* of the Parabola, which will ſatisfy the Problem in every Caſe. For it will cut the given Circle in the Points K, of which the higheſt and laſt will belong to the Problem of *Alhazen*, and the reſt to ſome other Problem.

Another Parabola may alſo be given, as I have hinted already, which will do the ſame as this, and whoſe Deſcription may eaſily be deduced from this, ſo that there is no Occaſion for any other. For let $A\delta$ be taken directly to D A, and equal to it; and $A\omega$ directly to O A, and alſo equal to it. Then biſect P δ in ξ, and through ξ let the Right Line $\nu\xi\beta$ be drawn perpendicular to X B, meeting with $\delta\nu$, perpendicular to O A, in ν, and upon which let the Perpendicular $\omega\beta$ fall. Then let it be as $\nu\xi$ to $\xi\beta$, ſo is $\xi\beta$ to $\beta\lambda$. Then λ will be the Vertex, $\lambda\xi$ the Axis, and $\nu\xi$ the *latus rectum* of the Parabola, which will cut the given Circle in the ſame Points with the former.

To find the principal Focus of Optick Glaſſes univerſally; by Mr. Edm. Halley. N. 205. *p.* 960.

Fig. 96.

X. Let B E β be a double Convex Lens, C the Center of the Segment E B, and K the Center of the Segment E β; B β the Thickneſs of the Lens, D a Point in the Axis of the Lens; and it is required to find the Point F, at which the Beams proceeding from the Point D are collected therein, the Ratio of Refraction being as m to n. Let the Diſtance of the Object D B $=$ D A $= d$, (the Point A being ſuppoſed the ſame with B, but taken at a Diſtance therefrom, to prevent the Coincidenee of ſo many Lines) the Radius of the Segment towards the Object C B, or C A $= r$, and the Radius of the Segment from the Object K β, or K $\alpha = \rho$, and let B β, the Thickneſs of the Lens, be $= t$, and then let the Sine of the Angle of Incidence D A G, be to the Sine of the Refracted Angle H A G, or C A φ, as m to n; and in very ſmall Angles the Angles themſelves will be in the ſame Proportion; whence it will follow, that, as d to r, ſo the Angle at C to the Angle at D, and $d+r$ will be as the Angle of Incidence G A D; and again, as m to n, ſo $d+r$ to $\frac{dn+rn}{m}$, which will be as the Angle G A H $=$ C A φ. This being taken from A C D, which is as d, will leave $\frac{m-nd-nr}{m}$ analogous to the Angle A φ D; and the Sides being in this Caſe proportional to the Angles they ſubtend, it will follow, that as the Angle A φ D is to the Angle A D φ, ſo is the Side A D or B D, to A φ or B φ: that is, B φ will be $= \frac{mdr}{m-nd-nr}$, which ſhews in what Point the Beams proceeding from D would be collected by means of the firſt Refraction: but if nr cannot be ſubtracted from $m-nd$, it follows, that the Beams after Refraction do ſtill paſs on diverging, and the Point φ is on the ſame Side of the Lens beyond D. But if nr be equal to $m-nd$, then they proceed parallel to the Axis, and the Point φ is infinitely diſtant.

The Point φ being found as before, and B φ — B β being given, which we will call δ, it follows by a Proceſs like the former, that β F, or the Focal Diſtance ſought, is equal to $\frac{\delta\rho n}{m-n\delta+m\varrho}=f$. And in the room of δ ſubſtituting B φ — B $\beta = \frac{mdr}{m-nd-nr}-t$, putting p for $\frac{n}{m-n}$, after due Reduction this Equation will ariſe, $\frac{mpdr\varrho-nd\varrho t+npr\rho t}{mdr+md\varrho-mpr\varrho-m-ndt+nrt}=f$.

Which *Theorem*, however it may ſeem operoſe, is not ſo, conſidering the great Number of Data that enter the Queſtion, and that one half of the

Terms ariſe from our taking in the Thickneſs of the Lens, which in moſt Caſes can produce no great Effect; however, it was neceſſary to conſider it, to make our Rule perfect. If therefore the Lens conſiſt of Glaſs, whoſe Refraction is as 3 to 2, 'twill be $\frac{6dr\rho - 2d\rho t + 4r\rho t}{3dr + 3d\varrho - 6r\rho - dt + 2rt} = f$. If of Water, whoſe Refraction is as 4 to 3, the *Theorem* will ſtand thus: $\frac{12dr\rho - 3d\rho t + 9r\rho t}{4dr + 4d\rho - 12r\varrho - dt + 3rt} = f$. If it could be made of Diamant, whoſe Refraction is as 5 to 2, it would be

$$\frac{\frac{10}{3}dr\rho - 2d\varrho t + \frac{4}{3}r\varrho t}{5dr + 5d\varrho - \frac{10}{3}r\varrho - 3dt + 2rt} = f.$$

And this is the univerſal Rule for the Foci of double Convex Glaſſes expoſed to diverging Rays. But if the Thickneſs of the Lens be rejected as not ſenſible, the Rule will be much ſhorter, *viz.* $\frac{pdr\rho}{dr + d\rho - pr\varrho} = f$; or in Glaſs, $\frac{2dr\rho}{dr + d\rho - 2r\rho} = f$. All the Terms wherein t is found being omitted, as equal to nothing. In this Caſe, if d be ſo ſmall, as that $2r\rho$ exceed $dr + d\varrho$, then will it be $-f$, or the Focus will be Negative, which ſhew that the Beams after both Refractions ſtill proceed diverging.

To bring this to the other Caſes, as of Converging Beams, or of Concave Glaſſes, the Rule is ever compoſed of the ſame Terms, only changing the Signs of $+$ and $-$; for the Diſtance of the Point of Concourſe of Converging Beams from the Point B, or the firſt Surface of the Lens, I call a Negative Radius, or $-r$ if it be the firſt Surface, and $-\varrho$, if it be the ſecond Surface. Let then converging Beams fall on a double Convex of Glaſs, and the *Theorem* will ſtand thus $\frac{-2dr\varrho}{-dr - d\varrho - 2r\varrho} = +f$, which ſhews, that in this Caſe the Focus is always Affirmative.

If the Lens were a *Meniſcus* of Glaſs, expoſed to diverging Beams, the Rule is $\frac{-2dr\varrho}{-dr + d\varrho + 2r\varrho} = f$: Which is Affirmative, when $2r\varrho$ is leſs than $dr - d\rho$, otherwiſe Negative: But in the Caſe of converging Beams falling on the ſame *Meniſcus*, 'twill be $\frac{+2dr\varrho}{+dr - d\varrho + 2r\rho} = f$. And it will be $+f$, whilſt $d\varrho - dr$ is leſs than $2r\rho$; but if it be greater than $2r\varrho$, it will always be found Negative or $-f$. If the Lens be double Concave, the Focus of converging Beams is Negative, where it was Affirmative in the Caſe of diverging Beams on a double Convex, *viz.* $\frac{-2dr\varrho}{+da + dr - 2r\varrho} = f$: which is Affirmative only when $2r\varrho$ exceeds $dr + d\varrho$: But diverging

Beams paſſing a double Concave, have always a Negative Focus, *viz.* $\frac{2dr\varrho}{+dr+d\varrho+2r\varrho}=-f$.

The *Theorems* for converging Beams are principally of uſe to determine the Focus reſulting from any ſort of Lens placed in a Teleſcope, between the Focus of the Object-Glaſs and the Glaſs itſelf; the Diſtance between the ſaid Focus of the Object-Glaſs and the interpoſed Lens being made $=-d$.

In caſe the Beams are parallel, as coming from an infinite Diſtance (which is ſuppoſed in the caſe of Teleſcopes) then will d be ſuppoſed infinite, and in the Theorem $\frac{pdr\varrho}{dr+d\varrho-pr\varrho}$ the Term $pr\varrho$ vaniſhes, as being finite, which is no Part of the other infinite Terms; and dividing the Remainder by the infinite Part d, the *Theorem* will ſtand thus $\frac{p\varrho r}{r+\varrho}=f$, or in Glaſs $\frac{2r\varrho}{r+\varrho}=f$.

In caſe the Lens were plano-convex expoſed to diverging Beams, inſtead of $\frac{pdr\varrho}{dr+d\varrho-pr\varrho}$, r being infinite, it will be $\frac{pd\varrho}{d-p\varrho}=f$, or $\frac{2dr}{d-2\varrho}=f$, if the Lens be Glaſs.

If the Lens be double-convex, and r be equal to ϱ, as being formed of Segments of equal Spheres, then will $\frac{pd\varrho r}{dr+d\varrho-pr\varrho}$ be reduced to $\frac{pdr}{2d-pr}=f$; and in caſe d be infinite, then it will be yet farther contracted to $\frac{1}{2}pr$, and p being $=\frac{n}{m-n}$, the Focal Diſtance in Glaſs will be $=r$, in Water $1\frac{1}{2}r$, but in Diameter $\frac{1}{3}r$.

This is not only uſeful to diſcover the Focus from the other propoſed Data; but from the Focus given, we may thereby determine the Diſtance of the Object, or from the Focus and Diſtance given we may find of what Sphere it is requiſite to take another Segment, to make any given Segments of an-her Sphere caſt the Beams from the Diſtance d to the Focus f: As ewiſe from the Lens, Focus, and Diſtance given, to find the Ratio of efraction, or of m to n, requiſite to anſwer theſe Data. All which, it bvious, are fully determined from the Equation we have hitherto uſed, $pd\varrho r=drf+d\varrho f-+pr\varrho f$. For to find d, the *Theorem* is $\frac{pr\varrho f}{+\varrho f-p\varrho r}=d$, the Diſtance of the Object; for ϱ, the Rule is $\frac{drf}{dr+df+prf}=\varrho$; but for p, it will be $\frac{drf+d\varrho f}{d\varrho r+f\varrho r}=p$; which lat- determines the Ratio of Refraction, m being to n as $1+p$ to p.

I ſhall not expatiate on theſe Particulars, but leave them for the Exerciſe of thoſe that are deſirous to be informed in Optical Matters, which I am bold to ſay are comprehended in theſe three Rules, as fully as the moſt Inquiſitive can deſire them, and in all poſſible Caſes, Regard being had to the Signs $+$ and $-$, as in the former Caſes of finding the Focus. I ſhall only ſhew two conſiderable Uſes of them; the one to find the Diſtance whereat an Object being placed, ſhall by a given Lens be repreſented in a Species as large as the Object itſelf, which may be of ſingular Uſe in drawing Faces, and other Things in their true Magnitude, by tranſmitting the Species by a Glaſs into a dark Room, which will not only give the true Figure and Shades, but even the Colours themſelves, almoſt as vivid as the Life. In this Caſe d is equal to f; and ſubſtituting d for f in the Equation, we ſhall have $pdr\varrho = ddr + dd\varrho - dp\varrho r$; and dividing all by d, $pr\varrho = dr + d\varrho - pr\varrho$, that is, $\frac{2pr\varrho}{r+\varrho} = d$; but if the two Convexities be of the ſame Sphere, ſo as $r = \varrho$, then will the Diſtance be $= pr$, that is, if the Lens be Glaſs $= 2r$; ſo that if an Object be placed at the Diameter of a Sphere diſtant, in this Caſe the Focus will be as far within as the Object is without, and the Species repreſented thereby will be as big as the Life; but if it were a Plano-Convex, the ſame Diſtance will be $= 2pr$, or, in Glaſs, to four Times the Radius of the Convexity.

A ſecond Uſe is to find what Convexity or Concavity is required to make a vaſtly diſtant Object be repreſented at a given Focus, after the one Surface of the Lens is formed; which is but a *Corollary* to our *Theorem* for finding ϱ, having p, d, r, and f, given; for d being infinite, the Rule becomes $\frac{rf}{pr-f} = \rho$, that is, in Glaſs, $\frac{rf}{2r-f} = \rho$; whence, if f be greater than $2r$, ρ becomes Negative, and $\frac{rf}{f-2r}$ is the Radius for the Concave ſought.

But to return to their firſt *Theorem*, which, accounting for the Thickneſs of the Lens, we will here again reſume, *viz.*

$$\frac{mpdr\rho - nd\varrho t - npr\varrho t}{mdr + md\varrho - mp\rho r - m - ndt - nrt} = f.$$

And let it be required to find the Focus, where a whole Sphere will collect the Beams proceeding from an Object at the Diſtance d. Here t is equal to $2r$, and $r = \varrho$; and after due Reduction, the *Theorem* will ſtand thus, $\frac{mpdr - 2ndr + 2nprr}{2nd + 2nr - mpr} = f$: But if d be infinite, it is contracted to $\frac{mpr}{2n} - r = \frac{2n-m}{2m-2n} r = f$; wherefore a Sphere of Glaſs collects the Sun's Beams at half the Semidiameter of the Sphere without it, and a Sphere of Water at a whole Semidiameter. But if the Ratio of Refraction m to n be as 2 to 1, the Focus falls on the oppoſite Surface of the Sphere; but if it be of greater Inequality, it falls within.

Another Example ſhall be, when a Hemiſphere is expoſed to parallel Rays, that is, d and ρ being infinite, and $t = r$, and, after due Reduction, the *Theorem* reſults $\frac{nn}{mm - mn} r = f$; that is, in Glaſs it is at $\frac{4}{3} r$, in Water at $\frac{9}{4} r$; but if the Hemiſphere were Diamond, it would collect the Beams at $\frac{4}{15}$ of the Radius beyond the Center.

Laſtly, As to the Effect of turning the two Sides of a Lens towards an Object; it is evident, that if the Thickneſs of the Lens be very ſmall, ſo as that you neglect it, or account $t = 0$, then in all Caſes the Focus of the ſame Lens, to whatſoever Beams, will be the ſame, without any Difference upon the turning the Lens: But if you are ſo curious to conſider the Thickneſs, (which is ſeldom worth accounting for) in the caſe of parallel Rays falling on a Plano-Convex of Glaſs, if the plane-ſide be towards the Object, t does occaſion no Difference, but the Focal Diſtance $f = 2r$. But when the Convex Side is towards the Object, it is contracted to $2r - \frac{2}{3} t$; ſo that the Focus is nearer by $\frac{2}{3} t$. If the Lens be Double Convex, the Difference is leſs; if a *Meniſcus*, greater. If the Convexity on both Sides be equal, the Focal Length is about $\frac{1}{6} t$ ſhorter than when $t = 0$. In a *Meniſcus*, the Concave Side towards the Object increaſes the Focal Length, but the Convex towards the Object diminiſhes it. A general Rule for the Difference ariſing on turning the Lens, where the Focus is Affirmative, is this $\frac{2at - 2\varrho t}{3r - 3 - t}$ for double Convexes of different Spheres. But for *Meniſci* the ſame Difference becomes $\frac{2rt + 2\varrho t}{3r - 3\rho + t}$; of which I need give no other Demonſtration, but that by a due Reduction it will follow from what is premiſed, as will the *Theorems* for all ſorts of Problems relating to the Foci of Optick Glaſſes.

XI. 1. Let D B and E C be oppoſite Hyperbola's, whoſe tranſverſe Axis is B C, Center A, and one of the Aſymptotes G P; alſo let O M be drawn through the Center at Right Angles to B C. Wherefore if the Hyperbola's
re converted about the Axis O M, it is plain, by ſuch a Revolution an Hy-
erbolical Cylindroidal Body will be generated, whoſe Baſes and Sections par-
llel to the Baſe will be Circles. I ſay moreover, if the ſame Body be cut
rough the Aſymptote G P, the Section will be a Parallelogram.

The Generation of an Hyperbolical Cylindroid; by Sir Chriſt. Wren. N. 48. *p.* 961. Jan. An. 1669. *Fig.* 97.

Let it be cut through the tranſverſe Axis by a Circular Section B N C,
ſo through O and M into equal Circles equally diſtant from the Center;
ſo through the Axis into a generating Figure, whoſe half is B D E C, in the

Plain of which will be the Aſymptote G P, through which at Right Angles let the Plain B D E be cut in the Plain F H P; laſtly let H O be joined.

Becauſe of the Right-angled Triangle O G H, the Square of O H or O D, ſubtracting the Square of O G, is equal to the Square of G H. And becauſe D O is parallel to B A, and cuts the Aſymptote in G, it will be (by the Properties of the Hyperbola demonſtrated in the Conics) the Square of O G together with the Square of B A, equal to the Square of O D. That is, the Square of O D leſſened by the Square of O G is equal to the Square of A B, or the Square of A N. Therefore the Square of G H is equal to the Square of A N, and thence G H is equal to A N, and they are at Right Angles to G A: And the like may be demonſtrated of all other Sections parallel to the Baſe. Wherefore the Hyperbolic Cylindroid is truly cut through the Aſymptote into a Parallelogram. *Q. E. D.*

Coroll. Hence it appears that in the Superficies of the Cylindroid, though it conſiſts of a double Flexure, innumerable Right Lines may nevertheleſs be drawn. It appears alſo, that there is another Way of generating this Body, that is, by the Revolution of a Parallelogram about the Axis at reſt, in an Angle to the Axis equal to G A O; or laſtly, the generating Line H R remaining immoveable, and forming or cutting the Maſs as it turns about.

And if the very ſharp and ſtreight Edge of the Chiſſel be diſpoſed in reſpect of the Axis after the manner of the generating Line, while the Mandrel turns about; it is plain, that by the Lathe as true Hyperbolas as Circles may be deſcribed, ſince nothing more is required for forming a Cylindroid than a Cylinder; unleſs that in the Cylinder the Edge of the Chiſſel muſt be parallel to the Axis, whereas it muſt be inclined to it to form a Cylindroid.

Therefore we muſt obſerve, that the Species of the Hyperbola will be varied, according to the different Inclination or Magnitude of the Angle G A O; ſo that it is more eaſily accommodated to a given Hyperbola, than to need any Demonſtration. But if the ſame Angle remaining, the generating Line approaches nearer to the Center, thence will ariſe a leſſer Hyperbola, but of the ſame Species as before.

The Application thereof to the Grinding of Hyperbolical Optick Glaſſes; by Sir Chriſt. Wren. N. 53. p. 1059. Nov. An. 1669. *Fig.* 98.

2. Let there be three Bodies fit for Grinding, P, Q, R; of which let P and Q be equal, and formed in the Shape of a Pillar, but let the Body R be in the Shape of a Lens. Let P have a Rotation about the Axis A B, Q about C D, and R about E G. But let A B and C D be in different Plains, yet ſo that E G produced may be at Right Angles to both A B and C D. Laſtly, let the Bodies approach to one another, as much as neceſſary, yet ſo as that the ſame Inclination of the Axes may be ſtill preſerved.

I ſay, that by the Revolution and mutual Attrition of the Bodies before ſuppoſed, new Geometrical Bodies will ariſe, of which P and Q will be equal Hyperbolical Cylindroids, and R an Hyperbolical Conoid, of a given Species and Magnitude.

We have the Demonſtration at hand, as alſo a Model of the Machine itſelf, which is intended for the grinding of Hyperbolical Lenſes. To deſcribe which by the laborious Apparatus of a Picture, and a tedious Explication,

tion, would be more troubleſome to me and the Engraver, than it would be to any ingenious Man to find the like. For after the Geometrical Principles are now explained, it is eaſy to gueſs what ſort of an Inſtrument it is. The Parts are three oblong Boards, plain, ſtrong, moveable, and placed upon one another. The loweſt and middlemoſt ſupport the unequal Puppets (or Handles that ſuſtain the Mandrel) placed alternately. This is required by the Obliquity and as it were Decuſſation of each of the Mandrels. The Puppets of the uppermoſt Board are equal, and diſpoſed according to the Length of the Board; and the Mandrel is let through the neareſt Puppet, being perforated for that End. I ſhall not mention the Wheels, Rollers, Thongs, Weights, Skrews, and other Things neceſſary for the expeditious Motion and Strength of the Machine. P belongs to the loweſt Board, Q to the middlemoſt, R to the uppermoſt. R is a Lens of Glaſs; Q is the Model grinding the Lens; P is the Pattern that corrects the Model; which as it is carried by an oblique Motion, and different from the Motion both of the Lens and the Model, continually grinds and wears away whatever Imperfection is communicated to the Model by the Attrition of the Lens and the Matter.

Wherefore ſince the Generation of the Hyperbolical Conoid is ſo ſimple and natural, being produced only by Circular Motions; and ſince the Motion is double and various; it is probable that Hyperbolical Lenſes may be formed upon theſe Principles, if upon any at all.

Why four Convex Glaſſes in a Teleſcope ſhew Objects Erect; by Mr. William Molineux. N. 183. *p.* 169. Jul. An. 1686.

XII. This Phænomenon appears very eaſily explicable, from the Conſideration of placing Glaſſes in a Tube; which is thus: After the Object-Glaſs, the firſt Eye-Glaſs is placed ſo much diſtant (towards the Eye) from the Focus of the Object-Glaſs, as is the Focus of the Eye-Glaſs; then the middle Eye-Glaſs is placed ſo much diſtant from the Focus of the firſt Eye-Glaſs, as is the Focus of the middle Eye-Glaſs: Laſtly, the neareſt Eye-Glaſs is placed ſo much diſtant from the Focus of the middle Eye-Glaſs, as is the Focus of this neareſt Eye-Glaſs; and the Eye looking through them all, is placed in the Focus of the neareſt Eye-Glaſs.

I ſay therefore, 1. That one ſingle Convex-Glaſs cannot properly be ſaid by itſelf to ſhew Objects Erect or Reverſe, but in reſpect of the placing of the Eye that looks through it: For if the Eye, that looks through a ſingle Convex-Glaſs, be placed nigher thereto than the Glaſs's Focus, the Objects are Erect; if the Eye be placed juſt in the Focus, the Objects are neither Erect nor Reverſed, but all in Confuſion between both; and if the Eye be placed arther from the Glaſs than the Focus, the Objects are Reverſed. I mean ere diſtant Objects, the Rays flowing from any Point whereof may be counted come Parallel towards the Object-Glaſs.

2. The Object-Glaſs of a Teleſcope reverſes the Object, both to the Eye-laſs and the Eye that looks through it: For the Eye-Glaſs is placed farther rom the Object-Glaſs than is the Focus of the Object-Glaſs: But the Eye-laſs does nothing towards the Rectification, or Reverſion; the Eye being aced juſt in its Focus.

3. If the ſecond Eye-Glaſs (the firſt being that next the Object-Glaſs) be placed, as it ought, in a Teleſcope, place the Eye nearer in this middle Eye-Glaſs than its Focus, and it ſees the Object inverted and confuſed: Place the Eye in the Focus, and it ſees the Object all in Confuſion, neither Erect nor Reverſed; for here again there is a diſtinct Repreſentation of the Objects to be received on a Piece of Paper, as in the Focus of an Object-Glaſs; and the Eye being placed at any Time at this Place (which is uſually call'd the Diſtinct Baſe) ſees all in Confuſion: But then let the Eye be placed farther from this middle Glaſs than its Focus, and it perceives the Objects erect and confuſed.

Laſtly, The third, or immediate Eye-Glaſs, does nothing towards the Erecting or Reverſing the Species, which it receives Erect from the middle Eye-Glaſs; no more than in a Teleſcope of two Convex-Glaſſes, the Eye-Glaſs does to the Species it receives from the Object-Glaſs; as we have ſhewn before. The Reaſon that this laſt or immediate Eye-Glaſs has nothing to do in the Erecting or Reverſing the Species, is the ſame as in the Teleſcope of two Convex-Glaſſes, *viz.* The Eye placed in its Focus, and therefore ſees the Species as 'tis repreſented in the Diſtinct Baſe; that is, the Species is inverted in the Diſtinct Baſe of the Object-Glaſs, and therefore a ſingle Convex Eye-Glaſs brings it to the Eye Inverted; but in the Diſtinct Baſe of the middle or ſecond Eye-Glaſs the Species is Erect, and therefore the third or immediate Eye-Glaſs brings it to the Eye Erect.

Wherefore we are to conſider the Teleſcope conſiſting of an Object-Glaſs and three Eye-Glaſſes, as two Teleſcopes, each conſiſting of two Convex Glaſſes. The firſt conſiſts of the Object-Glaſs and firſt Eye-Glaſs, and this inverts the Species; that is, the Species is inverted in the Diſtinct Baſe of the Object-Glaſs, and ſo brought into the Eye. The ſecond Teleſcope conſiſts of two immediate Eye-Glaſſes, and this Erects what the former Inverted; that is, the Species in the Diſtinct Baſe of the middle Eye-Glaſs is Erect, and is ſo brought into the Eye by the Eye-Glaſs; the Eye-Glaſſes themſelves in neither Caſe having any thing to do with the Erecting or Inverting, but merely in repreſenting in the ſame Poſture the Species immediately before them. So that one Convex-Glaſs, as poſited in a Teleſcope, Inverts; the ſecond (that is, the firſt Eye-Glaſs) does nothing towards the Erecting or Reverſing, but repreſents the Image as it is in the Diſtinct Baſe of the Object-Glaſs before it, that is Inverted; the third Glaſs Erects, or rather Reſtores, what was before Inverted; the fourth repreſents the Image as it receives it from the Diſtinct Baſe of the third, that is, Erect.

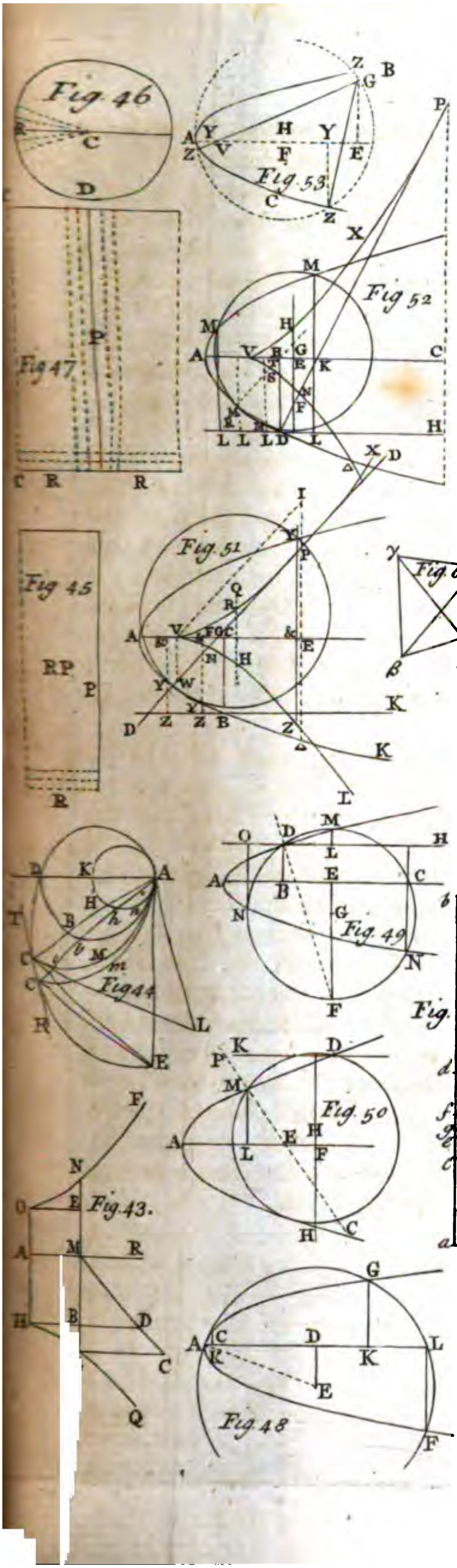
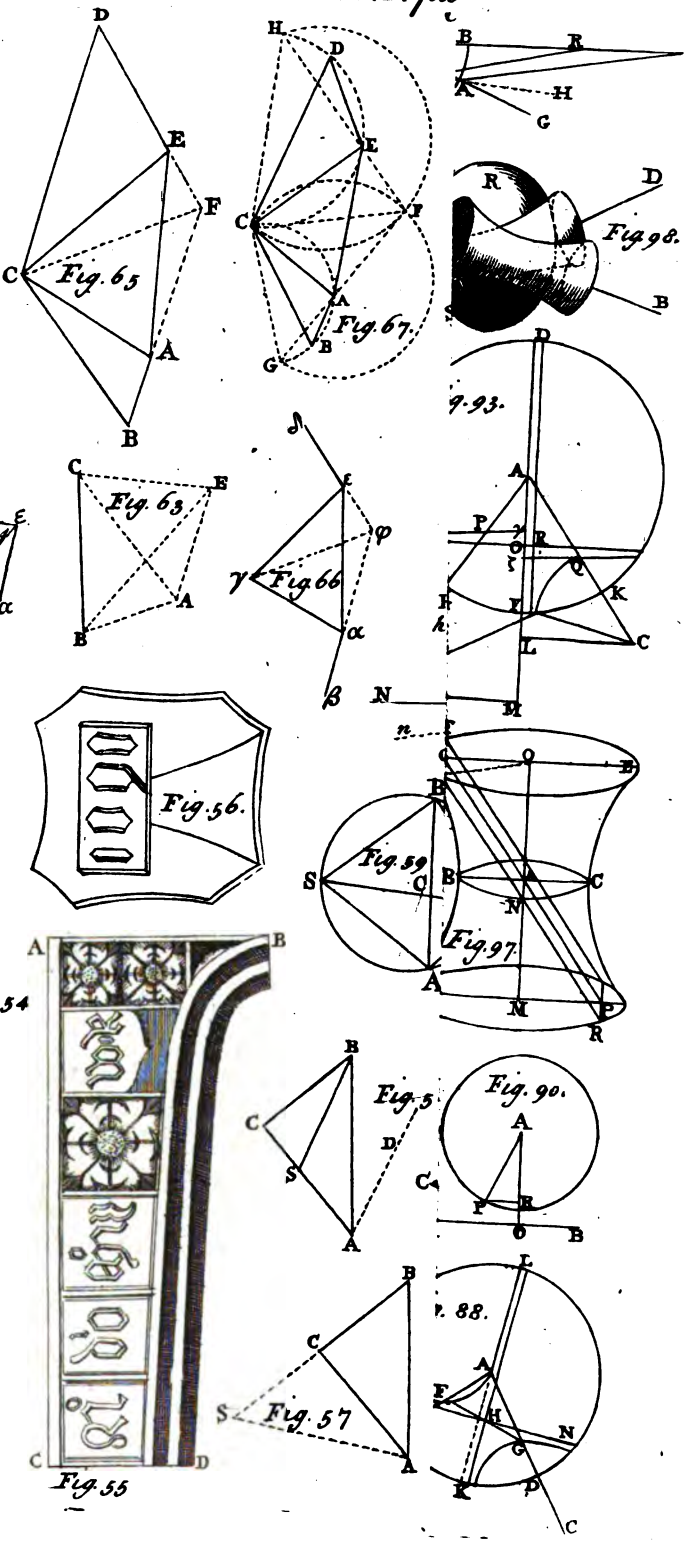


Plate II. Vol. 1. pa



XIII. 1. Mr. *Auzout* has found that the Apertures, which Optic-Glaſſes can bear with Diſtinctneſs, are in about a ſubduplicate Proportion to their Lengths: And, accordingly, he hath made the following *Table*.

The Apertures of Teleſcopes; by M. Auzout. N. 4. p. 55. Jun. An. 1665.

Lengths of Glaſſes.		*Apertures for* *Excellent*		*Good*		*Ordinary*	
Feet.	*In.*	*In.*	*Lin.*	*In.*	*Lin.*	*In.*	*Lin.*
	4		4		4		3
	6		5		5		4
	9		7		6		5
1	0		8		7		6
1	6		9		8		7
2	0		11		10		8
2	6	1	0		11		9
3	0	1	1	1	0		10
3	6	1	2	1	1		11
4	0	1	4	1	2	1	0
4	6	1	5	1	3	1	1
5	0	1	6	1	4	1	1
6		1	7	1	5	1	2
7		1	9	1	6	1	3
8		1	10	1	8	1	4
9		1	11	1	9	1	5
10		2	1	1	10	1	6
12		2	4	2	0	1	8
14		2	6	2	2	1	9
16		2	8	2	4	1	11
18		2	10	2	6	2	1
20		3	0	2	7	2	2
25		3	4	2	10	2	4
30		3	8	3	2	2	7
35		4	0	3	4	2	10
40		4	3	3	7	3	1
45		4	6	3	10	3	2
50		4	9	4	0	3	4
55		5	0	4	3	3	6
60		5	2	4	6	3	8
65		5	4	4	8	3	20
70		5	7	4	10	4	0
75		5	9	5	0	4	2
80		5	11	5	2	4	5
90		6	4	5	6	4	7
100		6	8	5	9	4	10
120		7	5	6	5	5	3
150		8	0	7	0	5	11
200		9	6	8	0	6	9
250		10	6	9	2	7	8
300		11	6	10	0	8	5
350		12	6	10	9	9	0
400		13	4	11	5	9	8

2. This Theory of Apertures ſeems to me not very clear: For the ſame Glaſs will endure greater or leſſer Apertures, according to the leſſer or greater Light of the Object; if it be for looking on the Sun and *Venus*, or for ſeeing the Diameters of the fixed Stars, then ſmaller Apertures do better; if for the Moon in the Day-light, or on *Saturn*, or *Jupiter*, or *Mars*, then the largeſt. Thus I have often made uſe of a 12 Foot Glaſs to look on *Saturn* with an Aperture of almoſt 3 Inches, and with a ſingle Eye-Glaſs of 2 Inches double Convex; but when, with the ſame Glaſs, I looked on the *Sun* or *Venus*, I uſed both a ſmaller Aperture, and ſhallower Charge.

Conſidered by Dr. Hook. ibid. p. 67.

XIV.

To meaſure Diſtances at one Station; by M. Auzout. N. 7. *p.* 125. Dec. An. 1665.

XIV. I have found long ſince a Way to meaſure, with a great Teleſcope, the Diſtance of Objects upon the Earth from one Station. The Practice indeed does not altogether anſwer the Theory, becauſe that the Length of the Teleſcopes admits of ſome Latitude; yet one comes near enough, and perhaps as juſt as by moſt of the Ways ordinarily uſed with Inſtruments. That which I am propoſing, I doubt not but Mr. *Hook* will ſoon underſtand, and ſee the Determination of all Caſes poſſible. I ſhall only ſay, That if we look upon the ſole Theory, we may make uſe of an ordinary Teleſcope, whereof the Eye-Glaſs is to be Convex: For by putting the Glaſſes at a little greater Diſtance than they are, proportionably to the Diſtance for which it is to ſerve, and, by adding to it a new Eye-Glaſs, the Object will be ſeen diſtinct, tho' obſcure; and if the Eye-Glaſs be Convex, the Object will appear Erect. They may be done two manner of Ways; either by leaving the Teleſcope in its ordinary Situation, the Object-Glaſs before the Eye-Glaſs; or by inverting it, and putting this before that. But if any will make uſe of two Object-Glaſſes, whereof the Focus's are known, the Diſtances of them will be known. If it be ſuppoſed, that the Focus of the firſt be B, and that of the ſecond C, and the Diſtance given, $B + 2D$, and that $D - C$ be equal to F; for this Diſtance will be equal to $B + C + F - r F^2 C^2$. And if you have the Focus of the firſt Object-Glaſs equal to B, the Diſtance where you will put the ſecond Glaſs, equal to $B + C + D$, the Focus of the ſecond Glaſs will be found equal to $\frac{CD}{C + D}$. And if you will that the Object ſhall be magnified as much with theſe two Glaſſes as it would be with a ſingle one, whereof the Focus ſhould be of the Diſtance given, having the Focus of the Object-Glaſs given equal to B, and the Diſtance given to $B + D$; the Diſtance between the firſt and ſecond Glaſs will be equal to $\frac{2B^2 + 2BD}{2B + D}$; whence, ſubducting B (the Focus of the Object given) there remains $\frac{BD}{2B + D}$: And if this Sum be compoſed equal to C, we ſhall eaſily know by the precedent Rule, the Focus of the ſecond Glaſs.

To make a Plano-convex Glaſs of a ſmall Sphere, collect the Rays at a great Diſtance; by Dr. Hook. N. 4. *p.* 66. N. 12. *p.* 102. June, An. 1665. May, An. 1666.

XV. Prepare two Glaſſes, the one exactly flat on both Sides, the other flat on the one Side, and convex on the other, of what Sphere you pleaſe. Let the flat Glaſs be a little broader than the other. Then let there be made a Cell or Ring of Braſs, very exactly turned, into which theſe two Glaſſes may be ſo faſtened with Cement, that the plain Surfaces of them may lie exactly parallel, and that the Convex Side of the Plano-convex Glaſs may lie inward; but ſo, as not to touch the Flat of the other Glaſs. Theſe being cemented into the Ring very cloſely about the Edges, by a ſmall Hole in the Side of the Braſs Ring or Cell; fill the interpoſed Space between theſe two with Water, Oil of Turpentine, Spirit of Wine, Saline Liquors, &c. then ſtop the Hole with a Screw: And according to the differing Refraction of the interpoſed Liquors, ſo ſhall the Focus of this Compound Glaſs be longer or ſhorter.

But this I would have only look'd upon as one Inſtance of many (for there may be others) of the Poſſibility of making a Glaſs, ground in a ſmaller Sphere, to conſtitute a Teleſcope of a much greater Length: Though (not to raiſe too great Expectation) I muſt add, That, of Spherical Object-Glaſſes, thoſe are the beſt which are made of the greateſt Sphere, and whoſe Subſtance hath the greateſt Refraction.

XVI. 1. *S. Campani* pretends to have found a Way to work great *Optic-Glaſſes* with a Turn-tool, without any Mould: And that he uſeth three Eye-Glaſſes for his great Teleſcopes, without finding any Rainbow Colours.

Teleſcopes and other Optic-Glaſſes; by Campani *and* Divini. N. 1. *p.* 2. Mar. An. 1665. N. 8. *p.* 131. Jan. An. 1666.

The Great Duke of *Tuſcany*, and Prince *Leopold* his Brother, upon Trial made of the Glaſſes of *Campani* and *Divini*, have found that thoſe of *Campani* excel the other; and with them they have been eaſily able to diſtinguiſh People at 4 Leagues Diſtance.

But *Euſtachio Divini* pretends, that in all the Trials made with them, his great Glaſſes have performed better than thoſe of *Campani*; and that *Campani* was not willing to do what was neceſſary for well comparing one with the other, *viz.* to put equal Eye-Glaſſes in them, or to exchange the ſame Glaſſes.

N. 12. *p.* 209.

2. 'Tis now above ten Years ſince I invented a peculiar Way of grinding Optic-Glaſſes, and reduced it alſo into Practice; by which 'tis eaſy, without any conſiderable Danger of failing, to make and poliſh Optic-Glaſſes of any *Conic Section*, and that (which is moſt notable) in any Diſh of any Section of a Sphere. I have already made ſeveral Glaſſes by it, which many learned Men have ſeen and tried.

By M. Hevelius. N. 6. *p.* 98. Nov. An. 1665.

Mr. *Huygens* alſo intends very ſhortly to try ſomething in that kind.

By M. Huygens. *Ibid.*

3. M. *du Sons* doth at preſent employ himſelf in *London*, to bring Teleſcopes to Perfection, by grinding Glaſſes of a *Parabolical* Figure. I have ſeen two Eye-Glaſſes of that Shape, about one Inch and a half deep, and one Inch and a quarter broad, wrought by this *eminent Artiſt* with a rare Steel Inſtrument of his own Contrivance and Workmanſhip, and by himſelf alſo poliſh'd to Admiration. And certainly it will be wondered at by thoſe, who ſhall ſee theſe Glaſſes, how they could be truly wrought to ſuch a Figure, with ſuch a Cavity; and yet more, when they ſhall hear the Author undertake to excavate other ſuch Eye-Glaſſes to above 2 Inches, and Object-Glaſſes of 5 Inches Diameter. He hath likewiſe already begun his Object-Glaſſes for the mentioned two ocular ones, of the ſame Figure of about 2 Inches Dia- ter, which are to be left all open, yet without cauſing any Colours.

By M. du Sons. *Ibid p.* 99. N. 7. *p.* 119. Dec. An. 1665.

4. The Optic-Glaſſes of M. *Burattini* in *Poland*, are perfectly well ought and poliſh'd. He hath ſent two to *Paris*, but they are only the e of 10, the other of 8 Foot. They bear a great Aperture in reſpect of ir Length.

By M. Burattini. N. 19. *p.* 348. N. 21. *p.* 374.

5. Mr. *Fr. Smethwick* having found a Way of grinding Glaſſes *not Sphe- al,* produced before the Royal Society (*Feb.* 27. 166$\frac{7}{8}$.) certain *Specimina* that Invention; which were, a Teleſcope, a Reading, and two Burning- ſſes.

By Mr. Francis Smethwick. N. 33. *p.* 631.

The Teleſcope was about 4 Foot long, furniſhed with four Glaſſes, whereof the three ocular ones, Plano-convex, were of this newly invented *not-Spherical* Figure, and the fourth a Spherical Object-Glaſs. This being compared with a common, yet very good Teleſcope, longer than it by about 4 Inches, and turn'd to ſeveral Objects, was found by thoſe of the ſaid Society that look'd through them both, to exceed the other in Goodneſs, by taking in a greater Angle, and repreſenting the Objects more exactly in their reſpective Proportions, and enduring a greater Aperture free from Colours.

The Reading-Glaſs of the ſame Figure being compared with a common Spherical Glaſs did far excel it, by magnifying the Letters to which it was applied up to the very Edges, and by ſhewing them diſtinctly from one Brim through the Center to the other; which the Spherical Glaſs came far ſhort of.

Laſtly, The two burning Concaves of this new-invented Figure, were the one of 6 Inches Diameter, its Focus 3 Inches diſtant from the Center thereof; the other of the ſame Diameter, but leſs Concave, and its Focus 10 Inches diſtant. Theſe, when approached to a large Candle lighted, did ſomewhat warm the Faces of thoſe that were 4 or 5 Foot diſtant at leaſt; and when held to the Fire, burned Gloves and Garments at the Diſtance of about 3 Foot from the Fire.

The Biſhop of *Salisbury*, Dr. *Seth Ward*, was by at another Time, when the deeper of the two Concaves turned a Piece of Wood into Flame in the Space of 10 Sec. of Time, and the ſhallower in 5 Sec. at moſt, in the Seaſon of *Autumn*, about Nine of the Clock in the Morning, the Weather gloomy. The Inventor adds, That the deeper Concave, when held to a lucid Body, would caſt a Light ſtrong enough to read by at a conſiderable Diſtance; and that expoſing the ſame to a Northern Window, on which the Sun ſhined not at all, or very little, he had perceived that it would warm one's Hand ſenſibly, by collecting the warmed Air in the Day-time, which it would not do after Sun-ſet.

By an Artiſt at Paris. N. 40. *p.* 795. Oct. An. 1668.

6. We have an Artiſt at *Paris* that poliſhes Optic-Glaſſes on a Turn. I have ſeen a Glaſs of his Workmanſhip, which is very good. He turns thoſe Glaſſes as he does Wood, that is, with the ſame Facility.

By M. Borelli. N. 128. *p.* 691. Aug. An. 1676.

7. M. *Borelli* hath found out a ſure and very eaſy Method to work all ſorts of great Glaſſes. He hath already made one of them very good of 200 Foot, wrought on both Sides on the ſame Rule. His Deſire of advancing Aſtronomical Diſcoveries hath induced him to make Preſents of them to ſeveral Perſons capable to make uſe of them. He hath entruſted the Secret to one of the *Royal Academy* of *Sciences.*

N. 140. *p.* 1005. July, An. 1673.

Campani and *Divini* have commonly ſold their Glaſſes at a Piſtole the Foot. Sometimes they have far exceeded that Price. One of *Divini*'s, of 12 Foot, was ſold for 400 Livres; and another of *Campani*'s, of 34 Foot, for 2000 Livres. Notwithſtanding which, *S. Borelli* is willing to part with the beſt of his own Glaſſes, of 50, 60, or 65 Foot, for 500 *(French)* Crowns; and the ſmall Glaſſes, from 6 to 12 Foot, at a *(French)* Crown a Foot; from 12 to 18, at half a Piſtole; and from 18 to 26, at a Piſtole.

Optic Lens of Rock Cryſtal; by Euſt. Divini. N. 23. *p.* 362. Dec. An. 1666.

8. Though it be commonly believed, that *Rock Cryſtal* is not fit for Optic-Glaſſes, becauſe there are many Veins in it; yet *Euſtachio Divini* made one of it, which he ſaid proved an excellent one, though full of Veins: But perhaps they were only ſuperficial Strictures and ſlight Scratches, not Veins.

Of Water; by Mr. Stephen Gray. N. 228. *p.* 539. May, An. 1697.

9. Drops of fair Water being let fall on a Piece of plain Glaſs, form themſelves into Plano-convexes, having a Convexity proportionable to the Heights from which they deſcend; from a greater Height a leſs, from a leſs a greater Degree of Convexity. I applied ſome of theſe as Reading-Glaſſes for ſingle Words of ſmall Letters, as on the Globes and Maps, and found no other Inconveniency, than that the Fluidity of the Water obliges one to keep the Glaſs Horizontal, which I after deviſed a Way to remedy. I took a ſufficient Quantity of Iſing-Glaſs, and diſſolved it in Water over the Fire, and whilſt it was warm I dipt a Stick into the Solution, and let ſome Drops of it fall on the Glaſs as before; and in a quarter of an Hour they acquire a Conſiſtency, that permits them to be held in any Poſition; and though they are not altogether ſo tranſparent, yet this is little or no Impediment to their Uſe. The Drops of this Solution are more exactly defined than thoſe of common Water, having their Edges exactly circular; and one may make them of a much longer Focus than thoſe.

A thin flat Ring of Braſs, not exceeding 4 Tenths of an Inch Diameter in its interior Circle, being cemented to a plain Piece of Glaſs, and filled with Water, or the Solution now mentioned; then by preſſing the Finger into it, 'till what is ſuperfluous be taken off, there will be formed a Plano-concave, which may ſerve as an Eye-Glaſs to a Perſpective, or to any other optical Uſe Concave Glaſſes are applicable to.

I have tried what would be the Succeſs of combining Portions of Water by the help of Braſs Rings, and plain Pieces of Glaſs, to give them their true Figure and requiſite Apertures, and inſerted them at the Ends of Tubes of ſeveral Lengths; and find, that though theſe *Natural Lentes* may ſerve as Eye-Glaſſes, yet when uſed as Object ones, either to Teleſcopes, or double Microſcopes, the Effects will not compenſate the Trouble there is in uſing them.

The Advantages of Reflection to Optic Inſtruments; by Mr. Newton. N. 80. *p.* 3079. Feb. An. 1672.

XVII. 1. When I had found, that *Light conſiſts of Rays differently refrangible*, I left off my Glaſs-works; for I ſaw, that the Perfection of Teleſcopes was hitherto limited, not ſo much for want of Glaſſes truly figured according to the Preſcriptions of Optic Authors, (which all Men have hitherto imagined) as becauſe that *Light* itſelf is an heterogeneous Mixture of *differently Refrangible Rays*: So that were a Glaſs ſo exactly figured as to collect any one ſort of Rays into one Point, it could not collect thoſe alſo into the ſame Point, which having the ſame Incidence upon the ſame Medium, are apt to ſuffer a different Refraction. Nay, I wondered, that ſeeing the Difference of Refrangibility was ſo great as I found it, Teleſcopes ſhould arrive to that Perfection they are now at: For, meaſuring the Refractions in one of my Priſms, I found, that ſuppoſing the common Sine of Incidence upon one of its Planes was 44 Parts, the Sine of Refraction of the utmoſt Rays on the red End of the Colours, made out of the Glaſs into the Air, would be 68 Parts, and the Sine of Refraction of the utmoſt Rays on the other End 69 Parts; ſo that the Difference is about a 24th or 25th Part of the whole Refraction. And conſequently, the Object-Glaſs of any Teleſcope cannot collect all the Rays which come from one Point of an Object, ſo as to make them convene at its Focus in leſs room than in a circular Space, whoſe Diameter is the 50th Part of the Diameter of its Aperture; which is an Irregularity, ſome hundreds of times greater, than a circularly figured *Lens*, of ſo ſmall a Section as the Object-Glaſſes of long Teleſcopes are, would cauſe, by the Unfitneſs of its Figure, were Light uniform.

This made me take Reflections into Conſideration; and finding them regular, ſo that the Angle of Reflection of all ſort of Rays was equal to their Angle of Incidence, I underſtood that by their Mediation Optic Inſtruments might be brought to any Degree of Perfection imaginable, provided a reflecting Subſtance could be found, which would poliſh as finely as Glaſs, and reflect as much Light as Glaſs tranſmits, and the Art of communicating to it a parabolic Figure be alſo attained. But theſe ſeemed very great Difficulties, and I have almoſt thought them inſuperable, when I farther conſider'd that every Irregularity in a reflecting Superficies makes the Rays ſtray five or ſix times more out of their due Courſe, than the like Irregularities in a refracting one: So that a much greater Curioſity would be here requiſite, than in figuring Glaſſes for Refraction.

Amidſt theſe Thoughts I was forced from *Cambridge*, *Anno* 1666, by the intervening Plague, and it was more than two Years before I proceeded further. But then having thought on a tender way of poliſhing, proper for Metal, whereby, as I imagined, the Figure would be corrected to the laſt, I began to try what might be effected in this kind, and by degrees ſo far perfected an Inſtrument (in the eſſential Parts of it like that I ſent to *London*) by which I could diſcern *Jupiter*'s four Concomitants, and ſhewed them divers times to two others of my Acquaintance. I could alſo diſcern the Moon-like *Phaſe* of *Venus*, but not very diſtinctly, nor without ſome Niceneſs in diſpoſing the Inſtrument.

From that time I was interrupted till this laſt *Autumn*, when I made another. And as that was ſenſibly better than the firſt (eſpecially for Day-Objects) ſo I doubt not, but they will be ſtill brought to a much greater Perfection by their Endeavours, who, as you inform me, are taking Care about it at *London*.

A new Catadioptrical Teleſcope invented by Mr. Newton.
N. 82. *p.* 4004. Mar. An. 1672.
Fig. 99.

2. This new Inſtrument is compoſed of two Metalline Speculums, the one Concave (inſtead of an Object-Glaſs) the other Plain: and alſo of a ſmall Plano-convex Eye-Glaſs; as in the Figure, where A B is a Concave Speculum, of which the Radius or Semidiameter is $12\frac{2}{3}$ or 13 Inches.

C D, another Metalline Speculum, whoſe Surface is flat, and the Circumference oval.

G D, an Iron Wire, holding a Ring of Braſs, in which the Speculum C D is fixed.

F, a ſmall Eye-Glaſs, flat above, and convex below, of the 12th Part of an Inch Radius, if not leſs.

GGG, the fore Part of the Tube (which is open) faſtened to a Braſs Ring HI. to keep it immoveable.

PQKL, the hinder Part of the Tube, faſtened to another Braſs Ring PQ.

O, an Iron Hook faſtened to the Ring Ring PQ, and furniſhed with a Screw N, thereby to advance or draw back the hinder Part of the Tube, and ſo by that means to put the Specula in their due Diſtance.

MQGI, a crooked Iron ſuſtaining the Tube, and faſtened by the Nail R to the Ball and Socket S, whereby the Tube may be turned every Way.

The Center of the flat Speculum CD, muſt be placed in the ſame Point of the Tube's Axe, where falls the Perpendicular to this Axe, drawn to the ſame from the Center of the little Eye-Glaſs; which Point is here marked at T.

And to give the Reader ſome Satisfaction to underſtand in what Degree it repreſents Things diſtinct, and free from Colours, and to know the Aperture by which it admits Light, he may compare the Diſtances of the Focus E from the Vertexes of the little Eye-Glaſs and the Concave Speculum; that is, EF, ⅙ of an Inch, and ETV, 6 ⅓ Inches; and the Ratio will be found as 1 to 38; whereby it appears, that the Objects will magnified about 38 times; and be repreſented bigger by 2 ½ times in Diameter, when ſeen thro' this, than thro' an ordinary Teleſcope of about two Foot long.

Thus far as to the Structure of this Teleſcope. Concerning the metalline Matter, fit for theſe Reflecting Speculums, the Inventor hath alſo conſidered the ſame, and gives this Caution, that whilſt Men ſeek for a white, hard, and durable metalline Compoſition, they reſolve not upon ſuch an one as is full of ſmall Pores, only diſcoverable by a Microſcope: For tho' ſuch an one may, to Appearance, take a good Poliſh, yet the Edges of thoſe ſmall Pores will wear away faſter in the poliſhing, than the other Parts of the Metal; and ſo, however the Metal ſeem polite, yet it ſhall not reflect with ſuch an accurate Regularity as it ought to do. Thus Tin-Glaſs mixt with ordinary Bell-Metal makes it more white, and apt to reflect a greater Quantity of Light; but withal, its Fumes raiſed in the Fuſion, like ſo many aërial Bubbles, fill the Metal full of the microſcopical Pores. But white Arſenick both blanches the Metal, and leaves it ſolid, without any ſuch Pores, eſpecially if the Fuſion hath not been too violent. What the Stellate-Regulus of *Mars* (which I have ſometimes uſed) or rather ſuch like Subſtance, will do, deſerves parti- ar Examination.

o this he adds this further Intimation, that Putty, or other ſuch like der, with which it is poliſhed, by the ſharp Angles of its Particles, fret- h the Metal, if it be not very fine, and fills it full of ſuch ſmall Holes as he eaketh of. Wherefore Care muſt be taken of that, before Judgment be given, hether the Metal be throughout the Body of it porous or not.

But not having tried, as he ſaith, many Proportions of the Arſenick and etal, he does not affirm which is abſolutely beſt, but thinks there may conveniently be uſed any Quantity of Arſenick equalling in Weight between a ſixth and an eighth Part of the Copper; a greater Proportion making the Metal brittle.

The Way which he uſed was this: He firſt melted the Copper alone, then put in the Arſenick, which being melted, he ſtirred them a little together, bewaring, in the mean time, not to draw in Breath near the pernicious Fumes. After this he put in Tin; and again, ſo ſoon as that was melted (which was very ſuddenly) he ſtirred them well together, and immediately poured them off.

He ſaith, he knows not, whether by letting them ſtand longer on the Fire after the Tin was melted, a higher Degree of Fuſion would have made the Metal porous; but he thought that Way he proceeded to be the ſafeſt.

He adds, that in the Metal, which he ſent to *London*, there was no Arſenick, but a ſmall Proportion of Silver; as he remembers, one Shilling in three Ounces of Metal. But he thought withal, that the Silver did as much harm in making the Metal ſoft, and ſo leſs fit to be poliſhed, as good in rendering it white and luminous.

At another time he mixed Arſenick one Ounce, Copper ſix Ounces, and Tin two Ounces; and this an Acquaintance of his hath, as he intimates, poliſhed better than he did the other.

As to the Objection, that with this kind of Perſpectives, Objects are difficultly found; he anſwers, That that is the Inconvenience of all Tubes that magnify much; and that after a little Uſe, the Inconvenience will grow leſs, ſeeing that himſelf could readily enough find any Day Objects, by knowing which way they were poſited, from other Objects that he accidentally ſaw in it. But in the Night, to find Stars he acknowledges it to be more troubleſome; which yet may, in his Opinion, be eaſily remedied by two Sights affixed to the Iron Rod, by which the Tube is ſuſtained, or by an ordinary Perſpective-Glaſs faſtened to the ſame Frame with the Tube, and directed towards the ſame Object; as *Des Cartes* in his *Dioptrics* hath deſcribed, for remedying the ſame Inconvenience of his beſt Teleſcopes.

Approved by M. Hugens de Zulichem. *Ibid. p.* 4008.

3. I ſee by the Deſcription you have ſent me of Mr. *Newton's* admirable Teleſcope, that he hath well conſidered the Advantage which a Concave Speculum hath above Convex-Glaſſes in collecting the parallel Rays; which certainly, according to the Calculation I have made thereof, is very great. Hence it is, that he can give a far greater Aperture to that Speculum, than to an Object-Glaſs of the ſame Diſtance of the Focus; and conſequently, that he can much more magnify Objects this Way, than by an ordinary Teleſcope. Beſides, by it he avoids an Inconvenience, which is inſeparable from Convex Object-Glaſſes, which is the Obliquity of both their Surfaces, which vitiateth the Refraction of the Rays that paſs towards the Sides of the Glaſs, and does more hurt than Men are aware of. Again, by the mere Reflection of the metalline Speculum there are not ſo many Rays loſt as in Glaſſes, which reflect a conſiderable Quantity by each of their Surfaces, and beſides intercept many of them by the Obſcurity of their Matter.

Mean time the main Buſineſs will be, to find a Matter for this Speculum that will bear ſo good and even a Poliſh as Glaſſes, and a Way of giving this

Poliſh without vitiating the Spherical Figure. Hitherto I have found no Specula that had near ſo good a Poliſh as Glaſs: And if Mr. *Newton* hath not already found a way to make it better than ordinarily, I apprehend his Teleſcope will not ſo well diſtinguiſh Objects as thoſe with Glaſſes. But 'tis worth while to ſearch for a Remedy to this Inconveniency, and I deſpair not of finding one. I believe that Mr. *Newton* hath not been without conſidering the Advantage, which a Parabolical Speculum would have above a Spherical one in this Conſtruction; but that he deſpairs, as well as I do, of working other Surfaces than Spherical ones with due Exactneſs; tho' elſe it be more eaſy to make a Parabolical, than Elliptical or Hyperbolical ones, by reaſon of a certain Propriety of the Parabolic Conoid, which is, that all the Sections parallel to the Axis make the ſame Parabola.

Ibid. p. 4009.

But though Mr. *Newton* (with M. *Hugens*) deſpairs of performing that Work by Geometrical Rules, yet he doubts not but that the thing may in ſome meaſure be accompliſhed by mechanical Devices.

A further Account of this Inſtrument; by Mr. Newton. *Ibid.*

4. In my laſt Letter I gave you occaſion to ſuſpect that the Inſtrument which I ſent you is in ſome reſpect or other indiſpoſed, or that the Metals are tarniſhed; and by yours I am fully confirmed in that Opinion: For, whilſt I had it, it repreſented the Moon in ſome Parts of it as diſtinctly as other Teleſcopes uſually do which magnify as much as that. Yet I very well know, that that Inſtrument hath its Imperfections both in the Compoſition of the Metal, and in its being badly caſt, as you may perceive by a ſcabrous Place near the Middle of the Metal of it on the poliſhed Side, and alſo in the Figure of that Metal near that ſcabrous Place: And in all thoſe Reſpects that Inſtrument is capable of further Improvement.

You ſeem to intimate, that the Proportion of 38 to 1, holds only for its magnifying Objects at ſmall Diſtances. But if for ſuch Diſtances, ſuppoſe 500 Feet, it magnify at that Rate, by the Rules of Optics it muſt for the greater Diſtance imaginable magnify more than 37 $\frac{3}{4}$ to 1, which is ſo inconſiderable a Diminiſhing, that it may be even then as 38 to 1.

Here is made another Inſtrument like the former, which does very well. Yeſterday I compared it with a ſix Foot Teleſcope, and found it not only to magnify more, but alſo more diſtinctly: And to Day I found, that I could read in one of the Philoſophical Tranſactions, placed in the Sun's Light, at 100 Foot Diſtance, and that at 120 Foot Diſtance I could diſcern ſome of the Words. When I made this Trial, its Aperture (defined next the Eye) was equivalent to more than an Inch and a third Part of the Object-Metal. This may be of ſome Uſe to thoſe that ſhall endeavour any thing in Reflections; ſ hereby they will in ſome meaſure be enabled to judge of the Goodneſs of tl Inſtruments.

The Apertures and Charges of theſe Inſtruments; by Mr. Newton. N. 82. *p.* 4032. April, An. 1672.

I know that the Aperture was 1 $\frac{1}{3}$ of an Inch, by trying that an Ob- ſl cle of that Breadth was requiſite to intercept all the Light which came from o Point of the Object.

ſhould tell you alſo, that the little plain Piece of Metal next the Eye-Glaſs is ot truly figured: Whereby it happens, that Objects are not ſo diſtinct at tl Middle as at the Edges. And I hope that by correcting its Figure (in which I find more Difficulty than one would expect) they will appear all over diſtinct, and diſtincter in the middle than at the Edges. And I doubt not but that the Performances will then be greater.

But yet I find, that there is more Light loſt by Reflection of the Metal which I have hitherto uſed, than by Tranſmiſſion through Glaſſes: For which Reaſon a ſhallower Charge would probably do better for obſcure Objects; ſuppoſe ſuch a one as would magnify 34 or 32 times. But for bright Objects at any Diſtance, it ſeems capable of magnifying 38 or 40 times, with ſufficient Diſtinctneſs. And for all Objects, the ſame Charge, I believe, may with Advantage be allowed, if the ſteely Matter, employed at *London*, be more ſtrongly reflective than this which I have uſed.

The Performances of one of theſe Inſtruments of any Length being known, it will appear by this following Table, what may be expected from thoſe of other Lengths by this Way, if Art can accompliſh what is promiſed by the Theory. In the firſt Column is expreſſed the Length of the Teleſcope in Feet; which doubled, gives the Semidiameter of the Sphere on which the Concave Metal is to be ground. In the ſecond Column are the Proportions of the Apertures for thoſe ſeveral Lengths. And in the third Column are the Proportions of the Charges, or Diameter of the Spheres, on which the Convex Superficies of the Eye-Glaſſes are to be ground.

Lengths.	*Apertures.*	*Charges.*	*Lengths.*	*Apertures.*	*Charges.*
$\frac{1}{2}$	100	100	8	800	200
1	168	119	10	946	211
2	283	141	12	1084	221
3	383	157	16	1345	238
4	476	168	20	1591	251
5	562	178	24	1824	263
6	645	186			

The Uſe of this Table will beſt appear by Example: Suppoſe therefore a half Foot Teleſcope may diſtinctly magnify 30 times with an Inch Aperture, and it being required to know, what ought to be the analogous Conſtitution and Performance of a four Foot Teleſcope: By the ſecond Column, as 100 to 476; ſo are the Apertures, as alſo the Number of Times which they magnify. And conſequently ſince the half Foot Tube hath an Inch Aperture and magnifieth 30 times, a four Foot Tube proportionally ſhould have $4\frac{76}{100}$ Inches Aperture, and magnify 143 times. And by the third Column, as 100 to 168; ſo have their Charges. And therefore if the Diameter of the Convexity of the Eye-Glaſs for a half Foot Teleſcope be $\frac{1}{5}$ of an Inch; that for a four Foot ſhould be $\frac{168}{500}$, that is about $\frac{1}{3}$ of an Inch; and ſo of other Lengths. But what the Event will really be, we muſt wait to ſee determined by Experience. Only this I thought fit to inſinuate, that they which intend to make Trial in other Lengths, may more readily know how to deſign their Inſtruments. Thus for a four Foot Tube, ſince the Aperture ſhould be 5 or 6

Inches, there will be required a Piece of Metal 7 or 8 Inches broad at leaſt, becauſe the Figure will ſcarcely be true to the Edges. And the Thickneſs of the Metal muſt be proportional to the Breadth, leſt it bend in the Grinding. The Metals being poliſhed, there may be Trials made with ſeveral Eye-Glaſſes, to find what Charge may with beſt Advantage be made uſe of.

Some Objections of Mr. —— Anſwered, by *Mr.* Newton. *ib. p.* 4034.

XVIII. 1. I doubt not but *M. A.* will allow the Advantage of Reflection in the Theory to be very great, when he ſhall have informed himſelf of the *different Refrangibility* of the ſeveral Rays of Light. And for the practick Part, it is in ſome Meaſure manifeſt by the Inſtruments already made, to what Degree of Vivacity and Brightneſs a metalline Subſtance may be poliſhed. Nor is it improbable but that there may be new Ways of poliſhing found out for Metal, which will far excell thoſe that are yet in Uſe. And when a Metal is once well poliſhed, it will be a long while preſerved from tarniſhing, if Diligence be uſed to keep it dry and cloſe ſhut up from Air: For the principal Cauſe of Tarniſhing ſeems to be the condenſing of Moiſture on its poliſhed Surface, which, by an acid Spirit, wherewith the Atmoſphere is impregnated, corrodes and ruſts it; or at leaſt at its exhaling leaves it covered over with a thin Skin, conſiſting partly of an earthly Sediment of that Moiſture, and partly of the Duſt, which, flying to and fro in the Air, had ſettled and adhered to it.

Where there is not occaſion to make frequent Uſe of the Inſtrument, there may be other Ways to preſerve the Metal for a long time; as perhaps by immerging it in Spirit of Wine, or ſome other convenient Liquor. And if they chance to tarniſh, yet their Poliſh may be recovered by rubbing them with a ſoft Piece of Leather, or other tender Subſtance, without the Aſſiſtance of any fretting Powders, unleſs they happen to be ruſty; for then they muſt be new poliſhed.

I am very ſenſible, that Metal reflects leſs Light than Glaſs tranſmits; and for that Inconvenience, I gave you a Remedy in my laſt Letter, by aſſigning a ſhallower Charge in Proportion to the Aperture, than is uſed in other Teleſcopes. But as I have found ſome metalline Subſtances to be more ſtrongly reflective, and to poliſh better, and be freer from tarniſhing than others; ſo I hope there may in time be found out ſome Subſtance much freer from theſe Inconveniencies than any yet known.

The Conſiderations of —— Anſwered by Mr. Newton. N. 88. *p.* 5084. Nov. An. 1672.

2. The *Conſiderer* is pleaſed to reprehend me for laying aſide the Thoughts mproving Optics by Refractions. If he had obliged me by a private ter on this Occaſion, I would have acquainted him with my Succeſs on Trials I have made of that kind, which, I ſhall now ſay, have been leſs than metimes expected, and perhaps than he at preſent hopes for. But ſince s pleaſed to take it for granted, that I have let this Subject paſs without Examination, I ſhall refer him to my former Letters, by which that jecture will appear to be ungrounded. For what I ſaid there was in pect of Teleſcopes of the ordinary Conſtruction, ſignifying, that their provement is not to be expected from the well figuring of Glaſſes, as Opti-

cians have imagined; but I deſpaired not of their Improvement by other Conſtructions, which made me cautious, to inſert nothing that might intimate the contrary. For although ſucceſſive Refractions that are all made the ſame Way, do neceſſarily more and more augment the Errors of the firſt Refraction; yet it ſeemed not impoſſible for contrary Refractions ſo to correct each other's Inequalities, as to make their Difference regular; and if that could be conveniently effected, there would be no further Difficulty. Now to this end, I examined what may be done, not only by Glaſſes alone, but more eſpecially by a Complication of divers ſucceſſive Mediums, as by two or more Glaſſes or Chryſtals with Water or ſome other Fluid between them; all which together may perform the Office of one Glaſs, eſpecially of the Object-Glaſs, on whoſe Conſtruction the Perfection of the Inſtrument chiefly depends.

To the Aſſertion, that Rays are leſs true, reflected to a Point by a Concave, than refracted by a Convex, I cannot aſſent; nor do I underſtand, that the Focus of the latter is a leſs Line than that of the former. The Truth of the contrary you will rather perceive by the following Table, computed for ſuch a reflecting Concave, and the refracting Convex, on Suppoſition that they have equal Apertures, and collect parallel Rays at an equal Diſtance from their Vertex; which Diſtance being divided into 15000 Parts, the Diameter of the Concave Sphere will be 60000 of thoſe Parts, and of the Convex 10000; ſuppoſing the Sines of Incidence and Refraction to be, in round Numbers, as 2 to 3. And this Table following ſhews, how much the exterior Rays, at ſeveral Apertures, fall ſhort of their principal Focus.

The Diameter of the Aperture.	*The Parts of the Axis intercepted between the Vertex and the Rays.*		*The Errors by*	
	Reflected.	*Refracted.*	*Reflection.*	*Refraction.*
2000	$14991\frac{2}{3}$	14865	$8\frac{1}{3}$	135
4000	14966	14449	33	551
6000	14924	13699	76	1301
8000	14865	12475	135	2525
10000	14787	9472	213	5528

By this you may perceive, that the Errors of the refracting Convex ſo far from being leſs, that they are more than 16 Times greater than the li Errors of the reflecting Concave, eſpecially in great Apertures; and tha without reſpect to the heterogeneous Conſtitution of Light. So that, how ever the contrary Suppoſition might make the Author of theſe Animadve ſions reject Reflections as uſeleſs for the promoting of Optics; yet I mu for this, as well as other Conſiderations, prefer them in the Theory befo Refractions.

Whether the *Parabolæ* be more difficult to deſcribe than the *Hyperbola*, or *Ellipſis*, may be a Query; but I ſee no abſolute Neceſſity of endeavouring after any of their Deſcriptions. For if Metals can be ground truly Spherical, they will bear as great Apertures, as I believe Men will be well able to communicate an exact Poliſh to. And for Dioptrique Teleſcopes, I told you, that the Difficulty conſiſted not in the Figure of the Glaſs, but in the Difformity of Refractions; which if it did not, I could tell you a better and more eaſy Remedy than the Uſe of the *Conic Sections*.

Objections; by M. ---------- N. 96. *p.* 6087. July, An. 1673.

3. We ſee that a Picture made by an Object-Glaſs of 12 Foot in a dark Room, is too diſtinct, and too well defined, to be produced by Rays, that ſhould ſtray the 50th Part of the Aperture.

Anſwered by Mr. Newton. N. 97. *p.* 6110. Oct. An. 1673.

To take away this Difficulty, I muſt acquaint you, that though I put the greateſt Lateral Error of the Rays from one another to be about $\frac{1}{50}$ of the Glaſs's Diameter; yet their greater Error from the Points on which they ought to fall, will be but $\frac{1}{100}$ of that Diameter: And then, that the Rays, whoſe Error is ſo great, are but very few in Compariſon to thoſe, which are refracted more juſtly; for the Rays which fall upon the middle Parts of the Glaſs, are refracted with ſufficient Exactneſs, as alſo are thoſe that fall near the Perimeter, and have a mean Degree of Refrangibility; ſo that there remain only the Rays which fall near the Perimeter, and are moſt or leaſt refrangible, to cauſe any ſenſible Confuſion in the Picture. And theſe are yet ſo much further weakened by the greater Space through which they are ſcattered, that the Light which falls on the due Point, is infinitely more denſe than that which falls on any other Point round about it. And by this Exceſs of Denſity, the Light which falls in or inviſibly near the juſt Point, may, I conceive, ſtrike the *Senſorium* ſo vigorouſly, that the Impreſs of the weak Light, which errs round about it, ſhall, in Compariſon, not be ſtrong enough to be animadverted, or to cauſe any more ſenſible Confuſion in the Picture than is found by Experience. But if this ſatisfy not, *N.* may try, if he pleaſe, how diſtinct the Picture will appear, when all the Lens is cover'd, excepting a little Hole next its Edge on one Side only: And if in this Caſe he pleaſe to meaſure the Breadth of the Colours thus made at the Edge of the Sun's Picture, he will perhaps find it to approach nearer to my Proportion than he expects.

A Reply by M. ---------- *Ibid. p.* 6112.

4. I am ſatisfied with the manner whereby Mr. *Newton* reconciles the Effect of Convex-Glaſſes with his Theory; but then he is alſo to acknowledge, that this Aberration of the Rays is not ſo diſadvantageous to Optic-Glaſſes, as he ſeems to have been willing to make us believe. His Invention ry good; but the Defect of the Metal ſeems to render it as impoſſible execute, as the Difficulty of the Form obſtructs the Uſe of the *Hyperbole* M. *Des Cartes*.

Anſwered, by Mr. Newton. N. 96. *p.* 6091. July, An. 1673.

M. *N*—— pleaſes to compare the Errors of a Glaſs and Speculum collect Rays at equal Diſtances, he will find how much he is miſtaken; d that I have not been extravagant, as he imagines, in preferring Refle- ons. And as for what he ſays of the Difficulty of the *Praxis*, I know it is y difficult; and by thoſe Ways which he attempted it, I believe it impracticable.

practicable. But there is a Way inſinuated above, by which it is not improbable, but that as much may be done in large Teleſcopes as I have thereby done in ſhort ones; but yet not without more than ordinary Diligence and Curioſity.

A Cata-Dioptrical Teleſcope, by M. Caſſegrain. N. 83. *p.* 4056. May, An. 1672. Fig. 100.

XIX. 1. M. *Caſſegrain* has communicated the Figure of a Teleſcope, almoſt like that of Mr. *Newton.*

A B C D is a ſtrong Tube, in the bottom of which there is a great *Concave Speculum* C D, pierced in the middle E.

F is a *Convex Speculum*, ſo diſpoſed, as to its Convexity, that it reflects the Species, which it receives from the great Speculum, towards the Hole E, where is an Eye-Glaſs, which one looketh through.

The Advantage which I find in this Inſtrument above that of Mr. *Newton*, is, 1. That the Mouth or Aperture A B of the Tube may be of what Bigneſs you pleaſe; and conſequently you may have many more Rays upon the *Concave Speculum*, than upon that of which you have given us the Deſcription. 2. The Reflection of the Rays will be very natural, ſince it will be made upon the Axis itſelf, and therefore more vivid. 3. The Viſion of it will be ſo much the more pleaſing, in that you ſhall not be incommoded by the great Light, by reaſon of the bottom C D, which hideth the whole Face. Beſides, you'll have leſs Difficulty in diſcovering the Objects, than in that of Mr. *Newton.*

Conſidered by Mr. Newton. *Ibid. p.* 4057.

2. When I firſt applied myſelf to try the Effects of Reflections, Mr. *Gregory*'s *Optica Promota* (printed in the Year 1663.) being fallen into my Hands, where there is an Inſtrument (deſcribed *p.* 94.) like that of Mr. *Caſſegrain*, with a Hole in the midſt of the Object Metal, to tranſmit the Light to an Eye-Glaſs placed behind it; I had thence an Occaſion of conſidering that ſort of Conſtructions, and found theſe Diſadvantages in it; *viz.* 1. There will be more Light loſt in the Metal by Reflection from the little Convex Speculum, than from the oval Plane. For it is an obvious Obſervation, That Light is moſt copiouſly reflected from any Subſtance when incident moſt obliquely. 2. The Convex Speculum will not reflect the Rays ſo truly as the oval Plane, unleſs it be of an hyperbolic Figure; which is incomparably more difficult to form than a Plane; and if truly formed, yet would only reflect thoſe Rays truly which reſpect the Axis. 3. The Errors of the ſaid Convex will be too much augmented by the too great Diſtance, through which the Rays reflected from it muſt paſs, before their Arrival at the Eye-Glaſs. For which Reaſon, I find it convenient to make the Tube no wider than is ceſſary, that the Eye-Glaſs be placed as near to the oval Plane as is poſſi without obſtructing any uſeful Light in its Paſſage to the Object-Met 4. The Errors of the Object-Metal will be more augmented by Reflection fro the Convex than from the Plane, becauſe of the Inclination or Deflexion of t Convex on all ſides, from the Points on which every Ray ought to be incident. 5. For theſe Reaſons there is requiſite an extraordinary Exactn in the Figure of the little Convex; whereas I find by Experience that it much more difficult to communicate an exact Figure to ſuch ſmall Pieces

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Metal, than to thoſe that are greater. 6. Becauſe the Errors of the Perimeter of the Concave Object-Metal, cauſed by the Sphericalneſs of its Figure, are much augmented by the Convex, it will not with Diſtinctneſs bear ſo large an Aperture as in the other Conſtruction. 7. By reaſon that the little Convex conduces very much to the magnifying Virtue of the Inſtrument, which the oval Plane does not, it will magnify much more in Proportion to the Sphere, on which the great Concave is ground, than in the other Deſign; and ſo magnifying Objects much more than it ought to do in Proportion to its Aperture, it muſt repreſent them very obſcure and dark; and not only ſo, but alſo confuſed, by reaſon of its being over-charged. Nor is there any convenient Remedy for this. For if the little Convex be made of a larger Sphere, that will cauſe a greater Inconvenience, by intercepting too many of the beſt Rays; or if the Charge of the Eye-Glaſs be made ſo much ſhallower as is neceſſary, the Angle of Viſion will thereby become ſo little, that it will be very difficult and troubleſome to find an Object; and of that Object, when found, there will be but a very ſmall Part ſeen at once.

By this you may perceive, that the three Advantages, which Mr. *Caſſegrain* propounds to himſelf, are rather Diſadvantages. For according to his Deſign, the Aperture of the Inſtrument will be but ſmall, the Object dark and confuſed, and alſo difficult to be found. Nor do I ſee, why the Reflection is more upon the ſame Axis, and ſo more natural in one Caſe than in the other; ſince the Axis itſelf is reflected towards the Eye by the oval Plane, and the Eye may be defended from external Light, as well at the Side as at the Bottom of the Tube.

Mr. *Gregory* ſpeaking of theſe Inſtruments, in the aforeſaid Book, *p.* 95. ſaith; I ſhall ſay nothing about the Mechaniſm of theſe *Specula* and *Lenſes*, which has been tried in vain by others; and as to myſelf, I am but little verſed in Mechanics. So that there have been Trials made of theſe Teleſcopes, but yet in vain. And I am informed, that about 7 or 8 Years ſince, Mr. *Gregory* himſelf, at *London*, cauſed one of 6 Foot to be made by Mr. *Reive*, which I take to have been according to the aforeſaid Deſign deſcribed in his Book; but, though made by a ſkilful Artiſt, yet it was without Succeſs.

XX. S. *Salvetti* hath made a Proſpective-Glaſs according to Mr. *Newton*'s new Invention. It was not above half a Foot long, but had the ſame Effect of one of two. He is now making another after the Conceit of Mr. *Caſſegrain*,
ugh he agrees not with him in making Convex the little Speculum, which
looks into through the Eye-Glaſs, but believes the *French* Author only
iſed that to diſguiſe as much as was poſſible his pretended new Invention,
ch he endeavours to make anterior to Mr. *Newton*'s moſt noble one.

A Cata-Dioptric Teleſcope; by S. Salvetti. N. 87. *p.* 5065. Sept. An. 1672.

I. 1. Oppoſite to the Place or Wall, where the Apparition is to be,
a Hole be made of about a Foot in Diameter, or bigger: If there be a
h Window, that hath a Caſement in it, 'twill be ſo much the better.
ithout this Hole or Caſement opened, at a convenient Diſtance (that it may

To make the Picture of any thing appear in a Light Room; by Dr. Hook. N. 38. *p.* 741. Aug. An. 1688.

may not be perceived by the Company in the Room) place the Picture or Object, which you will repreſent inverted, and by means of Looking-Glaſſes placed behind, if the Picture be tranſparent, reflect the Rays of the Sun ſo, as that they may paſs through it towards the Place where it is to be repreſented; and to the End that no Rays may paſs beſides it, let the Picture be encompaſſed on every Side with a Board or Cloth. If the Object be a Statue, or ſome Living Creature, then it muſt be very much enlightened by caſting the Sun-Beams on it by Refraction, Reflection, or both. Between this Object and the Place where 'tis to be repreſented, there is to be placed a broad Convex-Glaſs, ground of ſuch a Convexity, as that it may repreſent the Object diſtinct on the ſaid Place; which any one, that hath an Inſight in the Optics, may eaſily direct. The nearer it is placed to the Object, the more is the Object magnified on the Wall, and the further off the leſs; which Diverſity is effected by Glaſſes of ſeveral Spheres. If the Object cannot be inverted (as 'tis pretty difficult to do with Living Animals, Candles, *&c.*) then there muſt be two large Glaſſes of convenient Spheres, and they placed at their appropriated Diſtances (which are very eaſily found by Trials) ſo as to make the Repreſentations erect, as well as the Object.

Theſe Objects, Reflecting and Refracting Glaſſes, and the whole Apparatus; as alſo the Perſons employed to order, change, and make uſe of them, muſt be placed without the ſaid high Window or Hole, ſo that they may not be perceived by the Spectators in the Room; and the whole Operation will be eaſily performed.

Whatſoever may be done by means of the Sun-Beams in the Day-time, the ſame may be done with much more eaſe in the Night, by the help of Torches, Lamps, or other bright Lights, placed about the Objects, according to the ſeveral ſorts of them.

The Magick Lanthorn improv'd, by Sir Rob. Southwell. N. 245. *p.* 364. Oct. An. 1698.

2. There are every where made of theſe Lanthorns to repreſent and magnify Figures upon a Wall; but then 'tis only in the Dark: wherefore to give Variety of Colours, take Oil of Spike, and therein mix the ſeveral Colours, wherewith you will have your Glaſs to be ſtained; paint them finely on, they dry preſently, and penetrate any Glaſs.

A Way to help Short-ſightedneſs. by Dr. Hook. *Ph. Coll.* N. 3. *p.* 59. Dec. An. 1681.

XXII. Having found by many Trials, that ſome ſhort-ſighted Perſons could find little or no Relief by the Uſe of Concave-Glaſſes for ſeeing Objects at any Diſtance diſtinct, and that any one may be made ſhort-ſighted, and to be able to diſtinguiſh nothing but what is placed very near his Eye, but within certain Limits of Diſtance, by putting on and looking through a very deep Pair of Spectacles, ſuch as ancient Men uſe: I concluded th what Glaſſes ſhould make this Man, whilſt looking thro' theſe Spectacles, ſee Things at a greater Diſtance, would alſo help any other Perſon that ſhou be ſhort-ſighted by Nature. I then conſider'd, that by the help of a Co vex-Glaſs, placed between the Object and the Eye, the Image of the O ject may be made to appear at any Diſtance from the Eye; and conſequen all Objects may thereby be made to appear in any convenient Diſtance fro the Eye: So that the ſhort-ſighted Eye ſhall contemplate the Picture of t

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Object, in the ſame manner as if the Object itſelf were in that Place. But then, becauſe the Pictures themſelves are ſo inverted, and therefore will be uncouth to one, not uſed to ſee them in that Poſture, I conſider'd of theſe Expedients to help that Defect alſo:

Firſt, If it be only for reading of a Book, or Writing, there needeth nothing but the Inverſion of the Book, and then holding the Convex at a due Diſtance; for the Picture of the Letters will appear erected in the due Place, for the Eye to ſee and diſtinguiſh them very plainly.

Secondly, For ſeeing to write, I thought this would be the beſt Expedient, That the Perſon ſhort-ſighted ſhould firſt learn to read with his naked Eye (both printed Letters and alſo written Hand) upſide downwards, which is quickly attained to by one that can do both the right Way.

Thirdly, For diſtinguiſhing Objects at a Diſtance, I can aſſert by my own Experience, that with a little Uſe of contemplating Objects inverted, one ſhall have as good an Idea, and as true a Knowledge of all manner of Objects, as if they were ſeen erected in their natural Poſture.

XXIII. 1. *Euſtachio Divini* hath made a Microſcope of a new Invention, wherein, inſtead of an Eye-Glaſs Convex on both Sides, there are two Plano-Convex-Glaſſes, which are ſo placed, as to touch one another in the middle of their Convex Surface. It hath this Peculiar, that it ſhews the Objects flat and not crooked; and although it takes in much, yet nevertheleſs magnifieth extraordinarily.

Microſcopes; by S. Divini. N. 42. *p.* 842. Dec. An. 1668.

It is almoſt 16½ Inches high, and adjuſted at four different Lengths. In the firſt, which is the leaſt, it ſhews Lines 41 times bigger than they appear to the naked Eye; in the ſecond 90 times; in the third 111 times; and in the fourth 143 times: Whence one may eaſily calculate how much it augments Surfaces and Solidities.

2. *S. Salvetti* lately ſhew'd one of his Microſcopes, made in Imitation of thoſe of *Divini* and *Campani*, to the Great Duke of *Tuſcany*, which was judged by all much better than any of the beſt his Highneſs hath. It was found, for Magnifying, Defining, and Clearneſs, to be very excellent.

By S. Piet. Salvetti. N. 87. *p.* 5065. Oct. An. 1672.

3. M. *Leeuwenhoeck* hath lately contrived Microſcopes, excelling thoſe that have been hitherto made by *Euſtachio Divini*, and others.

By M. Leeuwenhoeck. N. 94. *p.* 6037. May, An. 1673.

4. I have Microſcopes of the manner lately brought out of *Holland* by Mr. *Huygens*, of ſeveral Faſhions ready made. I have tried ſeveral Ways for the making of Glaſſes of the Bigneſs of a great Pin's Head and leſs; as in the F me of a Tallow-Candle, and of one of Wax. But the beſt Way of all I ve yet found, to make them clear and without Specks, is with the Flame o Spirit of Wine well rectified, and burned in a Lamp. Inſtead of Cotton I ake uſe of very ſmall Silver-Wire, doubled up and down like a Skein of T read; which being wet with the Spirit of Wine, and made to burn in t Lamp, giveth through the Veril of the Lamp, a very ardent Flame. T en take your beaten Glaſs, being firſt waſhed very clean, upon the Point o a Silver Needle filed very ſmall, and wet with Spittle. Hold it thus in t Flame till it be quite round, and no longer, for fear of burning it; and

By Mr. Butterfield. N. 141. *p.* 1026. Sept. An. 1678.

if the ſide of the Glaſs next the Needle be not melted, you may put it off, and take it up with the Needle on the round ſide, preſenting the rough ſide to the Flame, till it be every where round and ſmooth, then wipe and rub one or ſeveral of them together with ſoft Leather, which makes them much the better. Then put them between two Pieces of thin Braſs, the Apertures very round and without Bur, and that towards the Eye ſo big almoſt as the Diameter of the Glaſs, and ſo placed in a Frame with the Object conveniently for Obſervation.

By Mr. Stephen Gray. N. 221. *p.* 280. June, An. 1626.

5. I took a ſmall Particle of Glaſs, about the Bigneſs I deſigned my Globule, and laying it on the End of a Charcoal, I could, by the Help of a Blaſt-Pipe, with the Flame of a Candle, ſoon melt it into a Spherule; and by this means could make them indifferently clear, and the ſmalleſt very round, and I could make them much larger, than by the unaſſiſted Heat of the Candle: but theſe latter were attended with an Inconvenience; they were, on that ſide that reſted on the Coal, flatted, and received a rough Impreſſion from it. To remedy this Inconvenience, I was wont to grind them and poliſh them on a Braſs-Plane, and ſo reduce them to Hemiſpherules; but I found the clear ſmall Globules, not to mention that they magnify more, ſhew Objects more diſtinctly.

A Water Microſcope; by Mr. Stephen Gray. N. 221. *p.* 281. N. 223. *p.* 353.

Fig. 101.

XXIV. 1. A B, I call the Frame of the Microſcope. It may be about $\frac{1}{18}$ of an Inch in Thickneſs. At A there is a ſmall Hole, near $\frac{1}{10}$ of an Inch Diameter; this ſerves for the Aperture of the Water, being in the Center of a larger Spherical Cavity, about $\frac{1}{8}$ of an Inch Diameter, and in Depth ſomewhat more than half the Thickneſs of the Braſs. Oppoſite to this, at the other ſide, there is another Concave, but half the Breadth of the former; which is ſo deep, as to reduce the Circumference of the ſmall Hole in the Center, to almoſt a ſharp Edge. In theſe Cavities the Water is to be placed, being taken upon a Pin, or large Needle, and conveyed into them till there be formed a double Convex Lens of Water; which, by the Concave's being of different Diameters, will be equivalent to a double Convex, of unequal Convexities. By this means, I find the Object is rendered more diſtinct than by a Plano-convex of Water, or by a double one, formed on the plain Surface of the Metal.

C D E, is the Supporter, whereon to place the Object; if it be Water, in the Hole G; if a ſolid Object, on the Point F. This is fix'd to the Frame of the Microſcope by the Screw E, where 'tis bent upwards, that its upper Part may ſtand at a Diſtance from the Frame: 'tis moveable on the Sc
as a Center, to the end that either the Hole C, or the Point F, may be
poſed before the Microſcope; and that the Object may be brought to, a
fix'd in its Focus. There is another Screw, about half an Inch in Leng
which goes through the round Plate into the Frame of the Microſco
A E, the Screw and Plate taking hold of the Supporter about D, whe
there is a Slit ſomewhat larger than the Diameter of the Screw, which
requiſite for the Admiſſion of the Hole C, or Point F, according to t
Nature of the Object, into the Focus of the Glaſs; for by turning the Scre
G, th

G, the Supporter is carried to or from the ſame; which may be ſooner done, if whilſt one turns the Screw with one Hand, the other hold the Microſcope by the End B, and one continue looking through the Water till the Object be ſeen moſt diſtinctly.

The Supporter muſt be made of a thin Piece of Braſs well hammer'd, that by its Spring it may the better follow the Motion of the Screw. I choſe rather to fix the Supporter by the Screw E, than by a Rivet; becauſe it may now, by help of a Knife, be unſcrew'd, and by the other Screw G, be brought cloſe to the Frame of the Microſcope without weakening its Spring, and ſo become more conveniently portable. If the Hole at G be filled with Water, but not ſo as to be Spherical, all Objects that will bear it, are ſeen therein more diſtinctly.

Another. Ibid. N. 221. *p.* 282.

2. Having obſerv'd ſome irregular Particles in Globules of Glaſs, and finding them diſtinct, but prodigiouſly magnified, when held cloſe to the Eye; I concluded that if I convey'd a ſmall Globule of Water to my Eye, and that there were any opacous or leſs tranſparent Particles than the Water therein, I might ſee them diſtinctly. I therefore took on a Pin a ſmall Portion of Water, which I knew to have in it ſome minute Animals, and laid it on the End of a ſmall Piece of Braſs Wire (that lay then by me) of about $\frac{1}{10}$ of an Inch Diameter, till there was formed ſomewhat more than an Hemiſpherule of Water; then keeping the Wire erect, I applied it to my Eye, and ſtanding at a proper Diſtance from the Light, I ſaw them and ſome other irregular Particles, as I had predicted, but moſt enormouſly magnified; for whereas they are ſcarce diſcernible by my Glaſs-Microſcopes, or firſt Aqueous one, within the Globule they appeared not much different both in their Form, nor leſs in Magnitude than ordinary Peaſe. They cannot well be ſeen by Day-light, except the Room be darkened after the manner of the famous Dioptrical Experiment, but moſt diſtinctly by Candle-light; they may be very well ſeen by the full Moon-light. If the Water be conveyed into the Hole B (which may be about $\frac{1}{10}$ of an Inch Diameter) till there remain near N. 223. *p.* 355. an Hemiſphere of Water on each ſide of the Hole, the Objects are ſeen more diſtinctly; and the Spherical Form of the Water is this way better ſecured, than on the Point of a Pin-Wire:

N. 221. *p.* 285.

Fig. 102.

The Reaſon of this Appearance may be thus explained. Let the Circle D B B D repreſent a Sphere of Water, A an Object placed in its Focus, ſending forth a Cone of Rays, two of which are A B, A B, which, Opticians know, coming into the Water at B and B, will be refracted from their direct Courſe, and become B D, B D; at D they will, at their paſſing into the Air, be again refracted into D E, D E, and ſo run parallel to one another, and to the Axis of the Sphere A F C G. Now 'tis a known and fundamental Principle in Optics, that the Angle of Reflexion is equal to the Angle of Incidence: wherefore let the Rays B D, B D, be imagined to come from ſome Point of an Object placed within a Sphere of Water, by being reflected from the interior Surface of the Sphere at B B; C B D is the Angle of Reflexion; to which making C B F equal, F will be the Place where an Object ſending forth a Cone of Rays, two of which are F B, F B, which are reflected into the Rays B D, B D, and then coming to the other ſide the Sphere at D and D, they are refracted into D E, D E, as before; and conſequently be as fit for diſtinct Viſion, whether the Object be placed in F within, or in A without the Sphere, if its interior Surface be conſidered as a Concave Reflecting Speculum.

Microſcopes improv'd; by Mr. Newton. N. 88. *p.* 5096.

XXV. From the Diſtinction I have elſewhere given between *Compounded* and *Uncompounded Colours*, I take occaſion to communicate a Way for the Improvement of Microſcopes by Refraction; *viz.* By illuminating the Object in a darkened Room with Light of any convenient Colour not too much *Compounded*; for by that means the Microſcope will with Diſtinctneſs bear a deeper Charge and larger Aperture, eſpecially if its Conſtruction be ſuch as I may hereafter deſcribe; for the Advantage in ordinary Microſcopes will not be ſo ſenſible.

A Reflecting Microſcope; by Mr. Newton, N. 80. *p.* 3080.

Fig. 103.

XXVI. 1. I have ſometimes thought to make a Microſcope, which ſhould have, inſtead of an Object-Glaſs, a reflecting Piece of Metal. For theſe Inſtruments ſeem as capable of Improvement as Teleſcopes, and perhaps more; becauſe but one reflective Piece of Metal is requiſite in them, as you may perceive by the Diagram, where A B repreſenteth the Object-Metal, C D the Eye-Glaſs, F their common Focus, and O the other Focus of the Metal, in which the Object is placed.

By Mr. Stephen Gray. N. 228. *p.* 541.

Fig. 104.

2. A repreſents a ſmall flat Ring of Braſs, whoſe interior Circle muſt not much exceed $\frac{4}{10}$ of an Inch Diameter, and about $\frac{1}{10}$ of an Inch thick: This we may call the Frame or Cell of the Glaſs; it muſt be prepared for Uſe after the following manner. Take a ſmall Globule of Quick-ſilver, and diſſolve it in a few Drops of *Aqua Fortis*, to which you may add 10 Parts of common Water; dip the End of a Stick in this Liquor, and rub the inward Circle of the Ring with it; ſo as it will have acquired a mercurial Tincture, and being wiped dry, be fit for Uſe. Then let it be laid on the Table, and pour a Drop of Quick-ſilver within it, which preſs gently with the Ball of the Finger, and it will adhere to the Ring; then cleanſe it with a Hare's Foot, and you will have a Convex Speculum. Take up the Ring and Speculum carrying it Horizontal, and lay it on the Brims of the hollow Cylinder B; ſo will the Mercury become a Concave Reflecting Speculum, which from the Smallneſs of the Sphere of which it ſeems to be a Section, may be uſed as a Microſcope. The Cylindric Veſſel B has a Screw-Hole at the Bottom, by which it is ſcrew'd to the Top of the Pedeſtal C D; C E F G is the Supporter of the Object-Plate, which, as you ſee, may be raiſed higher, or let lower, as there is Occaſion, by the Screw on the Pedeſtal: The Object-Plate muſt be of Glaſs cemented to the Ring G.

This Inſtrument, with a little Variation, may be made a Microſcope of Water, if, inſtead of the Ring G, there be only a ſmall Arm with a Hole in it to receive a Drop of Water, and the Cylindric Veſſel B be either taken away, or ſcrewed on with its Bottom upwards, ſo as to make an Object-Plate.

This will be more convenient for viewing the Textures of opacous Objects, than that above deſcribed, which is more fit for fluid and tranſparent ones.

A Burning Concave at Lyons; *made by M. de* Vilette. N. 6. *p.* 95.

XXVII. 1. The Figure of it is round, being 30 Inches and ſomewhat better in Diameter. On one ſide it hath a Frame of a Circle of Steel, to the end that it may keep its juſt Meaſure. 'Tis eaſy to remove it from Place to Place, tho' it be above an Hundred Weight, and 'tis eaſily put in all ſorts of Poſtures. The Burning Point is diſtant from the Center of the Glaſs about 3 Foot. The Focus is about half a *Louis d'Or* large. One may paſs one's Hand through it, if it be done nimbly; for if it ſtay there the time of a Second of a Minute, there is danger of receiving much hurt. Green Wood takes Fire in it in an inſtant, as do alſo many other Bodies.

	Seconds.
A ſmall Piece of Pot-Iron was melted, and ready to drop down, in -	40
A Silver Piece of Fifteen Pence was pierced, in — — — —	24
A Groſs Nail (called *le Clou de Piſan*) was melted, in — — —	30
The End of a Sword Blade of *Olinde* was burned, in — — —	43
A Braſs Counter was pierced, in — — — — — — —	6
A Piece of Red Copper was melted ready to drop down, in — —	42
A Piece of a Chamber Quarry-Stone was vitrified, and put into a Glaſs-Drop, in — — — — — — — — — —	45
Steel, whereof Watchmakers make their Springs, was found melted, in — — — — — — — — — — —	9
A Mineral-Stone, ſuch as is uſed in Harquebuſſes *à rouët*, was calcined and vitrified, in — — — — — — —	1
A Piece of Mortar was vitrified, in — — — — — —	52

In ſhort, There is hardly any Body which is not deſtroy'd in this Fire. If one would melt by it any great Quantity of Metal; that would require much Time, the Action of Burning not being performed but within the Bigneſs of the Focus, ſo that ordinarily none but ſmall Pieces are expoſed to it. One M. *de Alibert* buys it, paying for it 1500 Livres.

Ibid. p. 97.

You incline to believe, that the Glaſſes of *Maginus* and *Septalius* do approach to that of *Lyons*; but I can aſſure you they come very far ſhort of it. You may conſult *Maginus* his Book, where he deſcribes his; and there are ſome Perſons here who have ſeen one of his beſt, which had but about 20 Inches Diameter; ſo that this of *Lyons* muſt perform at leaſt twice as much. As to *Septalius*, we expect the Relation of it from Intelligent and Impartial Men. It cannot well be compared with that of *Lyons*, but in Bigneſs; and in this Caſe, if it have five Palms (as you ſay) that would be but 3 ½ Foot *French*, and ſo it were a Foot bigger, which would make it half as much greater in Surface: But as to the Effects, ſeeing it burns ſo far off, they cannot be very violent. And I have heard one ſay, that had ſeen it, that it did not ſet Wood on Fire but after the time of ſaying a *Miſerere*. You may judge of the Difference of the Effects, ſince that of *Lyons* gathers its Beams together within the Space of 7 or 8 Lines; and that of *Septalius* muſt ſcatter them in the Compaſs of 3 Inches.

N. 49. *p.* 986. It was diſpoſed of to the King of *Denmark*.

Another by the ſame. N. 49. *p.* 986. The ſame M. *de Vilette* of *Lyons* hath made another Burning Concave. It is of 34 Inches Diameter, and melts all ſorts of Metals, and Iron itſelf of the Thickneſs of a Silver Crown, in leſs than a Minute of Time, and vitrifies Brick in the ſame Time; and as for Wood, whether green or dry, it ſets it on Fire in a Moment. The King hath ſeen it, and the Performances of it, with great Satisfaction; and his Majeſty is likely to make it his, and then to beſtow it on his *Royal Academy* of Philoſophers, for making of farther Experiments with it.

By - - - - - *Ibid. p.* 987. This kind of Concaves burning the moſt forcibly of any Fire we know of, would be of great Uſe, if they could be ſo contrived as to have a Focus of any conſiderable Largeneſs, to take in a good Quantity of combuſtible Matter at once.

By S. Settalla. N. 40. *p.* 796. 3. *S. Settalla* at *Milan* cauſeth to be made a Burning-Glaſs of 7 Foot in Diameter. He pretends to make it burn at the Diſtance of 50 Palms, which is about 33 Foot.

A burning Concave in Germany; *by* - - - - - - N. 188. *p.* 252. 4. The outer Circle of the Concave Burning Speculum, which I lately cauſed to be made in *Luſace*, is near 3 *Leipſick* Ells in Diameter, exceeding that great one at *Paris* by $\frac{3}{8}$ of ſuch an Ell. It is made of a Copper Plate ſcarce twice ſo thick as the Back of an ordinary Knife, and may therefore be eaſily removed from Place to Place, and order'd for Uſe: And the Workmanſhip of it may, by the Contrivances I have invented, be eaſily, and in little Time performed by one Man. The Poliſh thereof is very good, and repreſents by diſtinct Reflections all thoſe Appearances which ariſe from the Concave Figure thereof.

The Force of this Speculum is incredible. For, 1. A Piece of Wood put into the Focus (which is 2 Ells off) flames in a Moment, ſo as a freſh Wind can hardly put it out. 2. Water applied in an Earthen Veſſel preſently boils, ſo as to boil an Egg; and the Veſſel being held there ſome Time, the Water evaporates all away. 3. A Piece of Tin or Lead 3 Inches thick, as ſoon as it is put into the Focus, melts away in Drops; and held there a little Time, is in a perfect Fluor, ſo as in two or three Minutes to be quite pierced through. 4. A Plate of Iron or Steel placed in the Focus, immediately is ſeen to be red-hot on the Backſide, and ſoon after a Hole is burnt through: I have made three ſuch Holes in a Plate in ſix Minutes Time. 5. Copper, Silver, and the like, applied to the Focus, melt; which I have tried with ſeveral Sorts of Coin; among the reſt, with a *Rix-Dollar*; and the ſame happened to it as to the aforeſaid Iron Plate in five or ſix Minutes. 6. Things not apt to melt, as Stones, Brick, and the like, ſoon become red-hot like Iron. 7. Slate at firſt is red-hot, but in a few Minutes turns into a fine ſort of black Glaſs; of which, if any Part be taken in the Tongs and drawn out, it runs into Glaſs Threads. 8. Tiles, which had ſuffered the moſt intenſe Heat of Fire, in a little Time melt down in a yellow Glaſs: As do, 9. Pot-Shreads, not only well burnt at firſt, but much uſed in the Fire, into a blackiſh yellow Glaſs. 10. Pumice-Stone, ſaid to be that of Burning Mountains, in this Solar Fire melts into a white tranſparent Glaſs. 11. A Piece of a very ſtrong Crucible put into the Focus, in eight Minutes was melted into a Glaſs. 12. I have ſeen Bones turned into a kind of opake Glaſs, and a Clod of Earth into a yellow or greeniſh Glaſs.

Theſe Experiments were made in *Auguſt* and *September*, when the Sun has not the ſame Force as when he is about the Summer Solſtice. The Beams of the Full Moon, concentred by this Speculum, did not produce any Degree of Heat, tho' the Light was not a little increaſed.

By Dr. Hook. *Ibid. p.* 354.

5. Some Years ago, Dr. *Hook* made a Propoſal to the *Royal Society* concerning the ſame Thing. He conceives one may be made of many Foot Diameter, for a ſmall Price, being hammered out of a Copper Plate, and tinned over with a Mixture of Tin, Lead, and Tin-Glaſs, which is found to bear a very good Poliſh. Such a Speculum might be of great Uſe in perfecting the Art of Paſtes, or fictitious Jewels, which require the moſt intenſe Degree of Heat, to bring them to an exact Mixture.

Concave Specula nearly of a Parabolic *Figure attempted by Mr.* Stephen Gray. N. 228. *p.* 542. N. 235. *p.* 787.

XXVIII. A Linnen-Cloth, firſt being wet in fair Water, and then laid on a Concave Cylinder, as the Verge of a Sieve, Keeler, or the like, its central Parts will deſcend ſo as to form a very regular Concave Superficies. And a Thread, being firſt wet in common Water, and then ſuſpended with its two Ends, or any two Points nearer than their utmoſt Extent, ſo as it might touch the Center of the ſuſpended Cloth, and its two oppoſite Points on the Ring, was found to have the ſame Curvature. My Buſineſs was then to examine the Figure of the Thread thus ſuſpended; which I did in manner following. On the ſide of a Wall I deſcribed *Parabola's* of ſeveral Species, whoſe Axes were perpendicular, and Perimeter horizontal; to which the Line being applied, ſo as it might touch the Vertex, paſs'd very nearly through all the intermediate Points of the *Parabola*, much nearer than the Portion of a *Circle*, which paſſed through the Extremity of the Perimeter, and *Latus rectum*, would do.

From hence I conclude, That a ponderous and pliable Subſtance, being ſuſpended on a Ring or hollow Cylinder, ſo as that its central Parts may deſcend, will form itſelf into a Figure that is more commodious for Burning-Glaſſes than the Spherical, of which they are now made, being much nearer their moſt abſolute Figure, the *Parabola*.

Now if there may be a Way found to give to Cloth or Leather a metalline Surface, or a Varniſh that may bear a good Poliſh; or if this be found impracticable, perhaps Plates of Metal may be beat out ſo thin, as being ſuſpended on a large Ring, will by their own Gravity receive their true Figure; one may make Speculums of what Largeneſs he pleaſeth.

Upon this Conſideration, I deviſed the following Experiment. There was taken a ſufficient Quantity of Potter's Clay, of which there was formed a plain circular Plate, by help of an Iron Ring about 13 Inches Diameter. This was laid on a leſſer Ring, which was ſupported by four Feet; and it immediately became a very regular Concave on its upper, and Convex on its under Superficies; but notwithſtanding 'twas ſet to dry in the Shade, yet before it was dry enough, its central Parts extended ſo as to become almoſt plain, not without ſome Defects; if it had continued in its Regularity, I deſigned to have burned and glazed it in a Potter's Furnace.

To make the Globe Looking-Glaſs; by Sir R. Southwell. N. 245. *p.* 363.

XXIX. Take Quickſilver, Marchaſite of Silver, each three Ounces; Tin and Lead, each half an Ounce; theſe two firſt throw on the Marchaſite, and laſt of all the Quickſilver; ſtir them well together; but they muſt be taken from the Fire, and be towards cooling before the Quickſilver be added; let your Glaſs be well warmed, and pour in the Mixture, and roll it from Side to Side.

Note, This will do alſo when cold, but 'tis beſt when the Glaſs is heated and very dry.

Note alſo, That if at the Glaſs-Houſe your Ball be of yellow Glaſs, then all will ſhine like Gold.

XXX. *Papers (of leſs General Uſe) omitted.*

Optic-Glaſſes by a Turn-Lathe. N. 2. *p.* 31. N. 4. *p.* 56.

1. DR. *Hook* having (in his *Micrographia*) deſcribed a new Engine for *Grinding Optic-Glaſſes* of very great Lengths, M. *Auzout* (in a ſmall *French Tract*) objects ſeveral Difficulties to this Engine itſelf: But however he thinks it impracticable to make any Glaſſes of above 300 or 400 Foot at moſt (and fears that neither Matter nor Art will go even ſo far) which will be very far from ſhewing us *Plants* or *Animals* in the *Moon*; and then propoſes Remedies to ſome of the Inconveniencies of the Turn. To all this, Dr. *Hock* here replies; He anſwers the Objections, and rejects the propoſed Expedients. *Ibid. p.* 63.

Upon a Plane. N. 42. *p.* 838.

2. *Carlo Ant. Mancini* having, in his *Occhiale all' Occhio*, deſcribed a particular Way for making Convex-Glaſſes upon a Plane, his Method is here tranſlated from the *Italian* into *Engliſh*. But 'tis added, That though the Contrivance be ingenious, yet it is conceived by ſkilful Artiſts, that it will be very difficult to put it into Practice.

XXXI. *Accounts of Books omitted.*

N. 79. *p.* 3068. 1. P*Hyſico-Matheſis de Lumine, Coloribus, & Iride*, &c. *Auth.* Franc. Maria Grimaldo, *S. J.* Bononiæ 1665. *in 4to.*

N. 71. *p.* 2163. 2. *Cogitationes Phyſico-Mechanicæ de Natura Viſionis. Auth.* Jo. Ott. *Schaphuſa Helvetio.* Heidelbergæ 1670. *in 4to.*

N. 32. *p.* 626. 3. *Synopſis Optica. Auth.* Honorato Fabri. *Soc. Jeſu.* Lugduni 1667. *in 4to.*

N. 42. *p.* 837. 4. *L' Occhiale all' Occhio, overo Dioptrica Prattica del* Carlo Ant. Mancini, *in* Bologna 1660. *in 4to.*

N. 75. *p.* 2258. 5. *Lectiones* xviii; *Cantabrigiæ in Scholis publicis habitæ, in quibus Opticorum Phænomenωn genuinæ Rationes inveſtigantur, & exponuntur, ab* Iſaaco Barrow. Lond. 1669. *in 4to.*

6. *La Dioptrique Occulaire, par le Pere* Cherubin *d'* Orleans, *Capucin.* A Paris, 1671. *in Folio.* N. 78. *p.* 3045.

7. Chriſtiani Hugenij *Aſtroſcopia Compendiaria, Tubi Optici molimine liberata:* or, The Deſcription of an *Aërial Teleſcope.* Hague, 1684. *in 4to.* N. 161. *p.* 668.

8. A Treatiſe of Dioptrics. By *Will. Molineux,* Eſq; F. R. S. *in 4to.* N. 205. *p.* 967.

9. *Catoptricæ & Dioptricæ Elementa. Auctore* Davide Gregorio, *D. M.* Oxon. 1695. *in 8vo.* N. 219. *p.* 214.

CHAP. IV.

ASTRONOMY.

I. THE Iſland *Ween* (vulgarly termed the *Scarlet-Iſland*) famous for the Obſervations of *Tycho Brahe,* that renowned *Daniſh Aſtronomer* (with all Submiſſion to better Judgments) was none of the fitteſt for Aſtronomical Obſervations of all ſorts, ſuch as the taking the exact Time of the true Riſing and Setting of Celeſtial Bodies, together with their reſpective Amplitudes; becauſe the Iſland lies low, and is Land-lock'd on all the Points of the Compaſs, ſave three. Beſides, the ſenſible Land-Horizon of the *Ween* is extremely uneven and rugged, the North and Eaſtern Parts thereof being ſome riſing Hills in the Province of *Schonen*; and the Weſtern Part is moſtly overſpread with Trees on the Iſland *Zealand*; from the remoteſt of which Coaſts the *Ween* is not diſtant above three Leagues.

The Obſervatory of Tycho Brahe; *by Mr.* Gourdon.
N. 266. *p.* 692.

II. M. *Weighelius* hath invented an Inſtrument, which he calls *Aſtrodicticum,* by the means whereof very many Perſons ſhall be able at one and the ſame Time to behold one and the ſame Star. He hath alſo invented an exceeding great Globe of the World, capable of 10 Perſons to ſit in it all at once, and to behold the Motions of the celeſtial Bodies, &c.

A new Aſtronomical Inſtrument by M. Weighelius.
N. 74. *p.* 2219.

III. The Bigneſs of this Globe is only of four Inches Diameter. The Body of the Globe of burniſhed Steel, where all the Figures of the Conſtellations are deſigned in Silver-Colour, but the Stars themſelves of all Magnitudes are put on in emboſſed Gold.

A Celeſtial Globe, by M. Didier, P. Alleman.
N. 136. *p.* 905.

This Globe moves from Eaſt to Weſt in 24 Hours; and you may there ſee the Sun exactly riſe and ſet as in the great World, together with the Moon, as alſo the Stars of the Conſtellations; likewiſe, how the Sun of this Globe comes to his Meridian, with an admirable Regularity, conform to the *Primum Mobile.* And you may alſo there perceive the mean Motions of the Sun and Moon from Weſt to Eaſt, and all the Lunations; and by the Diurnal Motion of the Moon, it ſhews the Flux and Reflux of the Sea.

The Meridian ſerveth for a Needle to ſhew the Hours, which hath two main Rays, one whereof goeth directly Northward, the other Southward. That of the North marks the Way, or Degree, which the Sun maketh from Weſt to Eaſt upon the Signs of the Zodiack, and upon a Circle of Silver, where the 360 Degrees of the Circle are marked. The other Ray of the South, marks upon another Circle of Silver the Days of the Month, where the 365 Days are noted. The Circles of the Longitude of the Stars, which ſeparate the Signs, and which come from the Poles of the Zodiack, are marked by Gold-Wires; as alſo the Equator, the Tropicks, and the Polar Circles.

There is but one great Spring, the *Primum Mobile*, which puts all the reſt in Motion: It is wound up by the *Antarctick* Pole, and you may wind it up to the Right or Left Hand, without wronging any contrary Motion: And by the *Arctick* Pole, you may advance and retard this Movement, if you ſhould find any Inequality, without altering at all the great Spring.

A Way to meaſure the Diameters of the Planets, and the Parallax of the Moon; by M. Auzout. N. 21. *p.* 373. Jan. An. 1666.

IV. I applied myſelf the laſt Summer to the taking of the Diameters of the Sun, Moon, and the other Planets, by a Method which M. *Picard* and myſelf have, eſteemed by us the beſt of all thoſe that have been practiſed hitherto; ſince we can take the Diameters to Second Minutes, being able to divide one Foot into 24000 or 30000 Parts, ſcarce failing ſo much as in one only Part, ſo as we can in a manner be aſſured not to deceive ourſelves in 3 or 4 Seconds. I ſhall not now tell you my Obſervations; but I may very well aſſure you, that the Diameter of the Sun has not been much leſs in his *Apogee*, than 31 min. 37 or 40 ſec. and certainly not leſs than 31 min. 35 ſec. and that at preſent in his *Perigee* it paſſes not 32 min. 45 ſec. and may be leſs by a Second or two: That which is at the preſent troubleſome, is, that the vertical Diameter, which is the moſt eaſy to take, is diminiſhed, even at Noon, by 8 or 9 ſec. becauſe of the Refractions, which are much greater in Winter than Summer at the ſame Height; and that the horizontal Diameter is difficult, becauſe of the ſwift Motion of the Heavens.

As for the Moon, I never yet found her Diameter leſs than 29 min. 44 or 45 ſec. and I have not ſeen it paſs 33 min. or if it hath, it was only by a few Seconds. But I have not yet taken her in all the Kinds of Situations of the *Apogees* and *Perigees* which happen, with the Conjunctions and Quadratures. I do not mention all that can be deduced from thence; I ſhall only tell you, that I have found a Way to know the *Parallax* of the Moon, by the means of her Diameter: *viz.* If on a Day, when ſhe is to be in her *Apogee* or *Perigee*, and in the moſt Boreal Signs, you take her Diameter towards the Horizon, and then towards the South, with her Altitudes above the Horizon. For if the Obſervation of the Diameters be exact, as in theſe Situations the Moon changes not conſiderably her Diſtance from the Earth in 6 or 7 Hours, the Difference of the Diameters will ſhew the Proportion there is of her Diſtance with the Semidiameter of the Earth. I do not enlarge, becauſe that as ſoon as one hath this Idea, the reſt is eaſy. The ſame would yet be practiſed better in the Places where the Moon paſſes through the Zenith, than here; for the greater the Difference is of the Heights, the greater is that of the Diameters. I do not note (for it eaſily appears) that if one were under the ſame Meridian or the ſame *Azimuth*, in two very diſtant Places, and took at the ſame Time the Diameter of the Moon, one would do the ſame Thing; tho' this Method goes not to Preciſeneſs.

From what has been ſaid may be collected the Reaſon of the Obſervation, which M. *Hevelius* made in the laſt Eclipſe of the Sun (*July* 2. *St. N.* 1666.) touching the Increaſe of the Moon's Diameter about the End. I am exceeding glad, that a Perſon, who probably knew not the Cauſe of it, has made the Experiment; but it is ſtrange, that until now no Aſtronomer has foreſeen that that ſhould happen, nor given any Precepts for the Change of the Moon's Diameter in the Eclipſes of the Sun, according to the Places where they ſhould happen, and according to the Hour and Height the Moon ſhould have: For what happened in that Eclipſe, of Augmentation, would have fallen out contrarily, if it had been in the Evening; for the Moon, which in that Eclipſe that began in the Morning was higher about the End than at the Beginning, was nearer us, and conſequently was to appear bigger: But if the Eclipſe ſhould happen in the Evening, ſhe would be lower at the End, and therefore more diſtant from us, and conſequently appear leſſer. So alſo in two different Places, whereof one ſhould have the Eclipſe in the Morning, and the other at Noon, the Moon ſhould appear bigger to him that hath it at Noon: And ſhe muſt likewiſe appear bigger to thoſe who ſhall have a leſſer Elevation of the Pole under the ſame Meridian, becauſe the Moon will be nearer them.

An Account of M. Gaſcoigne's *Micrometer; by Mr.* Richard Townley. N. 25. *p.* 457. May, An. 1667.

V. 1. I ſhould be looked upon as a great Wronger of our Nation, ſhould I not let the World know, that I have, out of ſome ſcattered Papers and Letters that formerly came to my Hands, of one Mr. *Gaſcoigne*'s, found out, that before our late *Civil Wars* he had not only deviſed an Inſtrument of as great a Power as M. *Auzout*'s, but had alſo for ſome Years made uſe of it; not only for taking the Diameters of the Planets, and Diſtances upon Land; but had farther endeavoured, out of its Preciſeneſs, to gather many Certainties in the Heavens; amongſt which I ſhall only mention one, *viz.* *The finding the Moon's Diſtance*, from two Obſervations of her Horizontal and Meridional Diameters; which I the rather mention, becauſe the *French Aſtronomer* eſteems himſelf the firſt that took any ſuch Notice, as thereby to ſettle the Moon's Parallax: For our Countryman fully conſidered it before, and imparted it to an Acquaintance of his, who thereupon propoſed to him the Difficulties that would ariſe in the Calculation; with Conſiderations upon the ſtrange Niceties, neceſſary to give him a Certainty of what he deſired. The very Inſtrument he firſt made, I have now by me, and two others more perfected by him; which doubtleſs he would have infinitely mended, had he not been ſlain unfortunately in his late Majeſty's Service. He had a *Treatiſe of Optics* ready for the Preſs; but though I have uſed my utmoſt Endeavour to retrieve it, yet I have in that Point been totally unſucceſsful: But ſome looſe Papers and Letters I have, particularly about this Inſtrument for taking of Angles, which was far from perfect. Neverthel eſs, I find it ſo much to exceed all others, that I have uſed my Endeavours to make it exact, and eaſily tractable; which above a Year ſince I effected to my own Deſire, by the Help of an ingenious and exact Watchmaker: Since which time, I have not altogether neglected it, but employed it particularly in taking the *Diſtances* (as Occaſion ſerved) of the *Circum-jovialiſts*, towards a perfect ſettling their Motion. I ſhall only ſay of it, That it is ſmall, not exceeding in Weight, nor much in Bigneſs, an ordinary Pocket-Watch, exactly marking above 40000 Diviſions in a Foot, by the help of two Indexes; the one ſhewing Hundreds of Diviſions, the other Diviſions of the Hundred; every laſt Diviſion in my ſmall one containing $\frac{1}{10}$ of an Inch, and that ſo preciſely, that, as I uſe it, there goes about 2 $\frac{1}{2}$ Diviſions to a Second. Yet I have taken Land-Angles ſeveral Times to one Diviſion, tho' (for the Reaſon mentioned by M. *Auzout*) it be very hard to come to that Exactneſs in the Heavens, (*viz.* the ſwift Motion of the Planets.) Yet, to remedy that Fault, I have deviſed a *Reſt*, in which I find no ſmall Advantage, and not a little pleaſing thoſe Perſons who have ſeen it, being ſo eaſy to be made, and by the Obſerver managed without the help of another; which ſecond Convenience my yet nameleſs Inſtrument hath in great Perfection, and is, by reaſon of its Smallneſs and Shape, eaſily applicable to any Teleſcope.

A Deſcription of it; by Dr. Hook. N. 29. p. 542. Nov. An. 1667.

Fig. 105.

2. *a a a a*, is a ſmall oblong Braſs Box, ſerving both to contain the Screws and their Sockets, or Female Screws, and alſo to make all the ſeveral moveable Parts of the Inſtrument to move very true, ſmooth, and in a ſimple direct Motion. To one End hereof is ſcrewed on a round Plate of Braſs *b b b b*, about 3 Inches over; the extreme Limb of whoſe Outſide is divided into 100 equal Parts, and numbred by 10, 20, and 30, *&c.* Through the Middle of this Plate, and the Middle of the Box *a a a a*, is placed a very curiouſly wrought Screw, of about the Bigneſs of a Gooſe-Quill, and of the Length of the Box; the Head of which is, by a fixed Ring or Shoulder on the Inſide, and a ſmall ſpringing Plate *d d* on the Outſide, ſo adapted to the Plate, that it is not in the leaſt ſubject to ſhake. The other End of this Screw is by another little Screw (whoſe ſmall Points fill the Center or Hole made in the End of the longer Screw for this Purpoſe) rendered ſo fixed and ſteddy in the Box, that there appears not the leaſt Danger of ſhaking. Upon the Head of this Screw, without the ſpringing Plate, is put on a ſmall Index *e e*, and above that a Handle *m m*, to turn the Screw round as often as there ſhall be Occaſion, without at all endangering the diſplacing of the Index, it being put on very ſtiff upon a cylindrical Part of the Head, and the Handle upon a Square. The Screw hath that Third of it, which is next the Plate, bigger than the other two Thirds of it, by at leaſt as much as the Depth of the ſmall Screw made on it: The Thread of the Screw of the bigger Third is as ſmall again, as that of the Screw of the other two Thirds. To the groſſer Screw is adapted a Socket *f*, faſtened to a long Bar or Bolt *g g*, upon which is faſtened the moveable Sight *h*, ſo that every Turn of the Screw promotes the Sight *h*, either a Thread nearer, or a Thread farther off from the fixed Sight *i*. The Bar *g g*, is made exactly equal, and fitted into two ſmall Staples *k k*, which will not admit of any ſhaking. There are 60 of theſe

Threads, and anſwerable thereto are made 60 Diviſions on the Edge of the Bolt or Ruler *g g*, and a ſmall Index *l* fixed to the Box *a a a a*, denotes how many Threads the Edges of the two Sights *b* and *i* are diſtant; and the Index *e e*, ſhews on the circular Plate what Part of a Revolution there is more; every Revolution, as was ſaid before, being divided into 100 Parts. At the ſame Time that the moveable Sight *b* is moved forwards or backwards, one or more Threads of the coarſer Screw, is the Plate *p p*, by the means of the Socket *q*, to which it is ſcrewed, moved forward or backward, one or more Threads of the finer Screw: So that this Plate being fixed to the Teleſcope by the Screws *r r*, ſo as the middle betwixt the Sights may lie in the Axis of the Glaſs, however the Screw be turned, the midſt betwixt the Sights will always be in the Axis, and the Sights will equally either open from it, or ſhut towards it.

Fig. 106.

It is conceived by ſome ingenious Men, that it will be more convenient, inſtead of the Edges of the two Sights *b* and *i*, to employ two Sights *r* and *s*, fitted with the Hairs *t* and *v*, ſo that they may be conveniently uſed in the Place of the ſolid Edges of the Sights *b* and *i*.

Fig. 107.

The Inſtrument is thus applied to the Teleſcope. The Tube A D is divided into three Lengths, of which (as in ordinary ones) B C is to lengthen or contract, as the Object requires: But A B is here added, that at A you may put ſuch Eye-Glaſſes as ſhall be thought moſt convenient, and to ſet them ſtill at the Diſtance moſt proper from the Indexes or Pointers, which here are ſuppoſed to be at B, which Length alters alſo in reſpect of divers Perſons Eyes. E is a Screw, by which the great Tube can be fixed ſo, as by the help of the Figures any ſmaller Part of it can immediately be found, meaſuring only, or knowing the Diviſions on B C, the Diſtance of the Object-Glaſs from the Pointers. F is the angular Piece of Wood, that lies on the upper Screw of the Reſt.

Fig. 108.

Fig. 109.

This Reſt (by Dr. *Hook*'s Suggeſtion) may be rendered more convenient, if, inſtead of placing the Screw Horizontal, it be ſo contrived, that it may be laid parallel to the Equinoctial, or to the Diurnal Motion of the Earth; for, by that Means, the ſame Thing may be performed by the ſingle Motion of one Screw, which in the other Way cannot be done but by the turning of both Screws; as will eaſily appear to thoſe that ſhall conſider it.

Ibid. p. 556.

3. I have by me two or three ſeveral Ways of Meaſuring the Diameters of the Planets, whether Horizontal, Perpendicular, or Inclined, to the Exactneſs of a Second, by the help of a Teleſcope; as alſo of taking the Poſition and Diſtance of the ſmall fixed Stars one from another, or from any of the leſs bright Planets, if the Diſtance be not above two or three Degrees.

More Ways to meaſure ſmall Diſtances intimated; by Dr. Hook.
N. 25. *p.* 459.
May, An. 1667.

4. With a Micrometer and a Tube of 14 Feet; I have often meaſured the Diameters of the Planets, and their Diſtances from the fixt Stars, almoſt to Seconds; which, without having tried it, you would hardly believe.

Excellence of the Micrometer; by Mr. Flamſteed.
N. 96. *p.* 6099.

Plain Sights rejected; by Mr. Flamſtead. N. 89. *p.* 5119. N. 96. *p.* 6100.

VI. 1. It may be proved by many Arguments, that *Tycho* erred from the Truth, by two, three, nay ſometimes four or five whole Minutes, both in the Places and Latitudes aſſigned to ſome of the fixt Stars. We have heard that that great Man *John Hevelius* has undertaken the Reſtitution of the fixt Stars; yet as it is reported he makes uſe of Sights without Glaſſes, it is doubtful whether we ſhall have their Places much more correct from him than thoſe left us by *Tycho*, unleſs where he has been very much miſtaken.

Plain Sights preferred to Teleſtopic; by M. Hevelius. N. 102. *p.* 27. Apr. An. 1674.

2. I perceive that all your People do not agree with me, in the Buſineſs of Sights, concerning which I have treated in the Organographia of my *Machina Cæleſtis.* But tho' Mr. *Hook* and Mr. *Flamſtead*, and others are of a different Opinion, yet I have been taught by daily Experience, and am ſtill convinced, that the Matter is far otherwiſe in thoſe great Inſtruments, as Quadrants, Sextants, and Octants, and chiefly Azimuth Quadrants, and other Quadrants conſtructed by Rulers, which cannot ſo eaſily or indeed by any means be diſturbed and inverted (which happens to Teleſcopic Sights when they are examined) as thoſe Inſtruments of three or four Feet conſtructed with a Perpendicular. The Matter chiefly amounts to this, that they can undertake no Obſervation with their Teleſcopic Sights, till they examine and rectify them anew; in which Examination there is Room to miſtake perpetually, and different Ways, tho' you perform it never ſo induſtriouſly. And indeed I cannot underſtand by what Method this Examination can be performed, in Azimuthal Quadrants, Octants, and Sextants, at all Times, with Convenience, and without a great Loſs of Time.

I ſee there are ſome alſo (among whom is Mr. *Flamſtead)* who have undertaken to give Judgment upon our Obſervations, whatever they may be, before they have ſeen or examined them, or can know any Thing about them. I do not deſire to be a vain Boaſter of my own Affairs, nor did I ever imagine, that in this Attempt of the Reſtitution of the fixt Stars, knowing my own Weakneſs, that I ſhould be perfect in every thing. But this I am convinced of, that if I had undertook the Buſineſs with the help of Teleſcopic Sights, I muſt not only have waſted many Years with fruitleſs Examinations, but doubtleſs I ſhould have been diſappointed of my Hopes, and that on various Accounts, which it is not neceſſary to mention here. Hence I congratulate with myſelf, that I never could embrace that Opinion, but that I performed every Thing by my own Method, whatever by the Aſſiſtance of God I have performed. Now when we ſhall have Obſervations on both Sides, continued for the Space of 20 or 30 Years; that is, as well thoſe that are made with Teleſcopic Sights, as thoſe which are derived from the Heavens by our plain Sights, the Matter will then be brought to a fair Trial. In the mean Time let every one enjoy his own Opinion, and proceed in his own Way.

Why Celeſtial Objects appear greater when nigh the Horizon than when higher elevated;

VII, 1. It is well known that the mean apparent Magnitude of the Moon is 30 min. 30 ſec. we will take it *Numero rotundo* to be 30 min. at a Full Moon in the midſt of Winter, and when ſhe's in the Meridian, and at her greateſt Northern Latitude, and conſequently the utmoſt that ſhe can be elevated

vated in our Horizon: 'Tis as well known alſo, that when ſhe is in this Poſture, being looked upon by the naked Eye, ſhe appears (that we may accommodate all to ſenſible Meaſures) to be *Magnitudinis Pedalis*, about a Foot broad. But the ſame Moon being looked upon juſt as ſhe riſes, ſhe appears to be three or four Foot broad, and yet if with an Inſtrument we take her Diameter, both in one Poſture and the other, we ſhall find that ſtill ſhe ſhall be but 30 min. That this Matter of Fact is true, beſides the Authority of many Authors, I can aſſert that I have accurately tried it myſelf, and I have ſo found it. One of the Ways I proceeded was thus: I took a very good Teleſcope of about 6 Foot long, in the inward Focus of whoſe Eye-Glaſs I applied a very fine Lattice made of the ſingle Hairs of a Man's Head; then looking with this at the Moon, when ſhe was juſt riſen, and looked extraordinary big, I obſerved what Number of the Squares of the Lattice were occupied by her Body; and then obſerving her again, when more elevated and free from all extravagant Greatneſs, I ſtill found the ſame Squares of the Lattice poſſeſſed by her. This Way is equivalent to that now more uſed, of taking her Diameter by Mr. *Townley*'s Micrometers; but I have alſo tried and found the ſame Thing by an accurate Sextant, taking the Diſtance of the Moon's oppoſite Limbs.

Examined by Mr. Will. Molineux.
N. 187. *p.* 314.
Apr. An. 1687.

The celebrated *Des Cartes* attributes this Appearance rather to a deceived Judgment, than to any natural Affection of the Organ or Medium of Senſe: For the Moon (ſays he) being nigh the Horizon, we have a better Opportunity and Advantage of making an Eſtimate of her, by comparing her with the various Objects that incur the Sight, in its Way towards her; ſo that tho' we imagine ſhe looks bigger, yet 'tis a meer Deceit; for we only think ſo, becauſe ſhe ſeems nigher the Tops of Trees, or Chimneys, or Houſes, or a Space of Ground, to which we can compare her, and eſtimate her thereby; but when we bring her to the Teſt of an Inſtrument, that cannot be deluded or impoſed upon by theſe Appearances, then we find our Eſtimate wrong, and our Senſes deceived. Theſe Thoughts, methinks, are much below the accuſtomed Accuracy of the noble *Des Cartes*; for certainly if it be ſo, I may at any Time increaſe the apparent Bigneſs of the Moon, tho' in the Meridian; for it would be only by getting behind a Cluſter of Chimneys, a Ridge of a Hill, or the Tops of Houſes, and comparing her to them in that Poſture, as well as in the Horizon; beſides, if the Moon be looked at juſt as ſhe is riſing from an Horizon determined by a ſmooth Sea, and which has no more Variety of Objects to compare her to, than the pure Air, yet ſhe will ſeem big, as if looked at over the rugged Top of an uneven Town, or rocky Country. Moreover, all Variety of adjoining Objects may be taken off, by looking through an empty Tube, and yet the deluded Imagination is not at all helped thereby.

The famous *Thomas Hobbes* gives this Solution. Let the Point G be the Center of the Earth, and F the Eye on the Surface of the Earth; on the ſame Center G let there be ſtruck the two Arches E H, determining the Atmoſphere, and A D to repreſent that blue Surface in which we imagine the Fixed Stars, and let F D be the Horizon: Divide the Arch A D into three

Fig. 110.

equal Parts by the Lines B F, C F; it is manifeſt that the Angle A F B is greater than the Angle B F C, and this again is greater than the Angle C F D. Wherefore (ſays he) to make the Angle C F D equal to the Angle C F B, the Arch C D muſt be greater than the Arch C B; and conſequently, that the Moon may in the Horizon appear under the ſame Angle, as when elevated, ſhe muſt cover a greater Arch, and therefore ſeem greater; that is, the Moon in the Meridian appearing under the Angle B F C, that ſhe may appear under an equal Angle in the Horizon, as ſuppoſe C F D, 'tis neceſſary that the Arch C D ſhould be greater than C B; and conſequently, tho' ſhe appear to ſubtend a greater Arch when in the Horizon than when elevated, yet ſhe appears under the ſame Angle; and all this without Refraction. The Geometry of this Figure is moſt certainly true and demonſtrable. At this I quarrel not; but it makes no more in our preſent Difficulty than if nothing had been ſaid: For he has made the Circle G F, repreſenting the Earth, very large in Proportion to the Circle A D; and then indeed taking the Point F in the Earth's Surface, and by Lines from thence dividing the Angle A F D into whatever equal Parts, the intercepted Arches A B, B C, C D, ſhall be unequal. But if he had conſidered, that the Earth is, as it were, a Point in reſpect of the Sphere of the Fixed Stars, nay the very annual Orbit of the Earth is almoſt imperceptible, he would have found that the Lines F B, F C, F D, muſt be all conceived as drawn from the Point G, and then equal Angles will intercept equal Arches, and equal Arches equal Angles: And ſo it happens (at leaſt beyond the Poſſibility of the Diſcovery of Senſe) to the Eye on the Surface of the Earth; ſo that his drawing his Lines ſo far from G as F is, and to another concentric Circle ſo nigh as A D, deceived him in this Point.

The famous *Gaſſendus* has written four large *Epiſtles* on this Subject, the Subſtance of all which is, That the Moon being nigh the Horizon, and looked at through a more foggy Air, caſts a weaker Light, and conſequently forces not the Eye ſo much as when brighter; and therefore the Pupil does more enlarge itſelf, thereby tranſmitting a larger Projection on the Retina. In this Opinion I find he is not alone; for this Diſquiſition being lately revived by a *French Abbé*, he therein follows the Sentiment of *Gaſſendus*, with this Addition, That this contracting and enlarging of the Pupil cauſeth a different Shape in the Eye; an open Pupil making the Cryſtalline flatter, and the Eye longer, and the narrower Pupil ſhortening the Eye, and making the Cryſtalline more convex: The firſt attends our looking at Objects which are remote, or which we think ſo; the latter accompanies the viewing Objects nigh at hand. Likewiſe an open Pupil and flat Cryſtalline attends Objects of a more ſedate Light, whilſt Objects of more forcible Rays require a greater Convexity and narrower Pupil. From theſe Poſitions the *Abbé* endeavoured to give an Account of our *Phænomenon*, as follows: When the Moon is nigh the Horizon, by Compariſon with interpoſed Objects, we are apt to imagine her much farther from us than when more elevated; and therefore (ſays he) we order our Eyes as for viewing an Object farther from us; that is, we ſomething enlarge the Pupil, and thereby make the Cryſtalline more flat:

moreover, the Duſkineſs of the Moon in that Poſture does not ſo much ſtrain the Sight; and conſequently the Pupil will be more large, and the Cryſtalline more flat. Hence a larger Image ſhall be projected on the Fund of the Eye, and therefore the Moon ſhall appear larger. And this Diſpoſition of the Eye that magnifies her, magnifies alſo the Diviſions of our forementioned Lattice, and conſequently ſhe by her Body ſhall poſſeſs no more of the Diviſions than when ſhe ſeems leſs. Theſe two forementioned Accidents, *viz.* The Moon's imaginary Diſtance and Duſkiſhneſs, gradually vaniſhing as ſhe riſes, a different Species is hereby introduced in the Eye, and conſequently ſhe ſeems gradually leſs and leſs, till again ſhe approaches nigh the Horizon. Theſe two Opinions of *Gaſſendus* and the *Abbé* being ſo near a-kin, I ſhall conſider them both together: And firſt I aſſert, That a wider or narrower Aperture of the Pupil increaſes not, neither diminiſhes the Projection on the Retina. I know, *Honoratus Faber*, in his *Synopſis Optica*, endeavours to prove the clear contrary to this my Aſſertion, and that after this manner: A B is an Object, E F the greater Aperture of the Pupil, admitting the Projection K I on the Retina, whereas the leſſer Aperture C D, admits only the Projection G H; but G H is leſs than K I, wherefore a leſſer Aperture diminiſhes the Projection. *Fig.* 111. I admire that any Man that undertook (as *Honoratus Faber*) to write of *Optics* more accurately than all that went before him, ſhould be guilty of ſo very groſs an Error; and I do more admire, that the celebrated *Gaſſendus*, and with him the noble *Hevelius*, ſhould be of the ſame Opinion: For tho' the aforeſaid Demonſtration hold moſt certainly true in direct Projections, as in a dark Room with a plain Hole; yet it will not hold in Projections made by Refraction, as it is in thoſe on the Retina in the Eye, by means of the Cryſtalline, and other Coats and Humours of the Eye. *Fig.* 112. For let A B be a remote Object, and E F the Cryſtalline at its large Aperture, projecting the Image I M on the Rtina. Let then C D be the leſſer Aperture of the Pupil before the Cryſtalline: I ſay, the Image I M ſhall be projected as large as before; for the Cone of Rays E A F conſiſts partly of the Cone of Rays C A D; therefore where the former E A F is projected, the latter C A D, as being a part of the former, ſhall be projected alſo. So that no more is effected by this narrow Aperture, but that the Sides of the radiating Cones are intercepted, and conſequently the Point I ſhall be affected with leſs Light, but it ſhall ſtill be in the ſame Place: What is ſaid of that Cone and that Point, may be ſaid of all other Cones and other Points of the Object. From hence appears, *Firſt*, The Invalidity of the Account given of the Moon's Appearance by *Gaſſendus* from this Reaſon. *Secondly*, The Reaſon appears why a Teleſcope's leſſer or greater Aperture, makes no Difference in the Angle it receives: For imagine E F to be an Object-Glaſs of a Teleſcope, and 'tis plain. *Thirdly*, 'Tis evident why a greater or leſs Aperture on a Teleſcope ſhould make the Objects appear lighter, or darker; for thereby more or leſs Rays are admitted to determine on the Projection of each Point. But all this by the by: And this is ſufficient for a Confutation of *Gaſſendus* and *Faber*. But our forementioned *Abbé* ſuperadds to a greater or leſſer Aperture of the Pupil, as a neceſſary Conſequent, a greater and leſſer Convexity of the Cryſtalline, as alſo a Lengthening and Shortening the Tube of the Eye. And this I muſt confeſs would do ſomething, if we find it true in our Caſe; and this let us try. *Firſt*, (ſays he) The Duſkiſhneſs of the Moon nigh the Horizon admits the Pupil to enlarge itſelf, the Cryſtalline to flatten, and the Eye to lengthen: But what if we change our Object, and inſtead of the Moon take the Diſtance between ſome of the Fixed Stars (as ſuppoſe thoſe of *Orion's Girdle*) we ſhall find the ſame *Phænomenon* in them, and yet I hope neither he nor *Gaſſendus* will aſſert, that they at one Time ſtrain the Eye more than at another, or that at any time their Fulgur ſtrains the Eye at all; if he do, let him take Stars of the leſſer Magnitudes, nay even thoſe that can but juſt be perceived, and then he will be convinced: Or let him conſider, whether this will hold in looking at the Sun through very dark Glaſſes, which render the Sight thereof as inoffenſive to the Eye as that of a green Field; but perhaps he will then ſay, that this other Reaſon holds, which is, *Secondly*, That the greater imaginary Diſtance, at which we think the Moon near the Horizon, than when more elevated, makes us contemplate her as if really ſhe was ſo, *viz.* with ample Pupils, *&c.* But this I have ſufficiently overthrown in my Remarks againſt *Des Cartes:* Therefore I paſs it over, only ſubjoining, that if there were any thing in this Surmiſe, methinks the horizontal Moon ſhould rather be fancied nigher to us than farther from us; for if we are for trying natural Thoughts, let us take Children to determine the Matter, who are apt to think, that could they go to the Edge of that Space that bounds their Sight, they ſhould be able (as they call it) to touch the Sky; and conſequently the Moon ſeems then rather nigher to us than farther from us.

After I had writ thus far, I accidentally caſt my Eye upon *Riccioli's* Treatiſe of *Refraction*, at the End of his Second Volume of *Almageſt*, *Lib.* 10. *Sect.* 6. *Cap.* 1. *Queſt.* 13. wherein he ſpeaks of our preſent Difficulty; but to my wonder I find him aſſert, That he and Father *Grimaldi* had often taken the horizontal Sun and Moon's Diameters by a Sextant, when to the naked Eye they appeared very large (*Grimaldus* directing his Sight to the left Edge, and *Ricciolus* to the right) and that even by the Inſtrument they always found the Diameters greater than when more elevated, the Sun often ſubtending an Angle of almoſt a Degree, and frequently 45 Minutes, the Moon alſo 38 or 40 Minutes. This is downright contrary to the Matter of Fact which I have before alledged, and directly repugnant to the Matter of Fact aſſerted by the forementioned *French Abbé:* Whether of us be in the right, I leave to accurate Experiment to determine, and ſubmit the Whole to the Deciſion of the illuſtrious *Royal Society*. Only give me leave to add one Word againſt *Riccioli*; for had his Experiments been accurately proſecuted, he ſhould have tried them when the horizontal Moon had looked ten times more large in Diameter than ordinary; and then, if it be true, that even by an Inſtrument ſhe will be found proportionally broader; then really ſhe ſhould ſubtend an Angle of 300 Min. or 5 Deg. for very often I have ſeen the Moon when ſhe appeared ten times broader than ordinary, which the ſmall Addition of 8 or 10 Min. to her uſual Diameter will never cauſe.

This Phænomenon conſider'd; by Dr. Wallis. *Ibid. p.* 323.

2. I diſcourſed of this Appearance near 40 Years ago with Mr. *Foſter*, then *Profeſſor of Aſtronomy* in *Greſham* College, who did then aſſure me (from his own Obſervation I ſuppoſe) that the apparent Magnitude taken by Inſtruments (however the Fancy may apprehend it) is not greater at the Horizon than when higher. Mr. *Caſwell* affirms the ſame Thing; and I do not doubt but the Thing is ſo: For though Refraction near the Horizon alters the Altitude of the Thing ſeen; yet it cannot alter the Azimuth at all. For ſince this equally reſpects all Points of the Horizon; let the Refraction be what it will, the whole Horizon can be but a Circle: So that there is no room for the Breadth of a Thing (as to the Angle at the Eye) to be made greater, whatever its Tallneſs may (the Refraction not equally affecting all Parts in the Circles of Altitude). Nor is there any Reaſon, why this ſhould rather thruſt the other, than the other thruſt this, out of Place. Whereas, in the Altitude, it is otherwiſe: For while what is near the Horizon is enlarged, that which is further off is thereby contracted: which, as to the Azimuth, or Horizontal Poſition, cannot be.

Suppoſing then that the Sun's apparent Horizontal Diameter, taken by Inſtrument, is the ſame near the Horizon, as in a higher Poſition; I take its imaginary Greatneſs, which is fancied near the Horizon, to be only a Deception of the Eye, or rather the Imagination from the Eye.

For ſure it is, that the Imagination doth not eſtimate the Greatneſs of the Object ſeen, only by the Angle which it makes at the Eye; but, by this compared with the ſuppoſed Diſtance. True it is, that, *cæteris paribus*, we judge that to be the greater Object which makes at the Eye the greater Angle; but not ſo, if apprehended at different Diſtances.

For if through a Caſement (or leſſer Aperture) we ſee a Houſe at 100 Yards Diſtance; this Houſe (though ſeen under a leſs Angle) doth not to us ſeem leſs than the Caſement through which we ſee it, (or this greater than that, becauſe it makes at the Eye the greater Angle) but the Imagination makes a comparative Eſtimate from the Angle and Diſtance jointly conſidered.

So that of two Things ſeen under the ſame or equal Angles, if to one of them there be ought which gives the Apprehenſion of a greater Diſtance, that to the Imagination will appear greater. Now ſure it is, that one great Advantage for eſtimating the Diſtance of a Thing ſeen, is from the Variety of intermediate Objects between the Eye and the Thing ſeen. For then the nagination muſt allow room for all theſe Things.

Now when the Sun or Moon is near the Horizon, there is a Proſpect of ills, and Valleys, and Plains, and Woods, and Rivers, and Variety of ields and Incloſures, between it and us; which preſent to our Imagination great Diſtance capable of receiving all theſe: Or if it ſo chance that (in ome Poſition) theſe Intermediates are not actually ſeen; yet having been acuſtomed to ſee them, the Memory ſuggeſts to us a View as large as is the viible Horizon.

But when the Sun or Moon is in a higher Poſition, we ſee nothing between us and them (unleſs perhaps ſome Clouds) and therefore nothing to preſent to our Imagination ſo great a Diſtance as the other is.

And therefore, though both be ſeen under the ſame Angle, they do not appear (to the Imagination) of the ſame Bigneſs; becauſe not both fancy'd at the ſame Diſtances: But that near the Horizon is judged bigger (becauſe ſuppoſed farther off) than the ſame when at a greater Altitude.

'Tis true, that as to ſmall and middling Diſtances (beſides this Eſtimate from Intermediates) the Eye hath a Means within itſelf to make ſome Eſtimate of the Diſtance. As, when we already know the Bigneſs of a thing ſeen, to which we have been accuſtomed; as, a Man, a Tree, a Houſe, or the like: If ſuch thing appear to us under a ſmall Angle, and indiſtinct, and faintly coloured, the Imagination doth allow ſuch Diſtance as to make ſuch a thing ſo to appear. And if this, through a Perſpective-Glaſs, be repreſented to us under a bigger Angle, and more diſtinct, it is accordingly apprehended as ſo much nearer. But the Caſe is otherwiſe, when we do not, by the known Bigneſs, judge the Diſtance; but, by the ſuppoſed Diſtance, judge of the Bigneſs, as in the Caſe before us. And accordingly, different Perſons, according to different fancied Diſtances, judge very differently.

Again: In our two Eyes (when the Object is ſeen by both) there is yet another Means of eſtimating how far off it is. (And it is this by which we judge of Diſtances.) Namely, there are from the ſame Object, two different viſual Cones, terminated at the two Eyes; whoſe two Axes contain, at the Object, different Angles, according to different Diſtances; an acuter Angle at a greater Diſtance, and more obtuſe when nearer.

Now, that ſuch Object may be ſeen by both Eyes clearly, it is requiſite that the Eyes be put in ſuch a Poſition, as that the Sight of each Eye receive the reſpective Axis at Right Angles; which requires a different Poſition of the two Eyes, according to the different Diſtance of the Object: As will manifeſtly appear, if we look with Attention on a Finger (or other ſmall Object) at 2 or 3 Inches Diſtance from the Eye, and then upon another like Object at 3 or 4 Yards beyond it (and this alternately ſeveral times.) For it will be manifeſt, that while we look intently on the one, we do not ſee the other (or but confuſedly) though both be juſt before us. And, as we change our View, from the one to the other, we manifeſtly feel a Motion of the Eyes (by their Muſcles) from one Poſture to another.

And according to the different Poſture in the Eyes, requiſite to a clear Viſion by both, we eſtimate the Diſtance of the Object from us.

And hence it is, that they who have loſt the Sight of one Eye, are at a great Diſadvantage, as to eſtimating Diſtances, from what they could do while they had the Uſe of both.

But now when the Diſtance grows ſo great, as that the Poſition of theſe viſual Axes become parallel, or ſo near to parallel as not to be diſtinguiſhable from it, this Advantage is loſt, and we can thenceforth only conclude, that it is far off; but not how far. Hence it is, that our View can make no Diſtinction of the Moon's Diſtance from that of the other Planets, or even of the fixed Stars: But they ſeem to us as equally remote from us; though we otherwiſe know their Diſtances from us to be vaſtly different; becauſe the Parallax (as I may ſo call it) from the different Poſition of the two Eyes, is quite loſt, and undiſcernable in Diſtances much leſs than the leaſt of theſe.

So that, though as to ſmall Diſtances we may make ſome Eſtimate from the known Magnitude of the Object; and as to middling Diſtances, from the Parallax (as I may call it) ariſing from the Interval of the two Eyes: Yet even this latter will hardly reach beyond, if ſo far as, the viſible Horizon, and all beyond it is loſt. And therefore, there being nothing left to aſſiſt the Fancy in eſtimating ſo great a Diſtance, but only the intermediate Objects; where theſe Intermediates appear to the Eye (as when the Sun or Moon are near the Horizon) the Diſtance is fancied greater, than where they appear not (as when farther from it) and conſequently (though both under the ſame or equal Angles) that near the Horizon is fancied the greater: And this I judged to be the true Reaſon of that Appearance.

An Experiment of the Refraction of the Air; by Mr. Lowthorpe. N. 257. *p.* 329. Mar. An. 1699.

VIII. We took a Cylinder of caſt Braſs, ABCD, and cut one End of it CD, perpendicular to the Axis *a c x*; the other End AB, inclined to it at an Angle of about 27 deg. 30 min. and therefore the Perpendicular to this inclining Plane *p c*, and the Axis of the Cylinder *a c x*, comprehended an Angle *p c a*, of about 62 deg. 30 min. Theſe Ends were ground very true upon a Glaſs-Grinder's Braſs Tool, and each of them was compaſs'd about with a narrow Feril of thin Braſs *b b b b*. Into the upper Side of the Cylinder, at E, was ſoldered the Braſs-Pipe EF, and into the under Side, at G, the other Braſs-Pipe GH; the former of theſe Pipes being about 3 Inches long, and the latter 6 Inches. Upon the Plate *d d d*, were fixed two other Plates, LL, perpendicular to it, and parallel to each other. Each of theſe two Plates had an Arch of a Circle (whoſe Diameter was equal to that of the Cylinder) cut out of its proper Edge; ſo that when the Pipe GH was let thro' a Hole near the Middle of the Plate *d d d*, the Cylinder fell into the Arches; and being faſtened there with Solder, the Axis *a c x* lay Parallel to the Plate *d d d*, and about an Inch and an half above it. The perpendicular End of the Cylinder, DC, was cloſed with an Object Glaſs of a $7\frac{1}{2}$ Foot Teleſcope *o o*, and the inclining End AB, with a well poliſhed flat Glaſs, *f f*; which was carefully choſen to tranſmit the Object diſtinct enough, notwithſtanding its Obliquity to the viſual Rays. The Ferils were filled with Cement round about the Edges of the Glaſſes, which lay flat, and every where touched the ſmooth Ends of the Cylinder, that they might firmly ſupport the Weight and Preſſure of the excluded Air.

Fig. 113.

Fig. 114.

Inſtead of a Ciſtern (as in the *Torricellian* Experiment) we made uſe of the inverted Syphon of Braſs MNO, ſolder'd to the Plate *g g g*. One of the Sides MN, ſtood perpendicular to the Plate *g g g*, and the other Side NO, inclined to it, and was ſupported near the upper End O, with a little Piece of Braſs, *k k*.

We then placed the Cylinder upon a Table, which was well faſtened to *Fig.* 115. a firm Floor: The Pipe G H, was let through a Hole in the Top of the Table; and the Plate *d d d*, was nailed down to it: The Tube of the Teleſcope *s s s*, with the Eye-Glaſs in it, was applied to the Object-Glaſs, and a Hair fixed at *x*, the common Focus of both Glaſſes, in the Axis of the Cylinder continued to it. Upon the Floor (under the Cylinder) we nailed the Plate *g g g*, with the inverted Cyphon upon it, and joined M to H, by the Inſertion of the Glaſs Tube T. The Joints were very carefully cloſed with Cement, and then covered over with Pieces of a Bladder wrapped hard with ſtrong Thread. There was alſo a Bladder tied below each Joint at *m*, and when it was filled with Water it was tied about at *n*; ſo that no Air could come to the Cement, to inſinuate itſelf through its Pores or Fiſſures, if any happened to be left uncloſed.

It will not (I hope) be thought more than neceſſary, that in this Account of the *Apparatus*, I have mentioned ſo many minute Circumſtances; for we found it difficult enough to exclude the Air, and almoſt impoſſible to diſcover the very little Holes through which ſo ſubtle a Fluid would freely enter and poſſeſs the Spaces deſerted by the *ſubſiding Mercury*. But, with all this Precaution, the Experiment ſucceeded at laſt, as I wiſhed; after this Manner:

We placed the Object *a* (which was a black Thread faſtened in a little Frame over a Piece of white Paper) in the Axis of the Cylinder *x c a*: We filled the Pipes and Cylinder with *Mercury*; and having ſtopped the upper End of the Pipe at F, with the little Iron Stopple K, and cloſed it, at the upper Part of the Tube and other Joints, we let the *Mercury* run out gently at O (into the Bladder *u*) till it remained ſuſpended at the uſual Height (as in the *Barometer*) leaving the upper Part of the Tube, and the Cavity of the Cylinder between the Glaſſes *o o*, and *f f*, void of Air. We then ſaw the Object, which before appear'd in the Axis at *x*, raiſed conſiderably above it; and we reduced it to appear again at *x*, by removing it from *a* to *α*. The Axis therefore of the viſual Ray (which was alſo the Axis of the Cylinder) *x c a*, falling perpendicularly on the void Space, paſſed through it without any Refraction: but emerging obliquely into the Air, it was refracted towards the Perpendicular *p c*, and received a new Direction to *α*. And therefore the Diſtance *a α*, ſubtended the Angle of Refraction *a c α*; all which we meaſured, and found as follows; *viz.*

	Inches.	*Dec. Parts.*
The Height of the Object above the Axis, or the unrefracted viſual Ray *a α* - - - - - -	000	425
The Diſtance of the Object from the refracting Plain *a c*, about 51 Feet, or — — —	612	000

	Deg.	*Min.*	*Sec.*
Therefore the Angle of Refraction *a c α*, was —	00	02	23
The Angle of Emerſion *p c a*, (by the Conſtruction of the Cylinder) was — — — —	62	30	00

There-

	Deg.	Min.	Sec.
Therefore the Angle of Incidence $pca = pca + aca$, was — — — — — —	62	27	37

And therefore *univerſally*, (according to the known Laws of Refraction)

The Sines of the Angles of Incidence being — — — —	100000
The Sines of the Angles of Emerſion are — — — —	100036
And the refractive Power of the denſe Air — — — — —	36

By the *refractive Power* of a pellucid Body, I mean that Property in it whereby the oblique Rays of Light are diverted from their direct Courſe, and which is meaſured by the proportional Differences (always obſerved) between the Sines of the Angles of Incidence and Emerſion.

This Property is not always proportional to the Denſity (at leaſt not to the Gravity) of the refracting Medium: For the refractive Power of Glaſs to that of Water is as 55 to 34, whereas its Gravity is as 87 to 34; that is, the *Squares* of their refractive Powers are (very near) as their reſpective Gravities. And there are ſome Fluids, which, tho' lighter than Water, yet have a greater Power of Refraction: Thus the refractive Power of Spirit of Wine (according to Dr. *Hook*'s Experiment, *Microgr. Obſ.* lviii. *p.* 220.) is to that of Water, as 36 to 23; and its Gravity *reciprocally* as 23 to 36, or $36\frac{1}{2}$. But the refractive Powers of Air and Water ſeem to obſerve the *ſimple* Proportion of their Gravities *directly*, as I have compared them in the following Table. The Numbers there expreſſing the Refraction of Water are taken from the Mean of Nine [a] Experiments, made at ſo many ſeveral Angles of Incidence, *Jan.* 25, $164\frac{0}{1}$, by Mr. *Gaſcoigne*, (the ingenious firſt Inventor of the Micrometer, and the Ways of meaſuring Angles by Teleſcopes) and thoſe of Air are produced by the preceding Experiment.

	Water.	Air.
The (aſſumed) Sines of the Angles of Incidence on the Void from — — — — — —	100000	100000
The Sines of the correſpondent Angles of Emerſion out of — — — — — — —	134400	100036
The refractive Power of — — — — —	34400	36
The ſpecifick Gravity (if as 900 to 1 at the Time of the Experiment) of — — — —	34400	38
Or (if as 850 to 1) of — — — — — —	34400	40

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[a] I am indebted for theſe Experiments to the Reverend and very accurate Aſtronomer Mr. *Flamſtead*; who copied them, together with many other Obſervations and ſeveral Paſſages relating to them, from Mr. *Gaſcoigne*'s *Letter* to Mr. *Crabtree*: They were happily preſerved, in the Time of our Civil War, by the late Sir *Jonas Moor*, and Mr. *Chr. Townley*; and they are now in the Hands of Mr. *Rich. Townley*, of *Townley* in *Lancaſhire*, by whom they were imparted ſome Time ago to Mr. *Flamſtead*.

From hence it ſeems very probable, that their reſpective Denſities and refractive Powers are in a juſt *ſimple* Proportion. And if this ſhould be confirmed by ſucceeding Experiments, made at different Angles of Incidence, and with Cylinders continuing exhauſted through ſeveral Changes of the Air, it would be more than probable that the refractive Powers of the Atmoſphere are every where, and at all Heights above the Earth, proportional to its Denſities and Expanſions: And then it would be no difficult Matter to trace the Light through it, ſo as to terminate the Shadow of the Earth, and (together with proper Expedients for meaſuring the Quantity of Light illuminating an opake Body) to examine at what Diſtances the Moon muſt be from the Earth to ſuffer Eclipſes of the obſerved Duration.

To find the Parallax of the Fixed Stars; by Dr. Wallis, *to Mr.* William Molyneux. July, An. 1693. N. 202. *p.* 844.

IX. Give me leave to ſuggeſt a Speculation, which hath been in my Thoughts theſe forty Years or more; but I have not had the Opportunity of reducing it to Practice: It is concerning the Parallax of the fixed Stars, as to the Earth's annual Orbit.

Galileo complains of it a great while ſince (in his *Syſtema Coſmicum*) as a Thing not attempted to be obſerved with ſuch Diligence as he could wiſh; and I doubt we have the ſame Cauſe of complaining ſtill. I know that Dr. *Hook* and Mr. *Flamſtead* have attempted ſomewhat that way, but have deſiſted before they came to any Thing of Certainty. What hath been done to that Purpoſe Abroad I know not.

Galileo hath ſuggeſted divers Things conſiderable in order to it; as, the Times of Obſervation, the Stars to be obſerved, and the Manner of obſerving them; which yet I doubt is not practicable. That which occurred to my Thoughts upon theſe Conſiderations, was to this purpoſe: That ſome circumpolar Stars (nearer to the Pole of the Equator than is our *Zenith*, and not far from the Pole of the Zodiack) ſhould be made choice of for this purpoſe. And in caſe the meridional Altitude be diſcernibly different at different Times, ſo will alſo be their utmoſt Eaſt and Weſt Azimuth, which may be better obſerved than their Riſing or Setting: And this will not be obnoxious to the Refraction, as is the meridional Altitude (for though the Refraction do affect the Altitude, yet not the Azimuth at all;) and we may here have choice of Stars for the purpoſe; which, in Obſervations from the Bottom of a Well, we cannot have; being there confined to thoſe only which paſs very near our Zenith, tho' very ſmall Stars.

I would then take for granted, as a Thing at leaſt very probable, that the fixed Stars are not all (as was wont to be ſuppoſed) at the ſame Diſtance from us, but the Diſtance of ſome vaſtly greater than of others; and conſequently, though as to the more remote, the Parallax may be undiſcernible, it may, perhaps, be diſcernible in thoſe that are nearer to us.

And thoſe we may reaſonably gueſs (though we are not ſure of it) to be neareſt to us, which to us do appear biggeſt and brighteſt, as are thoſe of the *Firſt* and *Second* Magnitude; and there are at leaſt of the *Second* Magnitude pretty many not far from the Pole of the Ecliptic (as that in particular in the *Shoulder* of the *Leſſer Bear:*) And, in caſe we fail in one, we may try again and again on ſome other; which may chance to be nearer to us than what we try firſt. And Stars of this Bigneſs may be diſcerned by a moderate Teleſcope, even in the Day-time; eſpecially when we know juſt where to look for them.

The Manner of Obſervation, I conceive, may be thus. Having firſt pitched upon the Star we mean to obſerve, and having then conſidered (which is not hard to do) where ſuch Star is to be ſeen in its greateſt Eaſt or Weſt Azimuth; it may be then convenient to fix (very firm and ſteddily on ſome Tower, Steeple, or other high Edifice, in a convenient Situation) a good Teleſcopic Object-Glaſs in ſuch Poſition as may be proper for viewing that Star. And at a due Diſtance from it, near the Ground, build on purpoſe (if already there be not any) ſome little Stone Wall, or like Place, on which to fix the Eye-Glaſs, ſo as to anſwer that Object-Glaſs: And having ſo adjuſted it, as through both to ſee that Star in its deſired Station (which may beſt be done while the Star is to be ſeen by Night in ſuch Situation, near the Time of one of the Solſtices) let it be there fixed ſo firmly, as not to be diſturbed (and the Place ſo ſecured, as that none come to diſorder it) and care be taken ſo to defend both the Glaſſes, as not to be endangered by Wind and Weather. In which Contrivance, I am beholden to Mr. *John Caſwel*, M. A. of *Hart-Hall* in *Oxford*, for his Advice and Aſſiſtance, with whom I have many Years ſince communicated the whole Matter.

This Glaſs being once fixed (and a Micrometer fitted to it, ſo as to have its Threads perpendicular to the Horizon, to avoid any Inconvenience which might ariſe from Diverſity of Refraction, if any be) the Star may then be viewed from Time to Time (for the following Year, or longer) to ſee if any Change of Azimuth can be obſerved.

This I thought fit to recommend to your Conſideration, who do ſo well underſtand Teleſcopes, and the Managery of them: But when I ſuggeſt (as a convenient Star for this purpoſe) the *Shoulder* of the *Leſſer Bear* (as being the neareſt to the Pole of the Zodiack of any Star that is of the Firſt or Second Magnitude) I do not confine you to that Star; but (without retracting that) ſuggeſt another; namely, the *Middle Star*, in the *Tail* of the *Great Bear*, which (though ſomewhat further from the Pole of the Zodiack) is a brighter Star than the other, and may be nearer to us.

But I do it principally upon this Conſideration; namely, That there is adhering to it a very ſmall Star (which the *Arabs* call *Alcor*, of which they have a proverbial Saying, when they would deſcribe a ſharp-ſighted Man, That he can diſcern the *Rider* on the *Middle Horſe* of the *Wayn*: And of one who pretends to ſee ſmall Things, but overlooks much greater, *Vidit Alcor at non Lunam Plenam*) which *Hevelius* in his Obſervations finds to be diſtant from it about nine Minutes and five or ten Seconds: So that beſides the Advantage of diſcovering the Parallax of the greater Star, if diſcernible; the Difference of the Parallax of that and of the leſſer Star (being both within the Reach of a Micrometer) may do our Work as well. For if that of the greater Star be diſcernible, but that of the leſſer be either not diſcernible or leſs diſcernible, their different Diſtances from each other at different Times of the Year may perhaps (without farther *Apparatus*) be diſcerned by a good Teleſcope of a competent Length, furniſhed with a Micrometer, if carefully preſerved from being diſordered in the Intervals of the Obſervations; and diſcover at once, both that there is a Parallax, and that the Fixed Stars are at different Diſtances from us; wherein, that I may not be miſtaken, my Meaning is not that the Inſtrument or Micrometer ſhould be removed for the obſerving of the Leſſer Star, but that (when the Azimuth of the Greater Star is taken) by a Micrometer (conſiſting of divers fine Threads parallel and tranſverſe) may (at the ſame Time) be obſerved the Diſtance of the two Stars, each from other, in that Poſition (both being at once within the Reach of the Micrometer) Which Diſtance (the Inſtrument remaining unmoved) if it be found (at different Times of the Year) not to be the ſame, this will prove that there is a different Parallax of theſe two Stars.

This latter Part of the Obſervation (of their different Diſtances at different Times) I ſuggeſt as more eaſily practicable, though not ſo nice as the former: For it may be done, I think, without any further *Apparatus* there than a good Teleſcope of ordinary Form, furniſhed with a Micrometer, (this being carefully kept unvaried during the Interval of theſe Obſervations.) And if this Part only of the Obſervation (without the other) be purſued, it matters not though the two Obſervations (near the two Solſtices) be, one at the Eaſtern, the other at the Weſtern, Azimuth (whereby both may be taken in the Night-time) for the Diſtance muſt (at both Azimuths) be the ſame. If, after obſerving the Azimuth of the greater Star, it be neceſſary to move the Micrometer for meaſuring its Diſtance from *Alcor*, that may be done another Night, (and it is not neceſſary to be done at one Obſervation) for that Diſtance cannot be diſcernibly varied in a Night or two.

Concerning the Diſtance of the Fixed Stars; by Mr. Francis Roberts. *N.* 209. *p.* 101.

X. Since the *Pythagorean Syſtem* of the World has been revived by *Copernicus*, (and now by all Mathematicians accepted for the true one) there ſeemed Ground to imagine, that the Diameter of the Earth's annual Courſe (which, according to our beſt Aſtronomers, is at leaſt 40000 Times bigger than the Semidiameter of the Earth) might give a ſenſible Parallax to the Fixed Stars, and thereby determine their Diſtance. But there are ſome Conſiderations which make us ſuſpect that even this Baſis is not large enough for that Purpoſe.

M. *Huygens* (who is very exact in his Aſtronomical Obſervations) tells us, He could never diſcover any viſible Magnitude in the Fixed Stars, though he uſed Glaſſes which magnified the apparent Diameter above 100 Times.

Now, ſince in all likelihood the Fixed Stars are Suns, (perhaps of a different Magnitude) we may, as a reaſonable Medium, preſume they are generally about the Bigneſs of our Sun.

Let us then (for Example) ſuppoſe the *Dog-Star* to be ſo. The Diſtance from us to the Sun being about 100 Times the Sun's Diameter, it is evident, that the Angle under which the *Dog-Star* is ſeen in Mr. *Huygens*'s Teleſcope, muſt be near the ſame with the Angle of its Parallax to the Sun's Diſtance, or Semidiameter of the Earth's annual Courſe; ſo that the Parallax to the whole Diameter can be but double ſuch a Quantity as even to Mr. *Huygens*'s nice Obſervation is altogether inſenſible.

The Diſtance therefore of the Fixed Stars ſeems hardly within the reach of any of our Methods to determine: But from what has been laid down, we may draw ſome Concluſions that will much illuſtrate the prodigious Vaſtneſs of it.

1. That the Diameter of the Earth's annual Orb (which contains at leaſt 160 Millions of Miles) is but as a Point in compariſon of it; at leaſt it muſt be above 6000 Times the Diſtance of the Sun: For if a Star ſhould appear through the aforeſaid Teleſcope half a Minute broad, (which is a pretty ſenſible Magnitude) the true apparent Diameter would not exceed 18''', which is leſs than the 6000th Part of the apparent Diameter of the Sun; and conſequently the Sun's Diſtance not the 6000th Part of the Diſtance of the Star.

2. That could we advance towards the Stars 99 Parts of the whole Diſtance, and have only 100th Part remaining, the Stars would appear little bigger to us than they do here: For they would ſhew no otherwiſe than they do through a Teleſcope which magnifies an hundred-fold.

3. That at leaſt 9 Parts in 10 of the Space between us and the Fixed Stars can receive no greater Light from the Sun, or any of the Stars, than what we have from the Stars in a clear Night.

4. That Light takes up more Time travelling from the Stars to us, than we in making a *Weſt-India* Voyage (which is ordinarily performed in ſix Weeks:) That a Sound would not arrive to us from thence in 50000 Years, nor a Cannon-Bullet in a much longer Time. This is eaſily computed, by allowing (according to Mr. *Newton*) 10 Minutes for the Journey of Light from the Sun hither, and that a Sound moves above 1300 Feet in a Second.

XI. Among your *Arabick* Books in the Library of *Merton* College (of which you have above forty, which abound with the Doctrine and Obſervations of the Heavens) in the *Ilchanic* Tables, by the Care of that famous *Perſian Choaga Naſirodinus Tuſius*, I have found it repreſented in a ſhort Page, what were the Latitudes and what the Longitudes of ſome of the principal Fixed Stars, according to the Obſervations of divers Aſtronomers. This little Canon I have enlarged, as you ſee, partly from your own Stock, and partly from others. I would not pretend to have done any great Matter in

The Places of the chiefeſt Fixed Stars, according to the beſt ancient Obſervers; by Dr. Edward Bernard, *to Dr.* Robert Huntington. N. 158. *p.* 567.

this, but that the preſent Age may have ſome Notion of the Aſtronomy of the Eaſt, where it is plain this Science had its firſt Original. Certainly there are many Things that recommend to us the Aſtronomy of the Eaſtern People: The peculiar Felicity and Serenity of the Regions in which they obſerved; the Largeneſs and Accuracy of their Machines, being ſo great that we can hardly conceive how they applied them to the Heavens; beſides the great Number of Obſervers and Writers, ten Times more than the *Greeks* or *Latins* can boaſt of; to which you may add, that their Princes were ten Times more powerful and munificent, who ſupplied ingenious Men with Wealth, and a neceſſary Apparatus for Celeſtial Obſervations. But what the *Arabian* Aſtronomers have juſtly reprehended in *Cl. Ptolomy*, the great Conſtructor of the Celeſtial Art; how diligently they diſtinguiſhed and meaſured the ſmalleſt Parts of Time by Drops of Water, by vaſt Sun-Dials, nay (which you will wonder at) by the Vibrations of a pendulous Thread; how ſkilfully and accurately they were converſant in that great Attempt of Human Ingenuity, concerning the Extent and Diſtances of the two great Luminaries and of our Earth; are too much for one Epiſtle to declare.

A Table of the principal Fixed Stars, according to the Obſervations of the Antients.

			Magnit.	By *Ptolomy*, in his *Magna Syntaxis*. A. C. 137.				
				Long.			Lat.	
				S.	°	′	°	′
1	34	The laſt Star of *Fluvius*.	1	0	10	30 al. 50	53	30 *s.*
2	12	The bright Star of *Caſſiopeia*'s Chair.	3	0	7	50	51	40 *n.*
3	12	*Perſeus*'s Head of *Meduſa*. *Algol.*	2	9	29	40 al. 20	23	0 *n.*
4	14	The Southern Eye of the *Bull*.	1	1	12	40 al. 20	5	10 *s.*
5	35	The Left Foot of *Orion*. *Rigel.*	1	1	20	50	31	30 *s.*
6	3	The little Kid of *Auriga*.	1	1	25	0	22	30 *n.*
7	2	*Orion's* Right Shoulder.	1	2	2	0	17	0 *s.*
8	1	The *Dog-Star*. *Alhabor.*	1	2	17	40 al. 20	39	10 *s.*
9	2	The *Little Dog*. *Algomeiſa.*	1	2	29	30 al. 10	16	10 *s.*
10	8	The *Lyon's* Heart. *Regulus.*	1	4	2	30 al. 10	0	10 *n.*
11	14	*Spica Virginis.*	1	5	26	40	2	0 *s.*
12	23	*Arcturus.*	1	5	27	0	31	30 *n.*
13	8	*Antares*. The *Scorpion's* Heart.	3	7	12	40 al. 20	4	0 *s.*
14	1	The Head of *Serpentarius*.	2	7	24	50	36	0 *n.*
15	1	The bright Star of the *Harp*.	1	8	17	20	62	0 *n.*
16	3	The bright Star of the *Eagle*.	2	9	3	50	29	10 *n.*
17	5	The bright Star in the Tail of the *Swan*.	2	10	9	10	60	0 *n.*
18	3	The bright Star in the Right Leg of *Pegaſus*. *Seat.*	2	11	2	10	35	0 *n.*
19	2	The laſt Star of *Pegaſus*'s Wing. *Algenib.*	2	11	12	10	12	30 *n.*
20	7	The bright Side of *Perſeus*.	2	1	4	50	30	0 *n.*
21	27	The *Lyon's* Tail.	1	4	24	30	11	50 *n.*
22	1	The Mouth of the *South Fiſh*.	1	10	7	0	23	0 *s.*
23	44	*Canopus.*	1	2	17	10	75	0 *s.*

A Table of the principal Fixed Stars, according to the Obſervations of the Antients.

	Ali Abolcaſimus. A. C. 938.					*Abdorahmanus Sophius.* A. C. 964.[a]			*Ebnolalamus.* A. C. 980.				
	Long.			Latit.		Long.			Long.			Latit.	
	S.	°	′	°	′	S.	°	′	S.	°	′	°	′
1	0	11	50	53	30	0	12	53	0	16	42	53	30
2	0	20	20	51	45	0	20	32	0	24	9	51	45
3	1	12	17	22	45	1	12	22	1	15	1	22	45
4	1	24	35	5	15	1	25	22	1	28	40	5	15
5	2	2	1	31	4	2	2	32	2	5	55	31	20
6	2	6	49	22	50	2	7	42	2	10	50	22	50
7	2	13	16	16	45	2	14	42	2	17	21	16	45
8	2	29	30	39	20	3	0	22	3	3	40	39	20
9	3	10	40	16	0	3	11	12	3	15	45	16	0
10	4	14	40	0	15	4	15	12	4	18	45	0	15
11	6	8	23	2	6	6	9	22	6	12	33	2	6
12	6	8	50	31	12	6	9	42	6	12	55	31	12
13	7	24	35	4	24	7	25	22	7	28	40	4	24
14	8	6	53	36	0	8	7	32	8	11	2	36	0
15	9	0	40	61	45	9	0	2	9	4	45	61	45
16	9	15	53	29	12	9	16	32	9	20	58	29	14
17	10	20	4	59	36	10	21	52	10	24	9	59	36
18	11	14	0	31	10	11	14	52	11	18	54	31	10
19						11	24	52	11	24	52	12	30
20	1	16	15	30	8	1	17	32					
21	5	6	40	11	50	5	7	12					
22						10	18	12					
23						2	29	52					

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[a] He agrees with *Ptolemy* as to Latitude.

A Table

A Table of the principal Fixed Stars, according to the Obſervations of the Antients.

	From the Hacimic Tables of Johanides Ægyptius. A. C. 996.					Choaga Naſirodinus Tuſius, *in the Ilchanic Tables.* A. C. 1233.					*From the Sultanic of* Ologbec. A. C. 1437.				
	Long.			Latit.		Long.			Latit.		Long.			Latit.	
	S.	°	′	°	′	S.	°	′	°	′	S.	°	′	°	′
1	0	16	20	53	28	0	24	55	51	45	0	15	40	53	45
2	0	24	34	51	50	0	24	35	51	40	0	28	1	50	48
3	1	15	24	22	39	1	16	25	23	0	1	18	54	22	0
4	1	29	7	5	15	1	29	22	5	13	2	2	31	5	15
5	2	5	30	31	35	2	6	35	31	30	2	9	25	31	18
6	2	11	32	22	3	2	11	10	22	40	2	14	43	22	42
7	2	17	46	16	50	2	18	0	16	50	2	21	13	16	43
8	3	4	2	39	30	3	3	50	39	10	3	6	19	39	30
9	3	15	52	16	2	3	15	45	16	5	3	18	22	16	0
10	4	19	10	0	10	4	19	14	0	17	4	22	13	0	9
11	6	12	58	2	10	6	13	25	1	52	6	16	10	2	9
12	6	13	49	31	33	6	13	0	31	25	6	16	31	31	18
13	7	29	9	4	25	7	29	0	4	10	8	2	16	4	30
14	8	11	34	35	59	8	11	15	35	55	8	15	13	35	51
15	9	5	0	61	55	9	4	40	61	50	9	8	19	62	0
16	9	20	34	29	10	9	20	40	29	15	9	24	10	29	15
17	10	23	31	59	38	10	24	30	59	50	10	28	46	59	42
18	11	18	44	31	12	11	18	44	31	12	11	21	37	30	51
19						11	18	55	31	1	0	1	22	12	34
20						1	21	35	30	0	1	25	7	29	21
21						5	11	55	11	50	5	13	49	12	0
22						10	23	45	23	0	10	20	40	21	30
23						3	3	55	7	0			*		

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* The Grandſon of *Timurus* never ſaw the bright *Canopus*, nor any other Aſtronomer of this Catalogue, except the *Alexandrines*.

A Table

A Table of the principal Fixed Stars, according to the Obſervations of the Antients.

	Abdolgalilus Segazius *in his* Genethliacs. A. C. 1261.			*From the* Perſian *Tables of* Chryſococca A. C. 1115.			*From the* Perſian *Tables*, *Pemb.* A C. 1346. Melixa 338.			*From the* Arnoldine Code *in* Gaſſendus, *for* A. C. 1364.		
	Long.			Long.			Long.			Long.		
	S.	°	′	S.	°	′	S.	°	′	S.	°	′
1	0	6	45	0	15	10				0	18	20
2	0	25	2	0	22	50						
3	1	16	52	1	14	40		18	10	1	17	45
4	1	29	45	1	27	0	2	1	30	2	1	0
5	2	6	52	2	4	50		8	20	2	8	0
6	2	11	43	2	10	50		13	3	2	13	10
7	2	19	12	2	17	0		20	30	2	18	10
8	3	4	28	3	2	40				3	6	0
9	3	16	11	3	14	10		18	0	3	17	20
10	4	19	42	4	17	30		9	31	4	20	40
11	6	13	26	6	11	40		15	10	6	15	10
12	6	13	47	6	12	0		15	30	6	14	10
13	7	29	45	7	27	40	8	1	10	8	27	40
14	8	11	49									
15	9	5	38	9	2	20		5	0	9	5	30
16	9	20	56	9	18	50		22	20	9	22	0
17	10	26	15	10	24	10		10	20	9	27	18
18	11	18	58	11	17	10		20	40	11	20	20
19				11	27	10						
20	1	22	2	1	19	50						
21	5	11	38				5	13	2	5	12	40
22	10	23	53	10	22	00						
23	3	4	3	2	2	10						

A Table

A Table of the principal fixed Stars according to ſelect Obſervations.

	From the Books of Savil *and* Bodley, *about* A. C. 750.			*From* Mohammed Tizinius. A. C. 1533.				Sahebodinus Alepenſis, *in the Ilchanics, for* A. C. 1436.			
	Long.			Aſc. Rect.		Declin.		Aſc. Rect.		Declin.	
	S.	°	′	°	′	°	′	°	′	°	′
1	0	12	20	130	17	41	0	132	10	41	30
2				86	55	56	41	84	2	55	45
3	1	2	50	130	14	39	43	127	44	39	4
4	1	19	50	153	0	15	43	150	40	15	20
5				163	43	9	13	162	20	9	20
6	2	2	10	161	39	45	0	158	30	44	43
7	2	9	10	173	15	6	28	171	55	6	15
8	2	24	50	187	0	15	50	185	45	15	43
9	3	6	40	199	0	6	5	197	25	6	7
10	4	9	39	136	0	14	8	234	0	15	10
11	6	4	50	185	36	8	29	283	40	7	30
12	6	4	10	299	0	22	14	297	39	33	8
13	7	19	20	330	57	22	42	331	0	24	12
14				348	57	13	9	347	5	13	30
15	8	24		38	37	5	29	38	32	4	20
16	9	15	0	21	5	7	24	20	5	7	3
17				36	40	43	50	36	20	43	15
18				71	0	25	16	68	50	24	30
19				88	10	18	23	85	54	11	30
20				133	46	47	52	130	20	47	11
21	5	1	45	161	20	17	55	259	30	17	56
22				68	45	33	51	67	25	35	2
23				183	40	51	35	183	30	5	25

A Table of the principal fixed Stars according to select Observations.

	Olaodinus Saterides *in the* Damascene *Tables.* A. C. 1480.							*From the* Rodolphine *Tables*, A. C. 1600. *at the Considerations of* Tycho Brahe.				
	Long.			Ascens.		Declin.		Long.			Latit.	
	S.	°	′	°	′	°	′	S.	°	′	°	′
1	0	20	34	129	30	17	0	0	21	10	53	30
2	0	18	16	83	50	55	50	0	29	35½	51	14½
3	1	20	4	127	55	38	50	1	20	37	22	22
4	1	3	4	149	50	15	0	2	4	12½	5	31
5				167	50	5	10	2	11	37	31	11½
6	2	15	24	157	40	44	40	2	16	16	20	50½
7	2	22	24	170	10	6	10	2	15	23	16	36
8	4	4	4	184	20	15	45	3	8	35½	39	30
9	4	19	44	196	20	6	30	3	20	18½	15	57
10	4	24	44	233	30	15	5	4	24	17	0	26½
11	6	16	30	283	20	7	30	6	18	16	1	59
12	6	17	24	297	0	23	30	6	18	39½	31	2½
13	8	3	4	322	0	24	15	8	4	13	4	27
14								8	16	50	35	57
15	9	7	44	4	0	38	30	9	9	43	61	47½
16	9	20	14	20	0	7	5	2	26	9	29	21½
17				35	10	4	30	10	29	53½	59	56½
18	11	22	34	68	20	24	0	11	23	49½	31	7½
19	0	2	34	85	5	11	30	0	3	28½	12	35
20				127	20	1	40	1	26	17	30	5
21	5	14	54	259	5	18	10	5	16	3	12	18
22	10	24	24					10	28	11½	21	0
23				181	15	51	50					

A Table of the principal Fixed Stars, according to the Observations of the Antients.

By Ricciolus Bononienſis, *in his* Aſtronomia Reformata.

	Long.				Latit.			Aſcenſ.			Declin.		
	S.	°	′	″	°	′	″	°	′	″	°	′	″
1	0	22	0	14	53	30	0						
2	1	0	22	0	51	17	0	357	44	33	57	18	40
3	1	11	32	10	22	22	40	41	33	54	39	36	30
4	2	5	1	43	5	30	50	64	6	58	15	46	10
5	2	12	3	10	31	10	10	74	32	38	8	37	30
6	2	17	6	15	22	51	45	72	52	20	45	36	0
7	2	16	13	0	16	52	30	76	45	4	5	59	50
8	3	9	3	0	39	32	5	97	30	18	16	16	30
9	3	21	6	30	15	57	10	110	22	32	6	4	0
10	4	26	4	45	0	26	20	147	31	14	13	36	30
11	6	19	6	0	1	59	30	196	51	0	9	20	30
12	6	19	30	40	31	0	40	210	4	38	20	59	50
13	8	5	1	40	4	26	30	242	10	40	25	33	30
14	8	17	43	20	35	56	15	259	47	14	12	52	30
15	9	10	32	40	61	47	0	276	19	32	30	30	40
16	9	16	58	30	29	20	40	293	31	59	8	1	20
17	11	0	41	25	59	57	20	307	26	35	44	6	40
18	11	24	40	20	31	8	20	341	51	48	29	15	50
19	0	4	40	20	12	37	0	358	57	0	13	19	20
20	1	27	0	8	30	6	40	44	52	30	48	34	30
1	5	16	53	0	12	16	20	172	53	58	16	27	40
2	10	29	2	56	20	59	40	339	37	0	31	20	10
3													

The Diſtances of the principal Fixed Stars, according to ſelect Obſervations.

			°	′		°	′
1	The laſt Star of *Fluvius*.						
2	The bright Star of *Caſſiopeia*'s Chair.	from *Seat*.	33	4	from Tail of the *Swan*.	33	32½
3	*Perſeus*'s Head of *Meduſa*. *Algol*.	from the Side of *Perſeus*.	9	23¼	from *Capella*.	23	37
4	The Southern Eye of the *Bull*.	from *Sirius*.	45	58⅗	from *Algomeiſa*.	46	22
5	The Left Foot of *Orion*. *Rigel*.	from the *Bull*'s *Eye*.	26	33	from *Capella*.	54	15¾
6	The little Kid of *Auriga*.	from the ſame.	36	43½	from the Side of *Perſeus*.	19	11 1/12
7	*Orion's* Right Shoulder.	from the ſame.	15	47⅗			
8	The *Dog-Star*. *Alhabor*.	from *Algomeiſa*.	25	42	from *Rigel*.	23	40
9	The *Little Dog*. *Algomeiſa*.	from *Regulus*.	37	20	from the ſame.	38	37½
10	The *Lyon's* Heart. *Regulus*.	from *Spica Virginis*.	54	2	from the Tail of *Swan*.	24	39½
11	*Spica Virginis*.	from *Arcturus*.	33	2	from *Antares*.	45	52
12	*Arcturus*.	from *Regulus*.	59	49	from the ſame.	56	4½
13	*Antares*. The *Scorpion's* Heart.	from bright Star of *Eagle*.	60	9½	from the *Lion*'s *Tail*.	24	39⅔
14	The Head of *Serpentarius*.	from *Spica Virginis*.	36	14½	from bright Star of *Harp*.	29	33⅔
15	The bright Star of the *Harp*.	from *Seat*.	55	30⅔	from *Spica Virginis*.	9	20½
16	The bright Star of the *Eagle*.	from bright Star of *Harp*.	34	9	from Tail of the *Swan*.	38	4⅓
17	The bright Star in Tail of the *Swan*.	from the ſame.	23	52	from *Seat*.	32	57⅙
18	Bright Star in R. Leg of *Pegaſus Seat*.	from *Algenib*.	20	37	from the bright Star in the Chair of *Caſſiopeia*.	22	4
19	The laſt Star of *Pegaſ.* Wing. *Algenib*.	from the Side of *Perſeus*.	51	45⅙			
20	The bright Side of *Perſeus*.	from the *Bull's Eye*.	36	20⅓			
21	The *Lyon's* Tail.	from *Spica Virginis*.	35	2⅙			
22	The Mouth of the *South Fiſh*.						
23	*Canopus*.						

The Declinations of eight of the principal Fixed Stars, by the Industry of the Antients; the noble Tycho Brahe *being alſo conſulted.*

		By *Ariſtyllus.* Y. b. C. 300.	*Timocharis.* Y. b. C. 295.	*Hipparchus.* Y. b. C. 128.	*Menelaus.* A. C. 97.	*Ptolemy.* A. C. 137.	*Tycho.* A. C. 1600
4	The *Bull's Eye.*		8 45 n.	9 45 n.		11 0 n.	15 28 n.
6	*Capella.*	20 0 n.		40 24 n.		41 10 n.	25 28½ n.
8	*Sirius.*		15 20 s.	16 0 s.		16 45 s.	16 11 s.
10	*Regulus.*		21 20 n.	20 40 n.		19 50 n.	13 53½ n.
11	*Spica Virginis.*		1 24 n.	0 36 n.	0 40 s.	0 36 s.	9 1 s.
12	*Arcturus.*		31 30 n.	31 0 n.		29 50 n.	21 18½ n.
13	*Antares.*		18 20 s.	19 0 s.		20 25 s.	25 26 s.
16	Bright Star in *Aquila*		5 48 n.	5 48 n.		5 50 n.	7 54 n

Longitude of the Bull's Eye *according to*

	Hermes.	♓	25	17
	Hipparchus.	♈	10	0
A. C. 140.	*Ptolemy.*	♈	12	40
A. C. 851.	*Abomaſar.*	♈	19	15
A. C. 831.	*Thabetus.*	♈	21	17
	Arzachel.	♈	23	20
	Albatanius	♈	24	30
	R. Groſtet.	♈	28	40
A C. 1316.	*W. Eveſham.*	♈	29	0

Longitude of Spica Virginis *according to*

Y. b. C. 295.	*Timocharis.*	♍	22	20
Y. b. C. 283.		♏	22	30
Y. b. C. 128.	*Hipparchus.*	♍	24	0
A. C. 97.	*Menelaus.*	♍	25	45

Longitude of Regulus *according to*

Y. b. C. 128.	*Hipparchus.*	♋	29	50
A. C. 137.	*Ptolemy.*	♌	2	30
A. C. 879.	*Albatanius.*	♌	14	0

Therefore in the Space of 742 Years; 11°. 50′, or rather 11°. 30′, becauſe of the *Ptolemaic* Longitude o *Regulus* ♌ 2°. 30′, not 2°. 10′, as was read formerly by the Aſtronomer of *Racca*.

Added to the Ptolomaic *Longitudes of the Fixed Stars, by*

A. C. 1303.	*Alphonſus.*	17	8
A. C. 1320.	*Wymundus.*	15	52
A. C. 1440.	*Walter Vigorn.*	19	5

The firſt Star of *Aries* is diſtant from the Vernal Equinox 25°. 40′.

All theſe according to *Walter Vigornienſis, Cod.* 13. among thoſe of *Digby*.

The

The Fixed Stars go forward one Degree in Solar Years.

1. ACcording to *Hipparchus*, *Ptolemy*, *Theo*, *Proclus*, and *Alferganus*, 100
2. *Timocharis Alexandrinus*, who obſerved *Spica Virginis* in the Years of *Nabonaſſar* 454, 466; *Abdorahmanus Salchius*, and *D. Petavius*, 72; or $\frac{60\times12}{10}$; and 5″ in every Year.
3. By *Johannides Ægyptius*, the Compiler of the *Hacimic* Tables, $70\frac{3}{4}$
4. By *Jabias Abomanſor*, and other Authors of what they called the approved Aſtronomy; alſo by *Naſirodinus Tuſius*, *Cothodinus Siraſius*, *Ologbec* Prince of the *Mogols*, *Xacholgius*, *Abolphetachus*, *Abeneſdra*, *Maimonides*, and moſt of the Moderns — — — 70. and 51″. 26‴.
5. *Chryſococca* in *Perſic.* and *Aſtron. Anglic. An. Chr.* 1300.—68. and 52″. 23‴.
6. By moſt of the *Arabian* Aſtronomers under Prince *Mamon* — $66\frac{1}{2}$.
7. By *Abdorahmanus Sophius*, *Baboninus Chorcius*, King *Alphonſus*, *Albatanius* of *Racca*, (which is *Callinico's* of *Meſopotamia*) *Abdolgalilus Segazius*, *Levi* and *Zacutus* the *Jews*, and ſome of the *Maragenſian* Obſervers 66. and 53″. 33‴.
8. By *Copernicus*, *Mæſtlin*, and others upon their Credit,—near 71. and 50″. 12‴. 5⁗.
9. By ſome in *Chorcius* the *Arabian* — — — — 54″.
10. By *Tycho Brahe*, *Kepler*, *Bullialdus*, from the Obliquity of the *Zodiac* $23\frac{1}{2}$°. $70\frac{7}{12}$. and 51″.
11. By *Longomontanus* — — — — $72\frac{1}{3}$. and 49″. 54‴.
12. By *Gaſſendus* — — $70\frac{1}{3}$. and 51″. 19‴. 24⁗.
13. By *Ricciolus* in *Aſtr. Reform.* from the Obliquity of the *Zodiac* 23°. 30′. 20″. 71. $19\frac{1}{2}$°. 50″. 40‴.
14. By us, and the *Hierophants* of the *Ægyptians*, 71. $9\frac{2}{3}$ *menſ.* & 50″. $9\frac{1}{4}$‴. *fere.*

The Pleiades *obſerved, in* 1671. *by Mr.* Flamſteed. N. 79. *p.* 3061, 3062.

XII. I have a Tube of $13\frac{1}{2}$ Feet, furniſhed with Convex Lens's, and a moſt exact *Townley*'s Micrometer, with which, in the ſerene Nights of the Months of *October* and *November* laſt paſt, I have often meaſured the minute Diſtances of the *Pleiades*, and that with ſuch Succeſs, that my repeated Obſervations never differed from one another 20″, and very ſeldom 10″. They are confirmed by the preceding Obſervations of the deceaſed Mr. *Gaſcoigne*, and the late ones of Mr. *Townley* (what great Men!) which were performed in the ſame Manner. Theſe moſt correct Diſtances are as follows.

Stars.	*Diſtances according*		*Stars.*	*Diſtances according*	
	To Me	*To* Mutus		*To Me*	*To* Mutus
	′ ″	′		′ ″	′
a b	35 40		a d	18 30	22
a e	27 40	31	e d	26 10	30
e b	20 00	32	d i	16 25	
c b	21 45	22	a i	18 18	
e c	10 00	11	f i	29 04	
e g	14 40		f a	23 00	27
c g	11 55		h a	23 20	
b d	22 04	24	f a	04 45	04

Vinc. Mutus adds in his Epiſtle to the moſt learned *Ricciolus* (which he mentions in his Appendix to *Alm. No. Tom.* 1. *Pag.* 747) that the Weſtern brighter Star paſſed over the Meridian juſt at the ſame Altitude as the bright Star of the *Pleiades*. Relying upon which Notice, and the obſerved Diſtances, I have aſſigned their Places to thoſe Stars as below; firſt having given the ſame Place and Latitude to the middle bright Star, as the *Caroline* Author has thought fit; and the reſt alſo being ſettled from thence. All which however, if I might follow my own Opinion in this Matter, I ſhould advance three Minutes, or two at leaſt: And I ſhould aſſign them a greater Latitude from the Ecliptic. At the Beginning of the Year 1672 were conſtituted

The Stars of the Pleiades.	Long. ♉			N. Lat.			mag.
	°	′	″	°	′	″	
The Weſtern brighter Star — — —	24	45	15	4	08	51	5
Between this and the Northerly Teleſcopic Star	24	46	47	4	19	21	8
The Weſtern and more Northerly — —	24	54	48	4	28	19	6
The higheſt in the Quadrilaterum —	25	01	24	4	20	39	5
The loweſt Southern oppoſite — — —	25	02	18	3	53	59	6
The middlemoſt bright Star — — —	25	19	48	4	00	00	3
That in the Point to the Eaſt — — —	25	41	29	3	52	19	5
The upper Teleſcopic of the Eaſtern Stars	25	42	55	3	56	51	7
Another Teleſcopic Star — — — —	25	14	04	3	42	37	9

A Nebulous Star, by M. Caſſini. N. 123. *p.* 565.

XIII. 1. Between the *Great Dog* and the *Ship* I lately diſcovered a *Nebulous* Star, ~~which was very beautiful to behold, if it was viewed~~ with large Teleſcopes; being compoſed of Stars very cloſe together. With the *Leſſer Dog* it divides the Heaven into equal Parts.

By Mr. Flamſteed. *Ibid. p.* 567.

2. Viewing the Heavens below the *Leſſer Dog*, I found a *Nebulous* Star, broad and very thick, ſet with ſmall Stars. I ſuppoſe this to be the ſame as was obſerved by Mr. *Caſſini*.

The firſt of Aries *a double Star; by Dr.* Hook. *Phil. Coll.* N. 4. *p.* 108. *Marg.*

XIV. *Ann.* 1664. I diſcovered the firſt Star in the Head of *Aries* to be a double Star, made of two conſiderable Stars, ſo near as not to be diſcovered two, but by a Glaſs of ſix or eight Feet long.

Changes amongſt the Fixed Stars, by S. Montanari. N. 73. *p.* 2202. *Marg.*

XV. 1. There are wanting in the Heavens two Stars of the ſecond Magnitude, in the Stern of the Ship and its Hatches, according to *Bayer* β and γ, near the *Greater Dog*, which were obſerved and recognized by me and others, chiefly on occaſion of the Comet, *An.* 1664. To what Year we are to refer its Diſappearance ~~I cannot tell; but this is certain, that~~ from the 10th Day of *April*, 1668, I can no longer obſerve any Traces of them; while the other Stars about them, even of the fourth and fifth Magnitude, continue immoveable. I have taken Notice of above an hundred Changes in the other fixed Stars, but not of ſuch Conſequence.

By M. Caſſini. N. 73. *p.* 2201.

2. M. *Caſſini* hath diſcovered many new Stars; *viz.* One of the Fourth Magnitude, and two of the Fifth, in *Caſſiopeia*. He hath diſcovered two others towards the Beginning of *Eridanus*, where we were ſure they were not yet about the End of the Year 1664. conſidering that this Place of the Heavens, where paſſed the then appearing *Comet*, was diligently beheld by many, who perceived divers others ſmall Stars, without obſerving thoſe two. The ſame hath alſo obſerved, towards the *Arctick* Pole, four of the Fifth or Sixth Magnitude.

He hath alſo obſerved, That the Star which *Bayerus* puts near that which he marketh in the Figure of *Urſa Minor*, appears no more; that that which is marked A, in the Figure of *Andromeda*, is alſo diſappeared; that in lieu of that which is marked υ, at the *Knee* of the ſame Figure, there are two others more Northward; and that that which is ξ, is very much diminiſhed; the Star which *Tycho* placeth at the Extremity of *Andromeda's Chain*, and calls it of the Fourth Magnitude, is now ſo ſmall that one can ſcarce ſee it; and that which is in his *Catalogue* the 20th of the Conſtellation of *Piſces*, is now no more ſeen.

The New Star in Pectore Cygni; *by M.* Hevelius. N. 19. *p.* 349. N. 21. *p.* 372.

3. On the 24th of *Sept.* (*St. N.*) 1666, I have obſerved that New Star in *Pectore Cygni*, (which from the Year 1662 until this Time, hath been almoſt altogether hid) not only with my naked Eye, like a Star of the Sixth or Seventh Magnitude, but alſo with a very great Sextant. It is ſtill in the very ſame Place of the Heavens where it was from *Ann.* 1661 to almoſt 1662. For its Diſtance from *Scheat Pegaſi* hath been by me found 35° . 51′ . 20″, and from *Marcab*, 43° . 10′ . 50″ . which Diſtances are altogether equal to thoſe which I obſerved *Ann.* 1658. the Firſt of *November*. For the Diſtance from

Scheat at that Time was 35° . 51′ . 20″ . and from *Marcab*, 43° . 10′ . 25″ . where that former from *Scheat* exactly anſwers to the Recent; and that from *Marcab*, 'tis true, differs in a very few Seconds; but that Diſparity is of no moment, ſince it only proceeded from thence, that this New Star is not yet ſo diſtinctly to be ſeen as at that Time, when it was of the Third Magnitude. It is therefore certain, that it is the ſelf-ſame Star which *Kepler* did firſt ſee *Ann.* 1601, and continued till *Ann.* 1662. He that will obſerve this Star, muſt take care leſt he miſtake the Three more Southern ones of the Sixth Magnitude; the higheſt of which is diſtant from *Scheat Pegaſi*, 36° . 25′ . 45″ . the middlemoſt from the ſame 37° . 25′ . 20″ . and the loweſt 38° . 4′ . 30″.

N. 134. *p.* 855. *An.* 1662. *Nov.* 28. That new Star in the Swan's Breaſt, which for ſome Time, from *An.* 1662, was intirely hid, the Heaven being clear, ſeemed as it were to revive.

An. 1666. *Sept.* 21. It appeared to the naked Eye, even when the Moon ſhined. *Sept.* 24. It was leſs than thoſe three preceding in the Neck, and ſcarcely ſeemed of the ſixth Magnitude.

N. 65. *p.* 2089. *An.* 1670. *Aug.* 26. It ſeemed ſenſibly to increaſe, though not yet greater than Stars of the ſixth Magnitude. *Sept.* 3, It ſeemed ſtill to increaſe. 8, We found it ſtill increaſing. *Octob.* 13, It appeared plainly enough.

N. 134. *p.* 855. *An.* 1671. *Apr.* 29. It hardly appeared greater than in the foregoing Year, for it was equal to Stars of the ſixth Magnitude.

An. 1671. *Jun.* 26. It almoſt ſeemed greater.

An. 1672. *March* 29. It ſtill ſeemed to increaſe.

An. 1675. *July* 22. It appeared ſtill as a Star of the ſixth Magnitude.

N. 154. *p.* 854. *An.* 1677. It was not yet arrived to its former Magnitude, which was that of the third, nor had it attained its uſual Brightneſs and Splendor, as it appeared in the Years 1657, 1658, 1659; for it did not yet ſhine but as a Star of the ſixth Magnitude.

Ph. Coll. N. 5. *p.* 162. *Ann.* 1681. *Aug.* 18. The new Star in the *Swan*'s Neck was hardly to be ſeen by the naked Eye becauſe of its Smallneſs and Obſcurity, yet at laſt was found by the Teleſcope.

The New Star, ſub Capite Cygni. N. 65. *p.* 2002. N. 73. *p.* 2198. 4. 1. Don *Anthelme*, a *Carthuſian* at *Dyon*, on the 20th of *June*, *An.* 1670, diſcovered a Star of the Third Magnitude beneath the *Head* of *Cygnus*, ſituated in the Section of the two ſtreight Lines, one of which goeth from *Lyra* to the neareſt of the *Quadrangle* in the *Dolphin*, and the other from the *Eagle* to the Star, which is on the Top of the *Upper Wing* of *Cygnus*. He ſent the News of this Diſcovery to M. *L'Abbe Mariotte*, one of the *Royal Academy*, who communicated it to the reſt. They all agree 'tis a *New* Star, though M. *B*—— oppoſed it at firſt, affirming it to be in *Bayerus*'s Tables; but they prove that Star in *Bayerus* to be another; giving for Diſtinguiſhment theſe Meaſures.

	°	′	″
The bright Star *ad Roſtrum Cygni*, its *Aſcenſio Recta*	289	22	00
Declinatio Borealis	27	19	20
But this *New* Star's *Aſcenſio Recta* is —— ——	293	33	00
Declinatio Borealis	26	33	20

Longi-

	°	′	″
Longitudo ♒	1	55	
Latitudo	47	28	10
Its Diſtance from that *ad Roſtrum Cygni* towards *Jaculum*	3	47	30
From the Tail of *Cygnus*	20	54	30
And from the *Lucida Lyræ*	18	39	40
It came to the Meridian after the Star *in Roſtro Cygni*		16	44
And before the *Lucida Aquilæ* — — — —		0	27

In the Beginning of *July*, this Star was obſerved to decreaſe: *July* 11, it ſcarce appeared of the Fourth Magnitude.

Aug. 10. It was of the Fifth, and continued to decreaſe till it wholly diſappeared.

Anno 1671. *March* 17, D. *Anthelme* ſpied it again of the Fourth Magnitude.

April 4. M. *Caſſini* found it greater than the two Stars of the Third Magnitude that are below in the Conſtellation of *Lyra*, and a little ſmaller than that in the Beak of *Cygnus*, but more radiant.

April 9. He found it a little diminiſhed, and almoſt equal to the greateſt of the two Stars that are below in *Lyra*.

The 12th, it was equal to the leaſt of theſe two Stars.

The 15th, he perceived that it increaſed, and found it equal the ſecond time to the greateſt of theſe two Stars.

From the 16th unto the 27th, it appeared of different Magnitudes, being ſometimes equal to the biggeſt of theſe two Stars, ſometimes equal to the leaſt, and now and then between both.

But the 27th and 28th, it was become as big as the Star in the *Swan's Beak*.

The 30th, it appeared a little clearer; and the firſt ſix Days in *May* it was greater.

The 15th of *May*, it was ſeen ſmaller than the ſame Star.

The 16th, it was in Bigneſs between the two Stars that are below in *Lyra*: And ever ſince ſhe hath ſtill diminiſhed.

Thus this Star hath been twice in her greateſt Splendor; firſt on the 4th of *April*, and the ſecond time in the Beginning of *May*.

By M. Hevelius. N. 65. *p.* 2087.

I write you this to acquaint you with a certain remarkable Obſervation, and at the ſame Time to let you know my Mind of the Matter. I mean of that new fixed Star, almoſt of the third Magnitude, about and below the *Swan's Head*, and conſpicuous among the unformed Stars. Its Longitude is now 1° . 52′ . 26″ . ♒ and Latitude 47° . 25′ . 22″, as plainly appears by my Obſervations, *An*. 1670. *July* 25. There is no Reaſon to doubt but that this is intirely a new Star, and altogether inconſpicuous in the Heavens in the Year 1660. For it happened that in the Years 1659, 1660, 1661, I obſerved almoſt all thoſe Stars, that appear in the Conſtellation of the *Swan*, with the utmoſt Diligence and with proper Inſtruments; and ſo took Notice of all thoſe about the Neck and Head, and meaſured their Diſtances from ſeveral fixt Stars. But I found no Star of the third Magnitude in that Place where the above-mentioned new Star is now to be ſeen; which if it had been there I muſt have ſeen it. So that, firſt, I am ſure from hence, that in the Years 1660 and 1661 this Star was not yet viſible; and then it clearly appears from *Bayer*'s *Uranometria*, that this now mentioned new Star did not appear in the Year 1603, and conſequently not to *Tycho*, and much leſs to *Hipparchus.* For *Bayer* would have found a Star of that Magnitude, ſince he deſcribes one of the ſixth Magnitude not far from it; as may be ſeen in his Conſtellation of the *Swan.* But perhaps you will ſay, this is the very ſame that you call a *new one*; for ſince *Bayer* did not obſerve the Stars with proper Inſtruments, it may eaſily be, that he might err a Degree or two from the true Place. But this cannot be the Thing, ſince that ſmall Star ſtill continues in the ſame Place where *Bayer* puts it, nor is it greater than a Star of the ſixth Magnitude, as he obſerved it. For, as I have lately found, it is diſtant from *Pegaſus*'s Mouth 32°. 39′. 00″. and from *Pegaſus*'s Right Knee 39°. 23′. 45″. Hence its Longitude comes out 00°. 06′. 28″. ♒, and its Latitude 46°. 11′. 14″ North, to the current Year 1670, in *July.* But the new Star is diſtant from *Pegaſus*'s Mouth 32°. 31′. 25″. and from the Right Knee of *Pegaſus* 38°. 18. 50″. From which Diſtances the Longitude is found to be 1°. 52′. 26. ♒, and Northern Latitude 47°. 25′. 22″. So that this new Star is plainly different from that of the ſixth Magnitude obſerved by *Bayer*, though theſe two are not above two Degrees removed from each other. And from what is ſaid it is manifeſt, that this new Star did not ſhine among the other Stars, neither *An.* 1603, nor *An.* 1670.

When it was firſt obſerved by me, as to Magnitude and Brightneſs it was not inferior to the Star in the *Eagle's Breaſt*, unleſs that its Light was a little more obtuſe. As to its Situation in reſpect of the other Stars, it was placed in a Right Line with that in the bending of the upper Wing of the *Swan*, and that in the Shoulder of the *Eagle*; as alſo with the bright Star of the *Harp*, and that in the *Rhombus* of the *Dolphin*, which is the more Northern of the middle ones. It made an Equilateral Triangle with that in the Head and Beak of the *Swan.*

It wonderfully decreaſed in the Month of *September*, ſo that on the 14th N. 66. *p.* 2028.
of *October* I could not obſerve it at all with my Sextant, though I uſed all N. 132. *p.* 855.
my Induſtry for that Purpoſe.

An. 1671. *Apr.* 29, I obſerved again. It exceeded that in the *Swan*'s N. 73. *p.* 2197.
Beak, and likewiſe that in the bending of the lower Wing of the *Swan*, and N. 134. *p.* 856.
was almoſt equal to that in the *Swan*'s *Breaſt*, except that it ſhone with a Light a little more dull and reddiſh. But upon what Day it firſt began to ſhine I dare not affirm. This I am ſure of, that it was not conſpicuous in the Months of *December*, *January*, or even *February.* For after *Octob.* 14, when it ceaſed to be ſeen, I remember I often ſought for it in its Place, but it did not appear. Therefore as far as I can recollect, it hardly came into Sight again before the Beginning of *March*, or rather later. On *April* 30, I meaſured its Diſtance from the other fixed Stars. It is diſtant from the

Swan's Tail 20° . 55′ . 20″. From the bending of the upper Wing of the *Swan* 17° . 47′ . 50″. From the Head of *Serpentarius* 34° . 19′ . 40″. ſo that it ſtill continues in the ſame Place where it was.

N. 134. p. 856. *May* the 17th, it ſeemed ſomething leſs than the *Swan*'s Beak, and that in the Shoulder of the *Eagle*, as alſo duller in its Light; but greater than that in the Point of *Sagitta*, and almoſt equal to that following in the Body of the *Harp*.

May 25. It ſeemed leſs than on the 29th of *April*, when it was firſt ſeen, ſo that it ſeemed to decreaſe. Leſs than that in the *Swan*'s Beak, or than that in the bending of the Southern Wing; or even leſs than thoſe in the Belly of the *Harp* and the *Eagle*'s Shoulder. It appeared hardly greater than the leſſer of the two in the *Swan*'s Foot, and that in the Breaſt of the *Eagle*.

Jun. 26. It appeared leſs than that in the *Swan*'s Neck, ſo that it had decreaſed notably.

July 3. Leſs than that in the *Swan*'s Neck; and on the 18th it hardly ſeemed equal to Stars of the fifth Magnitude.

Aug. 2. It hardly appeared of the ſixth Magnitude, nay leſs than all the other Stars about the Neck and Head of the *Swan*. It only twinkled now and then.

Sept. 11. It was not to be ſeen any more.

N. 81. p. 4018. *Ann*. 1672. *Mar*. 6. I obſerved it again; but it can hardly be ſeen with the naked Eye.

N. 134. p. 857, 854. *Mar*. 29. It hardly appeared of the ſixth Magnitude; from which Time to *An*. 1667. it came no more into Sight, though I often ſought for it very diligently,

The Nebuloſa *in the Girdle of* Andromeda; *by M.* Bullialdus. 5. *Ann*. 1667. in *Jan*. The *Nebuloſa* in *Andromeda*'s Girdle (which may well enough be ſeen by the bare Eye) appeared much obſcurer than the Year before. In the Months of *February* and *March* I did not ſee it.

N. 25. p. 459. *The New Star in* Collo Ceti; *by M.* Bullialdus. *Ibid*. 6. 1. *Ann*. 1667. *Jan*. 20. The *New Star* in the *Neck* of the *Whale*, did approach to the Bigneſs of a Star of the ſixth Magnitude, and grew bigger afterwards.

Feb. 12. I ſaw it at leaſt of the Fourth Magnitude.

Feb. 24. It was equal to the Stars of the Third Magnitude, ſhining very bright.

Feb. 26. and 27. It appeared yet to increaſe.

By M. Hevelius. N. 25. p. 460. N. 134. p. 855. 2. *Ann*. 1667. In the Beginning of *January* this Star did not appear.

Jan. 23. I found a little Star of the Sixth or Seventh Magnitude about the ſame Place where the ſaid *New Star* uſes to appear. But it then ſeemed to me not the genuine *New Star*, but another; to wit, preceding the *New*, whoſe Longitude in *An*. 1660. was defined by me, ♈ 25° 43′ 3″, and the Latitude 14° 41′ 32″.

Feb. 2. It appeared very bright, and that, when the Moon ſhone, of the Bigneſs of that in the *Mouth* of the *Whale*, or *Nodo Lini*; from which time I always obſerved it to grow bigger.

Mar. 13. I did ſtill find it extremely bright, but could not by my naked Eye, becauſe of the vivid Crepuſcle, and the low Sight of the Star, accurately determine its Magnitude.

An. 1668. *Octob.* 26. The new Star in the *Whale*'s Neck was firſt ſeen, but like the ſmalleſt of the fixed Stars. N. 132. p. 855.

Nov. 7. The new Star in the *Whale*'s Neck was almoſt equal to the middle Star in the Mouth.

An. 1669. *Jan.* 28. It was leſs than that in the Mouth.

Sept. 26. It appeared like a Star of the ſixth Magnitude.

Oct. 16. It was greater and brighter than that in the Mouth.

Oct. 27. It was equal to the bright Star in the Jaw.

Nov. 19. It was greater than that in the Mouth, and leſs than that in the Jaw.

An. 1670. *Aug.* 27. It ſhined with very much Light, and was almoſt equal to the Stars of the ſecond Magnitude in the *Whale*'s Jaw.

Sept. 3. It was very bright, and on the 8th was equal to the *Whale*'s Jaw.

To the middle of the Month of *October* it was almoſt equal in Magnitude to that in the *Whale*'s Jaw, and nearly exceeded it in Brightneſs; ſo that this Year it was of the ſecond Magnitude, and greater than in the former Years, excepting *An.* 1660, when I found it to be even greater than the *Whale*'s Jaw. At other Times I do not remember that it exceeded Stars of the third Magnitude. It is therefore certain, that it did not always appear of the ſame Magnitude or Brightneſs, however it may be in its greateſt Increaſe. N. 66. p. 2028.

Dec. 5. It had ſo dwindled away, that it was hardly equal to a Star of the ſixth Magnitude. N. 134. p. 856.

An. 1671. *Aug.* 14. It was equal to the Star at the Cheek, nay it ſeemed to be ſomething larger.

Sept, 12. It was equal to that in the Mouth, of the fourth Magnitude.

Oct. 30. It appeared to be hardly of the ſixth Magnitude.

Nov. 3. It appeared no more.

An. 1672. *Aug.* 9. It ſhined with very bright Rays, and was greater than that in the Mouth, and leſs than that in the Jaw.

Sept. 17. Leſs than that in the Cheek, was hardly of the fourth, nay of the fifth Magnitude, and on the 25th was hardly of the ſixth Magnitude.

From about the Month of *October* to *December* 23, 1676, it did not once come to view, though I directed my Eyes towards it very attentively, whenever I applied myſelf to Obſervations on a clear Night. *Ibid.* p. 854.

An. 1676. *Dec.* 10. I remember very well, I could not ſee the new Star in the *Whale*'s Neck, though I obſerved very many little Stars in that Part of the Heavens. *Ibid.* p. 858.

Dec. 23. The Heaven being very clear, we ſaw very plainly the new Star in the *Whale*'s Neck, It ſhone with ſo much Brightneſs, and was of ſuch a Magnitude, that it was not only equal to, but even exceeded the Jaw of the *Whale*.

Dec. 31. It was rather greater than the Jaw, that is, of the ſecond Magnitude.

An. 1677. *Jan.* 1. It again ſhone very brightly, rather greater than the

Whale's Jaw, and rather greater than that in the Extremity of the Wing of *Pegaſus*, and *Marcab*, in Colour and Light not unlike to the Jaw. Yet I remember to have obſerved formerly, when it was of the ſecond Magnitude, that it was a little whiter and brighter.

Phil. Coll. N. 5. *p.* 162.

An. 1681. *Aug.* 18. The new Star in the *Whale*'s Neck this Night, tho' the Moon was full and ſhining, was greater than that in the *Whale*'s Mouth, but yet not equal to the bright Star in the Jaw.

By M. Caſſini. N. 123. *p.* 565.

3. *An.* 1676. *March.* I viewed the new Star in the *Whale*'s Mouth, which had diſappeared for ſome Years, being immerſed in the Sun's Beams, at the Time of its greateſt ſhining. Now it plainly exceeds Stars of the third Magnitude.

By Mr. Flamſtead. *Ibid. p.* 567.

4. For eight Months before, I have often ſeen the new Star in the *Whale*'s Breaſt, which was not leſs than Mr. *Caſſini* mentions.

A new Star in Eridanus; *by M.* Caſſini. N. 35. *p.* 683.

7. *March* 10. 1668. Not far from the Star in *Eridanus*, which is called the 14th by *Bayerus*, there appeared a Star equal to the brighteſt of the 4th Magnitude, almoſt in the ſame Place where was obſerved the Comet of *An.* 1664. *Dec.* 31. which Star was not then ſeen, nor at other Times elſewhere, nor is deſcribed in any Catalogue, on any Globe or Map, that I can learn; which therefore I deem to be a *New* one, that is, of *New Appearance*.

A New Star in Taurus; *by M* Caſſini. N. 82. *p.* 4046.

8. The Comet, *Anno* 1672, had (on the 1ſt of *April*, *N. S.*) paſſed 45′ beyond the moſt Northern Star of the *Head* of *Taurus*, and was diſtant 1° 43′ from the Star that was neareſt to that towards the South. M. *Caſſini* having conſidered theſe two Stars, obſerved that the Second is not leſs bright than the Firſt, and yet that *Bayerus* hath not marked it; and that at firſt ſight it ſeems that *Tycho* hath left it out in his Catalogue: For he puts four Stars in the Place he calls *in Quadrilatero Cervicis*; and he ſpeaks not of this, which is the fifth, and maketh, with the other four, an irregular Pentagon. This Omiſſion of *Bayerus*, and the Denomination which *Tycho* uſeth to denote theſe Stars, which ſuits not with the Number nor the Configuration that now appears, do adminiſter Cauſe to doubt whether the Star in queſtion be not one of thoſe that appear from Time to Time.

To find the Aphelia of the Planets directly; by M. Caſſini. *Conſidered, by Mr.* Nic. Mercator. N. 57. *p.* 1168. Mar. An. 1670.

XVI. 1. Mr. *Caſſini* ſuppoſes, that to the Planet moving in an Ellipſis two Right Lines are extended from the two Foci, one of which is the Line of mean Motion, and the other of true Motion. Now the Conſtruction is this.

Fig. 117.

L is the Center of the Concentric ABCDE.
BLD is the Diameter.
BA, BC, BP, are appearing Intervals.
DE, DF, DQ, are Intervals of mean Motion.
BF, BF, BQ, as alſo DA, DC, DP, are Right Lines.
BE cuts DA in H; BF cuts DC in G, BQ cuts DP in R.
RHG is a Right Line.

BI is perpendicular to RHG.
I is the Center of the Ellipſis.
LI is the Excentricity.
IO = LI.
O is the Focus; about which the mean Motion is performed; L about which the true.
IM = IN = LB.
M is the Apogeum, N the Perigeum, BLM the true Anomaly.

Demonſtration. 1. The moſt illuſtrious and Right Reverend *Seth Ward*, Biſhop of *Saliſbury*, in his Examination of the Philolaic Aſtronomy, has taught us a Method, from the mean Anomaly of the Planets being given, to find the true one, which is thus:

C is the Center of the Ellipſis A E P, F the Focus about which the mean Motion is performed, S the Focus of the true Motion, A the Apogeum, P the Perigeum, E the Planet, A F E the mean Anomaly, A S E the true Anomaly, F E T a Right Line, $ET = SE$, S T is a Right Line. Fig. 118.

In the Triangle S F T are given, 1. S F, the Diſtance of the Foci. 2. $FT = FE + ES = AP$. 3. A F T the external Angle, or the mean Anomaly, equal to the Sum of the Angles F S T and T. Therefore F S E may be found, or the true Anomaly, equal to the Difference of the Angles F S T and T; thus,

As half the Sum of the Sides F T *and* F S, *is to half the Difference of the ſame, ſo is the Tangent of half the Sum of the Angles* F S T *and* T *to the Tangent of half their Difference.*

But the Half-ſum of the Sides F T and F S is found, by ſubſtituting for F T its Equal A P, whoſe half is A C, which added to C S, half of F S, makes the Semi-ſum A S, the Planets greateſt Diſtance.

Then if from the Half-ſum A S be taken the leſſer Side F S, there remains the half-difference of the Sides F A equal to P S, the leaſt Diſtance of the Planet. Then from the mean Anomaly to find the true one, the Rule is,

As A S, *the greateſt Diſtance of the Planet, is to* P S *the leaſt Diſtance, ſo is the Tangent of half the mean Anomaly to the Tangent of half the true Anomaly.*

Corol. 1. If S E is continued as far as V, ſo that $EV = FE$, and the whole $SV =$ Axis A P; the Angle V of the Triangle F S V will be half the Proſthaphereſis F E S, and therefore equal to the half-difference of the Angles of the mean and true Anomaly, that is, of the Angles A F E and A S E. (And the external Angle A F V is equal to half the Sum of the ſame Angles A F E and A S E) by taking away the Semi-difference V F E from the greater A F E; whence ariſe theſe two Analogies,

1. *As the Sine of half the Sum of the mean and true Anomalies* A F V, *to the Sine of half the Difference of the ſame* V; *ſo is* S V, *equal to the tranſverſe Axis* A P, *to* S F *the Diſtance of the Foci.*

2. *As the Sine of half the Sum of the mean and true Anomaly* A F V, *to the Sine of the true Anomaly,* F S V; *ſo is* S V, *or the Axis* A P, *to* F V *the Subtenſe of the true Anomaly; and ſo is the Semi-axis* A C, *to the Semi-ſubtenſe* V X *or* F X.

Corol. 2. If in the ſame Triangle F S V, from the middle Point X of the Subtenſe F V, be raiſed the Perpendicular X E; it will divide S V into two Parts, one of which V E is equal to the Line of mean Motion F E, the other S E is the Line of true Motion itſelf.

2. Let *a* be the Center of the Concentric *c b f i*. Fig. 119.

c a d the Diameter, and likewiſe the Line of the *Apſids*.

c b the Arch of the true *Anomaly*, to which anſwers,

d i the Arch of mean *Anomaly*; therefore

c d b is the Angle of half the true *Anomaly*, and

d c i the Angle of half the mean *Anomaly*.

c i and *d b* are Right Lines, which interſect one another in *g*.

From the Point of Interſection g let fall gb perpendicular to cd;

Then $db \,.\, bg$: : Radius . Tang. bdg or cdb.

And $cb \,.\, bg$: : Radius . Tang. bcg or dci.

Therefore $db \times$ Tang. $cdb = bg \times Rad. = cb \times$ Tang. dci.

Wherefore $db \,.\, cb$: : Tang. dci . Tang. cdb.

That is, db will be to cb, as the Tangent of half the mean Anomaly to the Tangent of half the true Anomaly: And therefore by the Rule above-mentioned, as the greateſt Diſtance of the Planet to the leaſt Diſtance. Therefore db will be equal to the greateſt Diſtance of the Planet, cb will be equal to the leaſt, and ab to the Eccentricity..

And ſince the ſame Thing may be demonſtrated in the ſame Manner of all the other Points of Interſection, that is, that Perpendiculars from them to the Line cd will fall upon the Point b, 'tis neceſſary that a Right Line joining thoſe Interſections muſt be congruous with the Perpendicular bgf.

3. Drawing the Diameter bak, make the Arch $kl = id$, and draw kc and bl cutting one another in p. From b upon bgf let fall the Perpendicular br, and the ſame is parallel to the Line of Apſids cd. Then the Angle rbs will be the half-difference of the Arches of the true Anomaly cb and of the mean di. Then from the ſame Point b let the Right Line $b\beta$ be drawn, making an Angle with kb equal to the Angle rbs, and meeting the Line of Apſids in β; then the Angle βab of the Triangle $a\beta b$ will be the Meaſure of the Arch cb, or of the true Anomaly; and βba the half-difference of the true and mean Anomaly, by Conſtruction; and the external Angle $c\beta b$ (equal to the two internal and oppoſite Angles βab and βba, and therefore compounded of the true Anomaly and its half-difference from the mean) will be the ſemi-ſum of the true and mean Anomaly. Therefore by the firſt Analogy of the firſt Corollary, as the Sine of $c\beta b$ to the Sine of βba, ſo is Radius ab to the Excentricity $a\beta$. But we have demonſtrated above, that ab is equal to the Excentricity. Therefore the Point β is congruous with the Point b.

Then from b let bt be raiſed perpendicular to bb. I ſay, that this being continued will fall upon the Point of Interſection p. For the Triangles rbs and bbt are ſimilar by Conſtruction. Wherefore alſo the Triangle bpk is ſimilar to the Triangle bgi, ſince the Angles pkl and gib, inſiſting upon the ſame Arch cb, are equal; as alſo the Angles pbk and gbi are equal, as inſiſting upon equal Arches kl and id. Therefore the third bpk is equal to the third bgi. And from Equals pbk and gbi taking away Equals bbt and rbs, there remain Equals pbb and gbr. Whence I argue thus, $srb = tbb$, and $rbs = bbt$; therefore $bsr = btb$, and therefore the Complements of theſe to a Semicircle are equal, that is, $rsi = btk$, and $sig = tkp$. Therefore alſo $igs = kpt$, which being taken from Equals igb and kpb, there remains $bgs = bpt$, and $gbr = pbb$. Therefore alſo $brg = bbp$. But brg is a Right Angle, and therefore alſo bbp is a Right Angle. And ſince by the Conſtruction bbt is a Right Angle, thence tb will make a Right Line with bp. And ſince the ſame Thing may be demonſtrated after the ſame Manner of any other Interſection of Lines from b and k, drawn to congruous Points of the true and mean Anomaly; it is plain, that not only the Right Line that joins the Interſections, will paſs through the Point b, but alſo the Line bb will be perpendicular to the ſame joining Line. Q. E. D.

Corollary. If from any Point of the true Anomaly, ſuppoſe b, to the correſponding Point i of the mean Anomaly, a Right Line bi be drawn; bf perpendicular to cbd, being drawn from the Center of Excentricity b, will cut bi in s, in the Ratio of the Line of the mean Motion to the Line of the true Motion.

For by the latter Analogy of the firſt Corollary, bb is the Semi-ſubtenſe; therefore by the ſecond Corollary, a Perpendicular bt erected at b cuts the Diameter bk in t, in the ſame Ratio that the Line of mean Motion has to the Line of true Motion. Therefore rs or bf cuts the Line bi in the ſame Ratio in s; becauſe of the Similitude of the Figures $tbbkpbb$ and $srbigbr$ juſt now demonſtrated.

But from the above-quoted Method of the Right Rev. Dr. *Ward*, for finding the firſt Inequality, it will not be difficult to produce ſtill another Way of inveſtigating the Apogeum and the Excentricities, which is not leſs direct and Geometrical, and admits of any Number of Obſervations; which I ſhall explain in a few Words. The Lovers of Aſtronomy may find ſeveral Ways in the Right Rev. Author's *Aſtronomia Geometrica*, to which therefore I refer them. But now

Fig. 120. Let l and d be the two Foci of the Ellipſis, t and u two Points of the Planet's true Motion, tu an Arch of the Ellipſis, which from l is viewed under the Angle tlu, and from d under the Angle tdu; alſo the Diſtance of the Foci ld is viewed from t under the Angle dtl, and from u under the Angle dul. I ſay, the Difference of the Angles tlu and tdu is equal to the Difference of the Angles dtl and dul.

For ſince all the three Angles of the Triangle lux taken together are equal to all the three Angles of the Triangle dtx when taken together, if from each be taken away the equal Angles lxu and dxt, the Sum of the other two $ulx + lux$ will be equal to the Sum of the others $tdx + dtx$; and if from theſe equal Sums be taken away Unequals, that is ulx from the former and tdx from the latter; the Difference of the Remainder lux and dtx will be equal to the Difference of the Angles taken away ulx and tdx. Which was the Thing propoſed.

With Center l, and Diſtance of the tranſverſe Axis mn, let a Circle abc be deſcribed, whoſe Arch ab is again viewed from l under the Angle alb, and from d under the Angle adb. Alſo the Diſtance of the Foci ld is viewed from a under the Angle lad, and from b under the Angle lbd. Therefore again the Difference of the Angles alb and adb is equal to the Difference of the Angles lad and lbd. But by *Corol.* 1. the Angle lad is half the Angle lud, and the Angle lbd is half the Angle ltd. Therefore the Difference of theſe Angles lad and lbd is equal to half the Difference of the Angles lud and ltd. Therefore alſo the Difference of the Angles alb and adb is equal to the half-difference of the Angles ult and udt, the former of which is the apparent Interval of the two Obſervations, and the latter is the Interval of the mean Motion. Therefore the Difference of theſe Intervals being given, half this Difference is alſo given, or the Difference of the Angles alb and adb. But alb is the ſame as the given Angle ult; therefore the Angle abd is alſo given, under which the Arch ab is viewed from d.

In a like manner it may be ſhewn, that the Difference of the Angles tly and tdy is equal to the Sum of the Angles ltd and lyd; alſo the Difference of the Angles blc and bdc is equal to the Sum of the Angles lbd and lcd. And ſince lbd is half of ltd, and lcd half of lyd; the Sum of lbd and lcd will be equal to half the Sum of the Angles ltd and lyd; that is, the Difference of the Angles blc and bdc will be equal to the half Difference of the Angles tly and tdy, of which the firſt is the apparent Interval of the two Obſervations, and the latter the Interval of the mean Motion. Therefore the Difference of theſe Intervals being given, half of this Difference is alſo given, or the Difference of the Angles blc and bdc. But blc is the ſame with the given Angle tly; therefore the Angle bdc is alſo given, under which the Arch bc is viewed from d.

Whence it appears, that the mean and apparent Intervals of the Obſervations being given, the Angles alſo are given, under which any Arches of the Circle abc are ſeen from d, which Arches are intercepted by Lines of the true Motion. Therefore by *Herigon's Theory of the Planets*, *lib.* 1. *cap.* 3. *Prop.* 12. *Schol.* 1. ſo many Segments of a Circle may be deſcribed, capable of receiving Angles under which thoſe Arches are viewed from d, all which Segments will interſect one another in d. Therefore by this Method the Apogees and Excentricities of the Planets may be found by a Geometrical Delineation, and applying any Number of Obſervations: Nor is it more difficult to draw Circles than Right Lines.

But to grant what is true, that Mr. *Caſſini's* Geometrical Delineation is ſomething more expeditious; it is however to be feared, that if we purſue that Exactneſs required by Aſtronomers, it requires Diagrams of an enormous Size, and thereby becomes more operoſe than the Calculation itſelf. But if we make uſe of this, we ſhall find both Methods to be equivalent. Now leſt any one ſhould think, that the Difference from the Truth, of the Apogeum and Excentricity found by either Method, is to be imputed to an Error of Calculation; we ſhall now diſcuſs the Hypotheſis itſelf.

The Invention of the Elliptic Orbit without any doubt is owing to *Kepler*; but to determine by what Degrees of Acceleration and Retardation the Planets move, does not leſs belong to the completing the Hypotheſis, than to determine the Orbit itſelf. Though from Mr. *Caſſini's* Words, or thoſe of his Interpreter, it no where appears; yet it is plain from his Conſtruction of the Problem, and from his Analyſis, that he ſuppoſed the Planet, if ſeen from the upper Focus, to move with an equable Motion. This was becauſe *Kepler* thought the ſame, as may appear to thoſe who will conſult his Writings. But when he perceived that this by no means agreed with the Obſervations, he changed his Opinion, and eſpouſed this, that the Line of the Planet's true Motion deſcribed equal Elliptical Areas in equal Times. But that Point from whence the Planet may be ſeen to proceed exactly with an equable Motion, is no where to be found in the Univerſe, unleſs we have a Mind to make it fluctuating. Yet no Point comes nearer to this equable Motion of the Planet, than that of the upper Focus of the Ellipſis. Nor is any one yet found that will deny, but that *Kepler*'s Area's may ſatisfy the Phænomena; but ſince neither he himſelf, nor any one ſince his Time, has been able to exhibit them by a direct Calculation, ſome have been apt to blame *Kepler*, as laying too great a Streſs upon Phyſical Cauſes, and thereby departing from Geometry. As though Phyſical Cauſes can be contrary to Geometry, or as if the Problem is leſs Geometrical which is thus propoſed, without making any mention of Phyſical Cauſes: *The Area of the Trilinium being given, which is comprehended between the Lines of the Apſids, of the true Motion, and alſo the Elliptical Arch; to find the Angle at the Sun.* They that object to this, a departing from Geometry, may take *Kepler*'s Anſwer; Let them go and ſolve the Problem.

But though *Kepler* was ſo ſcrupulous, as not to give up an Hypotheſis which, he was fully perſuaded, obtained in Nature, yet it may be free for others to make a Tryal, whether any other Way may be found, by which the firſt Inequality of the Planets may be inveſtigated by a direct *Calculus*. Therefore the learned *Bullialdus* undertook to ſearch out by Geometrical Reaſonings, in what Path, and with what Degrees of Intenſion and Remiſſion the Planets might be urged, ſo that by the Rule of equable Motion which was aſſumed by the Aſtronomers before *Kepler*, we might be brought to that Inequality we behold. The Monuments of that great Man are ſtill in being, whence the Lovers of Aſtronomy may be informed of his whole Method. The Right Rev. *Seth Ward* has embraced the ſame, and is the firſt that has ſhewed, that it performs the ſame Thing as a Line of equable Motion revolving about the other Focus of the Ellipſis. He has alſo ſupplied it with a direct Method of Calculation, the ſame which we have recited a little before. So that nothing ſeems to be farther required, than that *Urania* would proſper ſuch happy Attempts. And in her Name it was that the illuſtrious Count *Pagan*, in a Treatiſe publiſhed two Years after, of the ſame Tenour, ventured ſo far to aſſert the Truth of this Hypotheſis, as to chuſe to aſcribe to the Ignorance of Aſtronomers that Diſcrepancy that was diſcovered about the Octants. But the learned *Bullialdus* judged it ſafer to hear Aſtronomy herſelf, ſpeaking as it were by the Mouth of Obſervers; and by his ſecond Endeavours he exterminated that Diſcrepancy, by a certain Limitation which he annexed to his former Diſcoveries. Whence we may finally conclude, that this Hypotheſis, on which Mr. *Caſſini* has founded his Inveſtigation of the Apogee's and Excentricities, is ſo far removed from the Truth, as to make *Bullialdus*'s Limitation neceſſary; and from that Defect proceeds the Diſagreement between the Calculation and the Heavens themſelves.

By Mr. Edm. Halley.
Aug. An. 1676.
N. 128. *p.* 683.

2. The Annual Motion of the Earth through the Ecliptic occaſions an Optical Inequality in the Motions of the other Planets, which is well known to Aſtronomers that embrace the *Copernican* Syſtem, by the Name of the

Parallax of the Orbit. And this Inequality, which without much Labour is derived from Obſervations, I lay down as the ſtrongeſt Foundation of the following Method. In which, beſides Obſervations, nothing elſe is ſuppoſed, than that the Orbits of the Planets are Ellipſes, and that the Sun is placed in the common Focus of all the Orbits; and laſtly, that the Periodical Times of them all are ſo well known, that no Error can be perceived at leaſt in two or three Revolutions. Theſe Things being granted, firſt I muſt begin with the Motion of the Earth, which is neceſſarily required to account for the Motions of the other Planets.

Fig. 121. Let S be the Sun, A B C D E the Orbit of the Earth, P the Planet *Mars*, which for many Reaſons is chiefly to be preferred for this Purpoſe. And firſt let the true Time and Place be obſerved, when *Mars* is oppoſite to the Sun; for then the Sun and Earth coincide in the ſame Right Line with *Mars:* or, if he has Latitude, (which almoſt always happens) with that Point where a Perpendicular from *Mars* falls upon the Plain of the Ecliptic. Thus in the Scheme S, A, and P are in a Right Line. Again, after 687 Days *Mars* returns to the ſame Point P where he was oppoſed to the Sun in the former Obſervation; but as the Earth does not return to A till after 730½ Days, in B it regards the Sun in the Line S B, but *Mars* in the Line B P; and the Longitudes of the Sun and *Mars* being obſerved, all the Angles of the Triangle P B S are given, and P S being ſuppoſed 100000, in the ſame Parts the Length of the Line S B is found. In like manner, after another Period of *Mars*, the Earth being in C, the Line S C is found, and ſo likewiſe the Lines S D, S E, S F; and the Differences of the obſerved Places of the Sun are the Angles at the Sun A S B, B S C, C S D, D S E. Thus we come at laſt to this Geometrical Problem, *Three Lines being given both in Length and Poſition, meeting in one of the Focus's of an Ellipſis, to find the Length of the tranſverſe Diameter, and the Diſtance of the Foci.* The Reſolution of this Problem may be extended alſo to the other Planets, if, after the Theory of the Earth's Motion is known, we find out (according to the Method propoſed by the Right Rev. the Biſhop of *Salisbury* in his Geometrical Aſtronomy, *Lib.* 2. *Part* 2. *Cap.* 5.) three Diſtances of any Planet from the Sun with its Poſitions. Now becauſe the Biſhop ſuppoſes the Planet ſo to move in its Orbit, that in equal Times it compleats equal Angles at the other Focus of the Ellipſis, and upon this Suppoſition builds his Calculation; it will not be improper to ſhew how the ſame Thing may be done without that Suppoſition; which Obſervation obliges us to reject.

Fig. 122. Let S be the Sun, A L B K the Earth's Orbit, P the Planet, or the Point in the Plain of the Ecliptic marked out by the Perpendicular from the Planet; A B the Line of the Earth's Apſids. Firſt let the Longitude and Latitude of the Planet be obſerved, and alſo the Sun's Longitude from the Earth in K; and after a Period of the ſame Planet, the Earth being in L, let two Poſitions of the Planet and the Sun be again obſerved as before. Now from the obſerved Longitudes of the Sun and the Earth's Aphelion are given the Angles A S K, A S L, and conſequently the Sides S K, S L. Now in the Triangle K S L are given the Sides K S, L S, and the Angle K S L;

required the Side KL, and the Angles SKL, SLK. Then in the Triangle KLP are given KL, KPL the Difference of the obſerved Longitudes of the Planet, and PKL the Difference of the Angles SKL laſt found, and of SKP the Planet's Elongation from the Sun in the firſt Obſervation; required LP. Then in the Triangle LSP are given the Sides LS, LP, and the Angle PLS, the Planet's Elongation from the Sun in the ſecond Obſervation; required the Side SP and the Angle LSP; which being found it is, as SP to LP, ſo is the Tangent of the Latitude obſerved at L, to the Tangent of Inclination or Latitude to the Sun; and as the Co-ſine of Inclination to Radius, ſo is SP, the curtated Diſtance, to the true Diſtance of the Planet from the Sun. So at laſt we have found the deſired Poſition and Longitude. It remains to ſhew, how from three given Diſtances from the Sun, with the intercepted Angles, to find the middle Diſtance with the Excentricity of the Ellipſis.

Let S be the Sun, and SA, SB, SC, three Diſtances in due Poſition, *Fig.* 143. and drawing AB, BC, let AB be the Diſtance of the Foci in an Hyperbola, and $SA - SB = EH$ be the tranſverſe Diameter. Theſe Things being premiſed, let that Hyperbolical Line be deſcribed, whoſe internal Focus is at the Point A, and the Extremity of the longer Line SA. In like manner let B, C, be the Foci of another Hyperbola, whoſe Diameter is $SB - SC = KL$; from which let the Hyperbolical Line be deſcribed, having its internal Focus at the Point B. I ſay theſe two Hyperbola's thus deſcribed will interſect one another in the Point F, which is one of the Foci of the Ellipſis required; and drawing the Line FA, FB, or FC, either $SA + FA$, $SB + FB$, or $SC + FC$, will be equal to the tranſverſe Diameter, and SF is the Diſtance of the Foci. Theſe being ſuppoſed, the Deſcription of the Ellipſis will be very eaſy. But whereas the Reaſon of this Conſtruction may not be obvious to every one, it will not be improper to give ſome Illuſtration of it. I ſay therefore, that from the moſt known Property of the Ellipſis, 'tis $SB + FB = SA + FA$, and tranſpoſing the Parts of the Equation $FB - FA = SA - SB$, ſo that though we did not know FB and FA, yet their Difference is equal to $SA - SB$, that is, to EH. And ſince it is from the Nature of the Hyperbola, that it has any two Lines from their Foci to any Point in the Curve, conſtantly differing by the Quantity of the tranſverſe Diameter; it is plain that the Point F is ſomewhere in the Curve of the Hyperbola, whoſe tranſverſe Diameter is equal to $SA - SB$, and its Foci A and B. In a like manner it may be demonſtrated, that the Point F is in an Hyperbola, whoſe Diameter is $SB - SC$, and its Foci B and C. Therefore it muſt neceſſarily be in the Interſection of thoſe two Hyperbola's, which, as they interſect one another in one Point only, plainly ſhew where the other Focus of the Ellipſis required muſt be.

Now that the ſame Thing may be performed Analytically, ſuppoſe it done, and let $FB = a$, $Sa - SB = FB - FA = b$, $AB = c$, $SB - SC = FC - FB = d$, $BC = f$, and let the Sine of the Angle $ABC = S$, and the Coſine of the ſame $= s$.

Then $c \,.\, b :: 2a - b \,.\, \frac{2ab - bb}{c}$; and $\frac{2ab - bb + cc}{2c} = BD$, by 36. 3. *Eucl.* And $f \,.\, d :: 2a + d \,.\, \frac{2ad + dd}{f}$; and $\frac{ff - 2ad - dd}{2f} = BG$, by the ſame. Now to abbreviate the Calculation, make $\frac{cc - bb}{2c} = g$, and $\frac{h}{c} = h$; alſo make $\frac{ff - dd}{2f} = k$, and $\frac{d}{f} = l$. Then BD $= g + ha$, and BG $= k - la$. And becauſe in every $\left\{\begin{matrix}\text{Obtuſe-angled}\\ \text{Acute-angled}\end{matrix}\right\}$ Triangle the Square of the Baſe is equal to the $\left\{\begin{matrix}\text{Sum}\\ \text{Difference}\end{matrix}\right\}$ of the Squares of the Sides, and of the double Rectangle of the Sides into the Coſine of the contained Angle; it will be $gg + 2gha + hhaa + kk - 2kla + llaa + 2gks - 2glsa + 2khsa - 2hlsaa$ is equal to the Square of DG. But DG is equal to the Sine of the Angle DFG or DBG drawn into FB or a; (for FBDG is a Quadrilaterum inſcribed in the Circle whoſe Diameter is FB) therefore $SSaa = gg + 2gha + hhaa + kk - 2kla + llaa + 2gks - 2glsa + 2khsa - 2hlsaa$: Which Equation is eaſily reſolved, ſince it does not exceed an affected Quadratick, and is always compoſed of thoſe Squares and Rectangles. Yet the Signs + and —, becauſe of the different Conſtitution of the three Lines, muſt be applied to the Rectangles with good Caution.

The Obliquity of the Ecliptick, from the Obſervations of the Ancients; by Dr. Ed. Bernard. Sept. An. 1684. N. 163. *p.* 721.

XVII. 1. *An.* 230. before the Birth of *Chriſt*, *Eratoſthenes* found the Obliquity of the Ecliptic, 23° . 51′ . 19″ . 31‴ . 5⁗.

For according to him the Diſtance of the Tropicks was $\frac{11}{83}$ of the Circle of the Meridian, or 47° $\frac{59}{83}$. *Ptolemy*'s *Magna Syntaxis*, *p.* 18. 21. Wherefore *Eratoſthenes*'s Obliquity was leſs than *Ptolemy*'s by only $\frac{3}{83}$ of a Second, which is a very contemptible Difference.

Eratoſthenes according to *Cleomedes* (as derived by *Ricciolus*) makes it 23 Degrees, and 46 Minutes beſides.

Eratoſthenes, as corrected by *Ricciolus*, 31′, 5″. (above 23°).

Hipparchus (140 Years before *Chriſt*) has retained the Obliquity of *Eratoſthenes*. *Ptol. Mag. Syntax*, *p.* 18. and *p.* 60.

Theo, as taken more exactly, 51′ . 19″ . 31‴ . 5⁗.

Yet the *Chovareſmic* Tables, compiled 830 Years after *Chriſt*, exhibit the Canonic Obliquity of the *Alexandrians*, according to a *Latin* Manuſcript of D. *Hatton*, 51′.

Pytheas Maſſilienſis, 324 Years before *Chriſt*, (in *Ricciolus*) 52′ . 41″.

Ariſtarchus, before *Chriſt* 280 Years, by the illuſtrious *Savil*'s Computation, 51′ . 20″.

Ariſtarchus, as *Ricciolus* has deduced it, 30′ . 00″.

Strabo the Geographer, *p.* 93. 30 Years after *Chriſt*, makes it $\frac{4}{60}$ of a Circle, or above 23°, yet 1 more, or 60′.

Nor otherwiſe *Geminus*, in *Chriſt*'s Time, *Chap.* iv. *Elem. Aſtron.* And *Tatius*,

Tatius, *cap.* 26. and *Proclus* of the Sphere. And the Aſtrologers in *Noddamus* the *Arabian*, *Abraham Abeneſdra*, *&c.*

Noddamus the Aſtronomer, who flouriſhed about *An. Dom.* 1200, takes Notice, that the Obliquity was never obſerved greater than 24 Degrees, nor leſs than 23° . 33′, and yet that it continually decreaſes.

Claud. Ptolemeus, after *Chriſt* 140. having often tried it with his Ring and his Table, always found it nearly the ſame with that of *Eratoſthenes*, 51′ . 20″.

For the Diſtance of the Tropicks was between $47\frac{2}{3}$ and $47\frac{3}{4}$. But he choſe for his little Table 47° . 42′ . 40″ . *Mag. Syntax*, *p.* 18. 20, 21, and 27. He took nearly the middle, 23° . 51′ . 20″; nor otherwiſe in his Hypotheſes of the Planets. But *Theo* in his ſhorter Tables, for the Sake of Expedition, omits the Seconds. But *Ricciolus* is miſtaken, when from the Climate of *Rhodes* he deduces the Meaſure of the Obliquity for *Ptolemy* to be 23° . 30′.

Pappus of *Alexandria*, (390 Years after *Chriſt*) *l.* 6. *Theor.* 35. by *Ricciolus* 30′.

Pappus, as *Fr. Commandinus* computes it, 50′ . 00″ . 00‴.

Theo, (after *Chriſt* 370 Years) *p.* 88. more accurately, 51′ . 20″ . 00‴.

Elſewhere in a round Number, as *p.* 57. and frequently in his ſhorter Tables not yet publiſhed, 51′ . 00″ . 00‴.

Almamon the Prince, *An. Chriſt.* 825. of the *Hegira* 210, makes it 23° . 35′. *Grav. p.* 44. from *Ebn-Shatir* of *Damaſcus*, MS. *Selden.* many Aſtronomers aſſiſting him. For ſo *Abeneſdras* relates, MS. *Lat.* in the Archives of *Digby*. Alſo an uncertain Aſtronomer in *Archiv. Seld.* affirms, that *Jahia Ebn Albimanſur*, with many other Philoſophers, in the Time of *Almamon*, found the Obliquity to be by Obſervation 23° . 35′.

The moſt learned *Al Noddam*, in his Commentaries upon the Aſtronomical Works of *Hoſein Niſaburienſis*, ſays the ſame Thing of the Obſervations of *Almamon*. To which he adds, that in the ſame Age *Beni Muſa* often obſerved the ſame Meaſure to be 23° . 35′. in the Neighbourhood of *Bagdad*. MS. *Arab. Coll. St. John Oxon.* And this was approved by moſt of the Aſtronomers that ſucceeded. At leaſt *Alferganus* acquieſces in it in his Aſtronomy, *Chap.* 5.

Mohammed Ebn Gaber Al Batanius (or *Bategnius*) at *Racca*; according to *Ricciolus*, *An. Dom.* 880. *Savil.* 890. *Gravius p.* 44. 882. of the *Hegira*, 269. He died in the Year of the *Hegira*, 317. *An. Dom.* 929. In the Hiſtory of *Abolfaragus*, *p.* 191. *Obliq.* 35′ . 00″.

Al Batanius in this Affair makes no Scruple of preferring his own Obſerations before the Words of *Ptolemy*, *cap.* 4, and ſays, that being aſſiſted ith a very long *Albidada*, or Parallactical Ruler, in Reſemblance of thoſe of *tolemy*, with great Care and Diligence he found at *Racca*, that the Diſtance f the Tropics was 47° . 10′. (that is, 59° . 36′ — 12° 26′) and therefore he Latitude of *Racca* was 35°; which yet *Ulocbeg* makes 36° . 10′; *Schickrd* in *Curtius*, (*p.* 33.) and *Ricciolus* make it 36°.

Thabet Ebn Corra, (with *Ricciolus*, *An. Dom.* 1210. more truly 901, in ne 289. of the *Hegira*) found the Obliquity to be 33′ . 30″.

Abul Hoſein Ebn Suphi, 35′ . 00″.

Abul Waffi Albuziani, and *Abn Hamed Saganienſis*, a very ingenious Perſon, (*A. D.* 987. *Heg.* 377.) at *Bagdad* found the Obliquity to be very near 35′.

So a *Perſian* Author in the Archives of *Selden*, 35′.

In like manner the *Perſic* Tables of *Cryſococca*, 35′.

Al Batrunius Abul Riban, (*A. D.* 995. of the *Hegira* 385. *Abolfaragius* puts him at 463. of the *Hegira*, or *A. D.* 1070.) made uſe of a Quadrant whoſe Radius was 15 Cubits (*Grav.* p. 54. from an *Arabic* Book of *Birunius*) 35′.

But *Abu Joafer Alchazan*, with his Aſſociate *Abufaldus Harwanenſis* at *Edeſſa*, and others of that Age, (*A. D.* 970.) obſerved that the Obliquity did not quite come up to 23° . 35′, but was ſomething leſs.

Almæon Son of *Almanſor* (*A. D.* 1140. *Ricc.*) 33′ . 30″. But according to *Clavius* and *Mæſtlin* he made it 33′ only.

Iſmael Abulfeda Prince of *Hama*, (*A. D.* 1311. of the *Hegira* 711.) in his Manuſcript Tables *Arab. Coll. St. John*, retains 35′ . 00″ . perhaps on the Authority of *Almamon*.

Prophatius the *Jew*, (*A. D.* 1300. *Ricc.* 1303. *Mæſtlin* in *Curtius* p. 40. 230 Years after *Arzachel*, ſaith *Copernicus*) and in *Ricciolus*, and a Manuſcript of *Merton* College, 32′ . 00″.

Abu Mahmud Al Chogandi, (*A. D.* 992. of the *Hegira* 382.) in the Time of *Fecrodaula*, with a Sextant whoſe Radius was 40 Cubits, and the Limb was divided into Seconds, found the Obliquity leſs than it had ever been taken by any of his Anceſtors; that is, 32′ . 21″.

Hence *Noddamus* the Aſtronomer affirms, (MS. *Coll. St. John*) that the Sun's greateſt Declination was hardly ever found leſs than 23° . 33′.

Arzachel of *Spain*, (*Grav.* p. 44. *A. D.* 1089. of the *Hegira* 482. *Ricc.* 1070. *Mæſtlin* in *Curtius*, p. 35. 1075. *Copernicus*, l. 3. c. 6. 190 Years after *Al Batanius*) has propoſed the Obliquity 23° . 33′ . 30″. So MS. *Coll. Mert. Oxon.* where it is ſaid, that there is a Difference of 17′ . 30″. between the Obliquity of *Ptolemy* and that of *Arzachel*.

At *Maraga* the moſt noble *Perſian Chojah Naſiroddinus Tuſenſis*, *A. D.* 1629. of the *Hegira* 668. (but *Grav.* p. 44. 1261. *Hegir.* 660.) obſerved the Obliquity moſt accurately, and found it 23° . 30′ . 00″.

This is the leaſt of the Sun's greateſt Declinations, that has been found to this Day, ſays the very learned Commentator upon the Aſtronomical Works of *Hoſein Niſaburienſis*.

Ebn Shatir Damaſcenus, MS. *Seld. A. D.* 1363. ſays that he corrected the Obliquity, not neglecting the Sun's Horizontal Parallax, which was found 2° . 59′. Hence the Sun's greateſt Declination, 23° . 31′ . 00″.

Olocbegus the Prince, *A. D.* 1437. *Hegir.* 841. with *Ali Cuſhgius* and other Aſtronomers, with very great Care and the largeſt Inſtruments, (ſee *Graves*, p. 44.) found the Obliquity 23° . 30′ . 17″. So MSS. *Coll. St. John* and of the *Savilian* Library. For *Selden*'s Manuſcript makes it 23° . 30′ . 27″.

Rabbi Moyſes Ben Maimon, the moſt learned of the *Jews*, ſays in *Jad. de Conſecratione Calendarum*, c. *ult. ſect.* 4. that the greateſt Obliquity of the Zodiac, *A. D.* 1174, was 23° . 30′, very near.

I muſt let you know, that I have conſulted ſcarce half of the Oriental Aſtronomers, whoſe Writings are preſerved in the Libraries of the Univerſity of *Oxford*. But from theſe Obſervations, and ſome others which I keep to myſelf from the Vulgar, I conclude, that the Obliquity of the Zodiac has always been the ſame from the Beginning of the World. For the latter Ages, as you may perceive, by the Aſſiſtance of better Inſtruments, have truly corrected the Error and Exceſs of the antient Aſtronomy.

2. Whether the Poles and Axis of the Earth be really fixed in the Globe, or ſubject to be transferred from Place to Place, is an old Enquiry, though now lately revived by Mr. *Hook*, in his ingenious Eſſays upon the great Mutations and Cataſtrophes which, in all appearance, have happened to the Earth's Surface. A neceſſary Conſequence of ſuch a Tranſlation of the Poles would be the Change of the Latitudes of Places, which would increaſe in thoſe Regions towards which the Poles approach, and decreaſe in thoſe from which they recede: and under the Meridian, 90 Degrees removed from that in which the Poles ſhift, the Latitudes continuing the ſame, the Meridian Line would only alter; but not two Places conſiderably differing in Longitude can be ſuppoſed, wherein, if there be any ſenſible Motion of the Poles, it ſhall not be perceived by the Alteration of the Latitude of one or both of them.

The Obliquity of the Ecliptick and Elevation of the Pole continue unaltered; by - - - - - - Nov. An. 1687. N. 190. p. 403.

The accurate M. *Wurtzelbaur* has lately furniſhed us with the Means of examining this Hypotheſis by Obſervation, having ſent us the Meridian Altitude of the Sun taken at *Nuremberg* about the two Solſtices in the Year 1686. *June* 10. he found the Meridian Altitude of the Sun 64° 2′ 20″, and the next Day 64° 2′ 25″. And on *Decemb.* 14. (three Days after the Solſtice, wherein the Sun was got two Minutes higher) he found the Meridian Altitude 17° 9′ 10″. wherefore the ſolſtitial Altitude was 17° 7′ 10″. Theſe Heights were taken by an Inſtrument of 6 Foot Radius of Braſs; and the Skill and Diligence of the Obſerver is not to be doubted.

To compare with theſe, I find among *Bernard Walther*'s Obſervations, made in the ſame City of *Nuremberg* two hundred Years before, *viz.* in the Year 1487. that the Meridian Altitude of the Sun in the Summer Solſtice was obſerved by the parallactick Inſtrument of *Ptolemy*, whereby the Chord of the Sun's Diſtance from the Zenith was obſerved 44890 Parts of 100000 Radius; the ſame being obſerved by the Concurrence of the Obſervations of ſeveral Years both before and after. The Arch anſwering to this Chord gives the Sun's Diſtance from the Zenith 25° 56′ 30″, and conſequently the Meridian Altitude, its Complement to a Quadrant, 64° 3′ 30″. Again, the ſame Year 1487. the Chord of the Meridian Diſtance of the Sun from the Zenith, on the Day of the Winter Solſtice, was found 118790, confirmed likewiſe by many ſubſequent Obſervations; the Arch anſwering to this Chord is 72° 52′ 40″. and its Complement 17° 7′ 20″, the Meridian Height of the Sun in the Winter Solſtice.

Hence it appears, That the Solſtitial Heights were very nearly the ſame at *Nuremberg* 200 Years ago as now they are; that of the Summer Solſtice being but one Minute differing, the other only 10′; both which may poſſibly ariſe from the Defects of the Inſtruments of theſe Obſervers, being made with Plane Sights: But what I ſhall neceſſarily conclude from hence, is, That if there be ſuch a Motion of the Poles, it is either very ſlow, or elſe nearly at Right Angles to the Meridian of *Nuremberg*; in which latter caſe, the Latitudes of Places about *Tunking*, *Siam*, *Malacca*, and *Java*, on the one Side, and in our *American* Plantations of *New-England*, *Virginia*, *Jamaica*, &c. on the other, ought to change faſteſt: But I have never yet heard of any ſuch thing obſerved by any of our Navigators; whence, if there be ſuch a Change of the Earth's Poles, it muſt neceſſarily require a long time to become ſenſible.

Beſides, from theſe Obſervations, it appears, That the Obliquity of the Ecliptick has continued unaltered for theſe 200 Years laſt paſt; that is to ſay, that the Angle which the Earth's Axis makes with the Plane of the Ecliptick or Orb wherein ſhe moves annually round the Sun, has been without ſenſible Change in all that Time; which will be very hard to conceive, if we allow a Tranſlation of the Earth's Poles; for the Direction of the Axis being perfectly at Liberty, it muſt be purely caſual, if it ſo hit, that after ſuch Change, it make the ſame Angle with the Ecliptick as before.

A farther Argument of this Slowneſs of the Change of the Poles, is the Latitude of *Alexandria*, the Habitation of thoſe famous Aſtronomers of Antiquity, *Eratoſthenes*, *Timocharis*, *Hipparchus*, and *Ptolemy*; and for that Reaſon it may be concluded, that this, of all the Latitudes the Ancients have left us, ought to be one of the moſt Correct. This by *Ptolemy* is ſaid to be 30° 58′ North, (which he uſes in all his Computations in his *Almagiſt*, and ſeems derived from the Proportion of the Gnomon to its Equinoctial Shadow, as 5 to 3, but in his *Geography*, 31° juſt.) In the Year 1638, the curious and ingenious Mr. *Graves*, when he went to viſit the *Egyptian Pyramids*, of which he has given ſo good an Account, did, with a ſufficient Inſtrument, obſerve the Latitude of *Alexandria*, and found it 31° 4 or 6 Minutes more than it is reputed by *Ptolemy*, and before him by *Eratoſthenes*; ſo that in about 2000 Years, the Latitude of *Alexandria* has altered only a few Minutes; and ſo few, that the Accuracy of the Obſervations of the Ancients may well be queſtioned: But, both being granted, this Motion will amount to no more than a Degree in 20000 Years.

This is not ſaid with intent to invalidate what Mr. *Hook* hath from ſo good Grounds advanced, *viz.* That the Ball of the Earth, at leaſt the Fluids thereof, being neceſſarily of the Figure of a *Spheroides prolatus*, or flat Oval, whoſe ſhorteſt Diameter is the Axis, and greateſt Circle the Equinoctial; if the Poles be ſuppoſed changed, the Equinoctial will be ſo too; and conſequently the Water muſt riſe and cover thoſe Parts from which the Poles recede, and fall off and leave bare thoſe Places towards which the Poles approach. By this Means it may be accounted for, how ſuch ſtrange Marine things are found on the Tops of Hills, and ſo deep under Ground; and ſcarce any other Way. But from theſe, and the like Obſervations, it will follow, That if theſe Inundations are produced by any regular Motion of the Poles, it would require a prodigious number of Ages to effect the Changes we may be certain have been. Beſides, if the Acceſs and Receſs of the Sea were after ſuch a gradual manner, as when produced by ſuch an eaſy Tranſlation of the Poles, as can by Obſervation be admitted, thoſe Inundations could never be fatal to the Inhabitants; for that they would always give notice of their coming, ſo that the People might provide for their Safety. But the *Holy Scriptures*, and Pagan Tradition, do unanimouſly agree, That the laſt great *Deluge* was brought to paſs in a few Days, with no previous Notice; ſo that the Account we have thereof, could not, by this *Hypotheſis*, be made out, without the Suppoſition of a great and ſudden Alteration in the Poles of the Earth's Diurnal Revolution: For which, whether we ſhould have recourſe to the intelligent Powers, that firſt impreſs'd this whirling Motion on the Ball, or leave it to be performed naturally, by the caſual Shock of ſome tranſient Body, ſuch as a Comet, or the like, whereby the former Axis might be loſt, and a New Revolution produced, differing both in Time and Poſition from the Old; I ſhall not undertake to diſpute: Such a Suppoſition would include likewiſe a Change of the Length of the Year, and Eccentricity of the Earth's Orb; for which we have no ſort of Authority.

A ſuppoſed Alteration of the Meridian Line; by . . . . N. 255. p. 285. Jun. An. 1699.

1. As I was wondering how an ordinary Mathematician could miſs ſo eaſy a thing as the drawing a true Meridian, ſo far as in the inſtance of the old Meridian in the Church of St. *Potronio* in *Banonia*, which is found by M. *Caſſini* to vary 8 or 9 Degrees from the true Meridian of the Place; and in that of the Meridian of *Uraniburge*, which is found by M. *Picart*, and others, to vary 18′; I hit upon this Thought, that Meridians muſt needs vary. For you know, that (taking it for granted that the Earth moves, *&c.*) beſides the Diurnal and Annual Revolutions, there muſt be alſo a third to account for that ſlow Motion of the Fixed Stars upon the Pole of the Ecliptick, in about 25000 Years; which is ſolved by the Direction of the Earth's Axis from one Point to another of the polar Circle. And that Direction being nothing but a certain Wabble in the Earth's Motion, muſt needs make the Noon-ſhade of a Perpendicular not lie always in the ſame Line.

Conſidered by Dr. Wallis. *Ibid.* p. 289.

2. This being a new Suggeſtion, deſerves to be conſidered: For it is not probable that ſo careful a Man as *Tycho*, and thoſe concerned in the Church of St. *Petronio*, ſhould be ſo much miſtaken in the Meridian Line. But if there be ought of this Nature, it muſt ariſe from a Change of the Terreſtrial Poles (here on Earth) of the Earth's Diurnal Motion; (not of their pointing to this or that of the Fixed Stars:) For if the Poles of this Diurnal Motion remain fixed to the ſame Place on the Earth, the Meridians, (which paſs thro' theſe Poles) muſt remain the ſame.

The Parallax of the Sun; by Mr. Flamſteed. N 89. p. 1579. Nov. An. 1672. N. 96. p. 6100.

XVIII. I have had the good hap to meaſure the Diſtances of *Mars* from two Stars the ſame Night; whereby I find, that his Parallax was very ſmall, certainly not 30″: So that I believe the Sun's Parallax is not more than 10″.

To find the Sun's Ingreſs into the Tropical Signs; by Mr. Edmund Halley. N 215. *p.* 12. Feb. *An.* 1695.

XIX. It may perhaps paſs for a Paradox, if I ſhould aſſert, That it is an eaſier Matter to be aſſured of the Moments of the Tropicks, or of the Times of the Sun's Entrance into *Cancer* and *Capricorn*, than it is to obſerve the true Times of the Equinoctials or Ingreſs into *Aries* and *Libra*. But I here deſign to ſhew a Method to find the Moment of the *Tropicks*, capable of all the Exactneſs the moſt accurate can deſire; and that without any Conſideration of the Parallax of the Sun, of the Refractions of the Air, of the greateſt Obliquity of the Ecliptick, or Latitude of the Place; all which are required to aſcertain the Times of the Equinoctial, from Obſervation; and which, being faultily aſſumed, have occaſioned an Error of near three Hours in the Times of the Equinoctials deduced from the Tables of the Noble *Tycho Brahe* and *Kepler*, the Vernal being ſo much later, and the Autumnal ſo much earlier than by the *Calculus* of theſe famous Authors.

Now before we proceed, it will be neceſſary to premiſe the following *Lemmata*, ſerving to demonſtrate this Method; *viz.*

1. That the Motion of the Sun in the Ecliptick, about the Time of the *Tropicks*, is ſo nearly equable, that the Difference from Equality is not ſenſible, from 5 Days before the Tropick to 5 Days after, by reaſon of the nearneſs of the *Apogæon* of the Sun to the Tropick of *Cancer*.

2. That for 5 Deg. before and after the Tropicks, the Differences whereby the Sun falls ſhort of the Tropicks, are as the *Verſed Sines* of the Sun's Diſtance in Longitude from the Tropicks; which *Verſed Sines* in Arches under 5 Degrees, are beyond the utmoſt Nicety of Senſe, as the Squares of thoſe Arches. From theſe two follows a third;

3. That for 5 Days before and after the Tropicks, the Declination of the Sun falls ſhort of the utmoſt Tropical Declination, by Spaces which are in *Duplicate* Proportion, or as the Squares of the Times by which the Sun is wanting of, or paſt, the Moment of the Tropick.

Hence it is evident, That if the Shadows of the Sun, either in the Meridian, or any other Azimuth, be carefully obſerved about the time of the Tropicks, the Spaces whereby the tropical Shade falls ſhort of, or exceeds, thoſe at other times, are always proportionable to the Squares of the Intervals of Time between thoſe Obſervations and the true Time of the Tropick; and conſequently if the Line, on which the Limits of the Shade is taken, be made the Axis, and the correſpondent Times from the Tropick, *Expounded* by Lines, be erected on their reſpective Points in the Axis as Ordinates, the Extremities of thoſe Lines ſhall touch the Curve of a *Parabola*. Thus, *a*, *b*, *c*, *e*, being
Fig. 124. ſuppoſed Points obſerved, the Lines *a* B, *b* C, *c* A, *e* F, are reſpectively proportional to the Times of each Obſervation before or after the tropical Moment in *Cancer*.

This premiſed, we ſhall be able to bring the Problem of finding the true Time of the Tropic by three Obſervations, to this Geometrical one: *Having three Points in a* Parabola, A, B, C, *or* A, F, C, *given, together with the Direction of the Axis; to find the Diſtance of thoſe Points from the Axis.*

Of this there are two Caſes; the one, when the Time of the ſecond Obſervation B is preciſely in the Middle between A and C: In this Caſe, putting t for the whole Time between A and C, we ſhall have A c, the Interval of the remoteſt Obſervation A, from the Tropick, by the following Analogy.

As $2ac - bc$ to $2ac - \frac{1}{2}bc$, ſo is $\frac{1}{2}t$, or A E, to A c, the Time of the remoteſt Obſervation A, from the Tropick.

But the other Caſe, when the middle Obſervation is not exactly in the Middle between the other two Times, as at F, is ſomething more operoſe, and the whole Time from A to C, being put $= t$, and from A to F, $= s$, $ce = c$, and $bc = b$, the Theorem will ſtand thus, $\frac{ttc - bss}{2tc - 2bs} = \text{A}c$ the Time ſought.

To illuſtrate this Method of Calculation, it may, perhaps, be requiſite to give you one or two Examples.

Ann. 1500. *Bernard Walter*, in the Month of *June*, at *Nuremberg*, obſerv'd the Chord of the Diſtance of the Sun from the Zenith, by a large *Parallactick* Inſtrument of *Ptolemy*, as follows;

June	2.	45467.	and	*June*	8.	44975.
June	9.	44934.		*June*	12.	44883.
June	16.	44990.		*June*	15.	44990.

In both theſe Caſes the middle Term is exactly in the middle between the Extremes, and therefore in the former three, $ac = 533$, $bc = 477$, and t, the Time between, being 14 Days, by the firſt Rule, the Time of the Tropick will be found by this Proportion; as 589 to $827\frac{1}{2}$, ſo $\frac{1}{2}t$, or 7 Days, to 9 Days, 20^h. 2'. Whence the Tropick, *An.* 1500. is concluded to have fallen *June* 11. 20^h. 2'. In the latter three ac is $= 107$, and $bc = 15$, and the whole Interval of Time is 8 Days $= t$; whence, as 199 to $206\frac{1}{2}$, ſo is 4 Days to 4^d. 3^h. 37'; which, taken from the 16th Day at Noon, leaves 11^d. 20^h. 23'. for the Time of the Tropick, agreeing with the former to the third Part of an Hour.

Again; *Ann.* 1636. *Gaſſendus* at *Marſeilles* obſerved the Summer Solſtice by a Gnomon of 55 Feet high, in order to determine the Proportion of the Gnomon to the Solſtitial Shade, and he hath left us theſe Obſervations, which may ſerve for an Example for the Second Rule.

June 19	*St.* N. Shadow	31766	Parts, whereof the Gnomon was 89428.
20		31753	
21		31751	
22		31759	

Theſe being divided into two Setts, of three Obſervations, each, *viz.* The 19th, 20th, and 22d, and the 19th, 21ſt, and 22d, we ſhall have in the firſt three, $c = 13$, and $b = 7$, $t = 3$ Days, $s = 1$; and in the ſecond, $c = 15$, and $b = 7$, $t = 3$, and $s = 2$. Whence, according to the Rule, the 19th Day at Noon the Sun wanted of the Tropick a Time proportionate to one Day, as $ttc - ssb$ to $2tc - 2bs$; that is, as 100 to 64 in the firſt Sett, or 105 to 62 in the ſecond Sett; that is, $1^d. 17^h. 15'$. in the firſt, or $1^d. 17^h. 25'$. in the ſecond Sett: So that we may conclude the Moment of the Tropick to have been *June* $10^d. 17^h. 20'$. in the Meridian of *Marſeilles.*

Now that theſe two Tropical Times thus obtained, will be found to confirm each other's Exactneſs from their near Agreement, appears by the Interval of Time between them, *viz.* $1^d\ 2^h. 30'$. leſs than 136 *Julian* Years, whereof $1^d. 1^h. 8'$. ariſes from the Defect of the Length of the Tropical Year from the *Julian*, and the reſt from the Progreſſion of the Sun's *Apogæon* in that Time; ſo that no two Obſervations made by the ſame Obſerver in the ſame Place can better anſwer each other, and that without any the leaſt Artifice or Force in the Management of them.

What were the Methods uſed by the Ancients to conclude the Hour of the Tropicks, *Ptolemy* has no where deliver'd; but it were to have been wiſhed that they had been aware of this, that ſo we might have been more certain of the Moments of the Tropicks we have received from them; which would have been of ſingular Uſe to determine the Queſtion, Whether the Sun's *Apogæon* be fixed in the Starry Heaven; or, if it move, What is the true Motion thereof? It is certain, that if we take the Account of *Ptolemy*, the Tropick ſaid to be obſerved by *Euctemon* and *Meton*, *Junii* 27 *Mane*, *Ann.* 432. *ante Chriſtum*, can no ways be reconciled, without ſuppoſing the Obſervation made the next Day, or *June* 28 in the Morning. And *Ptolemy*'s own Tropick, obſerved in the 3d Year of *Antoninus*, *Ann. Chriſt.* 140. was certainly on the 23d, and not on the 24th Day of *June*, as will appear to thoſe that ſhall duly conſider and compare them with the Length of the Year deduced from the diligent and concordant Obſervations of thoſe two great Aſtronomical Genii, *Hipparchus* and *Albatani*; eſtabliſhed and confirmed by the Concurrence of all the Modern Accuracy. For the Obſervations give the Length of the Tropical Year, ſuch as to anticipate the *Julian* Account only one Day in 300 Years; but we are now ſecure, that the ſaid Period of the Sun's Revolution does anticipate very nearly 3 Days in 400 Years; ſo that the Tables of *Ptolemy*, founded on that Suppoſition, do err about a whole Day in the Sun's Place for every 240 Years. Which principal Error in ſo fundamental a Point, does vitiate the whole Superſtructure of the *Almageſt*, and ſerves to *convict its Author of want of Diligence, or Fidelity, or both.*

But to return to our Method: The great Advantage we have hereby, is, That any very high Building ſerves for an Inſtrument, or the Top of any high Tower or Steeple, or even any high Wall whatſoever, that may be ſufficient to intercept the Sun, and caſt a true Shade, and make the Spaces large and fair; though the Heighth and Diſtance of the Building, and Poſition of the Plane upon which you receive the Shade, and of the Line on which you meaſure the Spaces, be not exactly known. But it is convenient that the Plane on which you take the Shade be not far from perpendicular to the Sun, at leaſt not very oblique, and that the Wall which caſts the Shade, be ſtreight and ſmooth at the Top, and its Direction nearly Eaſt and Weſt. The principal Objection is, That the *Penumbra*, or Partile Shade of the Sun, is, in its Extremes, very difficult to diſtinguiſh from the true Shade; which will render this Obſervation hard to determine nicely. But if the Sun be tranſmitted through a Teleſcope, after the Manner uſed to take his *Species* in a *Solar Eclipſe*, and the upper Half of the Object-Glaſs be cut off by a Paper paſted thereon, and the exact upper Limb of the Sun be ſeen juſt emerging out of it, or rather continging the Species of the Wall, (the Poſition of the Teleſcope being regulated by a fine Hair extended in the Focus of the Eye-Glaſs) I am aſſured, that the Limit of the Shade may be obtained to the utmoſt Exactneſs. I ſhall only further advertiſe, That the Winter Tropick by this Method may be more certainly obtained than the Summer's; by reaſon that the ſame Gnomon does afford a much larger Radius for this manner of Obſervation.

XX. I have found it neceſſary to make new Solar Numbers, becauſe in my old I have neglected to apply Refractions in all the Altitudes above 30 Degrees; wherein yet Reaſon and ſome little Experience hath ſhewed me, they are not inſenſible. I found *S. Caſſini*'s Obſervations, which I took from *Ricciolus* his *Aſtronomia Reformata*, much more accurate than *Tycho*'s, and therefore ſought out Numbers that might anſwer them. The *Apogæum* I found it neceſſary to promote 44 Minutes; ſo that *Anno Ineunte* 1655, it might be in ♋ 7. 30. 00. and to make the greateſt Equation only $1^\circ. 54'. 13''$. whereby I found the *Phænomena* would be anſwered much more accurately than I expected, and as near, all things conſidered, as I could deſire.

The Solar Numbers corrected; by M. J. Flamſteed.
N. 110. *p.* 220.
Jan. An. 1675.

But ſtill I was uncertain, whether the Refractions in the ſaid *Caſſini*'s Tables were juſt Meaſures or not, and I had no Conveniencies for making Trial. At laſt I thought on this Expedient, which fully ſatisfied me; *viz.*

I conſidered, That if ſome of thoſe Obſervations of the Diſtances of ♀ from the ☉ by Day, and from the Stars in the Night preceding or following, were ſkilfully examined, they might ſhew me the true Quantity of the Equations of the Sun's Orb, or rather the Difference of his Mean and Equal Motion. I turned over his *Progymnaſmata*, and pitched on two: The firſt made *Ann.* 1585, *March* 5. $4^h. 42'$. and $7^h. 12'$. *poſt Meridiem*: whereby I found the ☉ at $4^h. 42'$. was $94^\circ. 47'$. in the Antecedence of the *Lucida Calcis* ♊; the ſecond made *Ann.* 1585, *September* 15. $5^h. 15'$. and $6^h. 55'$. *Mane.* Where-from (applying and conſidering the Refractions in both) I found the Sun at 6°. 35^{h}. to be 74°. 30′. in Conſequence of the *Lower Head* of ♊. The Difference of Longitude betwixt theſe two Stars is 17°. 59′: And therefore, now the Sun, in Conſequence of the *Lucida Calcis* ♊. 92°. 29′. So that the Sun's apparent Motion betwixt the Year 1582. *Mar.* 5. 4^{h}. 42′. and the Year 1585. *Septemb.* 15. 6^{h}. 55′. *Manè*, (beſides the whole Revolutions) was 187°. 16′. but the Mean Motion is 191°. 2′. greater than the Apparent by 3°. 46′. which parted in proportion to the Equation of the Earth's Motion, collected for thoſe Times from my New Tables, gives the greateſt Equation of the Orb, 1°. 54′. 15″. conſenting, to my wonder, without any wreſting of the Obſervations, with that, which I deduced from *Caſſini*'s correct Meridional Altitudes.

The Sun's Motion, by the Tables which I now uſe, grounded on this Equation, is leſs than *Tycho*'s by no leſs than 9. That great Equation made him commit no ſmall Errors, and put him upon ſtrange Shifts to hide and ſolve them. So that all his Obſervations of the Planets in their Oppoſitions to the Sun are to be corrected before we attempt to repreſent them by Numbers; for his Errors in the Sun's Place made him err ſometimes 5 or 6 Hours in the Time of the Oppoſition; which muſt be reformed.

The Equality of Natural Days; by . . . . Profeſſor of Mathematicks at Seville.
N. 118. *p.* 426. Oct. An. 1676.

XXI. There is ſtill no ſmall Diſagreement among Aſtronomers, how great the Proſthaphæreſis is in equating Time; ſo that *Longomontanus* confeſſes, that he has found no greater difficulty in all his Aſtronomical Labours: Which when I took notice of, I conſidered in ſome Obſervations of the Heavens made by me, what might be the reſult. And having a very exact Pendulum Clock, with a Meridian Line artificially contrived, I obſerved the Sun's Tranſit over the Meridian every Day, with which my Clock agreed exactly, and if at any time it diſagreed, it very ſeldom diſagreed two Minutes, which I rectified when there was occaſion. So that perſiſting for three Years, and duly obſerving the Sun in the Meridian, whenever it could be done, (which in this part of the World is very often,) I was ſatisfied at laſt, that no natural Day, at this or that Seaſon of the Year, had a longer Revolution than another Day; whence I can boldly pronounce, that all natural Days are equal, and that if there ſhould be any minute Difference among them, it is quite inſenſible. This I was willing to make known, that I might releaſe Aſtronomers from this Scruple, which has perplexed ſo many, and ſtill perplexes them every Day; though the Equation of *Tycho*, becauſe of the Obliquity of the Ecliptick, muſt not be rejected.

Refuted; by Mr. Flamſteed. *Ibid.* *p.* 430.

2. How Natural Days can be equal, and yet *Tycho*'s Equation be admitted, I confeſs I cannot comprehend. For becauſe of the unequal Right Aſcenſions of equal Parts of the Ecliptick, one Equinoctial Day will be ſhorter than a Tropical Day, by 40 Seconds of an Hour; and 14 Tropical Days are longer than ſo many Equinoctial ones by the ſixth Part of an Hour, or ten Minutes. But this Difference I think muſt be greater than that the Profeſſor of *Seville* ſhould not perceive it in his Obſervations; and therefore in examining them, I imagine he admitted *Tycho*'s Equation.

Now if we ſuppoſe that the Revolutions of the Primum Mobile are all equal, (which was never denied by any that received the *Ptolemaick Hypotheſis*,) it will however neceſſary follow, that the Equation of Time is not to be rejected, which proceeds from the unequal Motion of the Sun in his Orbit; for whereas the Apogee advances daily only 57′. 10″. but the Perigee 61′. 15″; the Apogee will return ſooner by 16″ (or in the Time in which the Primum Mobile revolves 4′. 5″.) from Noon to the Noon of the Day following, than the Perigee can. Yet ſometimes the Equation proceeding from this Cauſe admits of a ſlower daily increaſe, that is, 8″ daily at moſt when it is ſwifteſt, and in 15 Days it hardly amounts to the Quantity of two Scruples, which I believe is taken away in his Clock, by that Correction of two Scruples which he mentions: Which therefore this learned Man ſhould conſider.

But laſtly, if he is more inclined to admit the *Copernican Hypotheſis* than the *Ptolemaick*, in that alſo the ſame Equations will follow, ſuppoſing the Revolutions of the Earth to be Iſochronal. Indeed I confeſs, that the Equation of Time proceding from the unequal Motion of the Sun in its Orbit, may be removed, and directed the contrary Way, if we ſuppoſe the Earth, or the Primum Mobile, to make unequal Revolutions: But if he will conſider the Nature of Time, he will eaſily underſtand, that it is impoſſible all its Inequality ſhould be removed.

XXII.
An Equation Table; by M. Caſſini.
N. 214. *p.* 248.

A Table of the *Equation* of Days, to be entered with the Place of the Sun.

	♈		♉		♊		♋		♌		♍	
G.	'Sub."		'Add."		'Add."		'Sub."		'Sub."		'Sub."	
0	7	45	1	11	4	3	0	59	5	43	2	8
1	7	26	1	24	4	0	1	15	5	45	1	53
2	7	7	1	37	3	56	1	29	5	46	1	37
3	6	48	1	49	3	51	1	42	5	47	1	21
4	6	29	2	1	3	45	1	54	5	48	1	5
5	6	10	2	12	3	39	2	6	5	48	0	48
6	5	51	2	23	3	32	2	19	5	48	0	30
7	5	31	2	33	3	25	2	32	5	46	0	12
8	5	11	2	43	3	17	2	44	5	44	0 Add	7
9	4	51	2	53	3	9	2	56	5	40	0	26
10	4	31	3	3	3	0	3	8	5	36	0	35
11	4	11	3	13	2	51	3	20	5	31	1	3
12	3	52	3	22	2	41	3	32	5	25	1	21
13	3	33	3	30	2	31	3	43	5	19	1	40
14	3	14	3	37	2	21	3	54	5	13	1	59
15	2	55	3	43	2	10	4	4	5	6	2	19
16	2	37	3	48	2	0	4	14	4	58	2	40
17	2	19	3	53	1	49	4	26	4	49	3	1
18	2	1	3	57	1	37	4	34	4	39	3	22
19	1	43	4	1	1	25	4	43	4	30	3	44
20	1	26	3	5	1	13	4	51	4	20	4	6
21	1	9	4	8	1	1	4	59	4	9	4	29
22	0	52	4	10	0	49	5	6	3	57	4	51
23	0	35	4	12	0	37	5	13	3	45	5	13
24	0	19	4	13	0	24	5	19	3	32	5	35
25	0	3	4	11	0	10	5	24	3	19	5	57
26	0 Ad	12	4	9	0 Sub.	3	5	29	3	5	6	19
27	0	27	4	8	0	16	5	33	2	51	6	41
28	0	42	4	6	0	29	5	37	2	37	7	2
29	0	57	4	5	0	44	5	40	2	23	7	23
30	1	11	4	3	0	59	5	43	2	8	7	44

A Table of the *Equation* of Days, to be entered with the Place of the Sun.

	♎		♏		♐		♑		♒		♓	
G.	'Add.''		'Add.''		'Add.''		'Add.''		'Sub.''		'Sub.''	
0	7	44	15	34	13	25	0	59	11	48	14	36
1	8	5	15	42	13	7	0	27	12	4	14	22
2	8	25	15	48	12	48	0	Sub. 5	12	19	14	21
3	8	45	15	53	12	29	0	35	12	35	14	13
4	9	5	15	57	12	10	1	4	12	50	14	4
5	9	25	16	1	11	50	1	33	13	5	13	55
6	9	44	16	5	11	30	2	3	13	19	13	46
7	10	3	16	7	11	10	2	32	13	32	13	37
8	10	22	16	8	10	49	3	1	13	44	13	27
9	10	41	16	9	10	28	3	29	13	55	13	17
10	11	00	16	9	10	6	3	57	14	5	13	7
11	11	19	16	9	9	42	4	25	14	14	12	56
12	11	38	16	8	9	17	4	53	14	22	12	44
13	11	57	16	7	8	51	5	20	14	29	12	32
14	12	15	16	5	8	25	5	48	14	35	12	19
15	12	33	16	1	7	58	6	15	14	40	12	6
16	12	50	15	56	7	31	6	42	14	45	11	52
17	13	7	15	50	7	5	7	9	14	50	11	37
18	13	22	15	44	6	38	7	34	14	54	11	21
19	13	36	15	37	6	12	7	58	14	56	11	4
20	13	49	15	30	5	45	8	21	14	58	10	46
21	14	2	15	22	5	19	8	45	14	59	10	28
22	14	14	15	13	4	52	9	8	15	00	10	10
23	14	26	15	3	4	26	9	31	15	00	9	52
24	14	37	14	52	3	58	9	53	15	00	9	34
25	14	47	14	40	3	30	10	13	14	58	9	16
26	14	57	14	27	3	1	10	32	14	55	8	58
27	15	7	14	13	2	31	10	51	14	51	8	40
28	15	16	13	58	2	1	11	10	14	47	8	22
29	15	25	13	42	1	30	11	29	14	42	8	4
30	15	34	13	25	0	59	11	48	14	26	7	45

Spots obſerved in the Sun; by Mr. Boyle. N. 74. *p.* 2216.

XXIII. *Ann.* 1660. *April* 27. About 8 of the Clock in the Morning there appear'd a Spot in the lower Limb of the Sun, a little towards the South of its Equator, which was entered about $\frac{1}{40}$ of the Diameter of the Sun itſelf, being about $\frac{1}{160}$ in its ſhorteſt Diameter, of that of the Sun; its longeſt, about $\frac{1}{40}$ of the ſame. It diſappeared upon Wedneſday Morning, *May* 9. though we ſaw it the Day before about 10 in the Morning, to be near about the ſame Diſtance from the Weſtward Limb, a little South alſo of its Equator, that it firſt appeared to be from the Eaſtward Limb, a little South alſo of its Equator. It ſeem'd to move faſter in the middle of the Sun than towards the Limb. It was a very dark Spot, almoſt of a Quadrangular Form, and was encloſed round with a kind of duſkiſh Cloud.

We firſt obſerved this very ſame Spot both for Figure, Colour, and Bulk, to be re-entered the Sun, *May* 25, when it appeared to be in a Part of the ſame Line it had formerly traced; and was entered about $\frac{4}{33}$ of its Diameter about 7 o' Clock in the Afternoon. At the ſame time there appear'd another Spot, which was juſt entered, and appear'd to be entered not above $\frac{1}{131}$ Part of the Sun's Diameter. It appeared to be longeſt towards the North and South, and ſhorteſt towards the Eaſt and Weſt. There ſeem'd to be diſpers'd about it divers ſmall Clouds here and there.

Spots obſerved in the Sun; by M. Picard. N. 74 *p.* 2238. N 75. *p.* 2253. *By M.* Caſſini. N. 74. *p* 2238. N. 75. *p.* 2250.

XXIV. 1. *Ann.* 1671. M. *Picard*, at Sea near the *Texel*, obſerved a Spot in the Sun from *Aug.* 3. *St. N.* to the 19th. It appear'd at firſt like the Tail of a *Scorpion*, but on the 19th Day reſembling a Melon-Seed.

2. *Aug.* 11. (*S. N.*) 1671. About 6 o' Clock at Night, M. *Caſſini*, with a three-foot Glaſs, remark'd in the Sun's Diſk, two Spots very dark, diſtant from its apparent Center about the third Part of his Semidiameter. They were in the Southern Part of the Sun, and their Elongation from the *Parallel of the Equator* paſſing through the Center of the Sun, was about $\frac{1}{60}$ part of his Diameter. The Time which lapſed between the Tranſit of the Sun's Center and that of the firſt of theſe Spots, was 22″. or 23″. the Semidiameter of the Sun then paſſing in 66″. The firſt of theſe Spots, being looked upon with a Teleſcope of 17 Feet long, appeared with a ſomewhat Oval Figure; the other was oblong and a little curved, like the *Hebrew* Letter *Jod*; and both together were ſurrounded by a *Corolla*, or Coronet, made up of little dark Points, which conformed itſelf to the Figure of the Spots, conſidered as they were join'd together.

Aug. 12. He perceived that they were nearer his Center. The Time between the Paſſage of the Sun's Center, and that of the interior Edge of the *Coronet* which encompaſſed them both, was then of 16″. At 7 o' Clock it was but of 15″. and the Southern Limb of the *Coronet* touched the Parallel paſſing through the Sun's Center. The firſt Spot was compoſed of two others almoſt round, and conjoined. The ſecond repreſented the Shape of a *Scorpion*. The third was round. And they were all three environed with a *Coronet*, which was compoſed, as we ſaid above, of abundance of little obſcure Pricks. This *Coronet* appear'd to be clearer than the reſt of the Sun, when looked upon with the ſhort Glaſs, and darker when ſeen with the long. Without it there were other Points, but very black ones; *viz.* Five near the round Spot on the South-Side, and another near the *Scorpion*'s Tail on the North-ſide.

At 8 o'Clock 48′. the Figure of the *Scorpion* was ſeen divided into ſeveral Pieces, as if the Tail and Arms had been cut off. The Northern Point appear'd no more, there remaining none but thoſe on the South-Side; and the Length of the Incloſure of all the *Spots*, comprehended between the Extremities, was of 1′. 15″. and the Breadth of 30″.

The ſame 12th Day at 6 in the Evening, he found no great Change in the firſt *Spot*. The other two were ſevered into 5 diſtinct ones, compaſs'd about with a *Coronet*, together with 5 black Points, which ſtood in a ſtrait Row, and after another Manner than they did in the Morning. From 6 at Night unto 7, the Time when the Paſſage of the *Sun*'s Center, and that of the *Coronet*'s Limb, was found to be at one time of 8″. and another time of 7½, the Diſtance of the *Spots* unto the Parallel paſſing through the Sun's Center, was near the ſame on the North-ſide with what it had been obſerved to be in the Morning on the South-ſide.

Aug. 13. Between the Riſing of the Sun, and half an Hour paſt 6 in the Morning, the Edge of the *Coronet* was turned to a Point on the South ſide, and was diſtant from the *Æquator* on the North-ſide, half a Minute; and there was but a Second of Time from the Paſſage of the Sun's Center unto the Paſſage of the ſame anterior Edge of the *Coronet*.

At 8 o'Clock 30′. the fore Edge was in the ſame Horary Circle with the Center of the Sun: So that in one Day and Half, theſe Spots have run thro' very near the third Part of the Sun's apparent Semidiameter, which giveth an Arch of 19 Degrees and a Half of the Circumference of the Sun's Body; and conſequently their *Diurnal* Motion about the Sun's Axe hath been of 13 Degrees; and the time of their *Periodical* Revolution, as far as we could conjecture in ſo little time, muſt be about 27 Days and a Half.

Aug. 14. At 6 in the Morning, there paſſed 15″. of Time between the Paſſage of the anterior Limb of the *Crown*, and the Paſſage of the *Sun*'s Center through the ſame Horary Circle: And then the Southern Limb of the *Crown* was a Minute and an half diſtant toward the North from the Parallel of the *Æquator*, paſſing through the ſame Center of the Sun. The Figure of the firſt Spot was almoſt the ſame with that of the Day before. The ſecond had taken the Form of an Heart, the Point of which was turned to the North-ſide, and its Baſe between the South and the Eaſt. Three other ſmall Spots, diſpoſed triangle-wiſe, ſtood over the ſaid Baſe, and were accompanied with two others upon a Line turned Southward. And they were all encompaſſed by a Crown running out into a Point on the South-ſide; and on the North-ſide, Eaſtward, it had an *Appendix*.

Aug. 15. At 6 in the Morning, there paſſed 27″. between the Paſſage of the anterior Limb of the Crown, and that of the Sun's Center through the ſame Horary Circle. The Southern Limb of the ſame Crown was two Minutes and an half diſtant from the Parallel of the *Æquator*, paſſing through

the

the Center of the *Sun*, whoſe Diameter paſſed in 2′. 9″. through the ſame Horary Circle. The firſt *Spot* had a little changed the Figure; the ſecond was quadrangular, longer from Eaſt to Weſt than from North to South: It appeared bigger than ordinary, and had withal on its ſides, within the compaſs of the *Crown*, three other ſmall *Spots*. There were alſo ſeen four more without the ſaid *Crown* on the South-ſide.

Aug. 16. At 6 in the Morning, there was 27″. between the Paſſage of the *Sun's* anterior Limb, and the Paſſage of the anterior Limb of the *Crown* through the ſame Horary Circle; and 38″. between the Paſſage of the anterior Limb of the *Crown*, unto the Paſſage of the *Sun*'s Center. The Southern Limb of the *Crown* was 3½′ off from the Parallel of the *Æquator*, paſſing thro' the Center of the *Sun* towards the North: And the Obſervation having been made yet more exactly at half an Hour paſt 7 of the ſame Morning, this Diſtance was found of 3′ 33″. The Figure of the firſt *Spot* in the Beginning of the Obſervation, differ'd not much from that of the precedent Day; but afterwards it was ſeen divided into two. The ſecond, which likewiſe ſeemed to be the ſame in the Beginning, was afterwards divided into three, accompanied with black and dark Points without the *Crown* on the South-ſide. The ſame Day at 6 o' Clock, and 15′. at Night, the Figures of theſe *Spots* were much changed. There were 5 *Spots* encloſed in the *Crown*; The two foremoſt were Part of that which had been ſeen in the Morning as one; the two others following thoſe two firſt, were Part of the ſecond in the Morning; and without there were 5 Points on the South-ſide, and two more a little further to the North; which Points were ranged as in another Area made up of other Points, ſo ſmall that they could ſcarely be perceived.

Aug. 17. in the Morning, immediately after the Riſing of the *Sun*, there appeared three very dark *Spots*, which form'd in a manner theſe Letters *F n J*, poſited from Eaſt to Weſt, and included in their wonted *Crown*, which ſtretched out, as 'twere, two Arms, or two Handles, one to the South, and the other to the North. There paſſed 18″. between the Paſſage of the foremoſt Limb of the *Sun*, and that of the foremoſt Limb of the *Crown*, and 47½″. between the Paſſage of the anterior Limb of the *Crown*, unto the Paſſage of the Sun's Center. The Southern Limb of the ſame *Crown* was diſtant 11′. 17″. from the *Parallel* that touched the *Sun* on the North-ſide, and 4′. 38″. from the *Parallel* that paſs'd through the Center.

Aug. 18. at 7 in the Morning, the *Spots*, which appear'd through ſome Clouds, had almoſt the ſame Shape with thoſe of the Day before, only with this Difference, that they were a little cloſer together, drawing from Eaſt to Weſt. There lapſed 13″. between the Paſſage of the anterior Limb of the *Sun*, and that of the anterior Limb of the *Spot*, through the ſame Horary Circle, and 52½″. of the foremoſt Limb of the *Spot* unto the Paſſage of the Center. The Southern Limb of the *Spot* was 9′. 13″. diſtant from the *Parallel* that touched the Northern Limb of the *Sun*, and 6′. 41″. from the *Parallel* that paſſed through his Center. At 5 o' Clock and 55′. at Night of the ſame Day, there lapſed 11″. between the Paſſage of the anterior Limb of the *Sun* through the ſame Horary Circle, and the Paſſage of the anterior

Limb of the *Crown*; and from thence unto the Paſſage of the *Sun*'s Center $54\frac{1}{2}''$. The Limb of the *Crown* next the *Parallel* paſſing through the Center of the *Sun*, was diſtant from the ſame *Parallel* $7'. 40''$.

Aug. 19. from 4 to 5 in the Evening, the *Spot* appear'd oblong near the *Sun*'s Circumference, from which it was diſtant about the Breadth of the ſame *Spot*.

Aug. 20. in the Morning, which was not the full ſeventh from the Day that they were arriv'd to the middle of the Diſk, they diſappear'd.

The apparent Velocity nigh the Center was ſuch, that if it had continued the ſame, the *Spots* would have arrived almoſt in 4 Days to the Limb of the Diſk; but in this Hypotheſis, that the *Spots* were adherent to the *Sun*'s Surface, or at leaſt very nigh to it, this apparent Velocity was to leſſen according as they ſhould remove from the Center; as hath come to paſs in effect. The Diminution of the Length of the Miſty *Crown* was in a manner proportionable to the Diminution of the apparent Velocity; ſince that, when this Crown was in the Middle, and in a Situation wherein its true Figure could be beſt ſeen, it appear'd oblong, and of the Form of an human Ear, its greateſt Diameter reſpecting Eaſt and Weſt; but being nigh the Limb, the ſame Diameter ſeemed to ſhorten; and having appeared greateſt in its firſt Situation, it appear'd leaſt in this, becauſe it was almoſt in a Circle that paſs'd through the Center of the Sun, whoſe equal Arches are by ſo much the more oblique, by how much they approach more to the Limb of his Diſk, and conſequently appear leſs, according to the Rules of *Opticks*: mean time, the Diameter, that was turned from South to North, apparently kept the ſame Bigneſs it had near the Center, becauſe it was in a Circle almoſt Parallel to the Horizon of the *Sun*, which formed the Repreſentation of its Limb, and whoſe equal ~~Arches (by the ſame *Optical* Reaſons) do not appear contracted~~.

3. Several curious Obſervers at *London*, have ſeen one of theſe *Spots* recurr'd to the Sun's Eaſtern Limb, about *Aug*. 25. *St*. *N*. as M. *Caſſini* predicted they ſhould return.

By ſeveral at London. N. 75, *p*. 5253.

4. *Aug*. 30. 1671. I ſaw a large *Spot* in the Center of the *Sun*'s Face about Noon.

By Dr. Hook. N. 77. *p*. 2295.

Sept. 1. At 3 o' Clock, I ſaw the ſame *Spot* moved about a Quarter of the Diameter of the *Sun* Weſtward; it conſiſted of one greater and two leſſer black *Spots*, with a *duſky Cloud* encompaſſing them: The Diameter of the whole Phænomenon was about $\frac{1}{70}$ of the Diameter of the *Sun*, and it was diſtant from the next adjoining Limb $\frac{18}{72}$ (that is, exactly one Quarter) of the Diameter of the *Sun*.

5. The ſolar *Spots* have been obſerved by us at *Hamburg*, from *Aug*. 26, Old Style, (almoſt the firſt Day on which they began again to appear) as far as *Sept*. 5. on which they approached very near to the Limb.

By Mr. Hen. Siferus. N. 78. *p*. 3033.

XXV. *An*. 1676. *Jun*. 26. *St*. *N*. we have in the Sun a pretty large *Spot*, which was in the middle of the *Sun* at Four in the Afternoon, with South Latitude $4\frac{1}{4}'$. Its Diſtance from the ſouthern Pole of the *Sun* by many Obſervations I determined to be $78\frac{1}{4}$ Degrees. If it is of a ſufficient Conſiſtence to compleat its Circle, its Reſtitution may be expected at the Middle, in the Evening of *July* 25, with greater South Latitude.

Spots obſerved in the Sun; by M. Caſſini. N. 127. *p*. 665.

XXVI. 1.

Spots obſerved in the Sun; by Mr. Flamſteed, *and Mr.* Halley. N. 128. *p.* 687.

	At Greenwich, A. D. 1676.			At Oxford, A. D. 1676.		
Jul. *O. S.*	*Times of Obſervations.*	*Longit. from the* Sun's *Center.*	*South. Lat.*	*Times of Obſervations.*	*Long.*	*South. Lat.*
	h. ′	′ ″	′ ″	h. ′	′ ″	′ ″
25				6 46 p. m. Con.	13 40	2 8
27	10 3 a. m. Con.	9 34	3 25			
28	4 51 ------	5 40	2 50			
29	10 31 a. m. - -	3 05	3 27	6 21 a. m. ---	3 55	3 22
	2 54 p. m. - -	2 25	3 10			
30	9 15 a. m. Ant.	0 37	3 33	7 20 a. m. ---	0 00	3 30
31				7 40 a. m. ---	3 36	3 28
Aug. 1.	9 25⅚ a. m. ---	6 48	4 09	7 3 a. m. ---	6 54	3 50
				5 6 p. m. ---	8 7	3 53
2	8 8 a. m. ---	9 49	3 55	7 16 a. m. ---	9 57	3 40
3	9 36 a. m. ---	82 28	3 27	5 9 p. m. ---	13, 15	3 56
	4 16⅔ p. m. --	12 55	3 58	6 2 p. m. ---	13 25	3 26
4	7 38 a. m. ---	14 02	4 4	7 33 a. m. ---	14 7	3 14
				4 54 p. m. ---	14 43	3 23

Mr. *Halley* ſaith, That he ſaw a Spot again on the fifth Day, 8h. 20′. *mane*, very near the Limb of the Sun, ſo that it appeared only as a fine Line; but by reaſon of its Fineneſs and the too great Height of the Sun, he could not take any Meaſures to determine its Place and Latitude by; and that while the Spot continued one, as it was *July* 25. he meaſured to the middle of it; as alſo when the Pieces were divided, but not far disjoined: Afterwards, when they were ſeparated conſiderably, he obſerved the middle of the bigger Spot, which was to the South, apparently I ſuppoſe, but really North: for ſo only his Obſervations will agree with thoſe of Mr. *Flamſteed* exactly.

Hence it ſeems very evident, (ſaith Mr. *Flamſteed*) that the Spot's Way was not inclined to the *Ecliptick* ſix or ſeven Degrees, as *Scheiner* and ſome others make it, but much leſs, by the joint Conſent of the Obſervations of both our Obſervers. Mr. *Halley* adds, That conſidering the Motion of the Spot croſs the Sun's Diſk, as both their Obſervations give it, it appears that the Latitude was not ſo great at its Entrance into the Sun as in the middle of him. And by Mr. *Flamſteed*'s Obſervation, it was greateſt on the firſt of *Auguſt*, and then again inclining towards the Ecliptick. If you grant this, it will follow, (infers Mr. *Flamſteed*) that the Sun's Axis was inclined to the Plane of the *Orbis magnus*; but the quantity of this Inclination muſt not be very great. The *Nodes* of the Sun's Equinox and Eclipitck he gueſſes to be not far from the beginning of *Cancer* and *Capricorn* ; and that from *Cancer* to *Capricorn* the Earth is North of the Sun's *Æquator* ; from *Capricorn* to *Cancer*, South of the ſame : And the Period of the Sun's Revolution in reſpect of the fixed Stars 25 Days $9\frac{1}{2}$ Hours, ſufficiently exact. Of which things theſe two Obſervators ſay they might have been more certain, had not the *Spot* in its Paſſage broken into ſo many Parts, and thoſe often varied their Poſitions to each other.

By Mr. Caſſini, Ibid. p. 689.

2. We here obſerved the Solar Spot, from *Aug.* 6. to 14. and by comparing our Obſervations we have found, that it was in the middle of its courſe in the apparent Diſk of the Sun about Midnight after the 8th Day of *Auguſt*, at the apparent Diſtance of 3 Minutes from the Center towards the South. It is ſeparated into ſeveral Parts, which are disjoin'd from one another at a conſiderable Diſtance, daily towards the North and South ; ſo that beſides their common Motion about the Sun's Axis, each of the Parts have a direct Motion of their own among one another. I think this Spot to be different from that which we obſerved in the preceding Month of *June*. For ſince that had the middle of its Courſe in the apparent Diſk of the Sun on the 28th Day of the ſame Month, it muſt have return'd nearly to the ſame Situation, if it had been ſtill in Being, in the Night following the 25 of *July* ; as is deduced both from its Velocity, the obſerved Time of its Appearance, and alſo from the Courſe of the other Spots which we ſaw to finiſh their Period about the Sun in the Space of $27\frac{1}{3}$ Days, or $27\frac{1}{2}$. Beſides its Path is different from the foregoing ; for the firſt was ſomething more remote from the Equator of the Spots than the latter. And this, if it had Conſiſtence enough, return'd to the middle of the Sun on *Sept.* 5. in the Morning.

Spots obſerved in the Sun ; by Mr. Flamſteed: N. 159. p. 535.

XXVII. *Ann.* 1684. *Apr.* 25. About an Hour before Noon I diſcover'd a large Spot entred within the Sun's Diſk, a little diſtant from his following Limb. Theſe Appearances, however frequent in the Days of *Scheiner* and *Galileo*, have been ſo rare of late, that this is the only one I have ſeen in his Face ſince *December* 1676. By the obſerved Meridional Diſtances of it, and the Sun's Southern Limb from the *Vertex* at Noon, I found it to have $3'$. $40''$. more North Declination than the Sun's Center ; and at 3^h. $35'$. after Noon, I meaſured its Diſtance from his next Limb $40''$.

Next Morning, *April* 26. I ſaw it more remote from his Limb, and by the Obſervations then made (at 8^h. *mane*) determined its Longitude from the Sun's Axis $66\frac{3}{4}$ Deg. and its Declination from the Solar Æquator $9\frac{4}{5}$ Deg. South. Whence, ſuppoſing the Revolution of any Point of the Sun to the ſame fixed Star to be performed in 25 Days, 6 Hours, the Angle of his *Equator* and our *Ecliptick* 7 Deg. and the Longitude of his Northern Pole ♍ 16 Deg. I deſigned the Line of its Way or Trace over the Sun, and the Points in it where the Spot would appear every Morning after at the ſame Hour, till its Egreſs on the 8th of *May* ; which I found altogether confirmed by ſuch Obſervationsas I made till then ; ſo that I had no reaſon to doubt of the Theory.

When the *Spot* was near the Middle of the Sun, it appear'd very broad, and almoſt Square, the *Nucleus* of the ſame Figure about 40″ Diameter; but when it was near the Limb, much narrower, and almoſt oval: It ſeemed to have Conſiſtence enough to endure a ſecond Return; if it ſhall, it will enter the viſible Diſk of the *Sun* on the 21ſt of *May* in the Evening, and in its Paſſage over him deſcribe a Line nearly ſtreight, with greater Latitude from the *Ecliptick*.

To find in what Proportion the Planets are enlightned by the Sun; by M. Auzout. N. 4. *p.* 68.

XXVIII. One of the Means uſed by M. *Auzout* to enlighten an Object in what Proportion one pleaſeth, is by ſome great Object-Glaſs, by him called a *Planetary* one, becauſe that by it he ſhews the Difference of Light, which all the Planets receive from the *Sun*, by making uſe of ſeveral Apertures, proportionate to their Diſtances from the Sun, provided that for every 9 Feet Draught, or thereabout, an Inch of Aperture be given for the Earth. Doing this, one ſees (ſaith he) that the Light which *Mercury* receives, is far enough from being able to burn Bodies, and yet that the ſame Light is great enough in *Saturn* to ſee clear there, ſeeing that (to him) it appears greater in *Saturn* than it doth upon the Earth, when it is over-caſt with Clouds; which (he adds) would ſcarce be believed, if, by means of the Glaſs, it did not ſenſibly appear ſo.

The Æquinoxes; by M. Wortzelbaur. N. 265. *p.* 623.

XXIX. The *Æquinoxes* of this Year, 1699, according to the Obſervations of M. *Wortzelbaur* at *Nurenberg*, happened *March* 9. 20^h. 35′. 27″. and *Sept.* 12. 10^h. 22′. 42″. which, by his Tables, ought to have been *March* 9. 20^h. 40′. 30″. and *Sept.* 12. 10^h. 32′. 52″.

To obſerve Solar Eclipſes; by Mr. Flamſteed. N. 55. *p.* 1104.

XXX. For the well obſerving of Solar Eclipſes, caſt the Species of the Sun through a good Teleſcope of a competent Length, on an extended Paper, placed behind the Eye-Glaſs ſo far, as that the ſaid Species may appear at leaſt 6 Inches over; then divide both his *Periphery* into 360 Degrees for the better obſerving the Inclination of the *Cuſps* of each Phaſis, and his Diameter into Digits, and their Parts, by concentrick Circles, for meaſuring the Quantities of the obſcured Parts.

An Eclipſe of the Sun, An. 1666. June 22. *at* London; *by Mr.* Willoughby, *Dr.* Pope, *Dr.* Hook, *and Mr.* Phillips. N. 15. *p.* 295.

XXXI. 1. The *Eclipſe* began at 5^h. 43′.

					h.	′
It was darkned	$\frac{3}{11}$ Diam. - - - at	6	00	5 Dig. - - - at	7	6
	4 Digit. - - at	6	7	4 Dig. - - - at	7	13
	5 Dig. - - - at	6	13	3 Dig. - - - at	7	20
	6 Dig. - - - at	6	21	2 Dig. - - - at	7	26
	7 Dig. - - - at	6	39	1 Dig. - - - at	7	32
	6 Dig. - - - at	6	57	0 Dig. - - - at	7	37

Its *Duration* hence appears to have been one Hour and 54′. Its greateſt *Obſcurity* ſomewhat more than 7 Digits. About the middle, between the Perpendicular and Weſtward Horizontal Radius of the Sun, viewing it thro' Mr. *Boyle*'s

Mr. *Boyle*'s 60 Foot Teleſcope, there was perceived a little of the Limb of the Moon without the Diſk of the Sun; which ſeemed to ſome of the Obſervers to come from ſome ſhining Atmoſphere about the Body either of the Sun or Moon. They affirm to have obſerved the Figure of this Eclipſe, and meaſured the Digits, by caſting the Figure through a 5 Foot Teleſcope on an extended Paper, fix'd at a certain Diſtance from the Eye-Glaſs, and having a round Figure; all whoſe Diameters were divided by 6 concentrick Circles, into 12 Digits.

2. The *Eclipſe* began at $5^{h}.\ 44'.\ 52''$. *mane*. It ended at $7^{h}.\ 43'.\ 6''$. So that its whole Duration was $1^{h}.\ 58'.\ 14''$. The greateſt Obſcuration was 7 *Dig*. $50'$. but it ſeemed to have been greater by $3'$. which Mr. *Payen* imputes to a particular Motion or Libration of the Sun's Globe, which entertained that Luminary in the ſame Phaſis for the ſpace of $8'$. and ſome Seconds, as if it had been ſtopped in the midſt of its Courſe, rather than to a tremulous Motion of the Atmoſphere, as *Scheiner* would have it. The apparent Diameters were almoſt equal; for in the Phaſis of 6 *Digits*, the Circumference of the Moon's Diſk paſſed through the Center of that of the Sun, ſo as that two Lines drawn through the two Horns of the Sun, made, with the common Semidiameter, two Equilateral Triangles. *At Paris, M. Payen. Ibid. p.* 296.

The Beginning and Middle of the Eclipſe happened to be in the North-Eaſtern Hemiſphere, and the End in the South-Eaſtern. The firſt Contact (as 'twere) of the two Diſks, was obſerved in the ſuperior Limb of the Sun's Diſk, in reſpect to the Vertical Line; and in the inferior, in reſpect to the Ecliptick. But the Middle and the End were ſeen in the ſuperior Limb, in reſpect both to the Vertical and the Ecliptick: And, what to M. *Payen* ſeems extraordinary, both the Beginning and the End of this Eclipſe happen'd to be in the Oriental Part of the Sun's Diſk.

3. The *Eclipſe* began about 5 o'Clock in the Morning, at $5^{h}.\ 15'$. The Sun's Altitude was $6^{\circ}.\ 55'$. *At Madrid; by the Earl of* Sandwich. *Ibid. p.* 296.

The Middle of it was at $6^{h}.\ 2'$. The Sun's Altitude $15^{\circ}.\ 5'$.

The End was exactly at $7^{h}.\ 5'$. The Sun's Altitude $25^{\circ}.\ 24'$.

The Duration $2^{h}.\ 4'$.

Thirty-ſeven Parts of the Sun's Diameter remained light, and 63 were darken'd.

4. In this *Eclipſe* it is chiefly obſervable, That the Semidiameter of the Moon, from the very Beginning to about 5 or 6 Digits of the increaſing Phaſis, was almoſt equal to the Semidiameter of the Sun; but, after the greateſt Obſcuration, when I again contemplated the Moon's Semidiameter, I found it $8''$. or $9''$. bigger than that of the Sun; ſo that the Semidiameter of the Moon was not always, during the *Eclipſe*, conſtant to itſelf. Of this Variation the excellent *Iſmael Bullialdus* hath alſo obſerv'd ſomething at *Paris*: *For he hath written to me*, That in this ſame *Eclipſe*, the Semidiameter of the Sun to the Semidiameter of the Moon was, as $16'.\ 9''$. to $16'.\ 22''$. but that in another Phaſis of 6 Digits, the Semidiameters appeared equal. *At Dantzick by M.* Hevelius. N. 19. *p.* 1347.

Ibid. p. 349. N. 21. p. 370.

Order of the Phaſes.	*Quantity of the Phaſes.*	*Time eſtimated by a Watch.*	*Time by a Sun-dial.*	*Altit. of the Sun.*	*Time corrected.*	*Obſervations.*
		h. ′ ″	h. ′ ″	° ′	h. ′ ″	
		5 51 11	5 51 00	17 45	5 53 12	That the Sun-dial does not intirely agree with the corrected Time, is to be imputed only to the Meridian Line.
		5 57 05	5 57 00	18 37	5 59 28	
		6 00 00	6 00 00	18 55	6 1 28	
	Beginning.	6 55 30			6 57 30	The Beginning happen'd about 79 Degrees from the Zenith towards the Weſt
1	$0\frac{3}{8}$ Dig.	8 57 30			6 59 30	
2	$0\frac{3}{4}$	7 0 23	7 0 0		7 2 23	
3	$1\frac{1}{8}$	7 2 30	7 2 0		7 4 30	
4	$1\frac{1}{2}$ Dig.	7 4 50	7 5 near.		7 6 50	
5	$1\frac{3}{8}$ nearly	7 10 57	7 10		7 12 57	
6	$3\frac{3}{8}$	7 14 59	7 15		7 16 59	
7	$3\frac{3}{4}$	7 17 50	7 18 near.		7 19 50	
8	4 Digits.	7 21 35	7 21		7 23 35	
9	$4\frac{2}{3}$	7 23 43	7 23 near.		7 25 43	
10	$5\frac{3}{4}$	7 27 53	7 28		7 29 53	Hitherto the Semidiameter of the Moon was equal to that of the Sun.
11	6	7 31 50	7 32		7 33 50	
12	$6\frac{3}{4}$	7 36 55	7 37		7 38 55	
13	$6\frac{7}{8}$ſomet.mo.	7 38 5	7 38		7 40 0	
14	$7\frac{1}{8}$	5 39 45	7 39		7 41 45	
15	$7\frac{1}{4}$ſomet.mo.	7 42 30	7 42		7 44 30	
16	$7\frac{1}{2}$	7 44 6	7 44		7 46 6	
17	$7\frac{2}{3}$.	7 46 0	7 46		7 48 0	
18	8 nearly.	7 48 25	7 48 near.		7 50 25	
19	$8\frac{1}{5}$	7 51 15	7 51		7 53 15	
20	$8\frac{1}{4}$ſomet.mo.	7 53 37	7 52		7 55 37	The greateſt Obſcuration was Dig. 8. 25′. The Hour 8. 2′.
21	$8\frac{3}{4}$	7 55 45	7 56 near.		7 57 45	
22	$8\frac{3}{8}$ſomet. leſs	7 59 5	7 59		8 1 5	
23	$8\frac{1}{5}$	8 6 30	8 6		8 8 30	

Ord. of the Phaſ.	*Quantity of the Phaſes.*	*Time eſtimated by a Watch.*	*Time by a Sun-dial.*	*Altit. of the Sun.*	*Time corrected.*	*Obſervations.*
		h. ′ ″	h. ′ ″	° ′	h. ′ ″	
24	$7\frac{2}{4}$	8 11 25	8 12		8 13 25	Here the Semidiameter of the Moon appeared greater by 8″ or 9″.
25	$7\frac{1}{4}$ nearly	8 17 30	8 18		8 19 30	
26	7 nearly	8 19 41	8 19		8 21 40	
27	$5\frac{7}{8}$	8 28 8	8 28		8 30 8	
28	$5\frac{1}{2}$ nearly	8 30 14	8 30		8 22 14	
29	$4\frac{3}{4}$	8 36 25	8 36		8 38 25	
30	$3\frac{5}{8}$	8 43 19	8 43		8 45 19	
31	$3\frac{1}{4}$	8 46 12	8 46 near.		8 48 12	
32	3	8 47 32	8 47		8 49 32	
33	$2\frac{3}{4}$	8 50 57	8 50		8 52 57	
34	$2\frac{1}{2}$ nearly	8 54 15	8 54		8 56 15	
35	$1\frac{3}{8}$	8 58 24	8 58		9 0 24	
36	$1\frac{1}{8}$	8 59 35	8 59		9 1 35	
37	$0\frac{1}{6}$	9 1 38	9 1		9 3 38	
38	$0\frac{1}{2}$	9 3 20	9 3		9 5 20	
39	The End.	9 6 53	9 6		9 8 53	The Point of the End was diſtant from the Vertical 143° to the Eaſt.
		9 23 6		47 33	9 25 28	
		9 24 16		47 42	9 26 45	
		9 28 29		48 10	9 30 42	
		9 30 36		48 28	9 33 12	

Ordo

XXXII.

An Eclipſe of the Sun, June 23. (*St. N.*) 1675. *at* Dantzick; *by M.* Hevelius, N. 127. *p.* 660.

Order of the Phaſ.	*Obſervations.*	*Alt. of the S.*			*Time cor.*		
		h.	′	″	h.	′	″
	The Sun's Center in the Horizon - - - -				3	21	30
	Nothing yet in the Sun - - - - - - - -				4	42	0
	The Beginning - - - - - - - - - -				4	44	0
1	Almoſt 6½ Digits were obſcured - - - -				5	32	0
2	6⅓ Digits - - - - - - - - - - - - -				5	34	0
3	The greateſt Obſcuration was nearly 6 Dig. 42′.				5	38	50
4	6½ Digits - - - - - - - - - - - - -				5	43	30
5	6¼ Digits - - - - - - - - - - - - -				5	47	30
6	6 Digits - - - - - - - - - - - - -				5	49	30
7	5½ Digits. { The Diameter of the Moon was 14′. 37″. the Semidiameter of the Sun being ſuppos'd 15′.				5	55	0
8	5¼ Digits - - - - - - - - - - - - -				5	57	0
9	5 Digits nearly - - - - - - - - - -				5	59	30
10	4⅝ Digits - - - - - - - - - - - - -				6	2	0
11	4½ Digits - - - - - - - - - - - - -				6	4	15
12	4 Digits - - - - - - - - - - - - -				6	6	30
13	3⅝ Digits - - - - - - - - - - - - -				6	9	15
14	3¼ Digits - - - - - - - - - - - - -				6	14	35
15	2⅝ Digits - - - - - - - - - - - - -				6	18	10
16	2⅜ Digits - - - - - - - - - - - - -				6	20	0
17	2 Digits - - - - - - - - - - - - -				6	22	0
18	1½ Digit - - - - - - - - - - - - -				6	24	0
19	1¼ Digit - - - - - - - - - - - - -				6	25	25
20	1 Digit - - - - - - - - - - - - -				6	27	10
21	½ Digit - - - - - - - - - - - - -				6	30	30
	The End. - - - - - - - - - - - - -	°	′		6	33	30
	Altitude of the Sun - - - - - - - -	25	20		6	41	22
	Altitude of the Sun - - - - - - - - -	28	5		7	0	14

An Eclipſe of the Sun; June 1. 1676. *at* Weſtminſter; *by Mr.* Francis Smethwick. N. 126. *p.* 637.

XXXIII. 1. The Beginning of Obſcuration - - - - 7h. 50′.
The End - - - - - - - - - - - - 9 54¾.
The Duration of the whole Eclipſe - - 2 4¾.

The Time was obſerved by a Pendulum Clock, corrected by Obſervations. The Teleſcope made uſe of was a very good one, of 7½ Feet.

2. *At* Wapping; *by M.* Colſon. *ibid.*

The Time according to a Pendulum Clock.			*The Phaſes.*	*The Suns Altitude*		*The Time corrected by the Altitude.*			
h.	′	″		°	″	h.	′	″	
7	34	50		22	46	7	36	0	
7	37	14		33	10	7	38	40	
7	39	10		33	30	7	40	48	
7	50	40	$0\frac{1}{2}$ Digit.			7	51	51	Eſtimated by a Teleſcope.
8	8	34	$1\frac{1}{4}$ Digit.			8	9	45	
8	17	25	$2\frac{1}{8}$ Digits.			8	18	36	Meaſured by a Teleſcope.
8	27	10	$3\frac{1}{10}$ Digits.			8	28	21	
9	39		$1\frac{1}{2}$ Digits.			9	40		Eſtimated by a Tube.
9	43		$1\frac{1}{4}$ Digits.			9	44		
9	48		$0\frac{3}{4}$ Digit.			9	49		
9	54	25	Not ended.			9	55	36	
9	55	55	Ended.			9	57	6	
4	26	5		32	10	4	26	56	
4	28	58		31	53	4	29	52	
4	31	2		31	31	4	32	16	

3. *At* Greenwich; *by Mr.* Flamſteed. N. 127. *p.* 662.

For performing theſe Obſervations I had the Aſſiſtance of my Friend Mr. *Halley*. We had prepared two Tubes, one was 196½ Inches long, which had one of *Townley*'s Micrometers. I took myſelf the Meaſures of the eight firſt Phaſes. The other was only 103½ Inches long, with which, and with my Micrometer, Mr. *Halley* took the Meaſures aſſigned to them. But in the laſt two Obſervations, with the leſſer Tube and my Micrometer, (fitter for this Uſe than the other,) I took the Diſtance of the Azimuths falling by the lucid Limb of the Sun, and the neareſt Cuſpid of the Eclipſe; while Mr. *Halley* in the mean time meaſured the lucid Parts, and the Diſtance of the Cuſpids with the greater Tube. A little before the Beginning came the moſt noble Lord Viſcount *Brouncker*, Preſident of the *Royal Society*, who proved by his own Judgment the Meaſure of the Sun's Diameter taken with the longer Tube. At the Hour 7. 45′. the Sun firſt appeared through the Clouds. The Obſervations were thus.

Fig. 152.

Ord. of the Phases.	Hour by the Pen. Clock.	Corrected.	With the longer Telescope.	With the ſhorter.
	h. ′ ″	h. ′ ″		
	7 46 00	7 45 00	No Eclipſe as yet.	
1	7 54 50	7 53 50	The right Side of the Sun appeared Eclipſed, having got clear of the Clouds.	
2	7 58 24	7 57 24	I C —— 2040 = 10 10	
3	8 04 12	8 03 12	I C —— 2779 = 13 56	
4	8 13 40	8 12 40	I C —— 3580 = 17 52	PL—3198 = 26 18
5	8 18 37	8 17 37	PL —— 4975 = 24 50	I C—2334 = 29 13
6	1 21 06	8 21 06		PL—2989 = 24 35
			The Sun's Dia. 6360 = 31 43	3850 = 31 40
7	8 28 01	8 27 01		PL—2888 = 23 57
8	8 29 01	8 28 01	PL —— 4565 = 21 46	
9	8 35 12	8 34 12	PL —— 4478 = 22 18	AZ—2310 = 19 00
10	8 40 20	8 39 20	I C —— 4417 = 22 00	AZ—2070 = 17 02
	10 02 00	10 01 00	Afterwards the Sun got behind the Clouds, until the Limb emerging through the Clouds, appeared free from the Eclipſe. It came out clearer, and nothing	
	10 04 00	10 03 00	was found wanting in the Limb.	

For correcting the Clock, I had taken the Day before the Eclipſe, *May* 31. Morn.

The Hour of the Clock.		*The Sun's Altitude*	*The Time computed from thence.*	*The Error.*
h. ′ ″		° ′	h. ′ ″	′ ″
7 07 12	The Altitude of the inferior Limb.	27 47	7 06 09	1 03
7 10 16	Of the ſame Limb.	28 16	7 09 19	0 57
	Again *Jun.* 1. Afternoon.			
5 32 00	Altitude of the inferior Limb, &c.	22 06	5 31 06	3 56
5 35 23	Of the upper Limb	22 06	5 34 34	0 49
5 45 17	Of the Lower.	20 6	5 44 18	0 59
	Laſtly, *June* 2. Morn.			
8 9 44	Altitude of the inferior Limb, &c.	37 34	8 08 45	0 59
8 13 36	——	38 09	8 12 34	1 02
8 15 44	——	38 28		
8 17 51	——	38 47		
8 20 1	——	39 07	8 18 49	1 12

Whence it appears, that the Clock kept an even Motion, and was duly corrected in the Eclipſe.

The

4.
At Townley; *by Mr.* Rich. Townley. *Ibid.* 633.

The Hour of the Pend. Clock.			Corrected by a Merid. Line.			Measures.		Phases.		
h.	'	"	h.	'	"			'	"	' "
8	06	45	8	8	27	I C	1190	10	09	perhaps 1109 = 14 50
8	11	00	8	12	42	P L	1935	26	15	
8	18	00	8	19	42	I C	1405	19	04	
8	21	00	8	22	42	P L	1805	24	30	
8	26	14	8	27	56	I C	1504	20	47	
8	34	00	8	35	42	P L	1711	23	13	
8	42	15	8	43	57	I C	1551	21	03	exactly.
8	46	30	8	48	12	P L	1702	23	20	or - - - 1720 = 23 15
8	51	45	8	53	27	I C	1553	20	04	preciſely.
9	00	00	9	01	42	P L	1801	24	33	
9	12	34	9	14	16	I C	1357	18	25	
9	30	55	9	32	37	I C	872	11	50	
9	41	15	9	42	57					

Fig. 125.

The Eclipſe was ended, as near as I could perceive, becauſe of the Air's tremulous Motion. The Place of the Exit was ſo near the Vertex, that I could not well determine which way it inclined from it; though at 9h. 29'. by the Clock, the Cuſpids appear'd parallel to the Horizon.

At 9h. 10'. the Sun's Diameter was 2334, exactly enough as I thought.

Afterwards when the Sun came to the Meridian, by a long Meridian Line, the Clock was found too ſlow by 1'. 42". But by a great Equinoctial Dial, by which I could diſtinguiſh to half Minutes or leſs, the Clock was too ſlow all this Morning only 45". Yet I rather truſt to that Correction by the Meridian Line, than by the Sun-dial.

7h. 50'.	Nothing under the Sun.
7 50½	The beginning exactly.
7 52	The Defect was obſervable.
9 00	Digits 3½.
9 11	Digits 3$\frac{1}{10}$.
9 21	Digits 2$\frac{15}{100}$.
9 47½	Not finiſh'd, but the End juſt approaching.

5.
At Wingfield, *near* Derby; *by M.* Imman. Halton.
Ibid. p. 664.
At Paris, *by M.* Caſſini.

6. When the Sun appear'd out of the Clouds, approaching to an Altitude of 48 Degrees, I directed my Quadrant to him, and held it immoveable at this Altitude. From the Time that the upper Limb of the Sun *a*, touched the horizontal Thread *c d* in the Focus of the Teleſcope, to the coming of the Center *b*, there paſs'd 104" = *a b* or *b r*. From the Paſſage of the Center *b*, to that of the upper Limb of the Moon *o*, were 11" = *b s*. From the Tranſit of the Center *b* to that of the upper Weſtern Horn *e*, were 25"½ = *e b*. From the Tranſit of the Center *b* to that of the lower Eaſtern Horn *i*, were 93" = *i k*. Hence is determin'd the Line of the Horns *i e*, (ſetting aſide the Variation) and its Inclination to the Horizon *l k*; and the Point of Concourſe *p* of a Tangent to the Moon with the Secant *i e p*, and the Tangent it ſelf *p o*, being a mean Proportional between *p i* and *p e*. Alſo the Angles *n o e*, *t o i*; and hence the Angle *i o e*, and the Triangle *i o e* inſcribed in the Circumference of the Moon.

Ibid. Fig. 126.

From theſe and other Aſtronomical Principles I have deduced, that the beginning at *Paris* ought to be 7h. 55'. The End about 10h. 12'.

7.
At Dantzick; *by M.* Hevelius. *Ibid. p.* 666.

The Time according to a Sun-dial and Pendulum Clock.			*Altitudes of the Sun.*			*The Time corrected by the Sun's Altitudes.*			*The Order of the Phases.*	*Their Magnitudes.*	*Observations.*
h.	′	″	h.	′	″	h.	′	″			
7	58	10	36	17	0	7	58	18			
8	1	30	36	41	0	8	1	6			
8	3	30	37	3	0	8	3	39			
8	50	30				8	50	0			The Sun appear'd quite clear.
9	21	30				9	21	0			Nothing yet in the Sun.
9	22	30				9	22	0	1	Digit.	The Beginning of the Eclipſe.
9	24	10				9	23	40	2	$\frac{1}{2}$ almoſt.	
9	24	55				9	24	25	3	$\frac{1}{2}$ Digit.	
9	27	28				9	27	0	4	$\frac{3}{4}$ Digit.	
9	29	40				9	29	10	5	1 Digit.	
9	33	25				9	33	0	6	$1\frac{1}{4}$ Digit.	
9	36	30				9	36	5	7	$1\frac{5}{8}$ almoſt.	
9	39	35				9	39	10	8	2 Digits.	
9	45	49				9	45	25	9	$2\frac{1}{2}$ Digits.	
9	54	22				9	54	0	10	$3\frac{1}{8}$ Digits.	
10	3	44				10	3	22	11	$4\frac{1}{2}$ Digits.	
10	8	30				10	8	10	12	$4\frac{3}{4}$ Digits.	
10	18	17				10	18	0	13	$4\frac{1}{5}$ almoſt.	
10	22	42				10	22	22	14	$4\frac{1}{2}$ſomet.mo.	
10	26	19				10	26	0	15	$4\frac{1}{3}$ almoſt.	
10	35	24				10	35	6	16	4. 22′.	At the Hour 10h. 31′. 0″. was the greateſt Obſcuration.
10	38	53				10	38	38	17	$4\frac{1}{4}$ almoſt.	
10	47	34				10	47	20	18	4Dig.almoſt.	
10	53	49				10	53	30	19	$3\frac{5}{8}$ Digits.	
10	58	17				10	58	8	20	$3\frac{2}{8}$ Digits.	
11	5	27				11	5	20	21	$2\frac{7}{8}$ Digits.	
11	8	50				11	8	44	22	$2\frac{1}{4}$ Digits.	
11	22	13				11	22	8	23	$1\frac{2}{4}$ almoſt.	
11	29	14				11	29	10	24	$1\frac{1}{00}$	
11	35	25				11	35	20	25	$\frac{1}{2}$	
11	36	59				11	36	55	26	$\frac{1}{4}$ſomet.mo.	
11	39	15				11	39	15			The S. not yet clear of the Ecli.
11	39	40				11	39	40			The End of the Eclipſe.
4	18	10	33	13	0	11	18	19			
4	20	0	32	25	0	11	20	36			

	By Calc. from the Rudolphine Tables.	*By Observation.*	*Diff.*	*Time.*
	° ′ ″	° ′ ″	′ ″	h. ′
~~Semid. ☉~~	~~0 15 0~~			
Semid. ☾	0 15 3	0 13 53	1 10	10 0 0
		0 14 0	1 3	10 24 0
		0 14 50	0 13	11 0 0
		0 15 0	0 0	The last
	h. ′ ″	h. ′ ″		
Duration.	1 50 58	2 17 40		

At Avignon; by M. Gallet. N. 141. p. 1020.

8. On the preceding Days we chose a very fit Place, in which we enjoy'd a clear open Air; which was at the Convent of the *Barefoot Carmelites*, which in respect of the City of *Avignon* inclines to the East, and being close to the Walls enjoys a good Air, free from the Smoak and Vapours of the City. In the middle of the Garden we erected an Apartment, which we made dark with Tapeſtry Hangings, and in this we fixt our Inſtruments neceſſary for Obſervation.

We fitted our Teleſcope with a concave ocular Lens, and convex Object-glaſs, having a double Motion with a firm Support; that is, vertical and horizontal, carrying about with it a little Board made immoveable by ſtrong Skrews, cover'd over with a white Paper that was always parallel to the Eye-glaſs, on which we deſcribed the ſolar Image determin'd by the Diſtance of the Teleſcope. The Diameter of this we divided into twelve Digits by concentric Circles, and every Digit into 60 Parts.

Inſtead of a Quadrant, which requires much Caution, and is liable to wavering, we placed a Gnomon very well divided into 100 Parts, for taking the Sun's Shadow; ſo that it might freely move, and yet keep its vertical Situation by means of a Perpendicular. Laſtly, we prepared a Clock that ſhew'd Minutes and Seconds, and went with a Pendulum vibrating in a Cycloid.

On the 11th of *June*, which was the Day of the Eclipſe, at about an Hour after Sun riſing, to the beginning and end of the Eclipſe, we received his bright Image upon the Paper without Intermiſſion; and every one of us watch'd it with an Inſtrument appointed for him. M. *de Beauchamps*, the great Mæcenas of the Muſes at *Avignon*, and my ſelf were at the Teleſcope; the quick-ſighted M. *de St. Florent* was at the Gnomon; M. *Moutonier* was at the Clock, along with M. *Marin*, a Prieſt, who was very converſant in Mathematicks, but particularly in Clock-work.

Preſently and ſenſibly the Shadow began to enter the Diſk; over againſt the firſt Phaſis I put down the Quantity of the Parts obſcured, the Shadow in the Parts of the Gnomon, and the Hour told by the Clock; and thus I collected 39 Phaſes, which are contain'd in the following Tables.

Num. Phaſes.	*Digits obſcur.*		*The Shadow of the Gnomon in ſuch Parts as the Gnomon contains* 400	*The Sun's apparent Altitude.*			*The Sun's true Altitude.*			*The Hour by the Pendulum Clock*			*The Hour corrected by the Sun's Altitude.*			
	D.	'		°	'	"		'	'	h.	'	"	h.	'	"	
1	0	27	561	35	29	23	35	28	48	7	50	31	7	50	34	
2	1	0	536	36	44	0	39	43	28	7	57	25	7	57	28	
3	1	30	520	37	34	7	37	33	37	8	2	3	8	2	7	
4	3	0	478	39	55	23	39	54	57	8	15	14	8	15	13	
5	3	25	466	40	38	30	48	38	6	8	19	0	8	19	14	
6	4	30	438	42	24	14	42	23	53	8	29	19	8	29	6	
7	4	40	434	42	39	58	42	39	37	8	30	59	8	30	34	
8	5	0	424	43	19	53	43	19	32	8	34	34	8	34	18	
9	5	30	412	44	9	12	44	8	52	8	39	19	8	38	56	
10	6	0	394	45	10	57	45	10	39	8	44	54	8	44	44	The Horns were vertical.
11	6	40	375	46	35	50	46	35	33	8	53	19	8	52	45	
12	6	50	371	46	54	15	46	54	0	8	54	54	8	54	31	
13	7	0	466	47	17	30	47	17	14	8	56	44	8	56	44	
14	7	20	350	48	30	37	48	30	23	9	3	44	9	3	44	The greateſt Obſcuration.
15	7	8	339	49	29	10	49	28	58	9	9	14	9	9	15	
16	7	0								9	11	0				
17	6	35	325	50	39	22	50	39	10	9	15	54	9	16	12	
18	6	25	321	51	0	12	51	0	0	9	18	14	9	18	11	
19	5	25	296	53	10	29	53	10	19	9	31	5	9	31	1	
20	5	0	286	53	55	14	53	55	5	9	35	44	9	35	30	
21	4	40	283	54	27	6	54	26	57	9	38	39	9	38	43	
22	4	35								9	42	3				Horns were parallel to the Horizon.
23	4	0								9	47	19				
24	3	53	266	56	3	35	56	03	27	9	48	45	9	48	36	
25	3	35	262	56	28	29	56	28	22	9	51	29	9	51	11	
26	3	30	262	56	37	32	56	37	25	9	52	11	9	51	59	
27	3	26	260	56	43	34	56	43	27	9	52	34	9	52	45	
28	3	6	254	57	15	59	57	15	53	9	56	5	9	56	10	
29	3	0								9	57	40				
30	2	48	249	57	50	48	57	50	42	9	59	34	9	59	53	
31	2	35	246	58	9	32	58	9	26	10	1	34	10	1	53	
32	2	25	243	58	26	11	58	26	5	10	3	0	10	3	41	
33	2	0								10	6	46				
34	1	50	236	59	12	33	59	12	28	10	8	56	10	8	47	
35	1	0	226	60	16	59	60	16	55	10	15	51	10	16	0	
36	0	40	220	60	56	21	60	56	16	10	20	57	10	20	31	
37	0	30	217	61	16	11	61	16	6	10	22	54	10	22	50	
38	0	20	214	61	36	12	61	36	8	10	25	0	10	25	12	Weſtern Horn was vertical with the Center of the Sun.
39	Finis		209	62	6	23	62	6	19	10	28	41	10	28	50	

The Proportion of the Diameters appear'd equal, in the Eclipſe of ſix Digits; for then the vertical Horns of the Sun were diſtant on each Side from the Sun's Vertical about 30 Degrees. Whence it appears the Center of the Moon was then found in the Sun's Periphery, and that the Line through the Center was equal to the Sun's Semidiameter. But after the middle of the Eclipſe we found ſome Change in the Diameter of the Shadow; for the Shadow appear'd a little more convex, and therefore the Diameter was ſhorter, but almoſt inſenſibly.

XXXIV.
An Eclipſe of the Sun, July 2*d*. 1684. *at* Greenwich; *by M.* Flamſteed. N. 162. p. 691.

Time by the Pendulum Clock.			*Time corrected by Obſervation.*			*Obſervations.*	*Parts obſcured, of which 32 were equal to the Sun's Diam.*	*Parts free in the Micrometer.*	
h.	′	″	h.	′	″			′	″
2	12	30	2	03	45	The Sun was perfect; then Clouds.			
2	21	2	2	12	17	The S. eclipſed very lit. at low. Limb			
2	21	25	2	12	40	Parts obſcured above the Shadow - -	0½		
2	32	20	2	23	35	- - - - - - - - - - - -	4	22	19
2	46	40	2	37	55	- - - - - - - - - - - -			
2	51	37	2	42	52	- - - - - - - - - - - -	11		
3	5	23	2	56	38	- - - - - - - - - - - -	15		
3	9	5	3	00	20	The Center.	16		
3	10	4	3	1	19	- - - - - - - - - - - -		15	07
3	11	47	3	3	2	- - - - - - - - - - - -		14	36
3	12	15	3	3	30	- - - - - - - - - - - -	17		
3	13	23	3	4	38	The Horns were horizontal.			
3	17	43	3	8	58	- - - - - - - - - - - -	18		
3	21	48	3	13	3	- - - - - - - - - - - -		12	30
3	23	0	3	14	15	- - - - - - - - - - - -	19		
3	25	40	3	16	55	- - - - - - - - - - - -		12	11
3	29	51	3	20	6	- - - - - - - - - - - -	19½		
3	33	54	3	25	9	- - - - - - - - - - - -	19½		
3	40	8	3	31	23	- - - - - - - - - - - -	19		
3	43	16	3	34	31	- - - - - - - - - - - -	18½		
3	45	35	3	36	50	- - - - - - - - - - - -	18		
3	49	15	3	40	30	The low. Cuſp. was at the S.'s Nadir - - -			
3	50	2	3	41	17	- - - - - - - - - - - -	17		
3	54	20	3	45	35	The Center.	16		
3	56	45	3	48	00	- - - - - - - - - - - -	15		
4	2	58	3	54	13	- - - - - - - - - - - -	13		
4	6	0	3	57	15	- - - - - - - - - - almoſt	12		
4	8	45	4	00	00	- - - - - - - - - - - -	11		
4	16	32	4	7	47	- - - - - - - - - - - -	8		
4	17	44	4	9	00	Vertical Cuſpids were diſtant 84°.			
4	24	8	4	15	23	Between the Cuſpids - - - - -		17	32
4	24	20	4	15	35	- - - - - - - - - - - -	5		
4	27	14	4	18	29	Between the Cuſpids - - - -		15	35
4	29	36	4	20	51	- - - - - - - - - - - -	2½		
4	30	32	4	21	47	Between the Cuſpids - - - -		12	22
4	33	06	4	24	21	- - - - - - - - - - - -	1		
4	36	22	4	27	37	The End exactly - - - - -			

Then to find the Error of the Clock.

Hour by the Clock.			*Diſtances from the Vertex.*				*The H. thence computed.*			*The horary Error.*	
h.	′	″		°	′	″	h.	′	″	′	″
4	43	38	Of the lower	60	11	00	4	34	56	8	42
4	45	48	Limb of the	60	31	20	4	37	7	8	41
4	46	56	Sun.	60	41	50	4	38	15	8	43
4	49	3		61	1	40	4	40	22	8	41

At Paris; by M. Bullialdus. N. 162. *p.* 693.

2. The Altitude of the Sun was 50°. The beginning was elapſed, the Sun being cover'd with Clouds, and about a Digit was eclipſed.

The Sun's Altitude was 41°. 15′. a little more than 7 Digits. It reached 8 Digits.

The Altitude of the Sun was 29°. 30′. The End.

At the Obſervatory; *by M.* Caſſini. N. 162. *p.* 693. N. 163. *p.* 715.

3. The Beginning of the Eclipſe could not be ſeen, but was deduced from the following Phaſes. The apparent Diameter of the Moon appeared leſs than that of the Sun. It was judged that the Dilatation of the Sun's Light, might make the Moon's Diameter ſeem leſs.

Phaſes.	*Time.*			*Phaſes.*	*Time.*		
	h.	′	″		h.	′	″
Begin.	2	25	30	7 $\frac{7}{8}$ Digits	3	35	00
or	2	25	55	7 Digits	3	55	50
1 Digit	2	32	50	6 Digits	4	4	10
2 Digits	2	40	00	5 Digits	4	12	25
3 Digits	2	47	40	4 Digits	4	19	15
4 Digits	2	54	10	3 Digits	4	25	50
5 Digits	3	2	00	2 Digits	4	32	15
6 Digits	3	10	5	1 Digit	4	37	40
7 Digits	3	20	10	End.	4	43	23

4. *By M.* de la Hire *and* Pothenot. *Ibid. p.* 716.

Phaſes.	*Time.*			*Phaſes.*	*Time.*		
	h.	′	″		h.	′	″
Beginning.	2	25	24	7 Digits	3	53	34
1 Digit	2	33	2	6 Digits	4	3	53
2 Digits	2	40	30	5 Digits	4	11	8
3 Digits	2	47	47	4 Digits	4	17	42
4 Digits	2	54	41	3 Digits	4	25	14
5 Digits	3	2	41	2 Digits	4	31	56
6 Digits	3	12	6	1 Digit	4	38	11
7 Digits	3	20	54	End.	4	43	27
7 Dig. 5′	3	36	27				

The Beginning was deduced from many Obſervations made ſoon after it: The Moon's Diameter appear'd then not to be more than about 30′; though by the Obſervations of her Diameter ſome Days before and after, it was judged to be 31′. 30″. But the Extremities of the Horns, on which depended the Exactneſs of the Determination, appear'd a little blunted.

5. *At the* College *of* Lewis the Great; *by* R. P. Fontenay. *Ib. p.* 717.

Phaſes.	*Time.*			*Phaſes.*	*Time.*		
Leſs than	h.	′	″		h.	′	″
½ Digit	2	29	30	7 Digits	3	51	20
1½ Digit	2	37	40	6 Digits	4	2	25
2 Digits	2	40	25	5 Digits	4	10	50
3 Digits	2	48	34	3 Digits	4	24	31
4 Digits	2	54	30	2 Digits	4	29	54
5 Digits	3	3	00	0⅓ almoſt.	4	41	00
6 Digits	3	12	40				
7 Digits	3	22	18				
7¾ Digits	3	38	4				

6. The Beginning was at 2^h. 54′. 30″. The End at 5^h. 9′. 9″. The Greatneſs of the Eclipſe 8½ Digits. *At* Aix; *by* M. Gautier. *Ib. p.* 718.

7. *At* Lyons; *by* R. P. Paul Hoſte. *Ib.*

Phaſes.	*Time.*					
	By Fix'd Stars.			*By the Sun.*		
	h.	′	″	h.	′	″
1 Digit	2	45	3	2	50	3
8½ Digits	3	53	52	3	58	52
1 Digit	4	59	20	5	4	20

At ^{h}. 26′. 14″. (by the *Stars*) the Diameter of the Sun and Moon 30′. 58″. But at 4^h. 20′. 34″. the Diameter of the Sun 30′. 58″. of the Moon 30′. 5″.

8. *At the* Bay de Roſes; *by* M. Chaſſelles. *Ib. p.* 719.

Phaſes.	*Times.*		
	h.	′	″
The Beginning of the Eclipſe	2	40	00
The Edge of the Moon at the Sun's Center	3	25	00
The Horns Horizontal	3	40	00
The Horns Vertical	4	15	00
The End of the Eclipſe	5	1	30

The Greatneſs of the Eclipſe about ¾ of the Sun's Diameter, at which Time *Venus* might be ſeen without Pain.

9. The

At Honſieur ; *by M.* de Glos. *ibid.* 9. The Beginning was at $2^h. 15'. 2''$; the End, at $4^h. 34'. 35''$. The Greatneſs more than 8 Digits, but leſs than 9.

At Pau ; *by* R. P. Richaud. *ib.* 10. At $1\frac{3}{4}^h$. the Eclipſe was not begun : At $3\frac{2}{4}^h$. at 10 Digits : The End at $4\frac{3}{4}^h$.

11. *At* Avignon ; *by* R. P. Bonfa. *ibid.*

Phaſes.	*Time.*	*Phaſes.*	*Time.*
	h. ′ ″		h. ′ ″
The Beginning	2 43 27	Horns Vert.	4 24 32
1 Digit	2 51 58	1½ Dig.	5 1 16
9 Digits	4 2 00	The End.	5 4 37

The Sun's Diameter $31'. 38''$: The Moon's $30'. 6''$.

At Oxford ; by *Dr.*Ed. Bernard. N. 164. *p.* 747. 12. The careful Hand of Dr. *Wallis* determined the 40 Phaſes of this Eclipſe. Alſo the learned Mr. *Caſwell*, and Dr. *Rooke* obſerved the true Time of the ſame, by taking ſome Altitudes of the Sun, where our Clocks and Pendulums were deficient.

Obſcure Digits & their Tenths.	*Time by the Clock correct. by the Heavens.*	
	h ′ ″	
	2 03 00	The beginning of the Eclipſe.
0 6	2 07 44	
1 0	2 10 44	
1 3	2 13 19	
1 9	2 15 44	
2 2	2 21 34	
2 6	2 23 44	
2 9	2 25 39	
3 5	2 29 54	
4 0	2 33 04	
4 4	2 36 34	
4 6	2 40 04	
5 0	2 43 14	
5 3	2 48 19	
5 6	2 50 09	The middle of the Sun obſcure.
6 5	2 57 30	
6 7	2 59 14	
7 4	3 08 24	The greateſt Obſcurity.
7 1	3 11 09	
6 8	3 29 19	
6 7	3 31 39	Darkneſs again at the Cent. of the Sun
6 0	3 37 24	
5 7	3 39 29	

3 5 5

		h.	′	″	
5	5	3	44	29	
5	0	3	47	59	
4	4	3	50	34	
3	9	3	53	54	
2	9	4	2	49	
2	5	4	4	39	
2	2	4	6	19	
2	0	4	7	54	
1	4	4	13	9	
1	0	4	15	4	
0	7	4	16	49	
0	3	4	19	39	
		4	21	14	The End of the Eclipſe.

13. Mr. *Jacobs* at *Liſbon* noted At Lisbon. ib. p. 749.
The Beginning of the Eclipſe at 1ʰ. 30′ exactly.
The Ending at - - - - - - 4 12.

14. Mr. *Aſh* and *Molyneux*, toward the middle of the Eclipſe, having a short View of the Sun, they judged that about 8 Digits were covered; at the Ending alſo having a faint View thereof, they aſſigned its End at 3ʰ. 56′. p. m. In Ireland. ib.

The ſame Eclipſe was obſerved by one Mr. *Oſburn*, nigh *Tredagh*. The Beginning 1ʰ. 37′. 30″. The End 3ʰ. 56′. 20″.

15. *Jo. Ludovicus Donellus*, and *Nic. Ign. Joanettus*, meaſured the Sun's Diſtances from the Zenith; we were three of us employ'd in determining the Phaſes, D. *Jo. Galeatius Manzius*, *Hercules Vanottus*, and myſelf. The Obſervations were wrote down by D. *Greg. Maliſardus*. And the Time was obſerv'd at the Clock by D. *Bart. Ferrarius*. At Bononia, by S. Domin. Gulielmini. N. 203. p. 858.

Time by the Clock			*Corrected by the obſerved Diſtances.*			*Diſt. of the Cent. of the S. from the Zenith.*		*The Phaſes.*		
h.	′	″	h.	′	″	′	″			′
3	34							Dig.	2	30
3	37	30						Dig.	3	
3	48	14	3	47	47	51	31	Dig.	4	20 altogeth. doubtf.
3	52	00						Dig.	5	Very good.
3	57	00	3	56	2	52	58	Dig.	5	30
4	2	00	4	2	29	54	10	Dig.	6	
4	4	45	4	5	59	54	44	Dig.	6	30 Very good.
4	9	20	4	9	1	55	17	Dig.	7	exact.
4	14	40	4	15	43	56	28	Dig.	7	
4	19	00						Dig.	7	20 doubtful.

h.	′	″	h.	′	″	′	″			′	
4	27	44	4	28	42	58	39	Dig.	7	20	
4	32	35						Dig.	7		exact enough
4	34	20						Dig.	6	45	
4	44	30						Dig.	6	30	
4	47	15						Dig.	6	30	
4	51	30						Dig.	6		Very good
4	54	38	4	53	53	63	16	Dig.	5	30	exact
4	58	2	4	57	38	63	56	Dig.	5		exact
5	1	55	5	1	2	64	32	Dig.	4	30	exact enough
5	4	40	5	5	28	65	19	Dig.	4		altoge. diligent
5	7	0	5	6	23	65	28	Dig.	3	30	exact
5	10	30	5	9	9	65	58	Dig.	3		exact
5	12	35	5	13	28	66	43	Dig.	2	30	exact enough
5	16	15	5	16	14	67	13	Dig.	2		exact
5	19	50	5	20	24	67	57	Dig.	1	30	exact
5	22	30	5	23	21	68	28	Dig.	1		exact
5	25	0	5	25	14	68	48	Dig.	0	30	diligent
5	27	40	5	28	7	69	18	Dig.	0		exactly

It was to be obſerved, when the Quantity of the Eclipſe was Dig. 7. 20′. which ought to have brought on no ſmall Obfuſcation of the Air, (as hath been often obſerved in ſuch Eclipſes,) yet the State of the Air was not ſenſibly changed from what it uſed to be when the Sun is at Liberty. When many that did not view the Sun had a Suſpicion, either that the Sun was not at all eclipſed, or but very little. Of this it ſeems to me that no other Cauſe can be aſſigned, than the great Quantity of Clouds illuminated by the Sun, which were not very far from him. For the Rays of the Sun being multiply'd by the Reflexion and Refraction of theſe, and thereby become more intenſe, might eaſily make amends for the Light that was intercepted.

An Eclipſe of the Sun, May 1. 1687 at Oxford. N. 187. p. 329 *In divers other Places.* N. 189. p. 370.

XXXV. 1. Dr. *Wallis* writes from *Oxford*, that this Eclipſe of the Sun was obſerv'd there about ½ a Digit, between one and two o'Clock after Noon.

2. This Eclipſe, tho' it was but a ſmall one, and could not be perceived by the naked Eye, yet ſeems to be very convenient for the accurate Determination of the Parallax, and Latitude of the Moon.

At *London*, by the ſeparate Obſervations of Meſſ. *Hook* and *Halley*, tho' the Heavens were very clear, yet becauſe of the oblique Incidence of the Moon, the Moment of the beginning could not be rightly determined: But at 1^{h}. 16′. the Eclipſe had begun very remarkably. The middle of the Eclipſe was about 1^{h}. 40′. The Chord of the Part eclipſed, or that between the Horns, was found to be 9′. 30″. to which anſwers an Arch of 36°; but in the Diameter only 1′. 30″. By the Agreement of both Obſervators the End happen'd exactly at 2^{h}. 3′. 0″.

At *Greenwich*, at the Royal Obſervatory, Mr. *Flamſteed*, for the ſame Reaſon, did not ſee the beginning. But he determined the End at 2^{h}. 4′. 15″. In the middle of the Eclipſe, or at the greateſt Obſcuration, the Chord of the eclipſed Part was 9′. 54″.

At *Totteridge* near *London*, Weſtwardly, Mr. *Haines*, F. R. S. ſaw the End at 2^{h}. 2′. And the greateſt Quantity was half a Digit from the South.

In the Iſland of *Barbados*, at *Bridg-town*, Mr. *Frank* found the End at 1′. 30″. before the Sun's height was 31°. 47′. to the Eaſt; that is, 7^h. 56′. 45″. By Eſtimation he judged the greateſt Quantity to be two Digits from the South.

At *Nuremberg J. P. Wurtzelbaur* obſerved the ſame Eclipſe. The Beginning was exactly 1^h. 58½′. About the Middle, that is at 2^h. 36½′. he ſaw the greateſt Quantity to be two Digits exactly. The End was at 3^h. 13′. 33″.

At *Ulm* in *Suevia*, the Beginning was obſerved by *Honoldus* at 1^h. 48′. The greateſt Quantity 2⅓ Dig. The End at 3^h. 16′.

At *Leipſic* by *Kirchius*, the Eclipſe was obſervable at 2^h. 20′. 10″. At 2^h. 47½′. the Digits were about 1⅓. The End exactly at 3^h. 15′.

At *Wratiſlaw* in *Sileſia*, *D. G. Schultzius* obſerved the greateſt Obſcuration to have been ſomething ſooner than 3^h. 12′: 1½ Dig. Eclip. The End was 3^h. 17′.

XXXVI. 1. I did not ſee the Beginning of the late Eclipſe, but the End happen'd here, preciſely 24′. 9″. after 10 o'clock in the Morning, apparent Time: The greateſt Obſcuration, which was 10 Digits and a Quarter, was about 7 Min. after 9.

An Eclipſe of the Sun, Sept. 3. 1699. *at* Oxford; *by* Dr. D. Gregory. N. 256. *p.* 336.

2. *At* Nuremberg; *by M.* Wurtzelbaur. N. 265. *p.* 6 9.

Phaſes.	*Quant. eclipſed.*		*Times by a Pendulum.*		
	Dig.	′	h.	′	″
Begin.			8	57	14
1	0	52	9	3	26
2	1	32	9	8	23
3	2	28	9	14	14
4	3	19	9	19	40
5	4	8	9	35	57
6	5	15	9	38	57
7	5	50	9	24	2
8	6	26	9	31	43
9	6	53	9	40	36
10	7	20	9	43	47
11	7	56	9	50	39
12	8	30	9	55	9
13	9	23	10	1	44
14	9	53	10	5	46
15	10	24	10	10	34
16	10	38	10	14	37
17	10	45	10	17	54
18	10	45	10	22	29
19	10	12	10	27	31
20	9	49	10	30	10
21	9	21	10	33	11
22	8	52	10	35	53
23	8	30	10	38	46
24	7	38	10	42	42
25	7	14	10	46	7
26	6	33	10	49	42
27	6	6	10	53	22
28	5	27	10	56	37
29	5	9	11	0	00
30	4	33	11	4	24
31	3	57	11	8	16
32	3	13	11	13	3
33	2	41	11	18	3
34	2	11	11	21	37
35	1	32	11	25	38
36	1	2	11	28	27
End.	0	00	11	33	56

From the 8th to the 12th Phaſis, the opake Limb of the Moon on the South-ſide was a little rough; but about the Northern Horn, to near a 4th part of the Segment, it was more ſmooth: But when the Horns of the Eclipſe were almoſt parallel to the Horizon, before and after the 15th Phaſis, the Extremity of the gibbous Limb of the Moon looking downward, was ſomewhat enlightned, and of a kind of Saffron Colour; but though the Sky was free from Clouds, yet no Stars were viſible. Nor was even *Venus* itſelf viſible in the open Air, unleſs by ſome more ſharp-ſighted than ordinary.

Amongſt many round Plates cut out of thick Paper of divers Magnitudes, differing from one another 5″. about the firſt Phaſis, and after, none agreed to the Limb of the Moon but that which was cut to a Radius or Semidiameter of 15′. 30″. (taking the Radius or Semidiameter of that of the Sun to be 16′. 4″.) and that gradually ſo ſwelled or augmented, that larger Plates were neceſſary to be made uſe of; and about the 36th Phaſis, none leſs than one deſcribed of a Radius of 16′. 5″. would agree with, or equal the Appearance; and conſequently, that the Diameter of the Moon, about the End of the Eclipſe, did equalize, if not exceed, that of the Sun. Beſides, in the 27th Phaſis (when the obſcure Part was 6 Dig. 6′.) the Body of the Moon did obſcure more than two 3ds of the Sun's Limb; which is an Argument that its Semidiameter at that time was equal to that of the Sun.

By others, ib. p. 623.

3. This Eclipſe, by the Obſervations of Mr. *Godfred Tuber* at *Ciza*, began at 9^h. and ended at 11^h. 35′. and increas'd to 11 Dig. By the Obſervations of Mr. *Jacob Honold* at *Harvelſing* near *Ulm* of *Suevia*, it began at 8^h. 55′. and ended at 11^h. 31′. and its greateſt Defect was 10 Dig. And by Obſervations at *Leipſick*, it began at 9^h. 11′. and ended at 12^h 38′. 30″. The greateſt Obſcurity was 11 Dig. 20′. which laſted from 10^h. 16′. 45″. for 6′. Ten Digits being obſcured, the Sky (being otherwiſe very clear) began to appear of a more livid or wan Complexion, and more ſad than it uſually looks with a clear Sky when the Sun is ſet, or below the Horizon. The Cocks alſo, which had hitherto crowed very frequently, as if ſilenc'd, going to Rooſt, left off crowing, and did not renew it, till, by the Recovery of the Sun's Light, they had recover'd their former Gaiety and Mirth: However, we cannot learn that any Star, beſides that of *Venus*, was diſcover'd by thoſe who were Spectators of it in the open Air.

Changes likely to be diſcovered in the Moon, by M. Auzout. N. 7. *p.* 120. Dec. Ann. 1665.

XXXVII. I ſometimes think that the Earth muſt appear, to the ſuppoſed Inhabitants of the Moon, to have a different Face in the ſeveral Seaſons of the Year; and to have another Appearance in Winter, when there is almoſt nothing green in a very great Part of the Earth; when there are Countries all cover'd with Snow, others all cover'd with Water, others all obſcur'd with Clouds, and that for many Weeks together; another in Spring, when the Foreſts and Fields are green; another in Summer, when the whole Fields are Yellow, *&c.* Methinks, I ſay, that theſe Changes are conſiderable enough in the Force of the Reflections of Light to be obſerv'd, ſince we ſee ſo many Differences of Lights in the Moon. We have Rivers conſiderable enough to be ſeen, and they enter far enough into the Land, and have Breadth capable to be obſerved. There are Fluxes in certain Places that reach into large Countries, enough to make there ſome apparent Change; and in ſome of our Seas there float ſometimes ſuch bulky Maſſes of Ice, as are far greater than the Objects which we are aſſured we can ſee in the Moon. Again; we cut down whole Foreſts, and drain Marſhes, of an Extent large enough to cauſe a notable Alteration; and Men have made ſuch Works as have produced Changes great enough to be perceived. In many Places alſo are *Vulcano's* that ſeem big enough to be diſtinguiſh'd, eſpecially in the Shadow. And when Fire lights upon Foreſts of great Extent, or upon Towns, it can hardly be doubted, but theſe luminous Objects would appear either in an Eclipſe of the Earth, or when ſuch Parts of the Earth are not illuminated by the Sun: But yet, I know no Man, who hath yet obſerved ſuch things in the Moon; and one may be rationally aſſured, that no *Vulcano's* are there, or that none of them burn at this time. This it is which all curious Men that have good Teleſcopes ought well to attend; and I doubt not, but if we had a very particular Map of the Moon, as I had deſigned to make one, with a *Topography*, as it were, of all the conſiderable Places therein, that we or our Poſterity would find ſome Change in her. And if the Maps of the Moon of *Hevelius*, *Divini*, and *Riccioli*, are exact, I can ſay that I have ſeen there ſome Places conſiderable enough, where they put Parts that are clear, whereas I there ſee dark ones. 'Tis true, that if there be Seas in the Moon, it can hardly fall out otherwiſe than it doth upon our Earth, where Alluviums are made in ſome Places, and the Sea gains upon the Land in others; I ſay, if thoſe Spots we ſee in the Moon are Seas, as moſt believe them to be, whereas I have many Reaſons that make me doubt whether they be ſo. And I have ſometimes thought, whether it might not be, that all the Seas of the Moon, if there muſt be Seas, were on the Side of the other Hemiſphere, and that for this Cauſe it might be, that the Moon turns not upon its Axis, as our Earth, wherein the Lands and Seas are as it were balanced: That thence alſo may proceed the Non-appearance of any Clouds rais'd there, or of any Vapours conſiderable enough to be ſeen, as there are rais'd upon this Earth; and that this Abſence of Vapours is, perhaps, the Cauſe that no Crepuſcle is there, as it ſeems there is none, my ſelf, at leaſt, not having been hitherto able to diſcern any Mark thereof: For, methinks, it is not to be doubted but that the reputed Citizens of the Moon might ſee our Crepuſcle, ſince we ſee that the ſame is without Compariſon ſtronger than the Light afforded us by the Moon, even when ſhe is full; for a little after Sun-ſet, when we receive no more the firſt Light of the Sun, the Sky is far clearer than it is in the faireſt Night of the Full-Moon. Mean while, ſince we ſee in the Moon, when ſhe is increaſing or decreaſing, the Light ſhe receives from the Earth; we cannot doubt but that the People of the Moon ſhould likewiſe ſee in the Earth that Light wherewith the Moon illuminates it, with, perhaps, the Difference there is betwixt their Bigneſs. Much rather therefore ſhould they ſee the Light of the Crepuſcle, being, as we have ſaid, incomparably greater. In the mean time, we ſee not any faint Light beyond the Section of the Light, which is every where almoſt equally ſtrong, and we there diſtinguiſh nothing at all, not ſo much as that cleareſt Part which is call'd *Ariſtarchus* or *Porphyrites*, as I have often tried; although one may there ſee the Light which the Earth ſends thither, which is ſometimes ſo ſtrong, that in the Moon's Decreaſe, I have often diſtinctly ſeen all the Parts of the Moon that were not enlighten'd by the Sun, together with the Difference of the clear Parts and the Spots, ſo far as to be able to diſcern them all. The Shadows alſo of all the Cavities of the Moon ſeem to be ſtronger than they would be, if there were a ſecond Light. For although afar off the Shadows of our Bodies, environed with Light, ſeem to us almoſt dark, yet they do not ſo appear ſo much as the Shadows of the Moon do; and thoſe that are upon the Edge of the Section, ſhould not appear in the like manner. But I will determine nothing of any of theſe Things.

To find the Parallax of the Moon; by . . . . N. 1. *p.* 151. Feb. An. 1666.

XXXVIII. At certain Times agreed on by two Obſervers, making uſe of Teleſcopes, large, good, and well-fitted for this Purpoſe, by a Meaſuring-Rod placed within the Eye-Glaſs at a convenient Diſtance, that it may be diſtinctly ſeen, and ſerve for meaſuring ſmall Diſtances by Minutes and Seconds, (which is eaſy enough in large Teleſcopes). Let each of ſuch Obſervers, thus furniſhed, obſerve the viſible Way of the Moon among the fix'd Stars (by taking her exact Diſtance from any fix'd Star that lies in or very near her Way, together with the exact Time of her ſo appearing) and the then apparent Diameter of her Diſk; continuing theſe Obſervations, every time for two or three Hours; that ſo, if poſſible, two exact Obſervations of her apparent Place among the fix'd Stars being made at two Places thus diſtant in Latitude, and, as near as may be, under the ſame Meridian, by theſe Obſervations concurring at the ſame time, her true and exact Diſtance may be hence collected, not only for that time, but at all other times, by any ſingle Obſervator's viewing her with a Teleſcope, and meaſuring exactly her apparent Diameter. It were likewiſe deſirable, that as often as there happens any conſiderable Eclipſe of the Sun, that this alſo might be obſerv'd by them, noting therein the exact Meaſure of the greateſt Obſcuration compared with the then apparent Diameter of his Diſk. For by this means, after the Diſtance of the Moon hath been exactly found, the Diſtance of the Sun will eaſily be deduced.

As for the Time fitteſt for making Obſervations of the Moon, that will be when ſhe is about a Quarter, or ſomewhat leſs illuminated; becauſe then her Light is not ſo bright but that with a good Teleſcope ſhe may be obſerv'd to paſs cloſe by, and ſometimes over ſeveral fix'd Stars, which is about four or five Days before or after the Change: Or elſe at any other time, when the Moon paſſes near or over ſome of the bigger ſort of fix'd Stars, ſuch as of the firſt and ſecond Magnitude; which may be eaſily calculated and foreſeen: Or, beſt of all, when there is any total Eclipſe of the Moon; for then the ſmalleſt Teleſcopical Stars may be ſeen cloſe adjoining to the very Body of the Moon.

A Method for obſerving Lunar Eclipſes; by Mr. Rook. N. 22. *p.* 388. Feb. An. 1666.

XXXIX. 1. Eclipſes of the Moon are obſerved for two principal Ends: One Aſtronomical, that by comparing Obſervations with Calculations, the Theory of the Moon's Motion may be perfected, and the Tables thereof reformed; the other Geographical, that by comparing among themſelves the Obſervations of the ſame Ecliptick Phaſes, made in divers Places, the Difference of the Meridians or Longitudes of thoſe Places may be diſcerned.

The Knowledge of the Eclipſes Quantity and Duration, the Shadow's Curvity and Inclination, *&c.* conduce only to the former of theſe Ends. The exact Time of the Beginning, Middle, and End of the Eclipſes, as alſo in total Ones, the Beginning and End of total Darkneſs, is uſeful for both of them.

But becauſe in Obſervations made by the bare Eye theſe Times conſiderably differ from thoſe with a Teleſcope; and becauſe the Beginning of Eclipſes, and the End of total Darkneſs, are ſcarce to be obſerved exactly, even with Glaſſes (none being able clearly to diſtinguiſh between the true Shadow and Penumbra, unleſs he hath ſeen, for ſome time before, the Line ſeparating them paſs along upon the Surface of the Moon); and laſtly, becauſe in ſmall partial Eclipſes, the Beginning and End, and in total ones of ſmall Continuance in the Shadow, the Beginning and End of total Darkneſs are unfit for nice Obſervations, by reaſon of the ſlow Change of Appearances, which the oblique Motion of the Shadow then cauſeth: For theſe Reaſons I ſhall propound a Method peculiarly deſign'd for the Accompliſhment of the Geographical End in obſerving Lunar Eclipſes, free (as far as is poſſible) from all the mention'd Inconveniencies.

For, *Firſt*, It ſhall not be practicable without a Teleſcope. *Secondly*, The Obſerver ſhall always have Opportunity, before his principal Obſervation, to note the Diſtinction between the true Shadow and the Penumbra. And, *Thirdly*, It ſhall be applicable to thoſe Seaſons of the Eclipſe when there is the ſuddeneſt Alteration in the Appearances. To ſatisfy all which Intents,

Let there be of the eminenteſt Spots, diſpers'd over all Quarters of the Moon's Surface, a ſelect Number generally agreed on, to be conſtantly made uſe of to this purpoſe, in all Parts of the World. As for Example, thoſe which M. *Hevelius* calleth *M. Sinai*, *M. Ætna*, *M. Porphyrites*, *M. Serorum*, *Inſ. Beſbicus*, *Inſ. Creta*, *Palus Mæotis*, *Palus Maræotis*, *Lacus Niger Major.*

Let in each Eclipſe, not all, but (for Inſtance) three of theſe Spots, which then lie neareſt to the Ecliptick, be exactly obſerv'd when they are firſt touch'd by the true Shadow; and again, when they are juſt compleatly entred into it; and (if you pleaſe) alſo in the Decreaſe of the Eclipſe, when they are firſt fully clear from the true Shadow. For the accurate Determinations of which Moments of Time (that being in this Buſineſs of main Importance) let there be taken Altitudes of remarkable fix'd Stars; on this Side of the Line, of ſuch as lie between the *Æquator* and *Tropick* of *Cancer*; but beyond the Line, of ſuch as are ſituate towards the other Tropick; and in all Places, of ſuch as at the Time of Obſervation are about four Hours diſtant from the Meridian.

By M. Ja. Caſſini. N. 236. *p.* 15.

2. The Eclipſe of the Moon, *Octob.* 29. *An.* 1697, was obſerved at *Rotterdam* with a Teleſcope of almoſt four *Paris* Feet, with a convex Eye-Glaſs, in whoſe Focus were four Threads croſſing in the Axis at right Angles and half right Angles, for meaſuring the Phaſes, and determining the ſituation of the Lunar Spots. This Teleſcope was ſupported by a Fulcrum having its Axis in a parallel Situation to the Axis of the World, that after it was directed to the Moon for the Obſervation of one Phaſis, for obſerving other Phaſes it might turn about to the Weſt along the Path of the Moon. So it was firſt directed to the Moon, that continuing immoveable the Northern Limb of the Moon, by its Motion to the Weſt, might touch one of theſe Threads, which therefore we ſaid was parallel; tho' becauſe of the Motion of the Moon in Declination, blended with the much ſwifter Motion of the Moon to the Weſt, it declined ſomething from the Equator, whilſt the Diſk of the Moon fell ſucceſſively upon the three other Threads. The middle one of

theſe three Threads, making right Angles with the Parallel, we may call the ſtrait, perpendicular, and vertical Thread; the other two oblique ones, of which we call that the firſt, upon which the Moon falls firſt, the ſecond oblique one upon which the Moon falls afterwards. At the beginning of the Eclipſe, when the moſt Northerly Point of the Moon was not yet immerſed in the Shadow, we adapted it to the parallel Thread. Then after this Point was immerſed in the Shadow, we adapted the moſt Southern Point of the Fig. 127. Moon to the ſame Thread. Whence it is that the Thread which at the beginning was firſt, in the determining the other Phaſes was laſt, and that became the firſt which before was laſt. Now when the Limb of the Moon deſcribed the parallel Thread, the Center of the Moon muſt be ſuppoſed to deſcribe the Lunar Path parallel to this Thread, which was cut by the other three Threads. And the Portions of this Path are ſuppoſed proportional to the times in which the Center of the Moon paſſes over them; for the inequality of the proper Motion blended with the univerſal Motion in a ſmall time is not perceivable. Therefore when the Limb of the Moon deſcribed the Parallel, by the help of a Pendulum-Clock which beforehand had been ſet to the Sun, the Time of the approach of ſome of the Lunar Spots was obſerved, and alſo of the Lunar Horns, to any of theſe three Threads; and it was found, that at the time of the ſaid Eclipſe the Moon's Diſk paſſed through the upright Thread in 2′. 24″. and through the oblique Threads in 3′. 24″. and therefore the Semidiameter of the Moon paſſed through the upright Thread in 1′. 12″. but through the oblique in 1′. 42″; the difference of each Fig. 128. Tranſit being 30″. Hence one Appulſe of the Moon being obſerved to [illegible] of theſe Threads, or one Egreſs, all the others are given to the other Thr[illegible] The Semediameter of the Moon AB, lying in the Lunar Path ABCD[illegible], paſſes through any one of its Points while the Center A runs over a Space AB equal to it; as another Semidiameter AK, making a right Angle with another Line NCK at the Point K, in which therefore it will touch the Moon in K, declining from its Path by the Angle KCA, paſſes through the Thread CK, while the Center of the Moon paſſes through the Line AC, which is the Hypothenuſe of the right-angled Triangle AKC. And the time of the Tranſit of the Semidiameter AB through the Perpendicular Thread that touches the Moon in B, to the Tranſit of the Semidiamer AK through the oblique Thread NCK, as AB or AK, the Sine of the Angle ACK, to AC the Sine of the right Angle or Radius. Therefore the Thread NCK making half a right Angle KCA with the Path of the Moon, alſo the Angle KAC in the right-angled Triangle will be half a right Angle, and therefore the Sides CK and KA will be equal; the right Tranſit according to AB will be to the Tranſit of the Semidiameter AK, through the oblique Thread NK, as the Sine of half a right Angle to the Sine of a right Angle, as 707 to 1000, or as 72″ to 102″, or 1′. 42″ nearly, as was obſerved. The Path of the Center of the Moon being AH, to the Semidiameter of the Moon perpendicular to AM, MNO being drawn parallel to AH, it will be congruous to the Thread which the Limb of the Moon touches by its Motion to the Weſt, which will be cut by the oblique Threads NCK and NGI, and by the upright Thread NEP in the Point N, where the Axis of the Teleſcope paſſes. And with theſe Threads the Lunar Orbit will make two right-angled Triangles NEC, NEG, which are ſuppoſed to have half right Angles at the Points N, C, G; they are therefore like and equal, and have their Sides CE, EG, EN, equal to the Semidiater of the Moon AM. If on each ſide of the Interſections C and G, be taken in the Threads CK, CS, GI, GR, equal to the Semidiameter, and CA, CF, GD, GH, in the Orbit equal to CN, and AK, FS, DR, HI, are joined; theſe will all be equal to one another, and at the Threads will make right Angles, at K, S, R, I. Wherefore the Center of the Moon being in A, the Moon will touch the firſt oblique Thread in K, and after the Center of the Moon comes from A to C, its Semidiameter will coincide with the Line CE, and therefore the Moon will touch the upright Thread in E. But after the Center of the Moon ſhall come from A to D, it will touch the ſecond oblique Thread in R. But AD is equal to the Moon's Diameter; for ſince GD is equal to CA, adding DC we ſhall have AD equal to GC, which is equal to the Moon's Diameter. And ſince GD is equal to CF, if from theſe are taken Equals, GE and EC, then FE will be equal to ED, and will be the double of DF; and will be only from the firſt Contact of the ſecond oblique Thread in R, to the laſt Contact of the firſt oblique Thread in S; and after the Center of the Moon has gone on to G, at the diſtance of one Semidiameter EG, the Moon will finally touch the upright Thread in E. The Center going on from G to H, the Moon herſelf will finally touch the ſecond oblique Thread in I. Suppoſing therefore the [illegible] Tranſit of the Moon to be 2′. 24″. as it is obſerved to be; and,

			Diff.	*Contact.*
	′	″	′	″
Suppoſing the Center at A, and the Contact of the firſt oblique Thread at K - - - - - - - - - - - - -	0	0	1	42
The Center of the Moon will be at C, and will touch (firſt) the upright Thread at E - - - - - - - - - - - - -	1	42	0	42
The Center will arrive at D, and will touch (firſt) the ſecond oblique Thread at R - - - - - - - - - -	2	24	0	30
The Center of the Moon will be at E, the intermediate perpendicular Thread - - - - - - - - - - - - -	2	54	0	30
The Center will arrive at F, and will touch (laſtly) the firſt oblique Thread at S - - - - - - - - - - - -	3	24	0	42
The Center will be at G, and will touch (laſtly) the upright Thread at E - - - - - - - - - - - - - -	4	6	1	42
Finally it will be at H, and will touch (laſtly) the ſecond upright Thread at I - - - - - - - - - - - - -	5	48		

Hence the Obſervations in this Eclipſe generally correſponded with the Calculation within one Second. Therefore it was ſufficient in one Phaſis to obſerve two of theſe Tranſits, in the reſt one, that all the reſt might be known. As to the Lunar Spots, the Tranſit of the preceding Limb of the Moon and of the Spot through the upright Thread, is compared, to have what *Fig. 129.*

is called the Difference of the Longitudes of the Spot from the preceding Limb. And the upright Tranſit of the Spot is compared with the Oblique, to have the Difference, which is equal to the Diſtance of the Way of the Spot from the Path of the moſt Northern or Southern Point touching the parallel Thread. For ſince the Way of the Spot ABC is parallel to the Way of Fig. 130. the Limb D E F, it makes the ſame half right Angles at A and C with the ſame Threads, and right Angles at B; whence the Angle at A is equal to the Angle at C, and the Side B A equal to the Side B E, which is the Latitude of the Spot B from the Thread F E D. But the Longitude and Latitude of the Spot being given, its Situation in the Moon is given alſo. For a Square being deſcribed about it whoſe Side is A B, let it be ſuppoſed to become congruous to the parallel Thread, and let it be divided into ſo many equal Parts, as the Moon paſſes through the upright Thread in Seconds; and let the Sides A C, B D, perpendicular to the Thread, be divided in like manner into as many equal Parts. The Longitude A E, C F, being taken in the Parallels, and F E being drawn, and in the Perpendiculars the Latitude A G, B H, which we ſay is equal to the way intercepted between the upright and oblique Threads; the Situation of the Spot M is determin'd by the common Interſection of theſe right Lines.

As to what concerns the Horns of the Moon in the Eclipſe, they may be determin'd by their Longitude alone, if it be but known in what Semicircle North or South they are. As the Horn I by the Longitude A E or C F. For the right Line F E cuts the Moon's Limb in two Points L and I, one of which is in the Northern Semicircle, the other in the Southern. It may alſo be determin'd by the Latitude alone A K or B M, if it be but known in what Semicircle Eaſtern or Weſtern the Point I is ſituate. But of the Lines of Longitude and Latitude, that will determine the Situation of the Horn more exactly, which is nearer to the Center; as there the Point I is more exactly de- Fig. 131. termin'd by the Longitude than by the Latitude. And on the contrary, the Point O is more exactly determin'd by the Latitude than by the Longitude; and that becauſe of the leſſer Obliquity of the right Line to the Circumference, by which it is brought about, that a ſmall Variation of Diſtance is more ſenſible in the Circumference. In another manner the Situation of the Spots and Horns of the Moon may be determin'd by the oblique Tranſits, if the Line A D, parallel to the Moon's Path P Q touching its Limb, be made the Diameter of a Square circumſcribed about the Moon, which is divided into ſo many equal Parts as the Moon paſſes through the oblique Thread in Seconds; as in this Eclipſe into 204. Of this Square the two Sides A C, B D, will repreſent the firſt oblique Thread, as being parallel to it; the others A B, C D, will repreſent the ſecond oblique Thread. Now the Difference being taken between the Tranſit of the preceding Limb of the Moon and Spot M through the oblique Thread, in horary Seconds from the preceding Angle from A to T, and through T the right Line E F being drawn parallel to the Side A C; and in like manner from the ſame Angle A taking the Difference between the Tranſit of the preceding Limb K and the Spot M, through the ſecond oblique Thread AB, as AV; through the Point V let the right Line GVH be drawn parallel to the Side AB; it will repreſent the ſecond oblique Thread cutting the former in the Point M, and will there determine the Situation of the Spot. In the ſame manner will be determin'd the Situation of the Horn E, by the Difference of its Tranſit and of the Limb, through the firſt oblique Thread, taken in the Diagonal as AT; the Situation of the Horn H, by the Difference of its Tranſit along the ſecond oblique Thread AB, as AV, and drawing through V a right Line GH parallel to the Side AD; if it be known whether the Horn be in the precedent or ſubſequent Semicircle.

XL. The Tables did not indicate an *Eclipſe of the Moon*, *July* 27. (*N. S.*) 1665. but tho' the Sky here was very clear, yet the Moon was not at all obſcured by the true Shadow, but entred only a little into the Penumbra, wherein it continued 50′. The Beginning of its touching the Penumbra did then almoſt happen, when *Aquila* was elevated 36°. 18′.

An Eclipſe of the Moon, 1665, *obſerved at* Dantzick; *by M.* Hevelius.
N. 19. *p.* 348.

XLI. In the *Eclipſe* of *June* 16, (*N.S.*) 1666, the firſt *Phaſis* of 1 Dig. 45′. appear'd in the Moon's Altitude of 2°. 30′. when the greateſt Obſcuration was already paſt. The End fell out 9^h. 27′. about 120°. from the Zenith Weſtward.

An Eclipſe of the Moon, June 6. *A.* 1666. *by M.* Hevelius.
Ibid. p. 348.
N. 21. *p.* 371.

XLII. On the 29th of *September* new Style, An. 1670. in the Morning, the Beginning of this Eclipſe happened at 2^h. 22′. tho' this could hardly be obſerved very accurately, becauſe of the very thin Shadow of the Earth. For during the Eclipſe the Shadow of the Earth was ſo very thin and dilute, that I could ſee very well through it all the principal Spots, with my twenty-foot Tube, and even with ſhorter.

The greateſt Obſcuration fell at 3^h. 50′. The End at 5^h. 21′. So that the whole Duration was 2^h. 59′, and the Quantity hardly more than 9 Digits. About the Middle of this Eclipſe, at 3^h. 40′. I ſaw very plainly a certain unknown little Star, which could be ſeen only with the Tube, covered by the Moon near the greater black Lake; but I could not ſee it come out again. When the Eclipſe was ended it was a very pleaſant Sight to ſee the two Luminaries both above the Horizon together: For the Sun aroſe before the Moon was ſet. The other Things that were remarkable may be found in the following Table.

An Eclipſe of the Moon, Sept. 19. *A.* 1670. *by M.* Hevelius.
N. 61. *p.* 2023.

Order of the Phaſes.	*Phaſes of the Moon obſerved with a Teleſcope.*	*The obſerved Altitudes.*	*Times corrected by the Altitudes.*			*Things obſervable.*
			h.	′	″	
		Swan's Tail 50°. 54′.	12	8	34	
		Swan's Tail 50 25	12	11	32	
		Of *Pollux* ♊ 26 39	1	28	56	
		Of *Pollux* ♊ 27 16	1	33	17	
	Beg. Penum.		2	10	0	
	Beg. Eclipſe.		2	22	0	Eclipſe began in upper Parts
						of the M. about Sin. *Hyper.*
1	1⅜ Dig.		2	31	0	Shadow ſkim'd by Sin. *Apol.*
						and Mar. *Porphyr.*
2	2¼		2	36	0	*Lac.Nig.Maj.* beg. to be cov.
3	3¼		2	41	40	Shadow reach'd the Iſle of
4	3¾ ſom.more		2	45	50	*Corſica.*
5	4⅜		2	49	0	*Palus Mæotis* and M. *Ætna*
6	5 Dig. nearly		2	53	0	plainly cover'd.
7	5¾		2	57	0	*Inſul. Beſbic.* and *Inſ. Macra*
8	6 Dig.		3	3	0	cover'd.
9	6¼		3	5	0	
10	6¾		3	8	30	*Inſ. Melos* beg. to be cover'd.
11	7⅛		3	11	0	
12	7⅝		3	16	0	*Inſul. Rhodus* cover'd.
13	7⅞		3	20	0	
14	8¼		3	25	30	
15	8½		3	29	0	*Inſul. Maj. Caſp.* was cover'd
16	8¾		3	33	40	
17	8⅞		3	37	50	
			3	40	0	

18	9 Dig.		3	47	0	Greateſt Obſcurat. extended
						itſelf thro' *J.Let.* & *M.Aber.*
19	9 ſomet. leſs		3	51	10	*Pal. Maræotis* under the Section
20	8¾		2	57	0	of the Shadow.
21	8½		4	1	40	
22	8¼ nearly.		4	6	0	
		Of *Pollux* ♊ 49°. 50'.	4	10	28	
23	8 ſome. more		4	11	0	Iſlands *Melos* and *Rhodes*
24	7⅞.		4	17	20	came out of the Shadow.
25	7⅜		4	22	25	*Mar. Porphyrites* began to
						be illuminated.
26	6¾		4	27	35	The Shadow had now left
27	6¼ nearly		4	33	40	Mount *Ætna.*
28	5¾		4	37	37	
29	4¾		4	44	46	*M. Argentar.* and *Inſ. Beſbic.*
30	4 Dig.		4	50	56	were illuminated.
31	3⅜		4	54	0	
32	2¾		4	57	0	
33	2¼ nearly.		5	1	23	Shadow had almoſt paſs'd
34	1⅝		5	4	45	the whole Iſland *Caſp.*Maj.
35	1¼ Dig.		5	7	43	
		Lion's Heart 25° 48'	5	10	50	
	End Eclipſe.		5	21	25	The end happen'd about the
	End *Penumb.*		5	24	0	*Riphean* Mountains.
		Sirius 18°. 54'.	5	28	32	

XLIII. 1. On *Sept.* 8. 1671. about ſix in the Evening, the Moon roſe totally obſcured at *Ecton* in *Northamptonſhire.*

An Eclipſe of the Moon, Sept. 8. 1671. *at* Ecton; *by Mr.* Palmer. N. 76. *p.* 2272.

It began to emerge out of the Shadow, the Center of the Moon being 9°. 35' high, or at 7ʰ. 18'.

At the End *Arcturus* was 16°. 30'. high, or at 8ʰ. 16'. 20". Whence the middle of the Eclipſe is computed to be at 6ʰ. 28'. 16".

	°	'	h.	'
2. The Emerſion: Alt. of the upper Edge of the Moon	10	30	7	21
The End of the Eclipſe: Alt. of *Arcturus*	16	20	8	16⅓

At London; *by Mr.* Street. *ib.*

3. 7ʰ. 27½', I firſt obſerved the Moon eclipſed when it began to be enghtned, the total Darkneſs being already paſt. The Shadow paſſed through e middle of the Spot called by *Hevelius, M. Porphyrius.*

By Dr. Hook. N. 77. *p.* 2296.

7ʰ 49'

7^{h}. 49′. The Shadow paſſed through the middle of *M. Sinai*, through the middle of the Eaſtermoſt of the three Lakes called *Mare Adriaticum*, and juſt touched the Ridge of the *Apennine* Mountains.

7^{h}. 54′. It paſſed the middle of the *I. Beſbicus* in the *Propontis.*

8^{h}. 0½′. It paſſed through the *Streights* of the *Pontus Euxinus*, at the Promontories *Acheruſia* and *Ariſtes*.

8^{h}. 6½′. It touched the *Palus Mæotis*, which *Palus Mæotis* was then diſtant from the Limb of the Moon, next adjacent, one third part of its ſhorter Diameter or Breadth.

8^{h}. 17′. The Shadow went off the Body of the Moon upon the innermoſt Limb-line of *Hevelius*'s large Chart of the Moon at the 29th Diviſion, juſt without the *I. Major* of the *Caſpian* Sea. The duſkiſh *Penumbra* left not the Limb of the Moon quite without ſome kind of Darkneſs till 8^{h}. 29′; at which time I found that that ſide of the Moon which the Shadow laſt left, was full as light and clear as the other.

About four or five Minutes after the Shadow was gone off, I perceived a faint Repreſentation of Colours upon that Part of the Body of the Moon which was moſt affected with the *Penumbra*, ſomewhat reſembling the Colours of a faint *Halo* about the Moon; this grew fainter and fainter, and after a few Minutes was no more viſible. It did not ſeem to be cauſed by any Clouds or Exhalations in the Air, the Sky near the Moon being very clear, and the ſaid Colours not appearing any where but upon the duſky part of its Phaſis. Poſſibly it might be cauſed by the Refraction of the Light of the Sun through the Atmoſphere about the Earth.

At Paris; by M. Bullialdus. N. 75. p. 2273.

	h. ′ ″	° ′ ″
4. the End was obſerved, when the Altitude of *Arcturus* to the Weſt was - - - - - - - - - - -		13 41
Subtracting the Refraction, his Height was - -		13 37 45
Hence is given the End obſerved - - - - - - -	8 29 16	
The End of the *Penumbra*, the Height of *Arcturus* being - - - - - - - - - - - - - -		13 0
Corrected - - - - - - - - - - - - - -		12 56
Whence is given - - - - - - - - - - - -	8 33 40	
Mæotis was obſerved wholly out of the Shadow	8 24 16	
The Altitude of *Arcturus* was - - - - - - -		14 26 0
Corrected - - - - - - - - - - - - - -		14 23 55

The Moon came out of the Shadow over againſt *Petra Sogdiana*, by *Hevelius*'s Map.

At Dantzick; by M. Hevelius. N. 78. p. 3028.

5. At 8^{h}. 30′. a-Clock, through the opening Clouds, which were pretty thick, we perceived the Moon ſomething obſcurely, and ſo much enlightned, that one would have thought the Eclipſe had been over already. Thence it was certain, that the total Obſcuration was then paſs'd at leaſt, or rather ſomething ſooner. For that the Moon was come again, we could all perceive plainly enough through the Clouds. So that the Eclipſe muſt have appear'd in the Heavens above half an Hour ſooner, than *Kepler*'s Calculation made it. At 8^{h}. 34′. I obſerved the Moon had freed herſelf from the Shadow a whole Digit at leaſt; and again at 9^{h}. 41′. the Light of the Moon had increaſed to 1½ Digit, as near as we could judge.

6. We

At Hamburg; *by Dr.* Fogelius. *Ibid. p.* 3033.

6. We found here, as Mr. *Hevelius* had done at *Dantzick*, that the *Rudolphine* Tables were out in the late Eclipſe. We found here alſo, that the Moon emerged out of the Shadow of the Earth before nine a-Clock.

An Eclipſe of the Moon, Jan. 1. 1674-5. *at* London; *by D*r. Hook. N. 111. *p.* 237.

		h.	′
XLIV.	1. The beginning of the true Shadow	5	22
	The Immerſion	6	19
	The Emerſion	7	58
	The End of the true Shadow	8	58

The Penumbra was ſeen to continue near half an Hour before it wholly quitted the Body of the Moon.

At Derby; *by M.* Flamſteed. *Ibid.*

2. Mr. *Flamſteed* obſerved the Beginning of the Entrance of the true Shadow h. 5. and 19′.

At Paris; *by M.* Bullialdus. *Ibid. p.* 238.

	°	′	h.	′	″
3. The beginning of the true Shadow, Height of *Capella*	52	26	5	32	29
The Immerſion, Height of *Capella*	62	8	6	33	3
The Emerſion, Height of Cap. *Polluc.*	43	46	8	9	30
End of the true Shadow, Height of *Sirius*	20	47	9	10	0

At Paris; *by M.* Caſſini, *M.* Picard, *and M.* Roemer. *Ibid p.* 238. N. 112. *p.* 257.

4. At 5h. 12′. in the Evening, in the *Royal Obſervatory*, they began to perceive that the Oriental Part of the Moon, by little and little loſt its Light; ſo that at 5h. 25′ they ſaw a manifeſt *Penumbra:* Then at 5h. 32′. 50″. the Limb over-againſt the Spot called *Hevelius* grew ſo dark that they all agreed that this was the true *Beginning* of the *Eclipſe.* At 8h. 7′. one of the Obſervers believed the *Emerſion*, another at 8h. 8′. and the third at 8h. 9′. 30″. but afterwards conſidering the *Emerſion* of the firſt Spots, they all eſteemed it at 8h. 8′. At 7h. 21′. the Southern Limb of the Moon was come cloſe to a Teleſcopick Star: At 8h. 9′. 20″. another Star yet leſs than the former, came out of the darkeſt Side, almoſt over-againſt the Spot *Langrenus.* At 9h. 9′. 40″. all the three Obſervers agreed, that the Moon then came out of the Shadow. The Diameter of the Moon, being meaſured before the Eclipſe, was of 32′. 15″.

The Times were noted by great *Pendulum Watches* that had been adjuſted by the Sun the ſame Day, and that were afterwards verified the next Day: Beſides that, before the Eclipſe, at 4h. 45′. 1″. by the Watches, the Star *Capella* was 45 Degrees high towards the Eaſt.

Time.			*Phaſes.*
h.	′	″	
5	32	50	*Beginning* over-againſt the Spot *Hevelius.*
5	36	00	The firſt Spot of *Grimaldi.* *Palus Maræotis.*
5	36	30	The ſecond Limb of *Grimaldi.*
5	45	00	The middle of *Ariſtarchus.* *Mons Porphyrites.*
5	46	00	*Merſennus.*
5	48	30	*Herigone.*
5	53	00	*Heraclides.*
5	53	15	The firſt Limb of *Copernicus.* *Ætna.*
5	54	15	The middle of *Copernicus.*
5	54	40	*Pitheas,* or *Hiera Inſula.*
5	55	05	The ſecond Limb of *Copernicus.*
5	57	40	The firſt Limb of *Timocharis.* *Corſica.*
5	59	35	The firſt Limb of the *Sinus Medius Æſtuum.* *Adriatick Sea.*
6	01	30	The middle of the *Sinus Medius.*
6	02	40	The firſt Limb of *Tycho,* or *Sinai:* and the firſt Limb of *Plato,* or the *Lacus Niger Major.*
6	03	50	The ſecond Limb of *Plato,* and the middle of *Tycho.*
6	04	30	The *Center* of the Diſk.
6	09	00	The middle of *Manilius,* or *Mons Beſbicus.*
6	12	00	The middle of *Menelaus,* or *Byſantium.*
6	13	45	*Dionyſius Areop.* or *Mons Amanus.*
6	14	30	*Plinius.*
6	15	45	*Vitruvius.*
6	20	35	*Endymion,* or *Lac. Hyperbor. ſuperior.*
6	21	00	*Promont. Heraclium.*
6	24	50	Betwixt *Alcuin* and *Taruntius.*
6	26	00	The firſt Limb of the *Caſpain Sea.* *Mare Criſium.* *Palus Mæotis.*
6	28	15	The middle of the *Caſpian Sea.*
6	29	40	The other Limb of the *Caſpian Sea.*
6	30	5	The firſt Limb of *Langrenus,* or *Inſula Maj.*
6	3	5	The middle of *Langrenus.*
6	35	46	*Total Immerſion,* betwixt *Langrenus* and the *Caſpain Sea.*
8	8	00	Firſt *Emerſion,* towards *Grimaldus.*
8	12	35	The firſt Limb of *Grimaldus.*
8	14	00	The ſecond Limb of *Grimaldus.*
8	20	20	*Merſennus.*
8	24	5	*Herigone.*
8	24	35	The middle of *Ariſtarchus,* and the middle betwixt *Herigone* and *Morin*
8	26	30	The middle of *Kepler,* or *Loca Paludoſa.*
8	28	30	The firſt Limb of *Tycho.*
8	29	50	The ſecond Limb of *Tycho.*

Time.

Time.			Phaſes.
h.	′	″	
8	34	5	The middle of *Copercnius.*
8	35	35	The ſecond Limb of *Copernicus.*
8	36	10	*Pitheas.*
8	36	30	*Heraclides.*
8	40	0	The firſt Limb of *Timocharis.*
8	42	35	The firſt Limb of *Plato.*
8	43	45	The ſecond Limb of *Plato.*
8	49	30	The middle of *Manilius.*
8	52	10	*Menelaus* and *Dionyſ. Areopag.*
8	55	0	*Poſſidonius.*
8	56	6	*Vitruvius.*
8	59	30	*Endymion.*
9	6	20	The firſt Limb of the *Caſpian Sea.*
9	7	10	The middle of the *Caſpian Sea.*
9	8	40	The other Limb of the *Caſpian Sea.*
9	9	40	The *End*, between the *Caſpian Sea* and *Langrenus.*

At Dantzick; by M. Hevelius. N. 113. *p.* 289.

5. Through the whole Duration of the Eclipſe, with a Tube of 20 Feet, and others better than that, I directed my watchful Eyes to four fixt Stars, neglecting others that were leſs, (which however I ſaw very well,) among which the Moon was at that Time. It was diſtant from the little Star *a* hardly 4 Minuutes, with its lower Limb in Conjunction with it: And the three other Stars *b*, *c*, *d*, the Moon cover'd entirely with her Body. Now of all theſe four remarkable little Stars, only one of them *e* has been hitherto taken notice of by Aſtronomers, and inſcribed upon the Globes; it is call'd the uppermoſt of the unform'd Stars between ♊ and ♋, at the Back of *Pollux*: whoſe Courſe with its Ingreſs, Progreſs, and Egreſs, is chiefly to be well obſerved. For from this kind of Obſervations eſpecially it may be eaſier to perfect the Theory of the Moon's Motion, and to fix its Nodes and Latitude, than in my Judgment can be done from mere Solar Eclipſes. The little Star *b* was cover'd nearly at Mount *Eous*, and *d* at the inferior Limb of the Moon itſelf. This paſs'd through the *Sinus Sirbonis*, the Iſle of *Rhodes*, and S. *Athenienſis*, the other through the Deſert of *Mingui.*

The Time according to Pend. Clock, correct. by Alt.	*Order Phaſes*	*The Altitudes of the fixed Stars, with things worthy to be noted.*	*Through what Spots the Sections of the Shadow paſſed.*
h. ′ ″			
6 22 18		The height of the *Swan*'s Tail 39°. 3′. 0″	
6 25 4		The height of the ſame 41′. 0″.	
6 35 0		Moon enter'd the *Penum.*	
6 41 50		Beginning of the Eclipſe.	It began about 50° from the *Nadir*, towards the Eaſt.
6 43 45	1	It reach'd the *Palus Maræotis.*	Juſt at the Beginning *Sinus Sigaricus*, the Iſlands of *Beſbica* and *Melos* were in one right Line.
6 44 55	2	*Palus Maræotis* quite obſcured.	
6 49 0	3		The Section of the Shadow paſs'd by Mounts *Pentadact.* and *Eous.*
6 52 30	4	*Mons Porphyrites* cover'd.	By the *Loca Paludoſa* and M. *Cataract.*
6 57 35	5		To *M. Baronius*, thro' *M. Petri*, *Athor*, and *M. Troicus.*
7 2 15	6	It began to cover Mount *Ætna.*	To the Bay of *Apollo*, the Iſland *Ficaria*, to the bot. of M. *Ætna*, the Iſl. *Didyma*, and the S. Lake.
7 2 55	7	Mount *Ætna* quite cover'd.	To the Iſle of *Sardinia*, by the Iſles *Hiera* and *Crete.*
7 7 40	8		By the greater *Atlantic* Mountains, the Iſle *Vulcania*, *Rhodes*, and Mount *Ann.*
7 10 40	9	Mount *Sinai* cover'd.	By the Iſles *Ophiuſa*, *Cyprus*, and Mount *Sinai.*
7 14 0	10	The greater black Lake cover'd.	By greater black lake, by the M. *Sipylus*, *Libanus*, and *Seir.*
7 16 40	11		By leſſ. black lake, the Iſle *Beſbica*, Mounts *Olympus* and *Didymus.*

7	20	20	12	The Star *c* was diſtant from the Limb of the Moon towards the Eaſt almoſt 30′.	The Section of the Shadow paſs'd by M. *Carpathus*, *Byzantius*, and *Taurus*.
7	25	0	13		By the Lake *Boryſthenes*, the Iſle *Apollonia*, Mount *Moſchus*, and *Sogdianus*.
7	28	30	14		By the Mountains *Macrocemnios*, *Promontory* of *Aries*, of *Hercules*, by the utmoſt Bay of *Pontus*, and Mount *Parapæmiſus*.
7	32	0	15		By the *Palud. Hyperboreæ*, the Iſle *Corocondametis*, and M. *Caucaſus*.
7	34	10	16		By the *Riphæan* Mountains, the *Palus Mæotis*, and the lower Bay of the *Caſpian* Sea.
7	42	44		The total Immerſion happen'd about 50°. of the Limb from the Zenith towards the Weſt.	
7	55	30		The little Star *a*, in its Conjunction with the Moon, was diſtant from its lower Limb about 4 Minutes.	
8	0	50		The little Star *b* cover'd at Mount *Eous*.	
8	35	20		The little Star *c* enter'd at the Southern Lake.	
8	51	20		The little Star *d* was hid at the lower Limb.	
9	9	10		The little Star *c* emerged again under Mount *Nevoſus*; ſo that it enter'd the Moon at 3′. 20″. It was truly a very pleaſant Sight, to find all theſe very bright, even almoſt under the greateſt Obſcuration, and the M's Conjunction.	
9	12	13		Emerſion of the Moon out of the Shadow happen'd almoſt at 30°. of the M's Limb, from the Nadir towards the Eaſt.	
9	20	20	17		Shadow paſs'd by *Pal. Maræotis*, *Fontes Amari*, and Mount *Eous*.
9	25	0	18		By Mount *Audus*, *Ajax*, and *Troicus*.
9	30	20	19		By M. *Pentadactylus*, *Carpathus*, S. *Sirbonis*, and Mount *Lion*.

9	32	40	20	Mount *Porphyrites* came out of the Shadow.		By Mount *Porphyrites*, *Caſſius*, and *Didymus*.
9	36	5	21	Mount *Sinai* began to be illuminated.		By M. *Baron*. at the Iſle of *Sicily*, by *Crete*, at the bottom of *Sinai*.
9	42	15	22	Mount *Ætna* under the very Section of the Shadow.		By the Promontory of *Apollo*, M. *Ætna*, by *Rhodes*, and the Mountains of *Seir*.
9	47	10	23			By the Iſle *Vulcania*, M. *Maſicytus*, *Cragus*, and *Antilibanus*.
9	51	0	24			By the greater black Lake, Bay of *Paſtanum*, Mount *Didymus*, and Mount *Coibacaroni*.
9	56	50	25	The Iſle *Beſbica* came out again into the Light.		By the leſſer black Lake, at the Iſle *Besbica*, by the Mount *Uxerii*.
9	59	35	26			By the Lake *Salmiderſa*, M. *Herminium*, *Moſchus*, and lake *Thoſpitis*.
10	5	0	27			By *Pontus*, Iſle *Cyanea*, Bay of *Athens*, and Mount *Caucaſus*.
10	7	0	28			By Lake *Boryſthenes*, at the Iſle *Apollonia*, by *Heracleum*, and M. *Tancon*.
10	19	35		The Shadow was not yet quite gone.		
10	20	0		The End of the Eclipſe.		
10	23	0		The *Penumbra*.		
10	52	58		Alt. of *Mars*.	37°. 12′	
10	58	35		Alt. Star *Aries*.	28 52	
11	11	33		Alt. of *Capella*.	70 11	

At Seville; by S . . . . , *Profeſſor of Mathematicks.* N. 118. *p.* 428.

6. The Obſervation of this Eclipſe was very exact; for the Sun being near the Horizon, by the Latitude of this Place the Minutes were eaſily found. I obſerved the reſt by my Clock, which that Day did not differ ſo much as a Minute; ſo that there is no Room for Doubt in the Obſervation.

	h.	′	
The Beginning of the true Shadow	4	56	
The Immerſion	6	1	
The Emerſion	7	33	
The End	8	39	or ſomething longer.

XLV. 1. *An Eclipſe of the Moon, June 27. 1675. at London; by Mr. Flamſteed, and Mr. Halley.* N. 116. p. 371. N. 118. p. 432.

	Hour of the Pend. Clock.			*The Phaſes.*
	h.	′	″	
1	1	1	00	Moon's Diam. 3190 = 31′ 40″. } The Meaſures were often repeated, both by my ſelf, and my Aſſiſtant Mr. *Halley*.
3	1	11	00	Moon's Diam. 3191 = 31 46
5	1	37	50	No Appearance of the Penumbra; and now the Moon went under ſome flying Clouds, under which ſhe ſtill lay hid.
6	1	46	40	Till a denſe Penumbra appear'd through their Openings, or perhaps the Beginning itſelf; but I could not be ſure.
7	1	51	40	The Moon's Limb being now raiſed out of the Clouds, was ſeen to labour under a very notable Defect, a ſixth Part, or at leaſt an eighth Part of her Circumference being obſcured.
8	1	55	15	*Pentadactylus* was cover'd.
9	2	2	20	*Porphyrites* was cover'd.
10	2	5	30	The firſt Limb of *Sinai*.
11	2	6	00	The next Limb of *Ætna*.
12	2	8	40	The remaining Parts illuminated were 2071 = 20′. 38″.
13	2	12	00	A ſmall Teleſcope Star, that was not viſible with the leſſer Tube, in the greater appear'd to be of almoſt half the Capacity of the ſame, or diſtant 15′ apparently from the lower Limb.
14	2	17	45	The remaining Parts illuminated 1655 = 16′. 27″.
15	2	23	5	The firſt Limb of *Beſbicus*.
16	2	26	3	*Horminius* cover'd.
17	2	29	00	*Æmus* cover'd.
18	2	30	45	The remaining Parts illuſtrated 1047 = 10′. 24″.
19	2	35	00	The remaining bright Parts 865 = 8′. 38″. And now in the longer Tube another ſmall fixt Star appear'd, being diſtant from the Limb of the Moon the Long. of the *Caſp.* Spot, its Lat. from the line drawn thro' the Cuſpids towards the right Hand.
20	2	37	15	The Shadow cover'd the Weſtern Shore of *Pontus*.
21	2	39	30	It touched the firſt Limb of *Corocandometis*.
22	2	45	00	It touched the *Palus Mæotis*.
23	2	50	45	*Mæotis* was wholly cover'd.
24	2	56	10	Doubtful whether any true Light remain'd.
25	2	56	55	Immerſion: For ſurely the primary Light had intirely left the Moon, for over againſt, or a little above the *Riphean* Mountains, and about a Deg. of the Limb of *Hevelius* 130, 2^h. 57′. 20″. the Limb appear'd thro' the Tube of a kind of Aſh-colour.

The Air was very ſerene from the ſeventh Obſervation to the Immerſion. And a certain faint whitiſh Light ſeem'd to take Poſſeſſion of the Cuſpids of the darkned Moon through the whole Time of the Eclipſe, which made the Moon conſpicuous after the Immerſion, on that Side which fell laſt into the Shadow. The Penumbra of this Eclipſe was very thin, nor was its breadth greater than that of *Sinai* or *Ætna*. The *Palus Mæotis* appear'd very broad, and removed as far as it could be from the Limb of the Moon. On the contrary, *Maræotis* was very much contracted, nor farther diſtant from the Moon's Limb than half its Length.

At Paris; *by M* Bullialdus. *Ibid. p.* 372.

	°	′	h.	′
2. Beginning of true Shadow, *Capella*'s height to Eaſt	18	45	1	55
Shadow reach'd *Palus Mæot. Lyra*'s Alt. to Weſt	50	51	2	55
Total Immerſion, *Lyra*'s Alt. to the Weſt	48	50	3	6

3. *By M.* Caſſini, *M.* Picart, *and M.* Roemer. N. 117. p. 388.

Time.			*Tranſit of the Shadow.*
h.	′	″	
1	56	45	The Beginning below *Grimaldus.*
1	57	20	Through the firſt Limb of *Grimaldus.*
1	58	50	The ſecond Limb of *Grimaldus.*
2	1	45	*Galilæus.*
2	2	15	The firſt Limb of *Merſennus.*
2	4	15	The Beginning of *Gaſſendus.*
2	4	40	The middle of *Gaſſendus.*
2	5	30	The other Limb of *Gaſſendus.*
2	6	15	*Herigon* and *Seleucus.*
2	7	45	*Morinus.*
2	8	18	The middle of *Kepler.*
2	11	35	*Ariſtarchus* and *Bullialdus.*
2	12	40	*Ariſtarchus* diſappears.
2	16	25	The Beginning of *Tycho.*
2	16	40	The Beginning of *Copernicus.*
2	17	25	The middle of *Tycho* and *Copernicus.*
2	18	12	The other Limb of *Tycho.*
2	21	45	*Pythea*, and the firſt of the three *Sin. Medii.*
2	24	0	The middle of the ſecond *Sinus Medii.*
2	24	53	*Heraclides* or *Virgo.*
2	26	10	The firſt Limb of *Timocharis.*
2	26	40	The middle of *Timocharis.*
2	31	40	The Promontory between *Virgo* and *Plato.*
2	32	20	*Abulfeda.*
2	34	15	The Beginning of *Manilius.*
2	36	15	*Dionyſius* the *Areopagite.*
2	38	10	The Shore of the Sea of Tranquillity.
2	38	45	The firſt Limb of *Menelaus* and *Plato.*
2	39	20	*Fracaſtor.*
2	39	53	The middle of *Plato.*

Time.	*Tranſit of the Shadow.*
h. ′ ″	
2 41 15	Through the other Limb of *Plato.*
2 44 15	Promontory between *Conſorinus* and *Beda.*
2 50 20	The firſt Limb of *Palus Somnii.*
2 50 45	The Horns were vertical.
2 53 55	The beginning of *Langrenus.*
2 55 20	The firſt Limb of *Mare Caſpium.*
3 1 10	The other Limb of *Mare Caſpium.*
3 1 45	*Endymion.*
3 3 15	*Meſhala.*
3 7 45	End, or tot. Immerſ. above *Mare Caſpium.*

The *Caſpian* Sea was then diſtant from the Weſtern Limb about ¼ of its Breadth. After the total Immerſion the whole Body of the Moon was ſtill diſtinguiſhable.

XLVI. 1. *An Eclipſe of the Moon*, Decemb. 22. 1675. *at* Greenwich; *by Mr.* Flamſteed. N. 121. *p.* 435.

Correct. H. of the Clock	*Phaſes.*
h. ′ ″	
2 29 30	Between the Cuſpids 2085 = 17′. 16″.
2 55 45	The Shadow almoſt touch'd *Hæmus.*
3 0 30	It had certainly touch'd *Hæmus.*
3 11 30	The right Cuſpid from *Maræotis* 1235 = 10′. 14″.
3 35 0	The lucid Parts were about 2800 = 23°. 11″. or perhaps ſomething more; for it was very difficult to diſtinguiſh the Limits of the true Shadow, becauſe the Air was foul with Vapours.
3 42 30	The Shadow near *Macra.*
3 52 45	Between the Cuſpids about 2288 = 18′. 57″.
4 7 15	The End: For the Limb appear'd, and there ſeem'd nothing to be wanting in the Roundneſs of the Moon.
4 8 0	The Limb very plainly ſeen through the Tube.
4 15 30	The Penumbra, which to the naked Eye reſembled an Eclipſe.
4 19 30	The Diameter of the Moon taken 3757 = 31′. 5″. but not much confided in, tho' I think it not far from Truth.
4 23 0	Still and afterwards, the Limb forſaken by the Eclipſe ſeem'd ſomething obſcure, and as it were another Eclipſe.

The Diſtance of *Corſica* from the remote Limb of the Moon 2732=22′. 37″.
Its neareſt Limb from the neareſt of the Moon 1045 = 8′. 39″.
The remoter Limb of *Sinai* from the neareſt of the Moon 599=4′. 58″. good.
The Middle of the greater black Lake from the neareſt Limb 452=3′. 45″.

I took notice beſides, that the Shadow always appear'd far more diſtinct at the Horns than elſewhere, in the Face of the Moon. In the firſt Obſervation, or a little before, the Horns were parallel to the Horizon.

Then alſo *Porphyrites*, and the greater black Lake, were equally out of the Shadow, about the Length of *Maræotis*.

Yet it never went beyond *Porphyrites* in this Eclipſe, yet I ſaw it deeply immerſed in the Penumbra.

In the Height of the Eclipſe the Shadow almoſt reached *Corſica*; yet I never ſaw it extinct, but immerſed ſo deeply in the Penumbra, that I could hardly perceive it.

Neither did the true Shadow ever go over the Iſle *Macra*, but only a denſe Penumbra, through which it was difficult to ſee it.

At 4^h. $5\frac{1}{2}'$. I could not ſee the Limb, nor at 4^h. $6\frac{1}{2}'$. But at 4^h. $7'$ I thought I ſaw the Light of the Limb very faint, but with much Difficulty. At 4^h. $7'$. $15''$. I was ſure it had emerged out of the Shadow, nor was any thing wanting to its Roundneſs. Therefore there I placed the End.

The Shadow went out near the upper *Lacus Hyperboreus*, the Penumbra remaining, which exhibited the Eclipſe to the naked Eye as far as 4^h. $15\frac{1}{2}'$. But the Limb left by the Eclipſe recover'd the Brightneſs of the other Limb not till 4^h. $28'$. or later.

The Times of the Phaſes were corrected by the Altitudes of *Arcturus*, and the bright Star of the Crown, taken by a Teleſcopic Quadrant of above three Feet Radius. To the taking of which Altitudes I apply'd my ſelf, whenever the Clouds came over the Moon, Thoſe Stars ſometimes ſhone out very bright in the other Quarter of the Heavens.

At London; *by Mr.* Edm. Halley. *Ibid. p.* 498.

2. The Eclipſe was begun before I came to my Inſtruments. But this was obſerved at *London* in *Winchester-ſtreet* by Mr. *Halley*, when the upper Limb of the Moon was diſtant from the Vertex

	°	′		h	′
	39	51	whence he reckon'd the Hour	2	16
from the ſame	41	1	The Horns parallel to the Horizon	2	25
	54	12		3	58

By Mr. Colſon. *Ibid.*

3. *John Colſon* at *Wapping*, near the *Hermitage*, ſaw the Limb of the Moon ſomething deficient at 2^h. $17\frac{1}{4}'$. But at 4^h. $9'$. $25''$. he found it had come out of the true Shadow, and that there was only a denſe Penumbra remaining.

At Paris; *by M.* Caſſini. N. 123. *p.* 561.

4. In this Eclipſe two of the chief things have been exactly determin'd by us, that is, the middle Time of the Eclipſe, and its Magnitude. The middle is deduced not only from a Compariſon of the Beginning and End, but alſo of two equal Phaſes, which are very eaſily determin'd, that is, when the Diſtance of the Horns was equal to the Semidiameter of the Moon, taken before the Eclipſe, $15'$ $28''$. That is, when the beginning of the Eclipſe was eſtimated at 2^h. $24'$. $35''$.

	h.	′	″
End of the total Eclipſe, a like Penumbra being left as was in the Determination of the Beginning	4	15	25
The Duration of the whole Eclipſe comes out	1	50	50
The Half	0	55	25
And the middle of the Eclipſe	3	20	0
A ſixth Part of the Circumference is cut off	2	38	5

	h.	′	″
And again	4	2	25
The Interval	1	24	20
The Half		42	10
Hence the middle of the Eclipſe	3	10	15

Agreeing to the former Determination within a fourth part of a Minute.

We agree entirely with Mr. *Flamſteed* in the Situation of the Shadow, and the Magnitude of the Eclipſe. For it is obſerved by both of us, the Shadow never went beyond *Porphyrites*, tho' it was deeply immerſed in the Penumbra.

Next to *Porphyrites* is a little whitiſh Mountain, which we then call'd the *Companion of Ariſtarchus*, becauſe it is hardly diſtant from *Porphyrites* its own Diameter. That little Mountain was immerſed in the Shadow at 2^{h}. 51′. 15″. And it emerged at 3^{h}. 8′. 25″, and all the Time between was in the Shadow next to *Porphyrites*.

We both of us obſerved alſo, that in the Height of the Eclipſe the Shadow almoſt reached to *Corſica*, yet it was never cover'd by it, but a ſmall Diſtance was left; the Diſtance of which Limit being taken from the neareſt Limb of the Moon was 8′. 17″. Whereas *Flamſteed* found the Diſtance of the Iſland itſelf a little more remote from the ſame Limb, to be 8′. 39″. We alſo obſerved the Iſland *Macra*, or rather *Peninſula*, to be very long adjacent to both the Shadows. We took notice this begun at 3^{h}. 28′. 15″. and that it continued at the ſame Diſtance for a quarter of an Hour.

5. By M. Bullialdus. N. 125. p. 610.

Capella diſt. from the Vert.		*Phaſes.*	*Mean Time.*		
°	″		h.	′	″
39	36	The Penumbra thin.	2	6	12
40	42	The Penumbra more denſe.	2	12	7
42	30	Senſible Begin. over againſt *Sinus Hyperb.* about 70°.	2	23	32
42	50	Almoſt $\frac{1}{8}$ of a Digit.	2	25	48
44	25	The Shadow touch'd the leſſer *Atlas.*	2	36	11
47	28	The Shadow a little above *Baronius*, above *Liguſtinus*, it had ſeized *M. Macr.*	2	56	27
48	56	The Shadow had almoſt reached *Catena Mundi.*	3	6	22
[illegible]	54	The Shadow touch'd *Montunial.*	3	12	54
[illegible]	30	Touch'd *Sin.Peront.* M.*Pyra*, and middle of *Pal.Hyper.*	3	17	41
2	7	It ſeiz'd *Sinus Sagaricus*, and *Peronticus*, and the Promontory of the Moon.	3	29	1
4	52	*Leucopetra* was out of the Shadow.	3	48	2

55 40

55	40	*Sinus Peronticus* out of the Shadow	3	54	6
55	48	*Sinus Sagaricus* out of the Shadow.	3	55	12
56	17	*Sinus Cercinites* was almoſt emerged.	3	58	29
57	16	Part under Shad. equal almoſt to breadth of *Pal. Mæot.*	4	5	38
58	30	The End over againſt Mont. *Macr.* about g. 355.	4	13	59
59	6	Penumbra.			

The Shadow did not reach *Corſica*, nor *Lacus Thraſymenus*; wherefore the Eclipſe did not exceed 3 dig. 30′. or rather leſs. The Beginning was ſooner by a Minute or 45″ than what is ſet down, ſo that it may be fixt more exactly at 2^{h}. 22′. 30″. Hence the whole Duration exactly enough was 1^{h}. 51′. 24. So that the greateſt Obſcuration happen'd at 3^{h}. 18′. 14″.

6.

At Straſburg; *by M.* Richelt. *Ibid.*

Alt. of Arctur.		*Mean Time.*			*Phaſes.*
°	′	h.	′	″	
30	30	2	48	48	Beginning.
36	0	3	20	8	Shad. thro' *M. Porphyrites*, and Promont. of the Moon.
39	50	3	45	44	The Shadow ſkim'd by *Lacus Thraſymenus*, Mount *Baronius*, and *Sinus Cercinites*.
44	15	4	13	20	Shadow paſs'd by Promont. of the Moon, and *M. Cimmerius*.
46	25	4	27	36	Shadow touch'd the leſſer black Lake, and M. *Carparthus*.
48	30	4	41	44	It ended in the middle Region of *Sinus Hyperboreus*.
Therefore the whole Duration was 1^{h}. 52′. 56″.					

At Dantzick; *by M.* Hevelius. N. 124. *p.* 589.

7. In this Eclipſe it is to be well obſerved, that all the Sections never intirely cover'd Mount *Porphyrites*, but it remain'd perſpicuous in the very Limit of the Shadow, even in the greateſt Obſcuration.

Time

Time cor-rect. by Alt.			*Altitudes of the fixed Stars.*			*Order of Phaſes.*	*Through what Spots the Sections of the Shadow paſs'd.*
h.	'	"	°	'			
2	28	38	26	12	Right Should. *Orion.*		
2	33	20	25	35	The ſame.		
2	36	40			Begin. of the *Penum.*		At Mount *Baronius.*
3	8	10					The *Penumbra* more denſe.
3	16	35					The *Penumbra* denſe.
3	24	20					The *Penumbra* very denſe.
3	30	0			Begin. of Eclipſe.	1	At the Bay of *Apollo.*
3	36	25				2	By M. *Alabaſt.* and *Sin. Hyperb.*
3	42	5				3	At *Apollo*'s Bay.
3	46	30				4	By M. *Baronius* and the greater black Lake.
3	52	10				5	By the Iſle *Ophiuſa.*
3	59	15				6	At M. *Porphyr.*& leſſ.blac lake
4	7	45			Star diſtant from the Limb 36′ or 40′.	7	By M. *Porph.* and M. *Serr.*
4	11	45				8	By M. *Porph.* Promont of the Moon, and M. *Carpath.*
4	18	5				9	By the Iſle of *Corſica*, M. *Arg.* and Mount *Macroceran.*
4	26	0				10	By M. *Porphyrit.* and *Lac. Thraſim.*
4	56	20				11	At *Apollo*'s Bay, *Mons Chriſti*, and *Inſul. Macra.*
5	55	28	32	33	Brt. Star of the harp.		
5	57	33	32	50	The ſame.		
5	59	0					The Penumbra.
6	8	0					Penumbra vaniſh'd, as well as we could gueſs by theClouds.

XLVII.
An Eclipſe of the Moon, Oct. 29. (*St. N.*) 1678. *At Paris*; *by M.* Caſſini. N. 141. p.1015.

The Phaſes of the Moon, and of the Spots, as determined by Ricciolus.	*In the Royal Obſervatory.*						*In the College of Clarmont.*		
	Below.			*Above.*					
	h.	'	"	h.	'	"	h.	'	"
he Shadow begins.	6	43	30	6	43	40	6	43	54
The ſubſequent Limb of *Grimaldus.*	6	45	0				6	45	29
Galilæus.	6	46	0	6	46	0			
The End of *Galilæus.*	6	47	0						

	h.	′	″	h.	′	″	h.	′	″
Merſennus.	6	48	20						
The Chord of the Eclipſe was 6 dig.				6	49	00			
The Beginning of *Gaſſendus.*	6	50	50	6	50	50			
The middle of *Gaſſendus.*				6	51	30	6	51	37
The Beginning of *Schikard.*	6	51	43						
11 Dig. Eclip.				6	52	00			
The Beginning of *Ariſtarchus.*	6	52	50						
The middle of *Ariſtarchus.*				6	53	10	3	56	7
The End of *Ariſtarchus* and *Morinus.*	6	54	00						
Capuanus, or the Dragon's Eye.	6	56	00						
III. Digits.				6	56	30			
A Chord of 9 Digits.				6	57	00			
The Begin. of *Terra Pruina* and *Copernicus*	6	58	54						
The Beginning of *Copernicus.*				6	59	10			
The middle of *Copernicus.*	7	00	00	7	00	00	6	59	30
The end of *Copernicus.*	7	00	55	7	00	55			
The beginning of *Pitheas.*	7	1	50	7	1	40			
The end of *Pitheas.*	7	2	30						
The Virgin's Head.	7	2	45	7	2	40	7	3	18
Harpalus.	7	2	55						
The beginning of *Tycho.*	7	4	20	7	4	20			
The middle of *Tycho.*	7	5	00				7	5	48
The end of *Tycho.*	7	5	55						
Eratoſthenes.	7	6	20						
V. Digits.				7	6	50			
The Promont. between the Virgin and *Plato.*	7	7	00						
The Iſland in the fartheſt of the *Sinus Medii.*	7	7	30						
A bright Spot following *Tycho.*	7	8	31						
VI. Digits.				7	11	20			
Tymocharis.	7	11	48						
The beginning of *Plato.*				7	13	20	7	13	29
The middle of *Plato.*				7	13	40			
The end of *Plato*, and the begin. of *Manilius.*	7	14	00						
The end of *Plato.*				7	14	40			
Manilius.				7	14	50	7	15	4
The end of *Manilius.*	7	15	12						
The beginning of *Dionyſius.*	7	17	15						
Dionyſius.				7	17	25			
Menelaus.				7	18	10	7	17	59
The end of *Dionyſius*, and begin. of *Menelaus.*	7	18	28				7	18	11
Pliny begins.	7	20	56						
Pliny.				7	21	10	7	21	51
Picolomineus, or the bri^t Spot above the Ring.	7	21	30						
The beginning of *Fracaſtoreus* or the Ring.	7	23	00	7	23	5			
The beginning of *Poſſidonius.*	7	23	55				7	24	38
The end of *Fracaſtoreus.*	7	24	25						

The

	h.	'	"	h.	'	"	h.	'	"
Bright Spot before the Angle of the acute Promontory				7	24	35			
The Angle of the acute Promontory	7	25	10						
IX Digits				7	26	10			
Palus Somni	7	28	00						
The beginning of *Endymion*	7	29	20						
The beginning of *Taurentius*	7	30	30						
Angle of the horns with the Parallel 77° 15'.				7	30	40	7	31	16
The beginning of *Hermes*	7	31	5						
End of *Taurentius*, or of the Serpent's Head	7	31	40						
The end of *Hermes*	7	32	00						
Proclus	7	32	20						
The Limb of *Mare Caſpium*	7	32	29	7	33	3	7	33	10
The lower Spot	7	33	5						
The beginning of *Langrenus*	7	34	24						
Meſſala	7	34	40						
The end of *Langrenus*	7	35	40						
The *Peninſula* in *Caſpia*	7	37	50						
The end of *Caſpia*	7	37	50	7	37	60	7	38	18
An oblong Spot	7	39	16						
The end	7	40	41	7	41	00	7	41	28
The beginning of the Emerſion	9	21	30	9	21	30	9	21	5
Grimaldus	9	22	44				9	22	10
The end of *Grimaldus*	9	23	40	9	23	40			
Galilæus	9	24	35						
The Sea of *Humours*	9	28	15						
The middle of *Schikard*	9	29	00						
The beginning of *Ariſtarchus*	9	29	44				9	29	41
The middle of *Ariſtarchus*				9	30	20			
The whole of *Ariſtarchus*	9	30	25						
The whole of *Gaſſendus*	9	31	00						
The beginning of *Kepler*				9	31	40	9	31	6
The whole of *Kepler*	9	32	24						
Morinus	9	30	00						
The end of the Iſland of *Kepler*	9	34	22						
The whole of *Capuanus*	9	34	40						
The beginning of *Terra Pruina*	9	35	43						
Virgo	9	37	00						
Golden Spot between *Pitheas* and *Kepler*				9	37	20			
Harpalus, & the bright ſpot before *Copernicus*	9	37	34						
III Digits				9	37	40			
Bullialdus	9	38	30						
The beginning of *Copernicus*				9	39	00	9	38	36
The beginning of *Tycho*	9	40	00				9	40	22
The whole of *Copernicus*	9	40	18						
Pitheas				9	40	30			

	h.	'	"	h.	'	"	h.	'	"
The middle of *Tycho*	9	41	00	9	41	10			
The end of *Tycho*				9	42	20			
IV Digits				9	42	40			
The whole of *Timocharis*	9	45	30						
Plato begins				9	45	30	9	45	00
V Digits				9	47	00			
The whole of *Plato*	9	45	00	9	47	30			
The Iſland under *Sinus Medius*	9	50	00						
The end of *Sinus Medius*	9	50	50						
Archimedes	9	51	37						
VI Digits				9	52	00			
Manilius				9	54	00	9	53	52
Fretum	9	54	22						
The beginning of *Abifeldea*	9	54	40						
The end of *Abifeldea*	9	55	30						
The whole of *Ariſtotle* and *Eudoxus*	9	56	00						
VII Digits				9	56	20			
The beginning of *Dionyſius*	9	57	12						
Menelaus	9	57	50	9	57	40	9	57	16
The middle of *Dionyſius*				9	58	30			
The whole of *Menelaus*	9	59	00						
The whole of *Dionyſius*	10	00	00						
VIII Digits	10	1	30						
The Mouth of *Lacus Mortis*				10	2	00			
Pliny.				10	2	15	10	1	28
The Promontory above *Dionyſius*	10	3	00						
The end of *Pliny*	10	3	40						
Poſſidonius				10	5	30			
The Acute Promontory	10	6	00						
Hermes				10	6	15			
IX Digits				10	9	30			
The Chord of the Shade of 8 Digits	10	11	00						
Meſſala	10	13	30	10	12	10	10	11	30
The beginning of the *Caſpian Sea*	10	16	00						
The whole of *Langrenus*				10	14	00			
A Chord of 6 Digits				10	16	50	10	16	44
The firſt end of the *Caſpian*	10	17	33	10	17	25			
The other end of the *Caſpian*	10	18	15						
The Whiteneſs in the *Caſpian Sea*	10	20	00	10	20	10	10	20	42
The total End									

XLVIII. 1. *An Eclipſe of the Moon*, Aug. 19. m. 1681. *at* Greenwich; *by Mr.* Flamſteed. *Ph. Coll. N.* 3. *p.* 67.

Time by the Pendulum Clock.			*Time corrected by Obſervat.*			*Obſervations.*
h.	′	″	h.	′	″	
1	5	00	1	00	00	The Moon's Diameter 6191 = 30′. 53″.
1	46	30	1	41	30	The Penumbra obſervable.
1	53	40	1	48	40	The Penumbra denſe.
1	55	40	1	50	40	The beginning.
1	58	00	1	53	00	The Shadow at Mount *Climax.*
2	3	20	1	58	20	*Sinus Sirbonis* begins.
2	5	50	2	00	50	*Lacus Maræotis* begins.
2	7	30	2	2	30	*Mons Cataractes* begins.
2	8	16	2	3	16	*Maræotis* wholly cover'd.
2	10	50	2	5	50	*Sinai* begins.
2	12	56	2	7	56	*Sinai* wholly cover'd.
2	18	00	2	13	00	*Audus.*
2	27	24	2	22	24	*Circinna.*
2	31	00	2	26	00	*Abulfeda* according to *Ricciolus.*
2	32	30	2	27	30	*Pentadactylus* begins.
2	33	32	2	28	32	The firſt Limb of *Ætna.*
2	34	26	2	29	26	The whole of *Pentadactylus.*
2	35	6	2	30	6	The middle of *Ætna.*
2	36	00	2	31	00	*Ætna* wholly cover'd.
2	40	00	2	35	00	The Shadow thro' the middle of *Porphyrites.*
2	44	50	2	39	50	Iſle *Hiera* and *Ficaria* together at the Shadow.
2	46	10	2	41	10	The Shadow at *Horminius.*
2	50	10	2	45	10	*Mons Hercules* begins.
2	51	10	2	46	10	*Mons Hercules* within the Shadow.
2	52	10	2	47	10	The Shadow through the middle of *Beſbicus.*
2	54	00	2	49	00	Through the middle of *Inſula Major.*
2	55	30	2	50	30	Through the middle of *Bizantium.*
2	57	40	2	52	40	Through the middle of the Iſle *Cyanea.*
3	1	50	2	56	50	At the firſt Limb of *Corocondometes.*
3	4	30	2	59	30	At the firſt Limb of *Mæotis.*
3	13	00	3	8	00	*Macra* begins to be obſcured : Doubtful.
3	14	00	3	9	00	*Mæotis* wholly cover'd.
3	19	12	3	14	12	The remaining lucid parts were 1092=5′. 27″.

2.
At Paris *; by M.* Caſſini. *Ibid. p.* 66.

Digits eclipſed.	*Phaſes.*	*Time.*
d. ′ ″		h. ′ ″
	The *Beginning* at	1 58 30
	The Arrival of the Shadow at *Grimaldus*	2 1 30
	At *Tycho*	2 8 00
	At the Center of the *Moon*	2 37 30
	At the middle of *Copernicus*	2 39 30
	At *Ariſtarchus*	2 40 00
8 16 00	At the middle of *Manilius*	2 54 30
	At *Plinius*	3 1 00
9 2 00	At the lower part of the *Caſpian Sea*	3 7 30
9 31 20	At the upper ſide of the *Caſpian Sea*	3 18 30
10 1 00	The *greateſt Obſcurity*	3 35 45
4 46 00	The *Return* of the Shadow to *Ariſtarchus*.	3 47 00
	To the Center of the *Moon*	4 29 00
	To *Tycho*	4 41 30
	The *End* of the *Eclipſe*	5 13 00
	The *Whole Duration*	3 14 30

At 5h. 13′ 56″. the *Apparent Altitude* of the upper edge of the *Sun*, was 17′. 10″. and that of the *Moon*, 1°. 11′. 30″.

3.
At Dantzick *; by M.* Hevelius. N. 5. *p.* 160.

Time by the Clock.	*Phaſes.*	*Alti-tudes.*	*Time cor-rected.*
h. ′ ″		° ′	h. ′ ″
1 1 03	The Altitude of the Bull's Eye	24 14	1 4 48
2 4 30	The ſame Altitude	32 58	2 6 32
2 9 00	Taken again	33 30	2 10 32
2 45 00	The Track of the Penumbra		2 46 30
2 53 00	Penumbra ſomething denſer		2 54 00
2 59 00	Still more denſe		3 00 00
3 1 00	Now very denſe		3 2 00
3 2 00	The beginning of the Eclipſe		3 3 00
3 6 00	The firſt Phaſis		3 6 30
3 12 00	The ſecond Phaſis		3 12 30
3 14 00	The third Phaſis		3 14 30
3 20 00	The fourth Phaſis		3 21 00
3 22 30	Mount *Sinai* wholly cover'd		3 23 00
3 29 00	The ſixth Phaſis		3 30 00
3 43 00	The Altitude of *Pollux*	30 00	3 43 00
5 2 00	The Center of the Sun ariſes		

The Colour of this Eclipſe was Aſh-colour, or duſky. At the Time of Obſervation *Mons Porphyrites*, and Mount *Ætna* were nearly in the ſame Perpendicular.

Phaſes I. The Shadow paſs'd over Mount *Eous*.
II. It came to the Lakes of *Arabia*.
III. It touch'd the lower Bank of *Palus Maræotis*, and the Extremity of *Sinus Sirbonis*.
IV. It paſs'd over the Iſland *Letoa* as far as M. *Sinai*, but the whole Mountain was yet viſible and uncover'd.
V. It ſeem'd to paſs over *Sinus Syrticus*, ſo that the Shadow now cover'd all Mount *Sinai*.
VI. It went above *Sinus Syrticus*, beyond the Iſland *Crete*, and over *Mare Mortuum*.

An Eclipſe of the Moon, Feb. 11. 1682. *by Mr.* Flamſteed. N. 145. *p.* 89.

XLIX. 1. *An.* 168½. *Feb.* 11. Hour 8. 1′. with a Tube of 16 Feet I took the Moon's Diameter 6702 = 33′. 25″. Then the Diſtance of its neareſt Limb from the neareſt Limb of *Maræotis* 145 = 0′. 43″. But the Diſtance of the Limb of the ſame Spot from the remoter Limb of the Moon was 6575 = 32′. 48″. Alſo by the Help of the ſame Tube I obtain'd the Times, when the Obſcuration reach'd the Center of the Moon, and when the Radius ſubtended its Arches in the Periphery that were either deficient or after they were reſtored: From whence the middle may be derived, perhaps not leſs accurately, than from the compared Obſervations of the Beginning and End, the Immerſion and Emerſion.

* * *

Phaſes.	*The Times of the Phaſes corrected by Pendulum Clocks.*		
	Obſervat. in *Greenwich*		At *London*
	By myſelf.	*By* Halley.	*By* Haines.
	h. ′ ″	h. ′ ″	h. ′ ″
Famus at the lower Limb; by our naked Eyes	8 48 38		
The Shadow	9 4 8		
A denſe Penumbra	9 11 44		
The Beginning	9 12 32	9 13 04	9 12 18
The middle of *Palus Maræotis* cover'd	9 14 02		perh.ſoon.
The whole of *Maræotis* cover'd		9 14 39	9 13 48
A ſixth part of the Periphery was obſcured	9 18 10		
Circinna within the Shadow	9 20 10		
The middle of *Porphyrites*		9 21 1	
The Shadow ſkims by *Syrbo*		9 21 9	
Firſt Limb of *Mons Cataractes*, or of *Gaſſendus*	9 21 32		
The Eye of the Dragon	9 27 58		
It ſkim'd by the nearer limbs of *Crete* and *Ætna*	9 28 22	9 28 44	9 28 19
Mount *Ætna* wholly cover'd; *Hiera* begins	9 29 48	9 30 16	9 30 12
Hiera wholly cover'd		9 30 44	
The Beginning of *Corſica*	9 33 46		
The middle of *Corſica*		9 34 48	
Mount *Sinai* begins	9 36 38	9 37 10	9 36 20
The middle of *Sinai*	9 37 22		
All *Sinai* cover'd	9 38 04	9 38 10	9 38 20
The Center of the Moon, or 6 Digits	9 38 48		
The greater black Lake begins	9 39 58	9 40 36	
The middle of the greater black Lake			9 40 48
The whole is cover'd		9 41 40	
The beginning of *Beſbicus*		9 43 48	
The middle of *Beſbicus*	9 43 42		9 44 13
Whole of *Beſbicus*, it glides by the *Euxine* Sea		9 44 36	
Byzantium begins	9 46 58		
Horminius begins	9 47 22		
Carpathes	9 47 50		
Mons Serrorum	9 48 30		
Apollonia	9 50 24	9 51 21	
Macra	9 52 44	9 53 33	
Mount of *Hercules*	9 54 26		
All *Macra* cover'd		9 54 53	
All Mount *Hercules* cover'd		9 55 33	
Corocondometes Palus	9 59 10		
Thro' the middle of the upper *Lacus Hyperbor.*		9 59 53	
Corocondometes wholly cover'd	10 00 38		
The Shadow over Mount *Corax*		10 1 17	
It glides by *Mæotis*	10 2 20	10 2 38	

Through

	h.	′	″	h.	′	″	h.	′	″
Through the lower *Lacus Hyperboreus*				10	3	53			
Through the middle of *Inſula Major*	10	5	14						
Lacus Mæotis almoſt wholly cover'd	10	6	54	10	8	00	10	6	48
A ſixth part of the Periphery, the reſt bright	10	7	28						
Immerſion - - - - - - - - - - - -	10	10	14	10	10	11	10	9	48
Emerſion - - - - - - - - - - - -	11	47	38	11	47	9	11	47	48
A ſixth part of the Periphery illuminated, with the firſt Limb of *Maræotis* - - -	11	50	02	11	50	7			
The middle of *Maræotis* - - - - - - -	11	50	42				11	50	48
All *Maræotis* illuminated - - - - - - -	11	51	6	11	50	58			
The Shadow thro' the middle of *Porphyrites*	11	52	50						
All *Porphyrites* cover'd - - - - - - -	11	53	34				11	52	54
All *Circinna* - - - - - - - - - - - -	11	58	18						
It glides by the *Sinus Sirbonis* - - - -				11	58	59			
Cataractes Sirbonis, or all *Gaſſendus*	12	1	00						
Hiera begins - - - - - - - - - - - -				12	3	14			
Ætna begins to emerge - - - - - - - -	12	3	20						
Thro' the middle of the greater black Lake -							12	3	48
Through the middle of *Ætna* - - - - - -				12	4	19			
Ætna wholly uncover'd - - - - - - - -	12	5	18	12	5	4	12	5	45
Crete is illuminated - - - - - - - -	12	7	34						
Sinai begins - - - - - - - - - - - -	12	13	38				12	13	43
The middle of *Sinai* - - - - - - - -	12	14	28	12	14	12			
All *Sinai* - - - - - - - - - - - - -	12	15	34	12	14	55			
The middle of *Beſbicus* - - - - - - - -	12	17	34				12	17	48
The Center of the Moon, 6 Digits uncover'd	12	18	28						
Thro' the middle of the upper *Hyperborean* lake				12	19	5			
The Shadow thro' the middle of *Bizantium*							12	20	18
Bizantium without the Shadow - - - -	12	20	50	12	20	25			
All *Macra* - - - - - - - - - - - -				12	22	36			
All the *Inſula Apollonia*, and *Horminius*	12	24	24	12	24	16			
The lower *Lacus Hyperboreus* - - - -				12	26	6			
Mount *Hercules* - - - - - - - - - - -				12	30	56	12	31	18
The middle of *Corocondometes* at the Shadow	12	31	38						
Mons Corax - - - - - - - - - - - -				12	32	37			
Mæotis begins to be enlightned - - - -	12	33	18				12	32	18
Mæotis wholly uncover'd - - - - - - -				12	37	57	12	38	3
h part of the Periphery, the reſt obſcure	12	39	6						
ddle of the greater Iſland of the *Caſp.* Sea				12	41	22	12	42	18
hole Iſland uncover'd - - - - - -	12	42	18						
d doubtful - - - - - - - - - -	12	45	10				12	44	48
d certainly - - - - - - - - - -	12	45	38	12	44	35	12	45	18

During the Eclipſe Mr. *Halley*, together with *Thomas Smith*, took the following Diſtances of the Moon from the fixed Stars with a Sextant.

Times corr. by the Clock.	*Obſervations.*	*Diſtances.*
h. ′ ″		° ′ ″
8 53 07	Of the remoter Limb of the Moon from *Regulus*	8 05 20
8 55 07	Repeated	8 06 20
9 2 34	Of the remoter Limb of the Moon from the bright Star in the Lion's Side	14 58 15
9 4 18	Repeated	14 57 55
9 8 42	Of the remoter Limb of the Moon from *Regulus*, again	8 12 50
	The Moon being intirely immerſed in the Shadow, Mr. *Halley* with me.	
10 19 21	Of the neareſt Limb of the Moon from *Regulus*	8 13 30
11 38 06	Of the remote Limb from *Regulus*	9 23 50
11 41 48	Repeated	9 25 35
	And when the Eclipſe was over,	
12 52 00	Of the remoter Limb from *Regulus*, again,	9 58 20
12 55 10	Once more	10 00 00

⁂

Phaſes

2\.
At Paris, and Copenhagen.
N. 146. p. 145.

Phaſes.	At *Paris.*		At *Copenhagen.*	
	In the Royal Obſervatory.		*College of Clarmont.*	By *Roemer. A reduction of Meridians being made, ſubt.* 41′. 40″.
	By *Caſſini.*	By *Picard* and *De la Hire.*	By Father *de Fonteney*	
	h. ′ ″	h. ′ ″	h. ′ ″	h. ′ ″
The Beginning	9 20 55	9 21 58	9 21 25	9 21 50
The middle of *Ariſtarchus*	9 29 57	9 29 33	9 29 28	9 29 50
The middle of *Copernicus*	9 38 45	9 38 25	9 38 40	9 38 48
The Beginning of *Tycho*	9 45 52	9 45 48	9 45 48	9 46 20
The end of *Tycho*	9 47 40	9 47 36	9 47 20	9 47 50
The Beginning of *Plato*	9 49 47	9 49 08	9 49 25	9 49 50
The Center of the Moon		9 51 10		9 50 20
The Center from a Compariſon of the beginning and end	9 50 24	9 50 44	9 50 10	9 50 30
The middle of *Manilius*	9 53 16	9 53 30	9 51 18	9 53 20
Menelaus	9 56 43	9 56 20	9 56 14	9 56 20
Dionyſius	9 57 09	9 57 40		9 57 20
The Acute Promontory	10 04 31	10 04 15	10 04 12	10 04 20
The Beginning of *Caſpia*	10 11 40	10 11 25	10 11 20	10 11 50
The End of *Caſpia*	10 16 27	10 16 30	10 15 50	10 16 20
The Moon intirely immerſed	10 19 53	10 19 30	10 18 55	10 19 50
The Beginning of Emerſion between *Grimald.* and *Galilæus*	11 57 51	11 56 00	11 58 00	11 55 20
The middle of *Ariſtarchus*	12 02 00	12 01 55	12 03 09	12 01 50
The middle of *Plato*	12 12 06	12 12 10	12 13 26	12 12 20
The middle of *Copernicus*	12 14 32	12 14 30	12 14 29	
The middle of *Tycho*	12 23 35	12 23 55	12 24 21	12 23 20
The Center of the Moon		12 25 50		12 26 20
The Center by Compariſon of the Emerſion of the Limbs	12 26 09	12 25 15	12 26 08	12 24 50
The middle of *Manilius*	12 26 22		12 26 20	12 26 20
The middle of *Menelaus*	12 29 08	12 29 20	12 29 22	12 29 05
Dionyſius	12 32 30	12 33 15	12 32 20	12 32 50
The Acute Promontory	12 39 45	12 39 50		12 39 50
The Beginning of *Caſpia*	12 41 53	12 41 30	12 41 11	
The End of *Caſpia*	12 47 15	12 46 30	12 48 20	12 47 20
The end of the Eclipſe	12 54 27	12 54 30	12 54 17	12 54 20

3. *At* Dantzick; *by M.* Hevelius. *Ib. p.* 146.

Time by the Watch	*Order of Phaſes.*	*Digits eclipſed*	*Altitudes of the Stars.*	By what Spots the Sections of the Shadow paſs'd, and what was obſerved beſides.	*Time corrected by Altitudes.*
h. ′ ″			° ′		h. ′ ″
8 41 18			Bull's Eye 39 53		8 40 46
8 44 9			Again 39 27		8 44 20
8 48 14			Again 39 0		8 47 59
10 8 28				Begin. Penumbra very dilute.	10 9 0
10 12 50				The Penumbra more denſe.	10 13 20
10 19 5				The Penumbra ſtill thicker.	10 19 36
10 23 50				The Penumbra very denſe	10 24 24
10 24 30				Begin. Eclipſe about 150° of the Limb, or 95° from the upper point of perpendicular Line of 90th Deg. towards the Eaſt.	10 25 5
10 29 45	1	1¼ Dig.		It proceeded by M. *Germanician* to *Mare Syrticum* and M. *Acabe.*	10 30 30
10 32 13	2	2 almo.		By M. *Porphyr. Loca Paludoſa*, Iſle *Circinna*, *Sin. Syrticus*, over M. *Sacer*, and *Mare Caſpium.*	10 32 50
10 36 30	3	2½ alm.		By M. *Baron.* the Iſle *Æthuſa*, thro' the middle of *Sin. Sirbonis*, and Mount *Pharan.*	10 37 10
10 43 25	4	3⅝		Thro' bay of *Apollo*, M. *Ætna*, the Iſle *Letoa*, to Iſle *Didyma.*	10 44 10
10 47 25	5	4⅝ Dig.		To Iſles *Majorca*, *Vulcania*, *Lemnos*, *Carpathia*, and M. *Horeb.*	10 48 15
10 52 20	6	5¾		To the greater black Lake, by M. *Argentariuſa*, *Sipylus*, *Maſicytus*; *Tabor*, *Sinai*, and the Deſert of *Raphidim.*	10 53 10
10 55 45	7	6½		By the *Hyperborean* Rocks, Iſle *Beſbica*, M. *Olympus*, *Didymus*, and M. *Antilibanus.*	10 56 40
11 0 33	8	7¾ Dig.		At the Iſland *Cyanea*, M. *Horminius*, M. *Uxii*, M. *Coibacarani*	11 1 30
11 4 30	9	8½		By M. *Ambonus* to Iſle *Apollonia*, and the *Athenian* Bay, by M. *Uxii*, and *Coibacarani.*	11 5 30
11 9 59	10	9¼		By the upper *Hyperborean* Lake, *Palus*, *Byces*, by M. *Herculeus*, and the utmoſt Bay of *Pontus.*	11 10 55

By

h.	′	″					h.	′	″
11	13	30	11	10 Dig.		By the lower *Hyperborean* Lake, *Mons Cimmerius*, and to the lower Bay of the *Caſpian* Sea.	11	14	27
11	16	30	12	10½ alm		By the *Ripbean* Mountains, *Pal. Mæotis*, and Iſle *Alopecia*, to the greater *Caſp.* Iſle, by M. *Neroſus*.	11	17	17
11	19	10	13	11¼		Bank of *Palus Mæotis*, and M. *Hippocus*.	12	20	10
11	22	0	14	11½		By Mount *Alaunus* to the leſſer Weſtern Lake.	11	23	0
11	26	30		12 Dig.		Total Obſcuration was obſerved about 297° of the Limb, or in 94° from the upper Point of the perpendicular Line of the 90th Deg. towards the Eaſt.	11	27	30
11	35	36			Of *Pollux* 52° 13′		11	37	4
11	37	35			Again 52 0		11	38	42
11	38	50			Again 51 41		11	39	35
12	58	20				The Recovery of Light was about 118° of the Limb, or in 88° from the upper point of the perpendicular Line of the 90th Degree towards the Weſt.	1	0	8
1	0	40	15	½ Dig.		At Mount *Alabaſtrinus*, Mount *Pentadactylus*, Mount *Audus*, and *Palus Maræotis*.	1	2	31
1	3	35	16	1¼		By M. *Porphyrites*, to *Mare Syrticum*, and by *Mare Eoum*.	1	5	32
1	8	10	17	2 almo.		By M. *Baronius*, the *Loca Paludoſa*, the Iſle *Circinna*, between *Mare Syrticum* and *Egyptiacum*, at the Bay of *Syrbon*.	1	10	32
1	11	48	18	2½ alm		By the Bay of *Apollo*, the Iſle *Taracinia*, the Iſle *Æthuſa*, and the Bay of *Syrbon*.	1	13	54
1	16	20	19	3½		At the Iſland *Majorca*, by Mount *Ætna*, M. *Neptune*, *Jacbynthus*, the Iſland *Letoa*, the Iſland *Didyma*, and Mount *Lyon*.	1	18	33
1	20	50	20	4½ Dig.		At the Iſland *Corſica*, by the Iſland *Vulcania*, and *Carpathus*, at *Mare Mortuum*, by the Deſert of *Sinai*.	1	23	6

h. ′ ″					h. ′ ″
1 24 39	21	5¼ Dig.		At *Mons Argentarius*, *Mare Pamphilium*, Iſle of *Cyprus*, by M. *Horminius*.	1 27 0
1 26 25	22	6 almo.		By M. *Apennine*, to the Lake *Thraſimenum*, at Mount *Sipulus*, by the lower M. *Libanus*.	1 28 40
1 28 32	23	6½.		To Mount *Carpathos*, by the Iſle *Beſbica*, M. *Olympus*, *Didymus*, and Mount *Dalangueros*.	1 31 0
1 31 12	24	7½ Dig.		By Mount *Perce*, *Byzantium*, to Mount *Horminius*, and by M. *Antitaurus*.	1 33 44
1 35 59	25	8¼		By *Sinus Circinites*, the Iſle *Macra*, the Iſle *Apollonia*, the middle of Mount *Moſchus*, and the *Sogdian* Mountains.	1 38 37
1 39 33	26	9 Dig.		By the lower *Hyperborean* Lake, the Marſh *Byce*, M. *Strobilus*, by the extream Bay of *Pontus*, and Mount *Paropamiſus*.	1 42 15
1 41 45	27	9⅛ Dig.		Between the Marſh of *Byce*, Lake *Corocondometes*, by M. *Hercules*, and the lower M. *Caucaſus*.	1 44 30
1 44 19	28	10		By M. *Cimmerius*, M. *Tancon*, and Mount *Neroſus*.	1 47 9
1 47 20	29	10½.		By *Palus Mæotis*, by the leſſer Iſle of the *Caſpian* Sea, and by the upper Mount *Neroſus*.	1 50 14
1 51 44	30	11 little more.		By Mount *Alaunus*, Mount *Sanctus*, and the Mounts *Hippocus*.	1 54 42
1 56 12		12 Dig.		End about 294° of the Limb, or 97° from the upper point of the Perpendicular of the nonageſim towards the Weſt.	1 59 17
1 57 10				A very denſe Penumbra.	2 0 17
2 1 15				Still very thick.	2 4 27
2 2 30				Something more dilute.	2 5 45
2 10 20				A very dilute Penumbra.	2 13 42
2 12 30				The End of the Penumbra.	2 15 0
2 15 39			Alt. *Pol.* 29° 12′		2 19 7
2 17 35			Same Alt. 28 46		2 22 8

In the Progreſs of the Eclipſe the Shadow was very dilute, and its Limb was uneven and ill terminated, ſo that it was very difficult to determine the Phaſes from the Beginning, nor could it be exactly diſtinguiſh'd thro' what Spots the Shadow paſſed; yet in proceſs of Time as the Eclipſe went on, all things became more diſtinct. The Colour from the Beginning was ſufficiently dull, obſcure, and gloomy, as if the Eclipſe would ſhew itſelf about the greateſt Obſcuration, like that in the Month of *April* in the Year 1642, ſo overſhadow'd as hardly to be perceived. But the thing fell out otherwiſe; for when the Moon was quite eclipſed, its whole Diſk was perſpicuous enough to the Eye. For then its Colour was quite ruddy, bloody, or like Ruſt, which continued till the Moon had half recover'd her Light, and then again it appear'd pretty dark and duſkiſh.

L. The Obſervation of this ſmall Eclipſe was very difficult, becauſe of the oblique Incidence of the Moon into the Earth's Shadow, and the Thinneſs of the Shadow, through which we could very plainly perceive the Limb of the Moon, even in the middle of the Eclipſe. We could not accurately define the Parts of the Diameter, which were then deficient from the true Shadow, becauſe the Limits were ſo confuſed. I therefore took the Diſtance between the Cuſpids tho' ill defined, about the middle of the Eclipſe, from whence the ſaid Parts might be eaſily deduced, and with leſs Danger of Error.

An Eclipſe of the Moon, June 17. *m.* 1684. *at* Greenwich; *by Mr.* Flamſteed. N. 162. *p.* 669.

Time by the Pendulum Clock.			*True Times correct. by Obſervat.*			*Obſervations.*		
h.	'	"	h.	'	"		'	"
1	40	40	1	31	8	Tendency to an Eclipſe		
2	4	30	2	54	58	Diam. of the M. by a Tube of 16 feet was 1605=	31	57
2	6	40	2	57	8	- - - - - - - - - - repeated 6430=	32	4
2	12	0	2	2	20	A denſe Penumbra, perhaps the beginning		
2	16	00	2	6	28	The Shadow had defaced the limb above M. *Sinai*		
2	18	00	2	8	28	The Shadow was certainly within the Diſk		
2	21	36	2	12	4	The Chord of the darkned Periphery — 1670=	8	28
2	26	00	2	16	28	- - - - - - - - - - repeated 2010=	10	1
2	30	00	2	26	28	- - - - - - - - - - Again 2290=	11	25
2	42	00	2	32	28	The Eclipſe ſenſibly decreaſed		
2	43	00	2	33	28	Between the darkned Cuſpids again 1895=	9	20
2	50	00	2	40	28	The End, but doubtful to me		
2	54	00	2	44	28	Certainly ended, my Aſſiſtant *Smith* agreeing		
2	57	00	2	47	28	A denſe Penumbra		
3	11	00	3	1	28	And ſtill		
3	20	00	3	10	28	The ſouthern Limb had hardly yet recover'd the Brightneſs of the northern Limb: But its Light was duller than in the northern Limb, as it appear'd in the firſt Obſervation.		

An Eclipſe of the Moon, Nov. 30. (*St. N.*) 1685. at Dantzick; *by M.* Hevelius. N. 178. *p.* 1256.

LI. It required no little Skill and Labour to manage our Tubes, tho' they were but ſhort, of 5, 6, and 7 Feet, and to keep them ſo ſteady to the Moon, ſo that we might truly diſtinguiſh the Penumbra, and exactly mark out all the Phaſes, through what Spots they paſs'd, or what they arrived at in every particular Inſtant of Time. But I perform'd this as well as I was able, and as far as the ſevere Seaſon and floating little Clouds would allow. Firſt it may be obſerved, that a very denſe Penumbra went before the true Beginning of the Eclipſe, ſo that I could hardly, if at all, diſtinguiſh the Beginning. As to the Colour, it is chiefly to be noted in this Eclipſe; for I have very rarely found ſo great a Diverſity in Colour. Sometimes it was ruſty, or of the Colour of a Weeſel; but when the total Obſcuration came on, the Limb of the Moon was up and down ſomething livid, and partly brightiſh and ruddy. But in the middle of the Moon a denſe and obſcure little Cloud was ſeen, ſo that we could not well diſtinguiſh the Spots in the Moon. This blackiſh Cloud advanced by degrees towards the Right Hand, and to the *Palus Mæotis*; ſo that about the Beginning of the Recovery of Light, the whole genuine Shadow appear'd very dark and black; and about the laſt Phaſis taken Notice of, the remaining part of the Moon ſtill obſcured, or its Limb, could not be ſeen at all.

Order of the Phaſes.	*Time accor. to Clock cor. by Alt.*			*Obſervables.*	*Alt. Sun, Moon, and fixt Stars.*			*Through what Spots the Sections of the Shadow paſs'd.*
	h.	'	"		°	'	"	
				Merid. Alt. of the Sun.	12	39	0	
	3	19	0	Alt. of the S.	1	54	0	
	4	28	0	Alt. Moon	9	0	6	
	5	2	30	Alt. of *Capella*	31	0	0	
	5	8	0	Alt. of *Capella*	31	2	0	Dubious
	9	32	25	Penumbra				
	9	42	35	Penum. more denſe				
	9	48	5	Penum. very denſe				
	9	50	10	Begin. as far as could be collected				
1	9	53	20	*Pal. Maræotis* wholly cover'd.				At Mount *Andus*, and *Pal. Maræot.*
2	9	57	0					By *Germanicianus*, to *Mare Syrticum*, and Mount *Acabe.*
3	10	0	15	M. *Porphyrit.* not yet cover'd.				To Mount *Porphyrites*, over *Mare Syrticum*, and Mount *Ajax*.

	h.	'	"		
4	10	3	5		By *Mons Petri*, between the Iſle *Circinna*, and Iſle *Terracinnia*, by *Sinus Sirbonis*, and the Marſhes of *Arabia*.
5	10	6	25		To the Iſland *Ficaria*, *Malta*, M. *Athos*, by Mount *Pharan*, and Mount *Troicus*.
6	10	10	25		To the Bay of *Apollo*, by the Iſle of *Error*, by Mount *Ætna*, the Iſles *Letoa* and *Didyma*.
7	10	14	55	Iſland *Melos* now wholly cover'd.	Above the Bay of *Apollo*, to the Lake of *Hercules*, the Iſle of *Sicily*, by M. *Parthenius* and *Taigetus*, Iſle *Melos*, and M. *Lion*.
8	10	19	55		To the Iſle *Minorca*, thro' the Bay of *Paſtanus*, M. *Micalo*, Iſle of *Cyprus*, M. *Hor*. to M. *Sinai*, and the Deſert of *Raphidim*.
9	10	24	5		To Mount *Liguſtinus*, *Panyæus*, between M. *Sipulus* and *Didymus*, by M. *Libanus* and *Seir*.
10	10	27	10		Above Mount *Niger the Leſs*, by Mount *Apennine*, Iſle *Beſbica*, Mount *Cimæus*, and Mount *Calcbaſtan*.
11	10	31	27		At M. *Serrorum*, and *Carpathos*, at *Byzantium*, by Mounts *Horminius*, *Taurus*, and *Delanguer*.
12	10	36	0		By Mounts *Macrocemnii*, the Iſland *Cyanea*, Mount *Amanus*, and *Antitaurus*.
13	10	41	20		By the Iſland *Macra*, by the *Erichtenian* Rocks, *Athenian* Bay, and *Palus Arceſa*.
14	10	45	35		By the upper *Hyperborean* Lake, *Palus Byce*, Iſle *Aca*, Mount of *Hercules*, the utmoſt Bay of *Pontus*, and M. *Parapomis*.
5	10	48	40		At the lower *Hyperborean* Lake, Mount *Immerius*, by M. *Corax*, *Taucan*, the lower Bay of the *Caſpian Sea*.
6	10	52	10		By the *Riphean* Mounts, *Palus Mæotis*, the greater *Caſpian* Iſland, and *Mons Neroſus*.

	h. ′ ″			
	10 56 35	Moon not yet fully darkned.		
	10 58 25	Total Immerſion.		
			° ′ ″	
	11 20 45	Altitude of *Procyon.*	31 8 0	
	11 25 45	The ſame Altitude.	31 38 0	
	11 36 0	That dark Shade or little Cloud plainly ſeen in the mid. of M's Diſk		
	12 43 0	The begin. of Emerſion, or Recovery of Light.		
17	12 46 40	*Pal. Maræot.* now entirely cover'd.		To Mount *Audus*, not far from *Palus Maræotis.*
18	12 51 0			By *Mare Eoum*, *Mare Syrticum*, and Mount *Caſius*.
19	12 57 0			By the leſſer Mount *Atlas*, between the Iſles *Circinna* and *Taracinia*, by the *Egyptian* Sea, the *Sin. Sirbonis*, and M. *Pharan*.
20	1 5 0			To the Iſlands *Majorca*, *Sardinia*, and *Hiera*; by M. *Ætna*, the Iſle *Melos*, to M. *Anna* and *Hajalon*.

The Beginning of the Eclipſe was a little above *Palus Maræotis*.
The total Obſcuration was at *Mons Sanctus*, below *Palus Maræotis*.
The Recovery of Light happen'd nearly in the ſame Place where it was loſt.

2.

At Nuremburg; by M. G. C. Eimmart. N. 182. p. 146.

h. ′ ″	
9 19 00	The *Penumbra* was very obſcure, and the *Beginning* of the *Eclipſe* was at hand.
9 23 30	The *Eclipſe* was *begun*, the Quantity almoſt half a *Digit*, and the Diſtance between the Cuſps was about 42 Degrees of the Moon's Limb, and *Palus Maræotis* was juſt all *eclipſed*; hence we may conclude the *Beginning* about 9^{h}. 21′. 30″.
10 23 30	As near as I can collect, was the time of the *Total Immerſion* into the Shadow, to verify which, the Azimuth of the Moon's Center was obſerved to the Eaſt, 41°. 18′; 2′ 12″ of Time after the ſaid *Immerſion*.
12 13 0	Or 10′ 13″ before the *Culmination* of the *Right Shoulder* of *Orion*, was the *Emerſion*, or firſt Appearance of the Moon out of the total Darkneſs.
1 14alm	Was the juſt *End* of the *Eclipſe*; being 2′ 20″ before the *Culmination* of *Syrius*.

Whence the *Middle* of this *Eclipſe* ſhould have happened at 11^{h}. 18′ *p. m.* at *Nuremburg*: the *Total Duration* 3^{h}. 52′ 4″; and the *Total Darkneſs* 1^{h}. 49′ 30″.

The Meridian Altitude of the Moon's upper Limb was obſerved 63°. 23′ 50″, and the Moon's apparent Diameter while totally eclipſed was found 30′ 07″.

By M. J. Ph. Wurtzelbaur. Ibid. p. 147.

3. M. *Wurtzelbaur* made uſe of a Pendulum Clock, corrected by Altitudes: According to his Obſervation,

h. ′ ″	
9 23 30	Was the *Beginning* of the *Eclipſe*; at about 119 Degrees of the Limb of the Moon in *Hevelius*'s *Selenography*.
9 24 50	*Palus Maræotis* was all covered.
10 25 20	The *Total Immerſion*, about the 299th Degree of the Limb of the Moon.
12 11 30	The Moon began to *emerge* out of the Shadow, about the 112th Degree of her Limb.
1 14 30	The *End* of the *Eclipſe*, about the 295th Degree of the Limb.

By theſe Obſervations, the *Middle* of the *Eclipſe* ought to have been about 11^{h}. 19′. *p. m.* at *Nuremburg*; differing but one Minute from M. *Eimmart*'s Obſervation.

e *Duration* will be 3^{h}. 51′, and the *Total Darkneſs* 1^{h}. 46″.

At Lisbon; by Mr. Jacobs. N. 184. p. 206.

	h.	′
The Beginning - - -	8	02
Immerſion - - - -	9	06
Emerſion - - - -	10	50
End - - - - -	11	57

LII.

An Eclipſe of the Morn, Nov. 19. 1686. *at* Dublin; *by Mr.* W. Molineux. N. 185. *p.* 236.

Phaſes.	*Times.*			
	h.	′	″	
A notable Penumbra	9	15	00	
Clouds interpoſing, the beginning could not nicely be determined; therefore with Uncertainty I ſuppoſe Ecl. began 9^h. 25′, or 9^h. 27′.	9	27	00	*
The Shadow was at *Palus Maræotis*	9	38	20	
Palus Maræotis was cover'd	9	40	20	*
Mount *Sinai* was cover'd	9	46	30	*
Mount *Thambes* was cover'd	10	01	20	*
Mount *Audus* was cover'd	10	08	00	*
Mount *Neptune* cover'd	10	14	00	*
The Shadow at *Mons Sypylus*	10	15	10	*
The Iſle *Circinna* cover'd	10	16	40	
At *Mons Didymus*	10	18	20	*
Mons Didymus is cover'd	10	20	10	
Mount *Audus* emerges	10	23	00	
The Shadow at *Palus Maræotis*	10	48	30	
Maræotis emerges	10	52	00	
At Mount *Sinai*	11	38	00	
Sinai emerges	11	39	30	
The end of the Eclipſe	11	04	00	

Obſervables. The Times were thoſe of a Pendulum Clock, rectified by the fixed Stars.

The Obſervations marked with an Aſteriſm were taken through the Openings of the Clouds, and therefore I cannot account them as exact. The Quantity of this Eclipſe I think was ſix Digits.

An Eclipſe of the Moon, Apr. 5. 1688. *at* Moſcow; *by M.* Timmerman. N. 192. *p.* 453.

LIII. Half a Quarter of an Hour after 7 in the Evening, the Moon aroſe clear, but of a deep Red Colour without any Sign of Eclipſe: At $7^h\frac{1}{2}$ the Moon went into a thick Cloud, but was again clear at 7^h. 38′. when the Under ſide of the Body of the Moon was *begun* to be *obſcured*, in a clear Sky; ſhe being then in the 25th Degree of *Libra*, and 6°½ above the Horizon. (Suppoſe the Center.) At 9^h. the whole Under Side of the Moon was *eclipſed*; and about 8′ after 9^h it was at the Height, or rather ſeemed to decreaſe. At $9^h\frac{1}{2}$ there was ſtill a third Part of the Moon eclipſed. (Sup
of her Circumference.) About 10^h. it *decreaſed* apace, and at $10^h\frac{1}{2}$ t
was but little to be ſeen: At 10^h. 45′. it was certainly ended, the Moo
ing then about 22° high.

Immerſi

LIV. 1. *An Eclipſe of the Moon,* Oct. 19. 1697. *at* Cheſter; *by Mr.* Edm. Halley. N. 235. *p.* 784.

Immerſions.	*Times.*	*Emerſions.*	*Times.*
	h. ′		h. ′ ″
The Beginning	6 8½	*Porphyrites*, and the middle	
Porphyrites immerged	6 16	of M. *Ætna*	8 7 00
North part of *Maræotis*	6 21½	*Horminius* - - - -	8 17 30
Lacus Niger Maj. and South		*Mons Herculis* - - -	8 18 30
End of *Maræotis*	6 26	*Beſbicus* - - - -	8 21 00
Beſbicus - - -	6 46⅔	*Apollonia* - - - -	8 26 15
Apollonia - - -	6 49½	*Byzantium* - - -	8 29 00
Byzantium - - -	6 53	*Lacus Niger Major*	8 32½
Horminius - - -	6 59	South part of *Mæotis*	8 35
North-part of *Mæotis*	7 2½	North part of *Mæotis*	8 43
Mons Corax - - -	7 3½	The End - - -	8 49½
Mons Herculis - - -	7 10		
South part of *Mæotis*	7 12½		

About the Middle there remained 9′ 26″ of the luminous Part, and conſequently the Digits eclipſed 8⅔.

2. *At* Rotterdam; *by M.* Ja. Caſſini. N. 236. *p.* 20. Vid. ſup. *Sect.* XXXIX. 2.

	Time corrected.	*Obſervations.*
	h. ′ ″	
A	6 20 36	The acute Promontory at the firſt oblique Thread.
B	6 21 27	The preceding Limb of the Moon at the perpendic. Thread.
C	6 22 3	The acute Promontory at the perpendicular Thread.
C — B	0 00 36	The Difference of the Tranſit over the perpendicular Thread, which is the Longitude of the acute Promontory from the preceding Limb.
C — A	0 1 27	The Difference of the Tranſit of the acute Promontory between the firſt oblique Thread and the Perpendicular, which is its Latitude from the Northern Limb.
	6 32 34	The Moon ſeen among the Clouds ſeem'd as yet intire.

In the firſt Phaſe.

	h.	′	″	
A	6	41	23	The Beginning of *Mare Criſium* at the firſt oblique Thread.
B	6	41	50	The acute Promontory at the firſt Oblique.
C	6	42	12	*Pliny* at the firſt Oblique.
D	6	42	25	*Menelaus* at the firſt Oblique.
E	6	42	33	*Manilius* at the firſt Oblique.
F	6	42	43	The firſt Margin at the Perpendicular.
G	6	43	00	*Proclus* at the Perpendicular.
H	6	43	20	The acute Promontory at the Perpendicular.
I	6	43	26	The ſubſequent Margin at the firſt Oblique.
K	6	43	30	*Menelaus* at the Perpendicular.
L	6	44	00	The preceding Horn of the Moon at the Perpendicular; ſhe herſelf touches the horizontal Thread.
M	6	44	21	The ſubſequent Horn at the firſt Oblique.
N	6	44	35	*Menelaus* at the ſecond Oblique.
O	6	44	57	The ſubſequent Horn at the Vertical.
P	6	45	07	The ſubſequent Limb at the Perpendicular.
Q	6	45	33	The ſubſequent Horn at the ſecond Oblique.
R	6	45	55	*Grimaldus* at the ſecond Oblique.
S	6	46	49	Subſequent Limb at the ſecond Oblique.
P — F	0	02	24	The Tranſit of the Moon over the perpendicular Thread.
S — I	0	03	23	The Tranſit of the Moon over the ſecond Oblique.
H — F	0	00	37	Longitude of the acute Promont. from the preced. Margin.
H — B	0	01	30	Latitude of the acute Promont. from the North. Margin.
K — F	0	00	47	The Longitude of *Menelaus* from the preceding Margin.
K — D N — K	0	01	05	The Latitude of *Menelaus* from the Northern Margin.
L — F	0	01	17	Long. of the preceding Horn from the preceding Margin.
O — F	0	00	00	The Latitude nothing.
O — M	0	02	14	Long. of the ſubſequent Horn from the preceding Margin.
Q — O	0	00	36	Latitude of the ſame Horn from the Northern Margin.

In the ſecond Phaſis.

	h.	'	''	
A	7	6	40	The preceding Margin at the perpendicular Thread.
B	7	7	8	The preceding Horn to the vertical Thread.
C	7	7	47	The ſubſequent Horn to the firſt Oblique.
D	7	8	19	The ſubſequent Margin to the firſt Oblique.
E	7	9	4	The ſubſequent Horn to the Vertical.
	7	12	22	The Shadow to *Manilius*.
B — A	0	0	28	Long. of the preceding Horn from the preceding Margin.
E — A	0	2	24	Long. of the ſubſequent Horn from the Eaſtern Margin.
E — C	0	1	17	Lat. of the ſubſequent Horn from the Southern Margin.

In the third Phaſis.

	h.	'	''	
A	7	19	30	The preceding Horn to the firſt Oblique.
B	7	21	9	The preceding Margin to the Vertical.
C	7	21	19	The preceding Horn to the Vertical.
D	7	21	51	The preceding Margin to the ſecond Oblique.
E	7	22	24	The ſubſequent Horn to the firſt Oblique.
F	7	22	47	The ſubſequent Margin to the firſt Oblique.
G	7	23	9	The preceding Horn to the ſecond Oblique.
H	7	23	31	The ſubſequent Horn to the Vertical.
I	7	24	40	The ſubſequent Horn to the ſecond Oblique.
	7	26	4	The Shadow to *Dionyſius*.
C — B	0	0	10	Long. of the preceding Horn from the Eaſtern Margin.
C — A	0	1	49	Lat. of the preceding Horn from the Southern Margin.
G — C	0	1	50	The ſame Latitude.
H — B	0	2	22	Long. of the ſubſequent Horn from the preceding Margin.
H — E	0	1	7	Lat. of the ſubſequent Horn from the Southern Margin.
I — H	0	1	9	The ſame Latitude.

In the fourth Phaſis.

	h.	′	″	
A	7	40	24	The preceding Horn to the firſt Oblique.
B	7	41	35	The preceding Horn to the Vertical.
C	7	42	18	The preceding Margin to the firſt Oblique.
D	7	42	44	The preceding Horn to the ſecond Oblique.
E	7	42	51	The ſubſequent Horn to the firſt Oblique.
F	7	43	14	The ſubſequent Margin to the firſt Oblique.
G	7	43	58	The ſubſequent Margin to the Vertical.
H	7	45	04	The ſubſequent Horn to the ſecond Oblique.
D — 2′ 24″ = I	7	41	34	The preceding Margin to the vertical Thread.
B — I	0	00	01	Long. of the preced. Horn from the preced. Margin.
B — A	0	01	11	Lat. of the preced. Horn. from the South. Margin.
D — B	0	01	09	
H — E	0	2	13	Diff. of the Tranſit of the ſubſ. Horn betw. the Obl.
K	0	1	$6\frac{1}{2}$	Half, or Lat. of the ſub. Horn from South. Margin.
E + K	7	43	$57\frac{1}{2}$	The ſubſequent Horn to the Vertical.
E + K — I	0	2	$23\frac{1}{2}$	Long. of the ſubſ. Horn from the preced. Margin.

In the fifth Phaſis.

	h.	′	″	
A	7	50	22	The preceding Horn to the firſt Oblique.
B	7	51	04	The acute Promontory in the Shadow.
C	7	51	18	The preceding Horn to the Vertical.
D	7	51	58	The preceding Margin to the ſecond Oblique.
E	7	52	29	The ſubſequent Horn to the firſt Oblique.
F	7	52	56	The ſubſequent Margin to the firſt Oblique.
D — 2′ 24″ = I	7	49	34	The preceding Margin to the firſt Oblique.
B — I	0	01	30	Longit. Obl. of the acute Promontory from 1 Obl.
D — 0′ 42″ = L	7	51	16	The preceding Margin to the Perpendicular.
C — L	0	00	02	Long. of the preced. Horn from the preced. Margin.
C — A	0	00	56	Lat. of the preced. Horn from the South. Margin.
E — I	0	02	55	Long. Obl. from 1 Obl. to greateſt Long. 3′. 24″.
	0	00	29	The Complement.
F — E	0	00	27	The ſame Complement. The Medium 0′. 28″.

In the ſixth Phaſis.

	h.	′	″	
A	7	54	06	The preceding Horn to the firſt Oblique.
B	7	54	44	The acute Promontory to the firſt Oblique.
C	7	54	59	The preceding Horn to the Perpendicar.
D	7	55	38	The preceding Margin to the ſecond Oblique.
F	7	55	52	The preceding Horn to the ſecond Oblique.
G	7	56	04	The ſubſequent Horn to the firſt Oblique.
H	7	56	36	The ſubſequent Margin to the firſt Oblique.
I	7	57	20	The ſubſequent Horn to the Vertical.
D—2′ 24″ = L	7	53	14	The preceding Margin to the firſt Oblique.
D—0 42 = K	7	54	56	The preceding Margin to the Perpendicular.
C — K	0	00	03	Long. of the preced. Horn from preced. Margin.
C — A	0	00	53	Lat. of the preced. Horn from Southern Margin.
F — C				
I — K	0	02	24	Long. of the ſubſ. Horn from the preced. Margin.
I — G	0	01	16	Lat. of the ſubſ. Horn from the South. Margin.
B — L	0	01	30	Obl. Long. of the acute Promont. from 1 Obl.

In the ſeventh Phaſis.

	h.	′	″	
A	8	6	58	The preceding Horn to the firſt Oblique.
B	8	7	39	The preceding Horn to the Perpendicular.
C	8	8	15	The preceding Margin to the ſecond Oblique.
D	8	8	21	The preceding Horn to the ſecond Oblique.
E	8	8	55	Middle of the Shad. almoſt at the Perpendicular.
F	8	9	14	The ſubſequent Margin at the firſt Oblique.
G	8	9	55	The ſubſequent Horn at the Vertical.
H	8	9	58	The ſubſequent Margin at the Vertical.
C — 2′ 24″ = I	8	5	51	The preceding Margin at the firſt Oblique.
C — 0 42 = K	8	7	33	The preceding Margin at the Perpendicular.
B — K	0—	0	06	Long. of the preced. Horn from preced. Margin.
B — A	0	0	41	Lat. of the preced. Horn from South. Margin.
D — B	0	0	42	
G — K	0	2	22	Long. of the ſubſ. Horn from the preced. Margin.

In the eighth Phaſis.

	h.	′	″	
A	8	23	53	The preceding Horn to the firſt Oblique.
B	8	24	24	The preceding Horn to the Perpendicular.
C	8	24	55	The preceding Margin to the ſecond Oblique.
D	8	24	57	The preceding Horn to the ſecond Oblique.
E	8	25	55	The ſubſequent Margin to the firſt Oblique.
F	8	26	26	The ſubſequent Horn to the Perpendicular.
G	8	26	38	The ſubſequent Margin to the Vertical.
H	8	28	20	The ſubſequent Horn to the ſecond Oblique.
C — 0′ 42″ = K	8	24	13	The preceding Margin to the Perpendicular.
B — K	0	00	11	Long. of the preced. Horn from South. Margin.
B — A	0	00	31	} Lat. of the preced. Horn from South. Margin.
D — B	0	00	33	
F — K	0	02	13	Long. of the ſubſ. Horn from the preced. Margin.
H — F	0	01	54	Lat. of the ſubſ. Horn from the Southern Margin.

In the ninth Phaſis.

	h.	′	″	
A	8	49	25	The preceding Horn to the firſt Oblique.
B	8	49	42	The Concavity of the Eclipſe to the firſt Oblique.
C	8	50	08	The preceding Horn to the Vertical.
D	8	50	41	The preceding Margin to the ſecond Oblique.
E	8	50	51	The preceding Horn to the ſecond Oblique.
F	8	51	34	The ſubſequent Horn to the Vertical.
G	8	51	43	The ſubſequent Margin to the firſt Oblique.
H	8	52	26	The ſubſequent Margin to the Vertical.
H — 2′ 24″ = I	8	50	02	The preceding Margin to the Perpendicular.
C — I	0	00	06	Long. of preceding Horn from preceding Margin.
C — A	0	00	43	} Lat. of preceding Horn from Southern Margin.
E — C	0	00	43	
F — I	0	01	32	Long. of ſubſequent Horn from preced. Margin.

In the tenth Phaſis.

	h.	′	″	
A	9	4	29	The preceding Horn to the firſt Oblique.
B	9	5	4	The Shadow withdraws from *Pliny*.
C	9	5	53	The ſubſequent Horn to the firſt Oblique.
D	9	6	17	The preceding Horn to the ſecond Oblique.
E	9	7	2	The ſubſequent Margin to the firſt Oblique.
F	9	7	47	The ſubſequent Margin to the Vertical.
E — 2′ 42″ = H	9	5	20	The preceding Margin to the Vertical.
D — A	0	1	48	Tranſit of the preced. Horn between the Obliques.
M	0	0	54	Half is Lat. of preced. Horn from South. Margin.
D — M = I	9	5	23	The preceding Horn to the Vertical.
I — H	0	0	3	Long. of preced. Horn from the preced. Margin.
E — 3′ 24″ = L	9	3	38	The preceding Margin to the firſt Oblique.
F — 4 06 = L	9	3	41	
C — L	0	2	25	Obl. Long. of ſubſ. Horn from the firſt Oblique.
	9	9	4	The Shadow withdraws from *Langrenus*.
	0	9	19	The End of the Sea of *Tranquillity*.
	0	13	40	*Ariſtotle*.
	0	14	39	The End of *Mare Criſium*.
	0	21	34	The End.

LV.

A Tranſit of the Moon *above* Venus, Oct. 11. (*St. N.*) 1670. *at* Dantzick; *by M.* Hevelius. N. 66. *p.* 2026.

Time by a Watch in the Morn.		*Diſtances and Altitudes.*	*Time corrected by Altitudes.*
h. ′ ″		° ′ ″	h. ′ ″
3 47 20	Alt. of the bright Star of *Aries*	41 45 0	3 54 10
3 50 35	The ſame Altitude - - - - - - -	41 21 0	3 57 28
5 47 44	Diſtance of *Venus* from *Jupiter*		5 54 44
5 48 35	Diſt. of Eaſt. Limb of M. from ♃	35 25 30	5 55 35
5 51 59	Diſtance of ♀ from ♃	35 50 0	5 59 0
5 53 40	Diſt. of Eaſt. Limb of M. from ♃	35 29 10	6 0 40
6 12 30	Diſt. of *Venus* from *Jupiter*	35 50 20	6 6 0
6 14 3	Diſt. of the M.'s Eaſt. Limb from ♃	35 37 35	6 8 0
6 37 0	Diſt. of ♀ from ♃, as near as could be judged by the naked Eye	0 6 0	6 51 0
7 12 0	Diſt. of *Ven.* from low. limb of the M	0 5 0	7 7 0
7 17 0	The ſame - - - - - - - - -	0 4 0	7 12 0
7 35 0	*Venus* appear'd clearly - - - -		7 30 0
9 11 0	*Venus* continued conſpicuous		9 12 0
9 15 48	The Altitude of the Sun	19 8 0	9 17 20
9 17 39	The ſame - - - - - - -	19 58 0	9 19 4

An Occultation of Saturn *by the* Moon, June 1. (*St. N.*) 1671. *at* Dantzick; *by* M. Hevelius. N. 78. *p.* 3027.

LVI. The Beginning of the Occultation happen'd at $3^h. 38'. 27''$. in the Morning, about the Mountain *Germanicianus*. The Line of Paſſage, as far as could be collected from the Ingreſs alone, was through Mount *Ætna*, the Center of the Moon nearly, by Mount *Horminium*, the Mount of *Hercules*, and the upper Part of *Mare Caſpium* As far as I remember, excepting this Year's Obſervation, within 41 Years I have ſeen *Saturn* cover'd by the Moon only twice. Firſt in the Year 1630, *Jun.* 29, 11^h. in the Evening, when I was at the Iſland *Huenna* in the *Daniſh* Sea. And again in the Year 1661, *Aug.* 3, 7^h. 58′. 20″. in the Evening, here at *Dantzick*.

A Tranſit of the Moon *above* Jupiter, Sept. 30. (*St. N.*) 1671. *at* Dantzick; *by M.* Hevelius. *Ibid. p.* 3031.

LVII. In a very clear Sky, with a Tube of 20 Feet, with great Eagerneſs we received the riſing Moon, and *Jupiter* a little after. We found by our great brazen Octant, of almoſt 9 Feet Radius, that *Jupiter* was yet 1°. 23′. 4 ″. remote from the Eaſtern Limb of the Moon, and that all his four Satel s were there to the Right Hand, from whence the Moon approached. An - look'd for and unhappy Accident prevented us from taking the very Mom t of Conjunction. For when *Jupiter* now approached to the Eaſtern Limb f the Moon, at 3′, and was diſtant only 6′ from the Line of Conjunction dra n throu h through both Horns; ſome little Clouds intervening deprived us of the Sight both of *Jupiter* and the Moon. The *Rudolphine* Tables promis'd us an Occultation, and much ſooner than it could happen; yet there was none at all, but only a very cloſe Tranſit, at almoſt two Digits; the Time of which was 7^h. 26'. 0''.

An Occultation of the Pleiades *by the* Moon, Feb. 23. 1671-2, *At* Derby; *by Mr.* Flamſteed. N. 86. *p.* 5034. *Fig.* 116.

LVIII. The Altitude of the Moon being 20°. 50', I took its Diameter 32'. 48''; and when its Altitude was 19°. 23', I took it again 32'. 47''. Therefore the true Semidiameter of the Moon in the Horizon was 16'. 19''. Yet it was ſtill at a greater Diſtance from the Weſtern Star of the *Pleiades*, than the Teleſcope could conveniently take in. Now at 11^h. 19½' after Noon, when the Altitude of the Star *b*, the more Weſtern of the *Pleiades*, was 9°. 50', I took the Diſtance of the ſame Star from the neareſt Horn of the Moon 11'. 58''; then ſuddenly turning to obſerve the Stars Altitude, (which was ſhewn by a Quadrant of 20 Inches Radius fixt to the ſide of the Tube,) and immediately returning back again, I could not find the Star, being then cover'd by the Moon. In the mean while the Moon had deſcended 10 Minutes, and the Star as much, which from the following Phaſis I gueſs had gone under the Moon at 11^h. 20½'. For at 11^h. 30½', when the Altitude of the Star *e* was 8°. 43', I ſaw the Star *c* cover'd by the Moon. When I had taken its Diſtance 16'. 35'' from the neareſt Horn, by my Computations I found the Space of Time between the Occultation of this and of the foregoing to be 9'. 37'', which being taken from the Time of this Phaſis, gives the Time of the foregoing Occultation as I have made it.

At 11^h. 37½', when the Altitude of the Star *c* was 11°. 37½', it went under the Moon, as in the mean time I was meaſuring its Diſtance 22'. 36'', from the Horn of the Moon, which was apparently the inferior, but really the ſuperior. When the Star vaniſh'd, the apparent Semidiameter of the Moon was 16'. 21''; and therefore its Occultation was at 87°. 25' of the Moon's Periphery from the upper Cuſp, whoſe Reclination was 1° 37' from a Line drawn through its Center perpendicular to the Ecliptic. Thus the Ingreſs of the Star was 4° 12' above a Line drawn through the Center of the Moon parallel to the Ecliptic, and the Center of the Moon was antecedent of the Star 16'. 18'', with leſs Latitude 1'. 12''.

According to the Author of the *Caroline* Tables, the Place of the fixt Star is 25°. 1'. 24'' in *Taurus*, and its perpetual Latitude 4°. 20'. 39''; ſo that at the apparent Hour at *Derby* 11^h. 37½' after Noon, the apparent Place of the Moon was 24°. 45. 6'' in *Taurus*, and its Latitude obſerved 4°. 19'. 27'' North.

Moreover it well deſerves to be remarked, that though almoſt all the Hypotheſes of all Aſtronomers aſcribe a greater Diameter to the Perigee full Moon in the Quadratures, and therefore a leſs Diſtance from the Earth than in the Syzygees or Oppoſitions; yet the contrary obtains in the Heavens. For the Perigean full Moon paſſing near the *Pleiades*, *Nov.* 6, 1671, had a greater Diameter than in this Tranſit, when in almoſt the ſame Place it was diſtant from the Sun 70 Degrees. The Moon's horizontal Diameter according to

	Bullialdus.	*Street.*	*by Obſervations.*
Nov. 6. 1671.	17′ 00″	16′ 30″	17′ 00″
Feb. 23. 1672.	17 50	17 13	16 19
	+ 50	+ 43	— 41

We can no longer wonder, that the Moon ſhould ſo long refuſe the Confinement of Numbers, and that the Times of Appearances, when computed from the Tables, ſhould hitherto ſo far deceive our Expectations; ſince it appears thoſe Tables are generally computed from erroneous Hypotheſes.

The Moon's *Place*, Mar. 23. 1671-2; *by M.* Caſſini. N. 82. *p.* 4047.

LIX. *April* 2. (*St. N.*) 6^{h}. 50′ *v.* A Line drawn through the Horns of the *Moon* paſſed through the Star that is at the Point of the *Northern Horn* of *Taurus*; and the Diſtance of this Star to the *Northern Horn* of the *Moon*, was by a Minute greater than the Semidiameter of the *Moon*.

An Occultation of a fixed Star by the Moon, Feb. 29. (*St. N.*) 1676. *at* Paris; *by M.* Caſſini. N. 123. *p.* 564.

LX. An Immerſion of the ſubſequent Star of the two in the Left hinder Foot of the Lion, was at 1^{h}. 19′. 34.″ The Coaſt of the Immerſion was near the End of *Schicard* towards *Phocilides* in the Selenography of *Ricciolus*.

The Emerſion was at 11^{h} 16′ 40″, in an equal Diſtance from the Right Line between *Vendelinus* and *Petavius*.

A Right Line being drawn through the Points of Immerſion and Emerſion, when carefully obſerved, will divide a Diameter perpendicular to it in the Ratio of 6′. 45″. to 26′. 5″.

But the Diameter of the Moon approaching to the Meridian was 32′. 50″.

At 12^{h}. 29′ the upper Margin of the Moon was in the ſame Parallel with the Star, which then preceded the Moon by 1′. 50″ in time.

At 12^{h}. 40′. 18″ the Star preceded the Weſtern Margin of the Moon by 2′. 11″ in time. The Diameter of the Moon paſs'd through 2′. 14″.

At 12^{h}. 52′. 35″ the Star preceded the ſame Margin 2′. 25″.

The Meridian Altitude of the inferior Limb of the Moon was taken 39°. 25′. 25″.

LXI.
A Tranſit of the Moon *above* Jupiter, Feb. 28. m. 1675-6. *at* Greenwich; *by Mr.* Flamſteed. *Ibid. p.* 566.

Hour of the Clock cor.			Altitudes and Diſtances.		
h.	'	''		'	''
4	20	15	*Jupiter* from the bright Limb of the Moon	26	9
4	47	0	The Diameter of the Moon was taken	31	30
4	49	30	*Jupiter* from the neareſt Cuſp	26	28
4	52	15	*Jupiter* had paſs'd over a Right Line drawn through the Cuſps, by a tenth Part of the Diſtance, or about 3', making a Conjecture by the Eye through the Tube		
4	56	0	*Jupiter* from the Cuſp	27	33
5	1	15	From a Right Line thro' the Cuſps	7	53
5	3	30	From the Cuſp	28	22
5	7	25	From the Right Line	9	58
5	10	50	From the ſame	11	55
5	15	50	From the Cuſp	30	27
5	21	20	From the remoter Limb, dubious	62	4
5	26	0	From the neareſt Cuſp	33	0
5	31	25	From the Right Line thro' the Cuſps	20	9
5	37	0	From the Cuſp	36	15
5	41	10	The Altitude of the Moon being 10½ Degrees, the Diameter was about	31	53
5	48	30	Diff. of the Alt. of the lower Limb of the Moon and of *Jupiter*	23	1
5	52	40	*Jupiter* was diſtant from the neareſt Cuſp	41	40
6	9	40	From the Cuſp, dubious.	47	29

LXII. 1. *Aug*. 21. A. 1676. before Noon. For the Correction of my Clock, I took theſe Altitudes of the Sun's Limb.

An Occultation of Mars by the Moon, Aug. 21. 1676. *at* Greenwich; *by Mr.* Flamſteed. N. 129, *p.* 723.

Hour of the Clock.			Altitudes.			Hour computed.			Error in Time.		
h.	'	''		°	'	h.	'	''		'	''
8	04	31	Alt. of the inferior Limb of the Sun	26	04	8	09	26	+	4	55
8	5	42	- - - - - - - - - - - - - -	26	14	8	10	35	+	4	53
8	7	58	- - - - - - - - - - - - - -	26	34	8	12	53	+	4	55
8	9	10	- - - - - - - - - - - - - -	26	44½	8	14	03	+	4	53
8	10	15	- - - - - - - - - - - - - -	26	54	8	15	12	+	4	57
8	17	15	- - - - - - - - - - - - - -	27	54	8	22	09	+	4	54

Then after Noon, the Sky being exceeding clear.

Hour of the Clock.	*Corrected.*	*Diſtances.*			
h. ′ ″	h. ′ ″				′ ″
10 45 03	10 49 58	*Mars* from the bright Limb of the M.	5125	=	42 08
11 06 11	11 11 05	The ſame Diſtance	3829	=	31 29
11 20 00	11 24 55	Again - - - - - -	3007	=	24 44
11 35 57	11 40 52	Over again - - - - -	1982	=	16 18
11 57 31	12 02 26	♂ from Zen. Diff. Alt. inf. lim. and of ♂	1912	=	07 35
		With tube of 16 feet, *Mars* from Limb	1158	=	5 47
12 05 00	12 09 55	Planet could no longer be perceived with the naked Eye			
12 9 44	12 14 39	Light of *Mars* confounded with the Light of the M. *Mars* from Zen.	1185	=	9 44
12 10 03	12 14 58	*Mars* intirely cover'd by North. Cuſp	3475	=	17 20
12 18 38	12 23 33	[a] 41 in *Taurus* appear'd in a right Line drawn thro' the Cuſps.			
12 20 36	12 25 31	41 in *Taurus* from the Limb or Cuſp; with a ſhorter Tube	3912	=	32 10
12 24 58	12 29 53	41 in *Taurus* from the Cuſp, again with the ſame Tube	3935	=	32 21
12 46 00	12 50 55	Diam. of the Moon with a longer tube	5971	=	29 47
1 04 30	1 09 25	Again with the ſame Tube	5973	—	29 48
1 10 56	1 10 51	*Mars*'s Emerſ. perhaps 4″ or 5″ ſooner			
1 13 29	1 18 24	*Mars* from the Northern Cuſp	3675	=	18 20
1 18 15	1 23 10	The ſame Diſtance	4035	=	20 08
1 22 00	1 26 55	The Moon's Altitude 23°, with the longer Tube. Her Diameter	5988	=	29 55
1 39 00	1 39 55	Moon's Diameter with a ſhorter Tube	3645	=	29 58

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[a] 41 in *Taurus*. According to *Tycho* its Place is now ♉ 17°. 58½′ Latitude 1°. 20′. South. Therefore the Places both of the Moon and *Mars* may be exactly deduced.

Time

2. *At* Oxford; *by Mr.* Halley. *Ibid. p.* 734.

Time corrected.			*Diſtances*			
h.	′	″			′	″
11	43	30	The Center of *Mars* from the neareſt Limb of the *Moon*	719½ =	12	40
11	49	2	Again - - - - - - - - -	571 =	10	3
11	54	48	Again - - - - - - - - -	409 =	7	12
12	3	25	The Center of *Mars* from the North Cuſp of the *Moon* - - - - - - - -	1118 =	19	41
12	10	28	The gibbous part of *Mars* touched the *Moon*'s Limb - - - - - - - - - - - -			
12	10	42	*Mars* was wholly covered, being diſtant from the Cuſp - - - - - - - - - - - -	963 =	17	14
1	10	41	*Mars* did emerge, I ſuppoſe, his Center			
1	12	45	*Mars* diſtant from the North. Horn of the *Moon*	1018 =	71	55
1	31	10	*Mars* paſſed over a Point noted in the Teleſcope			
1	33	15	The Southern Limb of *Ætna* paſſed by the ſame Point			
1	34	00	The lucid Limb paſſed over the ſame Point			
1	52	35	The *Moon*'s Diameter obſerved 1698 = 30′ 1″ *Alt. Moon* 31° circ.			
1	57	52	*Mars* from the Northern Horn of the *Moon*	2042 =	36	5
2	2	53	*Mars* from the Southern Horn of the *Moon*	2266 =	40	3

3. *At* Dantzick; *by M.* Hevelius. *Ibid. p.* 721.

Time by Pend. Clock			*Names of the fixed Stars.*	*Altitudes.*		*Time correct. by Alt.*			*Remarks.*
h.	′	″		°	′	h.	′	″	
1	1	25	Swan's Tail	57	10	1	0	24	
1	9	45				1	8	45	*Mars* was nearly as far diſtant from the bright Limb of the Moon, as Mount *Porphyrites* was from Mount *Ætna*.
1	36	39				1	33	42	*Mars* cover'd by the Moon.
1	45	25	Swan's Tail	51	17	1	44	7	
2	47	54				2	46	29	*Mars* ſhin'd out again, and made an end of the Occultation.
3	19	50	*Scheat* in *Peg.*	45	3	3	18	19	

Mars was cover'd about Mount *Audus*, proceeding as it were through the *Loca Paludoſa* of the Moon, by Mount *Ætna*, below the Iſland *Beſbica*, above the *Palus Acheruſia*, above Mount *Corax*, by the *Palus Mæotis*, and a little above the Iſle *Alopecia*, and the very Center of the Moon. And thus again going out at the greater Weſtern Lake.

If you aſk from whence I could thus exactly mark out his Paſſage, and that at the obſcure Part of the Moon, you are to know it proceeded from hence, that by thoſe Tubes of mine I could very diſtinctly perceive the chief greater Spots in the ſhaded part of the Moon; and thus I could obſerve very plainly, that *Mars* emerged near the middle of *Palus Mæotis*.

An Occultation of Saturn *by the* Moon, Feb. 27. ſt. n. 1678, *at* Paris; *by M.* Bullialdus. *n.* 139. *p.* 969.

LXIII. *Bullialdus* obſerved the Beginning, when the Altitude of *Andromeda*'s Head above the Weſtern Horizon was 18°. 11′; whence the Aſtronomical Time from Noon is given $7^{h}\ 30'$; but the mean Time was $7^{h}\ 29'\ 55''$. And he ſaw the End when the more Southern Star of *Andromeda*'s Girdle of the ſecond Magnitude was 21° 17′ high to the Weſt. Whence the Aſtronomical Time from Noon will be found to be $8^{h}\ 30'\ 22'$.

Here we are obliged to take notice, that the *Philolaic* Tables make *Saturn* farther advanced in Longitude, by at leaſt 19 Minutes, than he really is by Obſervation; ſo that *Saturn* was then in ♊ 3° 28′, with South Latitude 1° 38′.

Farther, in this Obſervation if we have Recourſe to the Deſcription of the Lunar Diſk, as deſcribed by the illuſtrious Mr. *Hevelius*; we found *Saturn* to emerge in that part of the Limb, which is ſituated in a Right Line drawn from the middle of Mount *Beroſus* through the *Riphæan* Mountains, a little above Mount *Alanus*, and below the Southern Limits of the *Hyperborean* Marſhes.

An Occultation of Jupiter, Jun. 5. (*ſt. n.*) 1679. *at* Dantzick; *by M.* Hevelius. *Ph. Coll. n.* 1. *p.* 29.

LXIV. 1. Tho' I have apply'd my ſelf to the Obſervation of the Heavens now for 50 Years, (for which I ought to give, and do give, all poſſible Thanks to Almighty God,) yet I have only once ſeen *Jupiter* cover'd by the Moon, which was in the Year 1646, *Decemb.* 24. (new Style) in the Evening, when the Sun was under the Horizon. Therefore I very much congratulate with my ſelf, that I could make this Obſervation, not only with a very ſerene Sky, but alſo according to my Wiſh, in the Preſence of my moſt wellcome Gueſt, the famous and moſt learned Mr. *Edmund Halley*.

It entered the Moon at Mount *Audus*, and as far as could be expected from its Exit, it paſs'd through the *Loca Paludoſa* of the Iſland *Cercinna*, over Mount *Ætna*, through the Iſland *Beſbica*, through *Byzantium*, the Iſland *Apollonia*, and the upper Part of *Palus Mæotis*; ſo that it went a little above the Center of the Moon; the Moon having then ſome South Latitude. Then, which is very rare, we very accurately meaſured (as it ſeems to me) the apparent Diameter of *Jupiter* in this Obſervation. I remember that I have ſeveral Times obſerved the Diameter of *Jupiter* by the Lunar Spots, and found that it came to 55 Seconds more or leſs. But now the Diameter of *Jupiter* came out much leſs. For the whole Duration of this Occultation being known to be 58′ 10″, and at the ſame time the Diameter of the Moon being given 32′ 40″; it preſently becomes known from that Mora of Time, when firſt *Jupiter* with his Limb touch'd the Limb of the Moon, and when again it was hid, (which was done in the ſame Space of 55 Seconds,) that the Diameter of *Jupiter* is 30″ 53‴.

Or

Ord Obſer.	*Time by the Pendulum Clock.*			*Altitudes of the fixed Stars and of the Sun, with the Diſtances of* Jupiter *from the Limb of the Moon.*				*Time corrected by Altitudes.*		
	h.	′	″		°	′	″	h.	′	″
	1	18	55	The Altitude of *Andromeda*'s Head	24	52	0	1	20	54
	1	29	0	The Altitude of *Arcturus*	31	3	00	1	31	24
1	1	52	0	The Moon aroſe	—	—	—	1	54	25
2	3	26	0	The Sun riſes	—	—	—	3	28	25
3	3	33	0	The Diſtance of *Jupiter* from the Limb of the Moon was nearly equal to the Moon's Diameter				3	35	0
4	3	41	0	*Jupiter* ſo far diſt. from the Limb of the Moon, as is the Diſtance of M. *Porphyrites* from Mount *Sinai*				3	43	0
5	3	58	0	*Jupiter* was as far from the Eaſtern Limb of the Moon, as Mount *Ætna* is from *Palus Maræotis*				4	00	0
6	4	13	50	*Jup.* diſt. from the limb of the M. two Diam. of *Jup.*				4	16	15
7	4	14	40	*Jupiter* was diſtant one of his Diameters				4	17	5
8	4	15	40	*Jupiter* grazed upon the Eaſtern Limb of the Moon				4	18	5
9	4	16	9	Half *Jupiter* was hid				4	18	34
10	4	16	35	All *Jupiter* was quite cover'd				4	19	0
11	5	14	0	A notable part of *Jupiter* now ſeem'd to come out				5	16	25
12	5	14	20	Half *Jupiter* was now come out				5	16	45
13	5	14	45	All *Jupiter* was now come out				5	17	10
	10	22	30	Altitude of the Sun	53	34	44	10	25	2
	10	27	16	Altitude of the Sun	53	59	45	10	29	42
	10	30	8	Altitude of the Sun	54	14	0	10	32	26
	10	38	0	Altitude of the Sun	54	53	40	10	40	26
	10	45	28	Altitude of the Sun	55	27	20	10	47	46

At Paris; by M. Caſſini. *ib.* p. 33.

2. At $3^h\ 0'\ 11''$, the firſt *Satellite* was hid by the Eaſt Limb of the Moon. At $3^h\ 2'\ 0''\frac{1}{2}$ the Eaſt Side of the Moon touched the Weſt-ſide of *Jupiter*: then I took the Height of *Jupiter*, which was 8° 01′, at $3^h\ 2'\ 51''$. At $3^h\ 2'\ 57''$, *Jupiter* was intirely hid by the Moon. It entered at equal Diſtance from the two Spots *Grimaldi* and *Ariſtarchus*; the laſt of which was in the Section of the Moon, which diſtinguiſhed the Light from the Dark Part. At $3^h\ 5'\ 1''$, the ſecond *Satellite* was hid by the Eaſt-ſide of the Moon, At $3^h\ 5'\ 48''$, the third *Satellite* was hid. At $3^h\ 56'$, we perceived by the Eye that *Jupiter* was parted from the obſcure Side of the Moon.

M. *De la Hire* took the Height of *Jupiter* two Minutes after parting, and found it 17° 17′.

LXV.

An Occultation of the Bull's Eye, *at* Greenwich, *Sept.* 4. 1680. *by Mr.* Flamſteed. *Ph. Coll.* n. 4. *p.* 99.

Time correct. in the Morning.			*Obſervations.*				
h.	′	″			′	″	
2	49	52	South Cuſp of the M. from the Bull's Eye	5300	=26	26	
2	53	10	The Bull's Eye from the neareſt Limb	1028	=05	08	
2	56	54	From the ſaid Cuſp ——— ——	4830	=24	06	
3	01	16	Repeated—— ——	4370	=21	48	
3	05	42	Immerſion by the Cuſp ——— ——	4049	=20	12	or ſomething leſs
4	08	20	The Diameter of the Moon	6595	=32	54	
			Repeated by my Aſſiſtant ——	6588	=32	52	
4	14	02	The Bull's Eye emerged —— ——				
4	15	12	Was diſtant from the Southern Cuſp	3411	=17	01	
4	18	52	Repeated———	3725	=18	35	
4	22	00	Again —— ——	3980	=19	51	
4	23	52	Once more ———	4098	=20	56	

3^{h} 05′ 40″, the fixt Star ſeem'd to adhere to the bright Limb of the Moon, and after two Seconds of Time nothing appear'd in the Limb. The Place of Immerſion was near the moſt Southern of three ſmall Spots lying in the middle between *Palus Maræotis* and Mount *Climax*.

At 4^{h} 13′ 45″, the firſt Star was not yet emerged; then, or a little afterwards, I know not on what Occaſion, I removed my Eye from the Teleſcope, and when I apply'd it again at 4^{h} 14′ 2″, I ſaw the Star emerged, and ſhining with full Light.

h.	′	″		°	′	″
4	53	20	Neareſt Limb of the M. from the bright Foot of *Orion*	26	12	25
4	56	20	The bright foot of *Orion* from the Bull's Eye	26	29	25
4	58	20	Neareſt Limb of the M. from bright Foot of *Orion* again	26	12	00
5	04	02	The neareſt Limb of the Moon from *Pollux*	44	09	10
5	06	33	Repeated——	44	08	15
5	11	55	The bright Foot of *Orion* from the Bull's Eye	26	29	40

LXVI.

An Occultation of the Bull's Eye *at* Greenwich, *Oct.* 28. 1680. *by Mr.* Flamſteed. *Ph. Coll.* n. 4. *p.* 100.

Time corrected.			*Obſervations.*			
h.	′	″			′	″
7	11	52	The Diameter of the Moon ———	6745	=33	39
7	16	26	Repeated———	6744	=33	39
7	19	46	The Bull's Eye from the bright Limb —	5895	=29	24
7	24	44	Repeated———	5095	=25	25
7	40	14	—— —— —— ——	3203	=15	59
7	51	2	—— —— —— ——	1817	=39	04

h. ′ ″			′ ″
7 59 34	——— ——— ——— ———	810	=04 02
8 02 09	——— ——— ——— ———	490	=02 26
8 06 09	It reached the Limb		
8 06 30	It diſappear'd at the Longitude of *Palus Miris* towards the North, at its Northern Extremity.		

The Northern Limb of *Mæotis*, alſo of *Ætna*, had the ſame Declination with the Place of Ingreſs.

The Difference of the Declinations of the Place of Immerſion, and of the Northern Limb of the Moon, was 2770=13′. 49″.

At 9ʰ 2′ 58″, it emerged from the obſcure Limb, at the Longitude of *Inſula Major*, from its Northern Limit.

Its Paſſage was through the Place of Emerſion, to its Diameter North from *Crete*, through the North Limb of *Sirbon*, and Mount *Climax*.

At 9ʰ 10′ 26″, the Moon's Diameter was 6791=33′ 52″.

2. At *London*, we noted the *Immerſion* at 8ʰ 6′ 00″, and that Star was newly *emerged* at 9ʰ 2′ 52″.

At London; *by Mr.* Halley, *and Mr.* Haines. *Ph. Coll.* n. 5. *p.* 124.

3. Mr. *Benj. Harry*, Maſter of the *Berkley Caſtle*, riding at Anchor in *Ballaſore* Road, about 20 Miles E. S. E. from the Town, obſerved that the *Moon* paſſed to the Northward of the *Bull's Eye*, about 24 or 25 Min. and by his Pendulum Watch, rectify'd by Altitudes and the Riſing and Setting of the Sun, he noted, that preciſely at 16ʰ 00′ the *Bull's Eye* was in equal Altitude with the *Moon*'s Center, and that at 16ʰ 30′ the Star was in equal Altitude with the lower Limb of the Moon, and at 17ʰ 12′ the Occidental Limb of the *Moon* was in a Right Line with the *Bull's Eye* and *Capella*.

At Ballaſore *in* India; *by Mr.* Benj. Harry, *ib.*

LXVIII.

An Occultation of the Bull's Eye *at* Dantzick, *Jan.* 1. ſt. n. 1681. *by M.* Hevelius.

Ph Coll. n. 3. *p.* 65.

Times.	*Obſervations.*	*Altitudes.*
h. ′ ″		
6 47 00	The Bull's Eye was ſo far diſtant from the Confine of Light and Shadow, as the Mount of *Chriſt* is removed from the lower Limb of the Moon.	
7 37 00	The Star was hid at *Mare Syrticum*, under the Iſland *Cercinna*, the Altitude of *Andromeda*'s Head being then	° ′ 50 32
7 46 00	The Altitude of *Andromeda*'s Head ——— ———	49 27
7 49 30	The ſame Altitude ——— ——— ———	48 55
8 46 00	The Bull's Eye again ſhone out at the greater Iſland of the *Caſpian* Sea.	
	Therefore its Paſſage, in reſpect of the lunar Spots, was through *Mare Syrticum*, Mount *Athos*, under the Iſland *Lemnos*, and Mount *Didymus*, under the *Athenian* Bay, through the *Pontic* Sea, and the greater Iſland of *Mare Caſpium*.	
8 51 00	The Altitude of *Andromeda*'s Head.	40 22

At Ballaſore; *by Mr.* Ben. Harry. *Ph. Coll.* n. 5. *p.* 125.

2. Mr. *Benj. Harry* in *Ballaſore* Road obſerved, that the *Moon* paſt to the Northward of the *Bull's Eye*, and that the Star and the *Moon's* under-Limb were in equal Altitude when they were both 13° 45′ high to the Weſt, which gives the Time 14ʰ 49′; and when the *South Horn* of *Taurus* was 23° 30′ high, which makes the Time 15ʰ 13′, the Weſtern Limb of the Moon was in a Line with *Capella* and the *Bull's Eye*.

At Avignon; *by M.* Gallet. ib.

3. The correct Time of the *Immerſion* was 6ʰ 18′ 22″, and the *Emerſion* at 7ʰ 19′ 46″.

A Tranſit of the Moon *below the three* Superior Planets, *and* Regulus, 1682, *at* Dantzick; *by M.* Hevelius. *n.* 143. *p.* 17. *n.* 151. *p.* 325.

LXVIII. *Sept.* 27. *N. S.* 1682, at Three in the Morning, I ſaw with my naked Eye the Moon and the three other Planets; but the Moon at that Time was ſtill diſtant about 7 Degrees, according to the Order of the Signs, towards the Weſt.

n. 143. p. 18. *min.* 31. *ſec.* 17. n. 151. *p.* 326. *min.* 31. *ſec.* 55.

But as far as may be collected from the Inclination of the Horns of the Moon in reſpect of the Tendency of the Planets, I preſently concluded, that there would be no Occultations but only Tranſits; ſo as that the Moon would proceed below thoſe ſuperior Planets. In this Opinion I was more and more confirmed, when the following Day, *Sept.* 28. in the Morning, *Regulus* was not cover'd by the Moon, which Star, in reſpect of the Latitude of each, ſhould rather have undergone an Occultation. For *Regulus* in the Conjunction, at 4ʰ 6′, was diſtant 31′ 17″ towards the North from the upper Horn of the Moon; as I obſerved exactly with an excellent Tube and a very good Micrometer; ſo that there was no Occultation at all of *Regulus*, but only a Tranſit of the Moon. The ſame alſo happen'd *Octob.* 25. And *Jupiter* and *Saturn*, as alſo *Mars*, on *Oct.* 26. were not at all cover'd by the Moon, but the Moon paſs'd far enough beneath the ſaid Planets.

An Occultation of Regulus *by the* Moon, *Feb.* 11. ſt. n. 1683. *at* Dantzick; *by M.* Hevelius. n. 151. *p.* 331.

LXIX. *Feb.* 11. *N. S.* 1683, at 9ʰ. When firſt the Moon approached my Eye, *Regulus* was at a ſufficient Diſtance towards the Eaſt. So that the Conjunction itſelf (as far as I could make a rude Eſtimate) happen'd at the Riſing of the Moon at Five or Six a-Clock. But whether *Regulus* had been cover'd, or whether it was only a Tranſit, I could not be intirely ſatisfied.

LXX.

An Occultation of two fixed Stars by the Moon, *and a Tranſit above of a Third,* Apr. 2. *ſt. n.* 1683. *at* Dantzick; *by M.* Hevelius. *ib.*

Time by the Watch.		*Diſtances and Alt.*
h. ′ ″		
9 53 30	The beginning of the Occultation of the greater little Star A, of the 5 Magnitude	
10 08 30	The Conjunction of the Moon and the little Star C was diſtant from the lower Horn of the Moon	° ′ ″ 0 04 00
10 29 36	The beginning of the Occultation of the little Star B of the ſixth Magnitude	
10 52 50	The end of the Occultation of the Star A.	
11 45 30	The Altitude of *Lyra* ——— ———	31 44 00
11 46 30	The ſame ——— ——— ———	31 55 00
11 47 50	Again ——— ——— ———	32 06 00

The Section of the Light and Shade this Day fell upon the Mounts *Serrorum* and *Carpathos*, through the *Sinus Peronticus*, between *Byzantium* and the Iſle *Cyanea*, through the Mountains *Amanus* and *Taurus*, and the *Urian* Mountains.

The firſt little Star A is not to be found in *Tycho*'s Catalogue, but in my new one it is call'd, The Subſequent under the Bull's Horn of the fifth Magnitude. At this Time it is ♊ 19° 11′ 35″, and in 4° 43′ 44″ of Southern Latitude. The other B, as far as I could collect from this Obſervation, is in ♊ 19° 17′ 0″, and in South Latitude 4° 47′ 0″. But the third C, which is hardly to be ſeen with the naked Eye, is now to be found in ♊ 19° 9′ 0″, and in Latitude 5° 2′ 0″ South. Now the Star A entered the Moon at Mount *Audus*, paſs'd over the Iſland *Cercinna*, Mount *Neptune*, the *Adriatick* Sea, between Mount *Horminium* and Mount *Amanus*, over Mount *Hercules*. So that it emerged again between *Palus Mæotis* and the greater Iſland of the *Caſpian*. Whence it is plain that this little Star A was nearly in a central Conjunction with the Moon.

The other Star B of the ſixth Magnitude entered the Moon at *Palus Maræotis*, paſs'd through *Sinus Syrticus*, to Mount *Athos*, by Mount *Latmus*, between the Mountains *Sipylus* and *Macyſitus*, below the Center of the Moon, by the upper Mount *Moſchus*, through the *Pontic* Sea, and ſo below the greater Iſland of the *Caſpian* Sea.

LXXI.

An Occultation of a fixed Star, and a Tranſit above another, May 2. *ſt. n.* 1683. *at* Dantzick; *by M.* Hevelius. *ib.*

LXXI. *May* 2. *n. ſt.* 1683, at $11^{h}\ 0'\ 0''$ in the Evening. The Moon paſſed above the little Star at the Root of the Tail of *Cancer*, which then was in ♋ $27^{\circ}\ 53'\ 37''$, in South Latitude $2^{\circ}\ 18'\ 42''$; ſo that in the very Conjunction it was not diſtant from the lower Horn of the Moon above $12'$.

At $12^{h}\ 0'\ 0''$, another fixt Star, but a very ſmall one, was cover'd; which otherwiſe is not in the Catalogue. As far as can be gueſs'd it was in ♋ $28^{\circ}\ 30'$, and in South Latitude $1^{\circ}\ 54'$.

An Occultation of Regulus *by the* Moon, *May* 4. ſt. n. 1683. *at* Dantzick; *by M.* Hevelius. *ib.*

LXXII. I obſerved moſt exactly the very Moment of Immerſion, which happen'd at $11^{h}\ 17'\ 20''$, in the Evening, according to my Watch. The Line of Paſſage went through *Mare Pamphilium*, below the Iſland *Carpathos*, the Iſle of *Cyprus*, below the utmoſt Bay of *Pontus*, and the lower Bay of *Mare Caſpium*. At $11^{h}\ 24'\ 42''$, according to my Watch, the Altitude of the bright Star of the Harp was obſerved to be $44^{\circ}\ 39'\ 00''$; from whence the beginning of the Occultation may be corrected. The Section of Light and Shade was through the greater black Lake, at the Iſland *Corſica*, Mount *Niconius*, thro' the *Strymonic* Lake, and the Iſle of *Rhodes*; by Mount *Sinai*, and Mount *Techiſandam*.

An Occultation of Jupiter *by the* Moon, *Mar.* 31, 1686. *at* London; *by Dr.* Hook, *and M.* Halley. n. 181. *p.* 85.

LXXIII. 1. At $9^{h}\ 26''$ the under Limb of the *Moon* was juſt riſen, and ſoon after *Jupiter* appeared near the Eaſtern Limb of the *Moon*, within a few Minutes of being *eclipſed*.

$9^{h}\ 33'$, as near as could be gueſſed, was the Time of the *Central Immerſion*, which was very difficult to be obſerved by reaſon of the Aſperity of the *Moon*'s Limb, which undulated and ſparkled very much, as it appeared through the Vapours near the Horizon: The *Ingreſs* happened much about the length of the Spot, called by *Hevelius*, *Palus Maræotis*, to the North of the ſaid Spot, or about the 124th Degree of the outer Limb of his *Selenography*, nearly in the ſame Latitude with the *Moon*'s Center.

$10^{h}\ 30'$. The Weſtern Edge of *Jupiter* began to *emerge* out of the dark Limb of the *Moon*.

$10^{h}\ 31'\ 20''$. The whole Diſk of *Jupiter* was entire, ſo that he was about a Minute and a Third in coming out from behind the *Moon*.

The *Emerſion* was exactly in a right Line with the *Moon*'s *Center* and the Northern Part of *Palus Mæotis*, or about the 324th Degree of the inner Limb of the *Selenographick Table* of *Hevelius*.

Time

Time corrected by Pend.Clock				
h. ′ ″				
9 32 30	*Jupiter*'s Limb touch'd the bright Limb of the Moon, the Diameter of *Maræotis* from its Northern Limit.			
9 33 42	*Jupiter* was wholly cover'd, as far as could be gueſs'd by means of the Vapours of the Horizon, and the Undulation of the Limb which was very turbid.			
10 11 12	The Difference of the Declinations of the Limb of the Moon which was truly Southward, and of the place of Ingreſs; by a Tube of 8 Feet and a Micrometer.	1546=	′ 12	″ 42
10 30 30	A little part of *Jupiter* emerged over againſt the Northern Limb of *Mæotis*.			
10 31 36	All *Jupiter* was free.			
10 35 50	The Difference of Declinations of the Center of *Jupiter*, and the Southern Limb of the Moon	3436=	28	15
10 41 40	Repeated——	3585=	29	28
10 44 00	The Diameter of the Moon ——— ———	3906=	32	07
10 45 44	Repeated——	3915=	32	11

2. *At Greenwich; by Mr.* Flamſteed. *n.* 184. *p.* 206.

At Nuremburg; *by M.* Zimmerman. *n.* 183. *p.* 177.

3. At 10h 19′ 56″, *M. I. Ja. Zimmerman* obſerved the firſt *Contact* of the Limbs of *Jupiter* and the Moon, and at 10h 20′ 47″, *Jupiter* was all *eclipſed*.

At 11h 22′ 51″, *Jupiter* was wholly *clear* from the *Eclipſe*.

The *Immerſion* was about the 117th, the *Emerſion* at the 321ſt Degree of the Limb, in the *Chart* of *Hevelius*.

At 11h 31′ 06″, the third Satellite of *Jupiter emerged*. Theſe Times were collected from the Culminations of fixed Stars, and the Vibrations of a Pendulum.

By M. Wurtzelbauer. *ib.*

4. At 10h 20′ 50″, *Jupiter* applied to the Limb of the Moon, over-againſt the *Loca Paludoſa Inſulæ Cercinnæ*.

At 10h 22′ 00″, he appeared about half *eclipſed*.

At 10h 22′ 30″, he was wholly *hid*.

At 11 19′ 40″, *Jupiter* began to *emerge*.

At 11h 21′ 20″, he was quite *free* from the Interpoſition of the Moon. The Point of the *Emerſion* was ſomewhat to the North of the *Palus Mæotis*. No
S t in the Moon was ſo near the apparent Magnitude of *Jupiter*'s Diſk as the
I la Beſbicus Hevelii.

lt 11h 40′ 00″, the Altitude of *Procyon* was 8° 37′; whence the Pendulum
C :k, which had been ſet by Altitudes of the Sun the Afternoon preceding,
m be examined.

At Dantzick, *by M.* Hevelius. *ib.* p. 178.

5. Tho' hitherto for 56 Years I have neglected no Obſervation of any Mo~~ment,~~ yet I have ~~not been able rightly~~ to take ~~and note down~~ above ~~three~~ Eclipſes of *Jupiter*. The firſt was *An.* 1646. *Dec.* 24. in the Evening; but then I could only obſerve the End. The ſecond was *An.* 1679. *June* 5. in the Forenoon; at which Time the whole ſucceeded as I could wiſh. The third was this preſent Year 1686. *Apr.* 10. in the Evening.

Among other things this is to be noted, that this Occultation did not happen when the Moon was quite in the full, but about a Day after the full Moon, in the Evening. And likewiſe at the ſame time, (which is very wonderful, and is a Coincidence which hardly ever happens) and with the ſame Appearance, as was ſeen in that Occultation *An.* 1646. *Dec.* 24. in the Evening. At which Time the Moon had now decreaſed for two Days, and without doubt it exhibited the ſame Libration alſo, as in this our laſt Obſervation. For the Section of Light and Shadow was juſt the ſame, and paſs'd through the ſame Spots, which I cannot ſufficiently admire; that is, at the greater and leſſer *Hyperborean* Lake, alſo at the *Ripbean* Mountains, through *Palus Mæotis*, by the greater Lake of the *Caſpian* Sea, and its lower Bay to Mount *Neroſus*.

On the contrary, the Occultation of *Jupiter* obſerved by me *An.* 1679. was very different, as it happen'd not about the full Moon, but the new Moon, about three Days before the Conjunction.

Hour by the Watch.		*Altitudes taken with a Quadrant*	*Time corrected by the Alt.*
h. ′ ″		Gr. ′ ″	h. ′ ″
5 10 10	The Sun's Altitude.	13 47 0	5 11 43
5 12 30	The Sun's Altitude.	13 28 0	5 13 55
5 17 40	The Sun's Altitude.	12 41 0	5 19 21
5 23 50	The Sun's Altitude.	11 46 0	5 25 43
8 7 10	The Altitude of *Arcturus*.	29 55 0	8 12 50
8 11 15	The Altitude of *Arcturus*.	30 32 0	8 17 4
8 15 10	The Altitude of *Arcturus*.	30 59 0	8 20 51
	The Moon riſes about——		9 24 0
9 44 50	*Jupiter* was diſtant from the Iſland of *Cercinna* about 43 Minutes.		9 5
10 21 30	*Jupiter*'s Diſtance was ſuch, as the Diſtance of M. *Sinai* from *Palus Marœotis*.		10 21
10 40 35	*Jupiter*'s Diſtance was nearly equal to the Diſtance between Mount *Ætna* and Mount *Porphyrites*.		10

h. ′ ″			h. ′ ″
10 51 30	The Limb of *Jupiter* was as far diſtant from the Limb of the M. as *Pal. Maræotis* from the Limb of the M.		11 2 0
10 56 9	*Jupiter* with his Limb began to touch the Limb of the M. and thus the beginning of the Occultation came on.		11 7 9
10 56 54	Half *Jupiter* was hid.		11 7 54
10 57 39	All *Jupiter* was wholly cover'd by the Moon.		11 8 39
11 8 31	The Occultation of the outmoſt of *Jupiter*'s Companions happen'd at *Mons Alabaſtrinus*. Only two of his Companions could be ſeen on the Eaſtern Side.		
11 15 54	The Altitude of *Lyra*.	32° 59′ 0″	11 26 0
11 19 0	The Iſland *Beſbica* and *Rhodes* were found under the ſame Perpendicular, which was removed about 35 Degrees from the vertical Line of the Moon.		
11 21 37	The Altitude of *Lyra*.	33 50 0	11 32 15
11 24 57	Again.	34 24 0	11 36 24
11 38 15	The beginning of *Jupiter*'s Emerſion.		11 49 15
11 39 0	Half *Jupiter* emerged.		11 50 0
11 39 45	All *Jupiter* appeared. Diam. of the M. obſerved with a Microm. was 31′ 0″.		11 50 45
11 54 10	The Diſtance of *Jupiter* from the Confine of Light and Shade was equal to the Diſtance of Mount *Ætna* from Mount *Porphyrites*.		12 5 40
11 57 20	Diſtance of *Jupiter* from the Confine of Light and Shade was equal to the Diſt. between the Iſland *Beſbica* and Mount *Ætna*. And the moſt remote of *Jupiter*'s Companions was as far diſtant from *Jupiter*, as that Companion from the ſaid Confine of Light and Shade.		12 9 20
12 6 9	The Altitude of *Lyra*.	40 19 0	12 18 39
12 9 18	The ſame Altitude again.	40 46 0	12 21 40
12 12 20	The Altitude of the Moon.	16 15 0	

it is plain from the Obſervation itſelf, that the Path or Line of *Jupiter*'s was by Mount *Alabaſtrinus*, by the Mount of *Chriſt*, Mount *Carpathos*, ount *Macrocemnios*, and by the lower *Hyperborean* Lake. Secondly, ſland *Beſbica* and the Iſle of *Rhodes* were under the ſame Perpendi- the Time of the Occultation, about 11ʰ 30′; ſo that 35 Degrees of oon's Limb culminated. Therefore *Jupiter* enter'd the bright Limb of

the

the Moon about the 61 Degree, that is, from the Perpendicular Line of the Nonageſm and the Point of the Zenith, towards the Eaſt. And he went out about the 31 Degree from the ſaid perpendicular Line of the Nonageſm towards the Weſt, at the obſcure Limb of the Moon. Therefore *Jupiter*'s Path-Line was a Subtenſe of almoſt 104 Degrees, in the North Part of the Moon.

Beſides it is very well worth obſerving, that from this Obſervation I could derive the Diameter of *Jupiter* exactly; which was 51″ 42‴. And of this Magnitude was the Diameter of *Jupiter*, or about 50″, whenever I meaſured it by the Spots of the Moon. But in the Year 1679, on the 5th of *June*, when I obſerved a like Eclipſe of *Jupiter*, it was much leſs than this, being only 30″ 53‴. Which I imagine to proceed from hence, becauſe that Obſervation was made in the Day-time, and while the Sun ſhined, when the adventitious Rays of the Stars and Planets are more diſpell'd by the Sun's Light, than in a dark Night. Now if you ſhould inquire which of theſe apparent Diameters I take to be the truer, I ſhould anſwer that which was obſerved on the 5th of *June* in the Year 1679, when the Sun ſhined. Not becauſe the laſt was not obſerved with the ſame Diligence and Exactneſs; but becauſe, as I ſaid before, the adventitious Rays in the Night-time are a greater Hindrance than in full Day-light.

At Paris; *by* *M.* Caſſini. *n.* 183. *p.* 175.

6. At 9^{h} 31′ 6″, *Jupiter* was in a Perpendicular falling on the Limb of the *Moon* over-againſt the Northern-part of the Spot *Grimaldi*, (*Maræotis*) near to *Riccioli* (*Stag. Miris*) and diſtant from the Limb about 4 Times as much as the ſaid Spot.

9^{h} 40′ 21″, *Jupiter* touched the Circumference of the *Moon*, which undulated by reaſon of the Vapours near the *Horizon*.

9^{h} 41′ 20″, He quite diſappeared in the Inequalities of the *Moon*'s Limb, the *Total Immerſion* might be ſome Seconds later. So the *Central Immerſion* was at 9^{h} 40′ 51″. *Jupiter* entered over-againſt that Part of *Grimaldi* next *Riccioli*.

10^{h} 30′ 2″, The *outermoſt Satellite* which preceded *Jupiter* appeared over-againſt the Middle of the *Caſpian Spot* (*Pal. Mæotis*) through which the *Section of Light and Darkneſs* paſſed, and made nearly an Equilateral Triangle, with the Extremities of that Spot.

10^{h} 40′ 24″, The firſt Limb of *Jupiter* began to come out of the dark Side of the *Moon*, over-againſt the North-part of the *Caſpian* Spot, about *Cleomedes*, (*ad Montes Riphæos*.)

10^{h} 40′ 56″, The Center of *Jupiter* did *emerge*. It was difficult to diſtinguiſh the Moment when *Jupiter*'s Diſk was fully clear, but at 10^{h} 41′ 36″, the Eclipſe was certainly paſt.

At the *Emerſion* of the Center, the Altitude of *Jupiter* was 11° 31′.

At 10^{h} 42′ 49″, The *Second Satellite*, being the neareſt of the thi l-
lowed the Planet, *emerged*.

At 10^{h} 45′ 1″, The *Innermoſt Satellite*, being near its greateſt ,
emerged.

At 10^{h} 50′ 40″, The *Third*, or *Penextimus Satellites*, being likev ts
greateſt *Elongation*, began to appear over-againſt the Northern-Edge ie
Caſpian Spot. it

At $11^h\ 45'$. The Diameter of the *Moon* was $32'\ 27''$; and according to the *Calculus* of M. *Caſſini*, her *Parallax* was 61 Min.

7. The Central *Immerſion* was at $9^h\ 42'\ 13''$; and the Central *Emerſion* at $10^h\ 45'\ 26''$, over-againſt the Southern-part of the *Caſpian* Spot. *At* Avignon; *by* R. P. Bonfa. *ib. p.* 176.

LXXIV. 1. The *Immerſion* was ſeen at *Totteridge* (which Place is about 9 Miles from *London*, and nearly $25''$ of Time to the Weſtward thereof) by Mr. *Ed. Haines*, who between a Gap of the Clouds obſerved the *Contact* of the *Moon*'s Limb and *Jupiter*'s, at $3^h\ 3'\frac{1}{2}$. *An Occultation of* Jupiter *by the* Moon, Apr. 28. 1686. *n.* 181. p. 87.

The Clouds cloſing again permitted him to obſerve no more: however from this we may conclude the *Central Immerſion* at *London*, to have been $3^h\ 4'\frac{1}{2}$, *mane*.

The *Emerſion* was obſerved at *London*, by Mr. *Edm. Halley*, to fall out at $3^h\ 49'$: for at $3^h\ 50'$, *Jupiter* was all out, and the Limbs ſo little ſeparated, that he judged, that a Minute before, the *Center* of *Jupiter* had been upon the *Moon*'s Edge: The Point of the *Emerſion* was over-againſt the Southern-part of the Spot, call'd by *Hevelius Inſula Macra*, or at the 342d Diviſion of the inner Limb of his *Map* of the *Moon*.

2. The *Immerſion* of the *Center* happened at $3^h\ 37'\ 23''$, on the Eaſt-ſide of the Spot *Xenophanes*. The *Emerſion* was at $4^h\ 28'\ 24''$, between *Seneca* and *Beroſus*, according to *Riccioli*, or *ad Montes Alanos Hevelii*, a little to the Northward of the *Palus Mæotis*. *At* Avignon; *by* R. P. Bonfa. *n.* 183. p. 177.

3. *At* Dantzick; *by* M. Hevelius. *ib.* p. 184.

By the Watch.			*Time corrected.*
h. ′ ″		° ′ ″	h. ′ ″
3 23 20	Altitude of *Arcturus*.	31 16 0	3 20 12
3 24 25	The ſame Altitude.	31 04 0	3 21 35
3 44 30	*Jupiter* was diſtant from the M.'s Limb by a greater Interval than that of Mount *Sinai* from Mount *Ætna*.		3 41 30
3 47 00	*Jupiter*'s Diſtance was as much as that of Mount *Porphyrites* from *Byzantium*.		3 44 00
3 52 00	Diſtance of *Jupiter* from the Limb of the M. was equal to the Diſt. of the Iſle of *Sardinia* from *Pal. Maræotis*.		3 49 00
3 59 00	*Jupiter* was diſtant from the Limb of the Moon ſomething more than *Palus Maræotis* from *Ætna*.		3 56 00
10	Diſt. of *Jupiter* from the Limb of the Moon was almoſt equal to that of M. *Porphyr.* from the Iſland *Cercinna*.		4 13 40
	The Planets were now ſet.		4 17 00

LXXV. 1. *An Occultation of* Saturn *by the* Moon, Mar. 19. 1686-7 *at* Totteridge; *by Mr.* Ed. Haines. *n.* 186. p. 268.

The Hour by a large Clock.					*The Time corrected.*		
h. ′ ″			°		h.	′	″
12 06 10	The Altitude of *Pollux.*	28	35	12	12	14
12 16 47	———————	26	59	12	22	47
1 18 00	The Limb of the Moon touched the Weſtern Anſa of *Saturn.*			1	24	00
1 18 30	The Immerſion of *Saturn*'s Center a little below *Palus Maræotis.*			1	24	30
1 19 00	Now *Saturn* was quite hid.			1	25	00
4 01 25	The Altitude of the Center of the Sun.	20	00	7	07	17
4 09 06	———————	18	55	4	14	43

In Ireland; *by* Dr. Aſh, *Biſhop of* Cloyn. *n.* 243. p. 293.

2. *March* 18. At Night I obſerved here the *Occultation* of *Saturn* by the *Moon*, which happened at 12ʰ 13′ 55″: it paſſed directly under the Midſt of the *Moon's Diſcus.*

Phaſes of Saturn, *An.* 1665. *at* Mainhead *near* Exeter; *by Mr.* William Ball. *n.* 9. p. 152. *Fig.* 132.

LXXVI. 1. *Oct.* 13. 1665. at ſix of the Clock, with a very good Teleſcope near 38 Foot long, and a double Eye-glaſs, *Saturn* appeared to me ſomewhat otherwiſe than I expected, thinking it would have been decreaſing, but I found it as full as ever, and a little hollow above and below.

An. 1666. *at* London; *by Dr.* Hook. *n.* 14. p. 246. *Fig.* 133.

2. *June* 29. 1666. between 11 and 12 at Night, I obſerved the Body of *Saturn* thro' a 60 Foot Teleſcope, and found it exactly of the Shape repreſented in the Figure. The *Ring* appeared of a ſomewhat brighter Light than the Body; and the black Lines *a a*, croſſing the *Ring*, and *b b*, croſſing the Body (whether Shadows or not I diſpute not) were plainly viſible; whence I could manifeſtly ſee, that the Southernmoſt part of the *Ring* was on this ſide of the Body, and the Northern-part behind or covered by the Body.

An. 1668. *at* Paris; *by M.* Hugens *and M.* Piccart. *n.* 45. p. 900.

3. *Aug.* 17. 1668. at 11½ʰ theſe *Pariſian* Obſervers, imploying a Teleſcope of 21 Foot, ſaw the *Globe* of *Saturn* in the Middle manifeſtly appearing above and below, beyond the Oval of his *Anſes*; which was hardly diſcernible the laſt Year. They meaſured divers ways the *Inclination* of the greater Diameter of the *Oval* to the *Æquator*, and found it of about 9 Degrees. By this Obſervation and other like ones of this and the preceding Year, M *Huygens* finds, that, inſtead of 23° 30′, the Angle of the Planes of the *Ri* [illegible] c *Ecliptick* muſt be of 31°, or thereabouts.

An. 1670. *at* Dantzick; *by M.* Hevelius. *n.* 65. p. 2089. *Fig.* 134.

4. *Aug.* 26. New Style, 1670. That Teleſcope of 50 Foot long, [illegible] u lately ſent me, ſhew'd me the Face of *Saturn* very plainly and cl [illegible] withſtanding the Preſence of the Moon. Now how it appear'd to [illegible] e ſeen by this Delineation. It had a very different Appearance from [illegible] is ſeen by Mr. *Huygens*, and you alſo, *An.* 1666. and likewiſe by [illegible] *is*

Obſervers in 1668. For the Ring which encompaſſes *Saturn* was now found to be much narrower and cloſer than at that Time, its Path now being much more oblique in reſpect of the Earth.

5. This Summer Mr. *Huygens* obſerved *Saturn* with his Teleſcope of 22 Feet, and ſaw his Figure to be very conformable to what it ſhould be according to his *Hypotheſis*; viz. the *Anſæ* or Arms to be very narrow, inſomuch that their Opening appeared not but very obſcurely.

At Paris; *by M.* Huygens. *ib.* p. 2093.

6. *Sept.* 16. Dr. *Hook* obſerved the Phaſe of Saturn as here repreſented, *Fig.* 135.

At London; *by* *Dr.* Hook. *ib.*

7. *Saturn*, according to the *Hypotheſis* of Mr. *Huygens*, was to have retaken his *Round Figure* in the Months of *July* and *Auguſt* 1671. But this Appearance hath been perceived ever ſince the End of *May*, at a Time when he was diſtant enough from the *Sun* and the *Horizon*, to be well obſerved. He hath remained in this Figure unto the 11*th* of *Auguſt*, and Mr. *Caſſini* did then obſerve him thus; but three Days after he ſaw him with *Arms*, though very narrow ones.

An. 1671. *at* Paris; *by M.* Caſſini. *n.* 78. p. 3024.

8. Our *Philoſophers* here know very well, that as ſoon as Mr. *Caſſini* had told me that the *Arms* of *Saturn* were returned in *Auguſt*, I ſaid that aſſuredly they would diſappear before the End of this Year. I ſtill obſerved them, *Nov.* 6. *ſt. n.* in the Evening, but they were ſo faint and obſcure, that it was hard to diſcern them; ſo that within a few Days they will appear no more at all. This confirms altogether my *Hypotheſis* of the *Ring*, which now diſappears in proportion as the Rays of the *Sun* do obliquely illuminate the flat Surface of it, obverted to our Sight.

By M. Huygens. *ib.* p. 3025, 3026.

9. How it appear'd lately, on *Sept.* 11. New Style, I have delineated very truly and carefully, and have here ſent you the Scheme. But in the Months of *June*, *July*, and *Auguſt*, that you ſhould ſee it quite round, as the *Pariſians* affirm, I can hardly imagine. For tho' the Arms of *Saturn* might appear very cloſe at the Sides, even in a Tube of 60 or 70 Feet, yet I can hardly think they could quite vaniſh, ſo that no Remains of them could be ſeen. Perhaps the *Pariſians* might view *Saturn* in ſhort Teleſcopes, during the Twilight, when the Moon was up.

At Dantzick; *by M.* Hevelius. *ib.* p. 3032. *Fig.* 136.

10. *Oct.* 12. with my leſs Tube I thought I ſaw ſomething on each Side of *Saturn*, amidſt the Colours of my Glaſs, and the ſpurious Rays of his Body. Directing my longer Tube (of 14 Feet) to him, I could ſee his *Anſæ* ſomewhat more diſtinctly, but very ſlender, and to one, that thought not of them, ſcarce diſcernible.

At Derby *by Mr.* Flamſteed. *ib.* p. 3034.

Nov. 30. I observed him with my 14 Foot Teleſcope, the Aperture being 1½ Inch, and its Eye-Glaſs drawing two Inches. He appeared perfectly *round*, free Rays and Colours, and no *Anſæ* to be ſeen. Mr. *Townly* in his laſt (N .. 20.) tells me, that he looked at him one Night, and could hardly diſting uiſh his Line of the *Anſulæ*, but plainly ſaw a dark Line through him nea upper Part.

At Paris; *by M.* Caſſini. *n.* 92. p. 5180. An. 1675.

1 Dec. 16. we found that *Saturn* had retaken his *round Figure*.

At London; *by* *Mr.* Flamſteed. *n.* 116. p 372.

1 ne 27. in the Morning 1675. we ſaw *Saturn* improved with his wid and open *Anſæ*.

At Dantzick; *by M.* Hevelius. *n.* 127. p. 661.

1 In *Auguſt*, the Figure of *Saturn* appear'd, as *Fig.* 137.

14. From

An. 1676. *at* Paris; *by M.* Caſſini. *n.* 128. 690. *Fig.* 137. *Fig.* 138.

14. From the Scheme of *Saturn*, as obſerved by Mr. *Hevelius* a Year ago, I perceived that he made uſe of Teleſcopes which were much inferior to ours. For at that Time, as alſo now *Aug.* 1676. in the Globe of *Saturn*, an obſcure Zone was ſeen by us, a little to the South of the Center, not unlike the Zones of *Jupiter*. Moreover the breadth of the Annulus was divided into two Parts, by an obſcure Line that to appearance was elliptical, but was really circular, as it were into two concentrick Rings, the Interval of which was brighter than the External. I ſaw this Phaſis immediately after the Emerſion of *Saturn* out of the Sun's Rays through the whole Year quite to his Immerſion; firſt with a Teleſcope of 35 Feet, and afterwards with a leſs of 20 Feet.

Places of Saturn *obſerved An.* 1670, *at* Dantzick; *by M.* Hevelius. *n.* 63. p. 2089.

LXXVII. 1. *Aug.* 26. *ſt. n.* 1670. I ſaw *Saturn*, the Altitude of *Aquila* being 24° 32′ 0″, at the Diſtance of 33° 48′ 0″, from the utmoſt Wing of *Pegaſus*, and 24° 51′ 40″ from his Mouth; in 4° 11′ of *Piſces*, and in 1° 53″ of South Latitude, being then in Oppoſition to the Sun.

An. 1677. *at* Paris; *by M.* Bullialdus. *n.* 139. p. 973.

2. *Dec.* 29. new Style, 1677. at 8^{h} 58′. In the ſame Azimuth, and in the nonageſm Degree of the Eckliptick from the Horizon, we ſaw *Saturn* and the Northern Eye of the Bull, which was below *Saturn*. Whence we found that the Planet and the fixt Star had the ſame Longitude in the Zodiack; that is, ♊ 3° 58′ 53″, according to *Tycho*.

The outermoſt Satellite *of* Saturn *diſcover'd by M.* Caſſini. *n.* 92. p. 5178.

LXXVIII. 1. About the End of *October* 1671. we diſcovered, by a Teleſcope of 17 Feet, 11 ſmall Stars near *Saturn*, one of which by its particular Motion ſhew'd itſelf to be a true *Planet*; which we found, by comparing it not only to *Saturn* and his ordinary *Satellite*, diſcovered 1655 by *M. Huygens*, but alſo to the Fix'd Stars. The Motion of it was very manifeſt in reſpect of the Fix'd Stars, but leſs ſenſible in reſpect of *Saturn*; yet it appear'd, that from *October* 25, unto *November* 1, his Diſtance from *Saturn* increaſed Weſtward, and from that time unto *Novem.* 6. it diminiſhed: So that his greateſt *Digreſſion* from *Saturn* happened in the Beginning of *November*.

Dec. 16. We found that on the Eaſt of *Saturn*, there was a ſmall Star, far diſtant, in a ſtreight Line to *Saturn*, and to his ordinary *Satellite*, which was Oriental alſo, but little diſtant from *Saturn*. And *Dec.* 24*th*. we ſaw this *Satellite* in the Weſt, and a Star, Oriental likewiſe, leſs diſtant from *Saturn* than that we had ſeen the 16*th*.

Dec. 13. and 17. 1672. We perceived, with an excellent Teleſcope, (of 35 Feet, made by *Campani*) an Occidental Star, remote from *Saturn*, which in both thoſe Obſervations had a Southern Latitude in reſpect of the Line of his *Wings*; but in the firſt it was further diſtant from *Saturn* than in the ſecond: ſo that, if this was the ſame Star, as I ſuppos'd it to be, it mov'd towards *Saturn* on the Eaſt, and conſequently (ſuppoſing it to be his *Satellite*) it was in the ſuperior Part of his Circle.

Feb. 6. 1673. We began to ſee him again, and we obſerved him almoſt all the Days following till the 20*th*. This new *Planet* did more and more remove from *Saturn* till the 19th of *Feb.* when we meaſured the Difference between his Paſſage and that of the Center of *Saturn* to be 30″ of an Hour, which gave at leaſt 10 Diameters of *Saturn*, and on the 20*th* the Diſtance was judged by Eſtimate to be yet greater.

his Digreſſion, being treble to that of the *Ordinary Satellite*, enabled us n. p. 5179, 5184.
t to judge the Time of this Revolution to be Quintuple, applying to
tellites that Proportion, which *Kepler* hath noted in the *Principal Pla-*
etween the Periodical *Times* and their *Diſtances.* We were after-
confirmed in this Opinion; for by the apparent Swiftneſs of his
, it was eaſy to ſee that this *Planet* had been in Conjunction with
eb. 3. 1673. and by his Motion on the Weſt, it appears, that he
e inferior part of his Circle: And becauſe during this time of 17
remov'd more and more from *Saturn*, 'tis certain that he remained in the ſame Quadrant of the inferior Occidental Circle above 17 Days, and that his whole *Revolution* is more than 68 Days. He was theſe laſt Days at a Diſtance almoſt equal to that which he had about the end of *October* 1671; ſo that in 480 Days or thereabout he made a certain Number of Intire *Revolutions*, which can be no more than 7; ſince each of them is without Queſtion of more than 68 Days. If you ſhould count 7 of them, each would be 68½ Days; if you count 6, each would be 80 Days; and if you count but 5, each would be 96 Days. But this laſt Suppoſition can by no means be made to agree with the two Obſervations of *Dec.* 1672; and the firſt doth not agree ſo well with them as the ſecond.

Mr. *Caſſini* has ſince found, that this *outermoſt Satellite* is diſtant from the Center of *Saturn* 10½ Diameters of his *Ring*; that the Period of his *Revolution* in 80 Days is ſo juſt, that he doth not anticipate 9 Revolutions, which are made in two Years, but by one whole Day; and that in the Conjunctions with *Saturn*, his Latitude encreaſes according as the *Ring* of *Saturn* enlargeth itſelf; though the Line of his Motion is not parallel to the Circumference of the *Ring*. n. 133. p. 831.

Mr. *Caſſini* hath alſo diſcovered, after many Revolutions, that this *Satellite* hath a Period of *Apparent Augmentation* and *Diminution*, by which Period he becomes viſible in his greateſt Occidental Digreſſion, and inviſible in his greateſt Oriental Digreſſion; he begins to *appear* two or three Days before his Conjunction in the inferior part, and to *diſappear* two or three Days after his Conjunction in the ſuperior part: So that he remains inviſible in every Revolution of 80 Days for a whole Month together.

This Viciſſitude of *Phaſes* makes it ſeem probable, that one part of his Surface is not ſo capable of Reflecting to us the Light of the Sun, which maketh it viſible, as the other part is. Whence we may conjecture, that the *Globe* of the *Satellite* hath ſome Diverſity of Parts analogous to that of the Earth, the one part of whoſe Surface is cover'd by the Sea; which is not ſo fit to reflect from all parts the Light of the Sun, as the Continent which maketh up the other part: So that this *Planet* by a Converſion about his *Axis*, or by an Expoſition of the ſame Hemiſphere to *Saturn* (much after the manner of the Hemiſphere of the *Moon* to the *Earth*) ſometimes turns to us the part analogous to the Continent, ſometimes that part which anſwers to the Sea.

The Third Satellite *of* Saturn *diſcover'd; by M.* Caſſini, *n.* 92. p. 5181.

LXXIX. *Dec.* 23. 1672. We found a ſmall Star Weſtward of *Saturn*, between him and his *Ordinary Satellite*, which was on the Weſt alſo, almoſt at a double Diſtance. *Dec.* 30. we ſaw a little Star, on the Eaſt of him and his *Ordinary Satellite*, which had paſſed alſo to the Eaſt of him.

Jan. 10. 1673. This little Star appeared to have returned almoſt to the ſame Poſition in reſpect of *Saturn* and his *Ordinary Satellite*, where it had been *Dec.* 23. *Jan.* 15. the *Ordinary Satellite* was Oriental, and the *New* one Occidental, as it had been in the precedent, but a little nearer to *Saturn*: We had that Evening time enough attentively to obſerve this *Planet* for a whole Hour together, during which we perceived, it approached to *Saturn* on the Weſt, and conſequently was in the ſuperior part of his Circle: Which did fully confirm us in the Suppoſition we were inclined to, that it was an *Interior Satellite.*

Comparing the Obſervations together, we began to find the Rule of the Motion of the new *Interior Satellite*. For the two laſt ſhewed us, that in 5 Days he had made more than a whole Revolution. The firſt Obſervation compared with the third made us judge, that in 18 Days he had made a Number of Revolutions, almoſt whole ones, which certainly were 4; each of them was of $4\frac{1}{2}$ Days: So that between the 10*th* and 15*th* it might be, that there had been one Revolution of $4\frac{1}{2}$ Days, or two Revolutions of $2\frac{1}{4}$ Days each. But the Combination of the firſt with the ſecond, made us ſeclude the Period of $2\frac{1}{4}$ Days. We therefore judged by theſe Obſervations, that this laſt *Planet* finiſhes his Reſolution about *Saturn* in $4\frac{1}{2}$ Days; that the Semidiameter of his Circle is $1\frac{5}{8}$ of the Diameter of *Saturn*'s *Ring*; and that he was towards his greateſt Occidental Digreſſion the 23d of *December*, and *Jan.* 1. about 7 a Clock in the Evening. We have ſince found, that his greateſt Digreſſion from the Center of *Saturn* is only $1\frac{2}{3}$ of his *Ring*, and the Period of his Revolution is 4 Days 12^h and 27′. His Latitude augments alſo according as the *Ring* enlargeth, and at the preſent that the largeneſs of the *Ring* is greater than the Diameter of the Globe of *Saturn*, he is to paſs in the Conjunctions without touching either *Saturn* or his *Ring*. Yet notwithſtanding we have not yet been able to diſtinguiſh him in the Conjunctions either in the upper or lower part of his Circle; but only in his greateſt, as well Oriental as Occidental, Digreſſions.

n. 133. p. 833.

Two Interior Satellites *of* Saturn *diſcover'd; by M.* Caſſini. *n.* 181. p. 79.

LXXX. Theſe two *Satellites* were firſt of all ſeen in *Mar. An.* 1684. by two excellent Object Glaſſes of 100 and 136 Feet; and afterwards by two others of 93 and 70 Feet; all made by *S. Campani*, after the Diſcovery of the 3d and 5th *Satellites*, which had been made by others of his Glaſſes of 47 and 34 Feet. We have ſince ſeen all theſe *Satellites* with that of 34 Feet, and continued to obſerve them with Glaſſes of *M. Borelli* of 40 and 70 Feet, and by thoſe which *M. Artouquel* hath lately made, of 80, 155, and 220 Feet. It was eaſy for us to ſee theſe two *Satellites* by theſe different ſorts of Glaſſes, after having found the Rules of their Motion, whereby we might with more particular Attention look upon the Places where they ought to be.

The *Firſt Satellite* was obſerved 45° diſtant from its *Perigee*, moving toward the Weſt, *Mar.* 11. 1686. *ſt. n.* at 10^h $40'$ at Night, and returned to the ſame Poſition on the 14th of *April* at the ſame Hour.

The *Second* was 36° diſtant from the *Perigee* to the Weſt, the 30*th* of *Mar.* 1686. *ſt. n.* at 8 a Clock in the Evening.

The *Firſt* or *Innermoſt Satellite*, is never diſtant from *Saturn*'s *Ring* above $\frac{4}{3}$ of the apparent Length of the ſame *Ring*; it makes one *Revolution* in 1^d 21^h $19'$; and the Circle of its Orb is nearly in the ſame Plane with the *Ring*.

The *Second* or *Penintime Satellite* is $\frac{3}{4}$ of the Length of the *Ring* diſtant therefrom, and makes his Revolution in 2^d 17^h $43'$.

After a great Number of choice Obſervations, it was concluded, that the Proportion of the *Digreſſions* of the *Second* to that of the *Firſt*, (counting both from the Centre of *Saturn*) is as 22 to 17; and of its *Revolution* as $24\frac{3}{4}$ to 17. This is that very ſame Proportion which *Kepler* obſerves between the *Diſtances* and *Period* of the *Primary Planets*, and which we have found between the other *Satellites* of *Saturn*, and is verified in the *Satellites* of *Jupiter*. There is nothing that better ſhews the admirable Harmony of the particular *Syſtems* with the great *Syſtem* of the World.

The antient Aſtronomers having tranſlated the Names of their Heroes among the Stars, whoſe Names have continued down to us unchanged, notwithſtanding the Endeavour of the following Ages to alter them; and *Galileo* after their Example, having honoured the Houſe of the *Medici* with the Diſcovery of the *Satellites* of *Jupiter*, made by him under the Protection of *Coſmus* II. (which Stars will be always known by the Name of the *Sidera Medicea*.) Wherefore the Diſcoverer concludes that the *Satellites* of *Saturn*, being much more exalted and more difficult to diſcover, are not unworthy to bear the Name of *Louis Le Grand*, under whoſe Reign, and in whoſe *Obſervatory* the ſame have been detected; which therefore he calls *Sidera Lodoïcea*, not doubting but to have perpetuated the Name of that King, by a Monument much more laſting than thoſe of Braſs and Marble, which ſhall be erected to his Memory.

LXXXI. The *Fourth* or *Penextime Satellite* of *Saturn*, firſt diſcover'd by M. *Huygens* 1655, I have of late frequently obſerv'd with a 24 Foot *Teleſcope*: And I perceiv'd that M. *Huygens*'s Numbers were conſiderably run out, and about 15° in 20 Years two ſwift; this made me reſolve more nicely to enquire into it's *Period*, and accordingly I waited till I had gotten a competent Number of Obſervations, the moſt conſiderable whereof are theſe.

M. Huygens's *Theory of the Fourth* Satellite *of* Saturn *corrected; by Mr.* Ed. Halley. *n.* 145. p. 82. *Mar. An.* 1683.

1612. *Nov.* 13. 13^h $00'$ *p. m.* the *Satellite* appear'd on the North ſide of *S* *urn*, and a Perpendicular let fall from it on the tranſverſe Diameter of the *R* *g*, fell upon the middle of the dark Space of the following *Anſæ*; and the ſa e Night 19^h $00'$, it had paſt the Conjunction, and the Perpendicular fell ex ctly on the Weſtern Edge of the *Globe* of *Saturn*: the Northern Latitude a Retrograde Motion made it evident, that the *Satellite* was then in *Peri* *eo*.

Again, *Nov.* 21. $16^h\ 15'$, this *Satellite* of *Saturn* was on his South-ſide; the Perpendicular on the Line of the *Anſæ* fell on the Middle of the dark Space of the Weſtern *Anſæ*, and the ſame Night $19^h\ 00'$, the Perpendicular fell preciſely on the Center of *Saturn*, and the Diſtance therefrom was ſomewhat leſs than one Diameter of the *Ring*. By this it was evident that the *Satellite* was in *Apogæo*.

I obſerved it in *Apogæo* again on the 24*th* of *Jan.* 1683. at $8^h\ 00'$ *p. m.* the Perpendicular on the Line of the *Anſæ* fell exactly on the Weſtern Limb of the *Globe* of *Saturn*; and at $9^h\ 30'$, *p. m.* the ſaid Perpendicular fell within the *Globe* more than half way to the *Center*, and the Diſtance from the Line of the *Anſæ* towards the South, ſeemed much about one Diameter of the *Ring*.

Laſtly, *Feb.* 9. 1683. $8^h\ 10'$, *p. m.* it was again in *Apogæo*, and I could by no means diſcern towards which ſide it inclined moſt, nor whether the Tranſverſe Diameter of the *Ring*, or the Diſtance of the *Satellite* therefrom, were the greater; ſo that at that time it was preciſely *Apogæon*.

To compare with theſe, I choſe two out of thoſe of *Huygens*, which ſeemed the moſt to be confided in; the firſt made 1659. *March* 14. *ſt. n.* $12^h\ 00'$ at the *Hague*; when the *Satellite* appeared about one Diameter of the *Ring* under *Saturn*; but it was gone ſo far to the Weſtward, that he concluded, that about 4 Hours before, or $7^h\ 40'$ at *London*, it had been in *Perigæo*.

Again, *March* 22. 1659. $10^h\ 45'$, the *Satellite* was a whole Diameter above the Line of the *Anſæ*, and the Perpendicular thereon fell nearly upon the Extremity of the Eaſtern *Anſæ*.

By the Firſt of my Obſervations it appears that the *Satellite* was in *Perigæo* 1682. *Nov.* 13. $17^h\ 00'$, *circiter*, at which time *Saturn* was $30^s\ 29^\circ\ 39'$ from the firſt Star of *Aries* in the *Ecliptick*, but the *Earth* reduced to *Saturn*'s *Equinoctial*, and the *Satellite* was $9^s\ 23^\circ\ 46'$, à 1^a ✱ ♈. And *March* 4. 1659. $7^h\ 40'$, *Saturn*'s Place in the *Ecliptic* was $6^s\ 0^\circ\ 41'$; but the *Earth* reduced, and conſequently the *Satellite*, in $11^s\ 28^\circ\ 18'$, *à Prima Stella Arietis*. The Interval of Time is 8655 Days, $9^\circ\ 20'$; in which the *Satellite* had made a certain Number of Revolutions to the Fix'd Stars, and beſides $9^s\ 25^\circ\ 28'$, or 295 Degrees $28'$, whoſe Complement to a Circle $64^\circ\ 32'$ is 2 Days $20^h\ 36'$ Motion of the *Satellite*, according to *Huygens*. So that 8655 Days $5^h\ 56'$, or 12467846 Minutes of Time, is the Time of ſome Number of intire Revolutions; and dividing that Interval by 15 Days $22^h\ 39'$, or $22959'$ (the Period of *Huygens*) the Quotient 543 ſhews the Number of Revolutions; and again dividing 12467876 by 543, the Quotient $22961\frac{1}{10}$ *min.* or 15 Days, $22^h\ 41'\ 6''$, appears to be the true Time of this *Satellite*'s *Period*. Hence the Diurnal Motion will be $22^\circ\ 34'\ 38''\ 18'''$, and the Annual, beſides 22 Revolutions, $10^s\ 20^\circ\ 43'$. Having made Tables to this Period, I found that in the *Apogæon* Obſervation of *Huygens*, the *Satellite* was above 3 Degrees faſter than by my *Calculus*, and that in the three other Obſervations of my own, being likewiſe in the ſuperior part, it was $2\frac{1}{2}$ *Deg.* ſlower than by the ſame Calculation. Now 'tis evident, that the Differences muſt ariſe from ſome

Eccentricity in the *Orbit* of this *Satellite*, and that in *Mar.* 1659. the *Apocronion* (as I may call it) was ſomewhere in the Oriental Semicircle, and that in *Nov.* 1682. it was in the Weſtern Semicircle; and ſuppoſing the *Apocronion* fix'd, it muſt neceſſarily be between 9^{s} 23° 46′, and 11^{s} 28° 18′, à 1^{s} * ♈, that being the common Part between thoſe two Semicircles; and becauſe the Difference was greater in *Huygens*'s Obſervation than in mine, 'twill follow that the *Linea Abſidum*, or *Apocronion*, ſhould be nearer to 9^{s} 23° 46′, than to 11^{s} 28° 18′. I will ſuppoſe 10^{s} 22° 00′ *à Prima Stella Arietis*, (which happens to be alſo the Place of *Saturn*'s *Equinox*) and the greateſt Equation about 2½ *Degrees*. Upon the Score of this Inequality the mean Motion of the *Satellite* will be found about 2° 45′ ſlower in 23½ Years, or 7 Minutes in a Year, whence I ſtate the Annual Motion 10^{s} 20° 36′, above 22 Revolutions, and the correct *Epocha* for the laſt Day of *December* 1682, at Noon in the Meridian of *London* 9^{s} 10° 15′ à 1^{s} * ♈; from which *Elements* I compoſe the following Table.

Table

Tab. of the Mean Mot. of the Satel. of *Sat.* found by *Huygens*, from the first * ♈

Current Year of Christ.	*Epoques.*		
	s	°	'
1641	8	29	17
1661	10	14	10
1681	11	29	3
1682	10	19	39
1683	9	10	15
1684	8	00	51
1685	7	14	2

Month common Year.	*Mean Mot.*		
	s	°	'
Jan.	0	0	0
Feb.	11	9	53
March	8	12	2
April	7	21	56
May	6	9	14
June	5	19	7
July	4	6	26
August	3	16	19
Sept.	2	26	12
October	1	13	31
Nov.	0	23	24
Dec.	11	10	43

Years.	*Mean Mot.*		
	s	°	'
1	10	20	36
2	9	11	12
3	8	1	48
4	7	14	59
5	6	5	35
6	4	26	11
7	3	16	47
8	2	26	57
9	1	20	23
10	0	11	9
11	11	1	45
12	10	14	56
13	9	5	32
14	7	26	8
15	6	16	44
16	5	29	54
17	4	20	30
18	3	11	6
19	2	1	42
20	1	14	53

Days.	*Mean Mot.*		
	s	°	'
1	0	22	35
2	1	15	9
3	2	7	4
4	3	0	18
5	3	22	53
6	4	15	28
7	5	8	2
8	6	0	37
9	6	23	12
10	7	15	46
11	8	8	21
12	9	0	55
13	9	23	30
14	10	16	5
15	11	8	39
16	0	1	14
17	0	23	48
18	1	16	23
19	2	8	58
20	3	1	32
21	3	24	7
22	4	16	42
23	5	9	16
24	6	1	51
25	6	24	25
26	7	17	00
27	8	9	35
28	9	2	9
29	9	24	44
30	10	17	18
31	11	9	53
32	0	2	28

H. Min.	*Mean Mot.*	
	° '	' "
1	0	56
2	1	53
3	2	49
4	3	46
5	4	42
6	5	39
7	6	35
8	7	32
9	8	28
10	9	24
11	10	21
12	11	17
13	12	14
14	13	10
15	14	7
16	15	3
17	16	0
18	16	56
19	17	52
20	18	49
21	19	45
22	20	42
23	21	38
24	22	35
25	23	31
26	24	27
27	25	24
28	26	20
29	27	17
30	28	13

Minutes	*Mean Mot.*	
	'	"
31	29	10
32	30	6
33	31	3
34	31	59
35	32	55
36	33	52
37	34	48
38	35	45
39	36	41
40	37	38
41	38	34
42	39	31
43	40	27
44	41	24
45	42	20
46	43	17
47	44	13
48	45	10
49	46	6
50	47	3
51	47	59
52	48	56
53	49	52
54	50	49
55	51	45
56	52	42
57	53	38
58	54	35
59	55	31
60	56	27

In Leap-Year after *February* add one Day, and the Motion belonging to it.

I here ſuppoſe the *Linea Apſidum* fix'd, as having no Argument from Obſervation to prove the contrary, though it be very probable that as the *Apogæon* of our *Moon* has a Motion about the *Earth* in about 9 Years, ſo that of this *Satellite* ought to have about *Saturn*, but with a much longer Period, which future Obſervation may diſcover.

The Diſtance of this *Satellite* from the Center of *Saturn* ſeems to be much about 4 Diameters of the *Ring*, or 9 of the *Globe*, and the Plane wherein it moves, very little or nothing differing from that of the *Ring*, that is to ſay, interſecting the Orb of *Saturn* 4^s 22°, and 10^s 22° à 1^a ✱ ♈, with an Angle of 23½ Degrees, ſo as to be nearly parallel to the *Earth*'s *Equator*; whence the *Latitude* of the *Apogæon* Semicircle from 4^s 22° to 10^s 22° of *Saturn*'s *Longitude* from the firſt Star of ♈, will be Northern, and of the other Semicircle Southern; and the contrary in the other half of *Saturn*'s *Longitude*, to wit, from 10^s 22° to 4^s 22° of his diſtance from the firſt Star of ♈.

It follows now to ſhew how by the help of this Table to compute the place of this *Satellite*, to any Time required.

Firſt we muſt have the true *Longitude* of *Saturn* from the *Earth*, and numbred from the firſt Star of ♈, (or rather the Place of the Earth viewed from *Saturn*, together with its Latitude from the Orb of *Saturn*; but that being never fully ½ of a Degree we neglect it as a Nicety) and therefrom ſubtract 10^s 22°, there remains the Diſtance of *Saturn* from this *Equinoctial* Point, with which Diſtance with the *Longitude* of the *Sun*, take out the *Right Aſcenſion* and *Declination* thereto (23½ Degrees being the Obliquity common to both) and to the *Right Aſcenſion* adding 10^s 22°, the Sum ſhall be the *Longitude* of the *Satellite*'s *Apogæon*. Then ſay, as *Radius* to *Sine* of the *Declination*, ſo 8 to the greateſt *Latitude* in *Apogæo*, or *Perigæo* in the parts of the Semidiameter of the *Ring*.

Next collect the *Middle Motion* of *the Satellite*, and from it ſubtract 10^s 22°, the Remainder ſhall be the mean Anomaly, with which in the Table of the *Moon*'s primary *Equation*, take out the *Equation* anſwering thereto, and the half thereof added or ſubtracted to or from the *Middle Motion*, according to the Table, gives the true Motion of the *Satellite*; from which ſubtract the *Apogæon*, and if the Remainder be more than 6 *Signs*, the *Satellite* is Occidental, if leſs, Oriental; and as *Radius* to *Sine* of the Remainder, ſo 8 to the Semidiameters of the *Ring*, or 18 to the Semidiameters of the *Globe*, that the *Satellite* is to the Eaſtward or Weſtward of the Center of *Saturn*, according to the foregoing Precept.

Laſtly, as *Radius* to *Coſine* of the ſaid Remainder, ſo is the greateſt *Latitude* from the Line of the *Anſæ*, to the *Latitude* ſought.

Here note, that I purpoſely neglect the Inequality of the Diſtance ariſing m the *Eccentricity*, as being too ſmall to be any way obſervable.

Laſtly, to clear all Difficulties that may ariſe to them that are but little ſed in this ſort of Calculation, I have added an Example of the Work, t where the Precept may ſeem obſcure it may be thereby illuſtrated.

An. 1657. *May* 9. *ſt. n.* M. *Huygens* obſerved the *Satellite* very near *Saturn* on the Weſtern Side, and very little above the Line of the *Anſæ*. I ſuppoſe this about 10^h *p. m.* Let us now *calculate* to that time.

1657, *May* 9^d 9^h 10′, at *London.*

The Place of *Saturn*		28°	57′
Sat. from firſt Star of *Aries*	5^s	0	32
Equinoct. ſubtract.	10	22	00
Saturn from the Equinox	6	8	32
Right Aſcenſion	6	7	50
Apogæon	4	29	50
Declination South.		3	23

Mean Motion of the Satel.	^{s}	°	′
1641	8	29	17
16	5	29	54
May	6	9	14
9^d	6	23	12
9^h 40′	0	9	5
Mean Long. of the Satellite	4	10	42
Apocronion	10	22	00
Mean Anomaly	5	18	42
Equation ſubtract.	0	0	31
True Long. of the Satellite	4	10	11
Apogæon	4	29	50
Remainder	11	10	21
That is, before the *Apogæon*	0	19	39

Therefore it is 2$\frac{7}{10}$ Semidiameters of the Ring, and $\frac{9}{20}$ to the North, agreeing exactly with the Deſcription and Figure of M. *Huygens*.

I here call the Plane of this *Satellite*'s Orb, which hitherto I ſuppoſe the ſame with that of the *Ring*, *Saturn*'s *Equinoctial*; not that any Diſcovery hath been able to prove that the *Axis* of that *Globe* is at right Angles thereto, but becauſe it hath pleaſed M. *Huygens* to call it ſo, and likewiſe becauſe it is ſo nearly parallel to our *Globe*'s *Equinoctial*; nevertheleſs, to ſpeak my Opinion, I believe that the *Axis* is inclined, and that not a little, to the Plane of the *Ring*: for as the Reflection of the *Sun*'s Light from the *Ring* is a great Convenience to that Hemiſphere of *Saturn*, which beholds its illuminate Side; ſo the other Hemiſphere is very much incommoded by the Shadow of the *Ring*, which for many Months, and in ſome Parallels for ſeveral Years, occaſions a continual Night by the Interception of the *Sun*'s Beams, which is a Conſequence that demonſtratively follows the Poſition of the *Ring*, in the Plane of *Saturn*'s *Æquator*. Now this great Inconvenience would be in ſome meaſure relieved by the oblique Poſition of the *Axis*; for then the Parallels of Latitude interſecting the Plane of the *Ring*, many, and in moſt caſes all of them, might for ſome time in every diurnal Revolution of the Globe, free themſelves from this *Eclipſe*, which otherwiſe were ſufficient to render this Globe of *Saturn* unfit for any ſettled Habitation; but this is but Conjecture.

The other two *Satellites* of *Saturn* diſcovered by *S. Caſſini* at *Paris*, *An.* 1672. and 1673. I muſt confeſs I could never yet ſee ; I have been told that they diſappear for about $3\frac{2}{3}$ of *Saturn*'s Revolution, and were only to be ſeen when the *Anſæ* were very ſmall, it being ſuppoſed that the Light which proceeds from the *Anſæ*, when conſiderably opened, might hide theſe *Satellites*.

The Theory of the 5 Satellites of Saturn corrected; by M. Caſſini. n. 187. p. 299.

LXXXII. 1. The Diſtance of the firſt Satellite from the Center of *Saturn*, appears to me to be variable, and its Motion is ſenſibly unequal, ſwifter at this Time in the Weſtern Semicircle than in the Eaſtern. I have finally determin'd its mean Diſtance to be $\frac{39}{40}$ of the Diameter of *Saturn*'s Ring, its daily Motion to be $6^{s}\ 10^{\circ}\ 41'\ 31''$. So that if its Motion were equal, the Duration of its Conjunction with *Saturn*, that is to ſay, all the Time it takes up in paſſing through the Ring, would be $7^{h}\ 46'$. It appear'd greater to me by immediate Obſervations ; but it is to be obſerved, that as yet I have not been able to ſee this Satellite nearer to *Saturn* than a quarter part of an *Anſa*.

I have calculated the Epoque of its Motion, for the laſt of *December* 1685, at Noon, at the Meridian of *Paris*, to be in ♑ 24° 50′.

The Diſtance of the ſecond Satellite from the Center of *Saturn* has ſeem'd to me to be more uniform. I have determined it to be $1\frac{1}{4}$ Diameter of the Ring. Its Motion alſo ſeems to be more equal. I have calculated the daily Motion, and find it $4^{s}\ 11^{\circ}\ 31'\ 30''$. Therefore the Duration of its Conjunction ſhould be $8^{h}\ 36'$. Nor have I yet been able to ſee this Satellite nearer the Ring of *Saturn* than $\frac{1}{4}$ of an *Anſa*. As this Satellite was ſeen, for the greateſt part of its Time, within the Confines of the Diſtance of the firſt, to which it is equal in Magnitude, and like to it in Colour ; the Difficulty to diſtinguiſh one from the other was very great ; ſo that without a conſtant Application to Obſervations, and a great Multitude of Combinations, I could by no means perform it.

I have determin'd the Epoque of this Satellite, for the 31ſt of *December* 1685, at Noon, to be ♍ 9° 10′.

The Diſtance of the Third from the Center of *Saturn* ſeems to be $1\frac{3}{4}$ Diameters of the Ring. Its daily Motion is $2^{s}\ 18^{\circ}\ 41'\ 50''$. Therefore its Conjunction muſt continue 10 Hours. The Epoque of its Motion for the laſt Day of the Year 1685 at Noon is ♏ 9° 39′.

The Diſtance of the fourth Satellite from the Center of *Saturn* ſeems to be 4 Diameters of the Ring. Its daily Motion is $22^{\circ}\ 34'\ 38.''$ The Duration of its Conjunction is $15^{h}\ 6'$. The Epoque of its Motion in the ſame Time and Place as the others, in ♓ 16° 19′. On theſe Principles Tables and Ephemeris's may be conſtructed.

By . . . . . *ib.* p. 300.

2. The following Tables are calculated from theſe Elements, and reduced to the Meridian of *London*.

A Table of the Mean Motion of the inmoſt Satellite of *Saturn*, diſcover'd by Mr. *Caſſini*, *An.* 1686.

Current Year of Chriſt.	Epoques.		
	•	°	'
1681	♒	19	34
1685	♑	10	30
1686	♉	13	4
1687	♏	15	37
1688	♑	18	11
1689	♐	1	26
1701	♌	4	14

Months common Year.	*Mean Mot.*		
	•	°	'
Jan.	0	0	0
Feb.	5	1	27
March	3	0	49
April	8	2	16
May	6	23	2
June	11	24	29
July	10	15	15
Auguſt	3	16	42
Sept.	8	18	9
October	7	8	54
Nov.	0	10	21
Decemb.	11	1	7

Years.	*Mean Mot.*			*Days.*	*Mean Mot.*			*H. Min.*	*Mean Mot*			*Minutes*	*Mean Mot.*	
	•	°	'		•	°	'		•	'	"		°	'
1	4	2	34	1	6	10	42	1	0	7	57	31	4	6
2	8	5	7	2	0	21	23	2	0	15	53	32	4	14
3	0	7	41	3	7	2	5	3	0	23	50	33	4	22
4	10	20	56	4	1	12	46	4	0	31	47	34	4	30
5	2	23	30	5	7	23	28	5	0	39	44	35	4	38
6	6	26	4	6	2	4	9	6	0	47	40	36	4	46
7	10	28	38	7	8	14	15	7	0	54	37	37	4	54
8	9	11	52	8	2	25	32	8	1	3	34	38	5	2
9	1	14	26	9	9	6	14	9	1	11	31	39	5	10
10	5	17	00	10	3	16	55	10	1	19	28	40	5	18
11	9	19	34	11	9	27	36	11	1	27	24	41	5	26
12	8	2	48	12	4	8	18	12	1	35	21	42	5	34
13	0	5	22	13	10	19	00	13	1	43	18	43	5	42
14	4	7	56	14	4	29	42	14	1	51	15	44	5	50
15	8	10	29	15	11	10	23	15	1	59	11	45	5	58
16	6	23	43	16	5	21	4	16	2	7	8	46	6	5
17	10	26	17	17	0	1	46	17	2	15	5	47	6	13
18	2	28	51	18	6	12	28	18	2	23	1	48	6	21
19	7	1	25	19	0	23	9	19	2	30	58	49	6	29
20	5	14	39	20	7	3	50	20	2	38	55	50	6	37
				21	1	14	32	21	2	46	52	51	6	45
				22	7	25	13	22	2	54	49	52	6	53
				23	2	5	55	23	3	2	45	53	7	1
				24	8	16	36	24	3	10	42	54	7	9
				25	2	27	18	25	3	18	39	55	7	17
				26	9	7	59	26	3	26	35	56	7	25
				27	3	18	41	27	3	34	32	57	7	33
				28	9	29	22	28	3	42	28	58	7	41
				29	4	10	3	29	3	50	25	59	7	49
				30	10	20	45	30	3	58	22	60	7	57

In Leap-Year after *February* add one Day, and the Motion belonging to it.

A Table of the Mean Motion of *Saturn*'s Satellite the inmoſt but one, diſcover'd by Mr. *Caſſini*, *An.* 1686.

Current Year of Chriſt	Epoques. s	°	′
1681	♓	20	41
1685	♐	28	42
1686	♉	5	20
1687	♏	11	57
1688	♑	18	35
1689	♎	6	44
1701	♒	0	48

Month common Year.	Mean Mot. s	°	′
Jan.	0	0	0
Feb.	3	27	16
March.	6	19	58
April	10	17	15
May	10	3	0
June	2	0	16
July	1	16	1
Auguſt	5	13	18
Sept.	9	10	34
Octob.	8	26	19
Nov.	0	23	36
Dec.	0	9	21

Years.	Mean Mot. s	°	′
1	4	6	37
2	8	13	15
3	0	19	52
4	9	8	1
5	1	14	39
6	5	21	16
7	9	27	54
8	6	16	3
9	10	22	40
10	2	29	18
11	7	5	55
12	3	24	4
13	8	00	42
14	0	7	19
15	4	13	57
16	1	2	6
17	5	8	43
18	9	15	21
19	1	21	58
20	10	10	7

Days.	Mean Mot. s	°	′
1	4	11	31
2	8	23	3
3	1	4	34
4	5	16	6
5	9	27	37
6	2	9	9
7	6	20	40
8	11	2	12
9	3	13	43
10	7	25	15
11	0	6	46
12	4	18	18
13	8	29	49
14	1	11	21
15	5	22	52
16	10	4	24
17	2	15	55
18	6	27	27
19	11	8	58
20	3	20	30
21	8	2	1
22	0	13	33
23	4	25	4
24	9	6	36
25	1	18	07
26	5	29	39
27	10	11	10
28	2	22	42
29	7	4	13
30	11	15	45

H. Min.	Mean Mot. Sex. °	′	″	*Minutes*	Mean Mot. °	′
1	0	5	29	31	2	50
2	0	10	58	32	2	56
3	0	16	26	33	3	1
4	0	21	55	34	3	7
5	0	27	24	35	3	12
6	0	31	53	36	3	17
7	0	38	22	37	3	23
8	0	43	51	38	3	28
9	0	49	19	39	3	34
10	0	54	48	40	3	40
11	1	00	17	41	3	45
12	1	5	46	42	3	50
13	1	11	15	43	3	56
14	1	16	44	44	4	1
15	1	22	12	45	4	7
16	1	27	42	46	4	12
17	1	33	11	47	4	17
18	1	38	39	48	4	23
19	1	44	8	49	4	28
20	1	49	37	50	4	34
21	1	55	6	51	4	39
22	2	00	34	52	4	45
23	2	6	3	53	4	50
24	2	11	31	54	4	56
25	2	17	00	55	5	1
26	2	22	29	56	5	7
27	2	27	58	57	5	12
28	2	33	26	58	5	18
29	2	38	55	59	5	23
30	2	44	24	60	5	29

In Leap-Year after *February* add one Day, and the Motion belonging to it.

A Table of the Mean Motion of the middlemoſt Satellite of *Saturn*, diſcover'd by *Caſſini*, *An.* 1686.

Current Year of Chriſt.	*Epoques.* s	°	'
1661	♎	22	50
1681	♍	16	3
1685	♒	2	41
1686	♏	17	10
1687	♍	1	39
1688	♊	16	9
1689	♊	19	20
1701	♌	9	15
Months common Year.	*Mean Mot.* s	°	'
Jan.	0	0	0
Feb.	9	9	37
March	10	23	8
April.	8	2	45
May	2	23	40
June	0	3	17
July	6	24	12
Auguſt	4	3	49
Sept.	1	13	25
October	8	4	20
Nov.	5	13	57
Decemb.	0	4	52

Years.	*Mean Mot.* s	°	'	*Days.*	*Mean Mot.* s	°	'	*H. Min.*	*Mean Mot Sex.* °	'	"	*Minutes*	*Mean Mot.* °	'
1	9	14	29	1	2	18	42	1	0	3	17	31	1	41
2	6	28	58	2	5	7	24	2	0	6	33	32	1	45
3	4	13	27	3	7	26	5	3	0	9	50	33	1	48
4	4	16	38	4	10	14	47	4	0	13	7	34	1	52
5	2	1	8	5	1	3	29	5	0	16	24	35	1	55
6	11	15	37	6	3	22	11	6	0	19	40	36	1	58
7	9	00	6	7	6	10	53	7	0	22	57	37	2	1
8	9	3	17	8	8	29	35	8	0	26	14	38	2	5
9	6	17	46	9	11	18	16	9	0	29	31	39	2	8
10	4	02	15	10	2	6	58	10	0	32	47	40	2	11
11	1	16	45	11	4	25	40	11	0	36	4	41	2	14
12	1	19	55	12	7	14	22	12	0	39	21	42	2	18
13	11	4	24	13	10	3	4	13	0	32	38	43	2	21
14	8	18	54	14	0	21	46	14	0	45	55	44	2	24
15	6	3	23	15	3	10	27	15	0	49	11	45	2	28
16	6	6	34	16	5	29	9	16	0	52	28	46	2	31
17	3	21	3	17	8	17	51	17	0	55	45	47	2	34
18	1	5	32	18	11	6	33	18	0	59	1	48	2	37
19	10	20	1	19	1	25	15	19	1	2	18	49	2	40
20	10	23	12	20	4	13	57	20	1	5	35	50	2	44
				21	7	2	39	21	1	8	52	51	2	47
				22	9	21	20	22	1	12	8	52	2	50
				23	0	10	02	23	1	15	25	53	2	54
				24	2	28	44	24	1	18	42	54	2	57
				25	5	17	26	25	1	21	59	55	3	00
				26	8	6	8	26	1	25	15	56	3	4
				27	10	24	50	27	1	28	32	57	3	7
				28	1	13	32	28	1	31	49	58	3	10
				29	4	2	13	29	1	35	6	59	3	13
				30	6	20	55	30	1	38	22	60	3	17

In Leap-Year after *February* add one Day, and the Motion belonging to it.

A Tabl

A Table of the Mean Motion of *Saturn*'s Satellite the outmoſt but one, diſcover'd by *Huygens*, *An.* 1673.

Current Year of Chriſt.	*Epoques.* s	°	'
1641	♑	24	43
1661	♓	11	19
1681	♈	27	56
1685	♐	13	15
1686	♏	3	56
1687	♍	24	37
1688	♌	15	19
1689	♋	28	34
1701	♊	14	32

Month common Year.	*MeanMot.* s	°	'
Jan.	0	0	0
Feb.	11	9	54
March	8	12	3
April	7	21	57
May	6	9	16
June	5	19	10
July	4	6	29
Auguſt	3	16	22
Sept.	2	26	16
October	1	13	35
Nov.	0	23	29
Dec.	11	10	48

Years.	*MeanMot.* s	°	'	*Days.*	*MeanMot.* s	°	'	*H. Min.*	*MeanMot.* ° '	' "	*Minutes*	*MeanMot.* '	"
1	10	20	41	1	0	22	35	1	0	56	31	29	10
2	9	11	22	2	1	15	9	2	1	53	32	30	6
3	8	2	3	3	2	7	44	3	2	49	33	31	3
4	7	15	19	4	3	0	18	4	3	46	34	31	59
5	6	6	00	5	3	22	53	5	4	42	35	32	55
6	4	26	41	6	4	15	28	6	5	39	36	33	52
7	3	17	22	7	5	8	2	7	5	35	37	34	48
8	3	00	39	8	6	0	37	8	7	32	38	35	45
9	1	21	20	9	6	23	12	9	8	28	39	36	41
10	0	12	1	10	7	15	46	10	9	24	40	37	38
11	11	2	42	11	8	8	21	11	10	21	41	38	34
12	10	15	58	12	9	0	55	12	11	17	42	39	31
13	9	6	39	13	9	23	30	13	12	14	43	40	27
14	7	27	20	14	10	16	5	14	13	10	44	41	24
15	6	18	1	15	11	8	39	15	14	7	45	42	20
16	6	1	17	16	0	1	14	16	15	3	46	43	17
17	4	21	58	17	0	23	48	17	16	0	47	44	13
18	3	12	40	18	1	16	23	18	16	56	48	45	10
19	2	3	21	19	2	8	58	19	17	52	49	46	6
20	1	16	36	20	3	1	32	20	18	49	50	47	3
				21	3	24	7	21	19	45	51	47	59
				22	4	16	42	22	20	42	52	48	56
				23	5	9	16	23	21	38	53	49	52
				24	6	1	51	24	22	35	54	50	49
				25	6	24	25	25	23	31	55	51	45
				26	7	17	00	26	24	27	56	52	42
				27	8	9	35	27	25	24	57	53	38
				28	9	2	9	28	26	20	58	54	35
				29	9	24	44	29	27	17	59	55	31
				30	10	17	18	30	28	13	60	56	27

In Leap-Year after *February* add one Day, and the Motion belonging to it.

hath thence deduced the Motion of *Jupiter* about his *Axis* and that M. *Caſſini* at firſt oppoſed it: *But that Spot was obſerved in* England *a good while before.*

By *M.* Caſſini. n. 8. *p.* 143. n. 10. *p.* 172. n. 35. *p.* 687. n. 82. *p.* 4039.

4. There are two Sorts of *Spots* at certain times to be ſeen in the Diſk of *Jupiter*. One Sort are nothing but the Shadows of his Satellites; but the other have ſome reſemblance to thoſe that are ſeen in the *Moon*; and they are perhaps of the ſame Nature with thoſe that are called *Belts*. They move from the Eaſtern to the Weſtern Limb; their apparent Motion is unequal, and ſwifter near the Center than the Circumference; and they never are ſo well as when they approach to the Center, they being very narrow and almoſt imperceptible when they approach to the Circumference; which makes us believe, that they are flat and ſuperficial to *Jupiter*.

Among theſe *Spots* there is none ſo ſenſible, as that which is ſituated in the Northern-part of the *Southern-Belt*. Its Diameter is about the tenth part of that of *Jupiter*; and at the Time that its Center is neareſt to that of *Jupiter*, it is diſtant from it about the third part of the Semidiameter of that *Planet*.

M. *Caſſini*, after he had made many Obſervations of this *Spot* during the Summer of the Year 1665, found, that the *Period* of its apparent *Revolution* is of $9^h\ 56'$. He continued to obſerve it till the Beginning of 1666, when *Jupiter* approach'd to the Beams of the *Sun*; but after he was got free of the *Sun-Beams*, it was difficult to be diſcern'd. This giving grounds to think that it might be of the Nature of the *Spots of the Sun*, (which after appearing for a while, diſappear for ever) M. *Caſſini* ceaſed at length to obſerve them.

But *Jan.* 19. 1672. (*ſt. n.*) when he obſerv'd *Jupiter*, at 4 $\frac{3}{4}$h. in the Morning, he perceived in the ſame Place of his Diſk the Figure of the ſame *Spot*, adhering to the ſame *Southern-Belt*. It was already gone beyond the Moiety of this *Belt*, and he ſaw it advance little by little towards the Weſtern Limb, to which it ſeemed to be very near at $6\frac{1}{4}^h$.

By the Celerity of its Motion near the Center, and by the Place where he had begun to ſee it, he judged that it might have been in the midſt of the *Belt* at $4^h\ 35'$ in the Morning. And as he prepared himſelf to make Ephemerides of its Motion for that *Year* 1672, he perceived that in thoſe he made for the Year 1666, this *Spot* had been in the midſt of *Jupiter* the ſame Day, namely the 19th of *January* at the ſame Hour. So that in ſix Years, of which one is a *Biſſextile*, it is found to have made, in reſpect of the Earth, at leaſt 5294 Revolutions, each of $9^h\ 55'\ 85''$, compenſating one Revolution by another; and at moſt 5294 Revolutions of $9^h\ 55'\ 51''$; foraſmuch as he was aſſured of the Preciſeneſs of one *Mean Revolution* to one eighth of a Minute.

Until then he had never ſeen an immediate Return of this *Spot* after 9 Hours and 56 Minutes; becauſe it had not happened, that *Jupiter* after the Apparition of the *Spot* had ſtayed, in one and the ſame Night, long enough above the *Horizon*, at leaſt a ſufficient Height, to obſerve him with due Diſtinctneſs. He had only concluded the Time of this *Revolution* by Returns obſerved after about 20, 30, and 50 Hours; and he had more preciſely limited it by Obſervations more diſtant. But the Night after *March* 1, at $7\frac{1}{2}^h$ in the Evening, he ſaw this *Spot* in the midſt of the *Belt*; and the ſame

Night at 5^h 26′, in the Morning, he ſaw it again returned preciſely to the ſame Place. *Mar.* 3. He together with M. *Buot* and M. *Mariotte*, began to ſee at 8^h 4′ the *Spot* already ſomewhat removed from the Oriental Limb, but yet obſcure and ſmall. At 8^h 47′, they ſaw it very diſtinctly advancing towards the middle of the *Belt*. At 9^h 15′ 40″, until 9^h 8′, they ſaw it in the midſt of the *Belt*. At 9^h 15′, it was paſt the middle, and was come nearer to the Occidental Limb. And a little after the Heavens being over-caſt, he could obſerve it no further.

LXXXV. 1. *An.* 167½. *Feb.* 16. 7^h 44½. The Altitude of *Jupiter* was 18° 10′. I meaſured with a long Tube its Diſtance from a fix'd Star of the fourth Magnitude, and found it 16′ 33″. The Latitude of this Star was 1° 40′ northerly. Its Place according to me was ♍ 10° 7′ 16″: But according to *Street* it was 14° 3′ 54″. The Difference of the Altitudes of the Centers of *Jupiter* and the fix'd Star was 1′ 1″.

Places of Jupiter obſerv'd; by Mr. Flamſteed *at* Derby, n. 82. p. 4036.

17 *Feb.* 7^h 25′. After Noon, the Altitude of *Jupiter* being 15° 54′, he was diſtant from the fix'd Star 21′ 50″; the Difference of Altitudes was 8′ 40″.

18 *Feb.* 7^h 0′. The Diſtance of the fix'd Star from the Center of *Jupiter* was 28′ 15″. The Difference of Altitudes was about 15′ 29″. In each Obſervation the Planet was higher than the fix'd Star, from whence it always ſtood towards the Meridian.

Hence making a Calculation for every Day and Hour of the Obſervations; I found

	d.	h.		d.	h.	′	d.	h.	′
February	16	7	44½	17	7	25	18	7	0
	°	′	″	°	′	″	°	′	″
The Longitude of *Jupiter* from the fix'd Star in Antecedentia	0	9	16	0	17	22	0	25	12
Latitude from it towards the South	0	13	30	0	13	14	0	12	45
Therefore *Jupiter*'s Northern Lat. was	1	26	30	1	26	46	1	27	15
Its true place in *Virgo* according to me	13	58	0	13	49	54	13	42	4
according to *Street*	13	54	38	13	46	32	13	38	42

2. *March* 15 in the Evening, I undertook to obſerve the Diſtances and Poſitions of *Jupiter* from the Star ♌ 38, whoſe Latitude is 1° 20½ North; its Place according to *Street* ♍ 9° 54′ 0″; but according to me is ♍ 9° 57′ 20″. At 7^h 25′ afternoon, the Altitude of *Jupiter* was 32° 52′. The Diſtance of its Center from the fix'd Star was 33° 50′ The Difference of Altitudes was about 20′ 2′.

March 16. At 7^h 8′. The Altitude of the fix'd Star being 36°, the Diſtance of *Jupiter* from it was 27′ 7″. The leſſer Altitude 16′ 3″.

March 19, at 6^h 45′, the Altitude of *Jupiter* was 29° 35′. The fix'd Star was 2′ 24″ higher than the Planet; from which at 6^h 55′, its Diſtance was 10′ 21″.

At 7^h 11′, the fourth Satellite was diſtant 7′ 28″ from the fix'd Star. Tho' the Planet always appear'd to be higher than the fix'd Star, yet it was really lower; afterwards it ſeem'd to be lower, but was really higher.

March 20. The Micrometer being better adapted to the taking of the Differences of Altitudes, I made the following Obſervations, which I take to be very exact.

Alt. of Jupiter.		*Times.*				
°	′	h.	′		′	″
30	0	6	44	*Jupiter*'s Center was really higher than the Star ——	2	13
30	47	6	51⅕	The ſame Difference of Altitudes taken over again	2	14
32	0	6	59	*Jupiter*'s Center was diſtant from the fix'd Star ——	7	0
38	30	7	54¾	The Center of *Jupiter* really higher than the fixt Star	3	14
40	50	8	18⅔	The Difference of Altitudes taken again ——	3	42
				Then the Diſtance of the Centers was ——	7	5

For obtaining the Place of *Jupiter* from theſe Obſervations, I computed

	h.	′	″	h.	′	″
	6	51	30	8	18	40
	°	′	″	°	′	″
The Angles of the Vertical Circle with the Ecliptick	35	39	—	46	15	—
The Diſtance of *Jupiter* from the fix'd Star was ——	0	7	0	0	7	0
The Difference of Altitudes ——	0	2	14	0	3	42
Therefore *Jupiter* was in conſequence of the fix'd Star	0	2	3	0	1	44
With a greater Latitude ——	0	6	42	0	6	47
So that the true Latitude of *Jupiter* was ——	1	27	12	1	27	17
The true Place ♍ according to *Street* ——	9	56	3	9	55	44
according to me ——	9	59	23	9	59	4

N. 86, p. 5037.

May	*Time.*		*Alt. of* Jupiter.					
d.	h.	′	°	′		°	′	″
24	10	00	14	10	The Center of *Jupiter* was lower than the ſaid Star ♌ 38 ——	0	7	46
					From which I took its Diſtance firſt ——	0	20	00
					Afterwards ——	0	19	54
26	8	46	33	30	Diſt. of the Cent. of *Jup.* from the firſt Star	0	10	4
Sky	9	00	31	50	The fix'd Star was higher than *Jupiter* ——	0	6	30
very	9	20	29	10	The Difference of Altitudes was ——	0	6	38
clear.	9	33	27	20	Diff. of Azimuth of *Jup.* and the fix'd Star	0	7	19
	9	36	27	00	Diſt. of *Jup.* from the fix'd Star taken again	0	10	

	h.	'	°	'		°	'	"	
27. Wind ſome-times moved the Tube.	8	59	31	24	Diſt. of the Center of ♃ from the fix'd Star	0	6	2	*
	9	7	30	30	The Difference of Azimuths—	0	1	50	
	9	16	29	13	The ſame Difference again	0	1	52	*
	9	23	28	10	Diſt. of the Center of *Jupiter* from the Star	0	6	1	
28. Wind blew hard, and ſhook the Tele-ſcope.	8	57	31	8	Again	0	6	2	*
	9	19	28	10	Diff. of the Azimuths of the Center, &c.	0	3	29	
	9	31	26	30	The ſame Difference again	0	3	32	
	9	34	26	8	The Diſtance again	0	6	7	
30. Sky was pretty clear.	9	36	26	15	*Jupiter*'s Diſtance from the fix'd Star	0	15	38	
	This and the foregoing Night *Jupiter* was in conſequence of the fix'd Star, but always before in Antecedence.								

Thoſe Obſervations which are mark'd with an Aſteriſk I look upon as the moſt certain. I imagine the Differences of Azimuths, which were obſerved on the 27 and 28 Days, were taken rather too ſmall, becauſe of the ſhaking of the Tube; therefore ſince I could not meaſure them as accurately as I deſired, I took care not to take them too large.

To obtain *Jupiter*'s Place from three Obſervations, I have computed

	d.	h.	'	d.	h.	'	d.	h.	"
May	26	0	0	27	9	7	28	9	19
	°	'	"	°	'	"	°	'	"
The parallactical Angles, or of the vertical Circle with the Ecliptick	80	47	—	79	49	—	78	36	—
The obſerved Diſt. of *Jup.* from the fix'd Star	9	10	4	0	6	2	0	6	3
The Difference of Altitudes	0	6	30	Azy.	1	52	Azy.	3	30
Therefore the Angle of Poſition	31	1		62	38	0	66	3	0
And the Planet in Antecedence of the fix'd Star	0	8	38	0	2	51	conſ.	2	27
With leſſer Northern Latitude	0	5	11	0	5	19	0	5	32
Tycho's Northern Latitude of the fix'd Star	1	20	30	1	20	30	1	20	30
The Place of the Star ♍ according to *Street*	9	54	9	9	54	9	9	54	9
according to me	9	57	30	9	57	30	9	57	30
Therefore *Jupiter*'s Place according to *Street*	9	45	31	9	51	18	9	56	36
according to me	9	48	52	9	54	39	9	59	57
With Northern Latitude	1	15	19	1	15	11	1	14	58

n. 94. p. 6033.

3. *An.* 1673. *March* 13. in the Evening. *Jupiter* in his *Aphelion*, going to his *Acronical* Phaſis, having paſs'd a little the Northern Limit of his Orbit, proceeded retrograde towards the 9th Star of ♍ of the fourth Magnitude, from which (his Altitude being about 6 Degrees) I took the Diſtance of his remoteſt Limb, with a ſeven-foot Tube, and *Townley*'s Micrometer, 4560= 52′ 34″.

March 17. About half an Hour after *Jupiter* aroſe, with the ſame Tube I again took the Diſtance of his remoteſt Limb from the fix'd Star, 2073= 23′ 54″.

March 20. I made the following Obſervations; the firſt with a ſhorter Tube of 85 Inches, the reſt with a longer of 164 Inches.

	Alt. of fixt Star.		*Hour by Calc.*				*Limb*		*Cent.*	
	°	′	h.	′			′	″	′	″
1	6	0	7	14	Diſt. of the remoter Limb of ♃ from the fixt Star	850	9	48	9	24
2	—	—	—	—	The ſame Diſtance taken with the longer Tube	1650	9	52	9	28
3	14	40	8	16	Lower Limb of ♃ more depreſs'd than the fixt *	784	4	41	4	17
4	15	40	8	23	The ſame Difference of Altitude repeated ——	786	4	41	4	17
5	—	—	—	—	The Diameter of *Jupiter* ——————	135	0	48		
6	16	25	8	26	The Diſtance of the Limb taken again ———	1665	9	57	9	33
7	—	—	—	—	Once more —— —— ———	1658	9	54	9	30
8	19	00	8	50	Diff. of Alt. of *Jupiter*'s Limb and the fixt Star	138	5	00	4	36

March 26. in the Evening. The Altitude of *Jupiter* being 15° 50′, I meaſured the Diſtance of his remoter Limb from the fix'd Star with the ſame leſſer Tube, 4205=48′ 30″.

For finding the Place of the Planet from theſe Obſervations, I made the following Calculations.

Mar.	h. 20	h. 8	′ 16		h. — 8	′ 50
	°	′	″	°	′	″
The Parallactical Angle was—— —— ——	34	44	0	37	30	0
The Center of *Jupiter* was diſtant from the fix'd Star	0	9	38	0	9	30
The Difference of Altitudes was ————	0	4	17	0	4	36
Therefore the Angle of Poſition was ———	80	6	0	78	21	0
And *Jupiter* in Antecedence of the fix'd Star ———	0	1	38	0	1	55
With leſſer Latitude ————	0	9	19½	1	9	18

According to me the Place of the fix'd Star was ♎ 13° 37′ 11″ (taking the annual Motion at 50″,) which the Author of the *Caroline* Tables takes to be 13° 33′ 47″. Its North Latitude 1° 45′. Therefore according to me the true Place of *Jupiter* will be

$$\left.\begin{array}{llllll} 8^{h} & 16' & ♎ & 13° & 35' & 33'' \\ 8 & 50 & ♎ & 13 & 35 & 16 \end{array}\right\}\begin{array}{l}\text{its true}\\ \text{Latitude}\end{array}\left\{\begin{array}{lll} 1° & 35' & 40''\tfrac{1}{2} \\ 1 & 35 & 42 \end{array}\right.$$

At

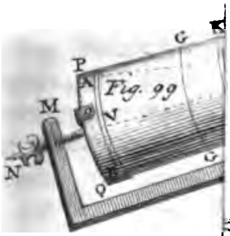
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Fig. 99
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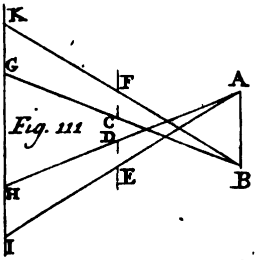
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Fig. 111
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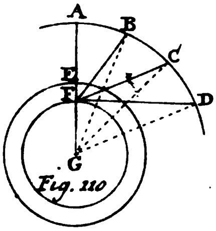
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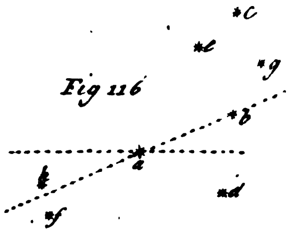
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Fig 116
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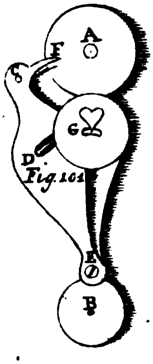
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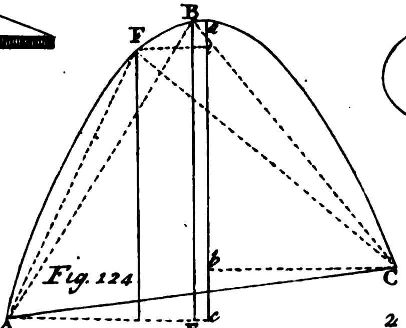
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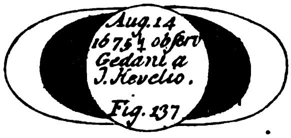
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Fig. 137

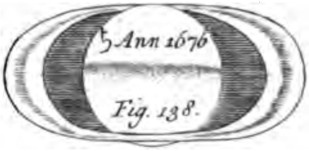
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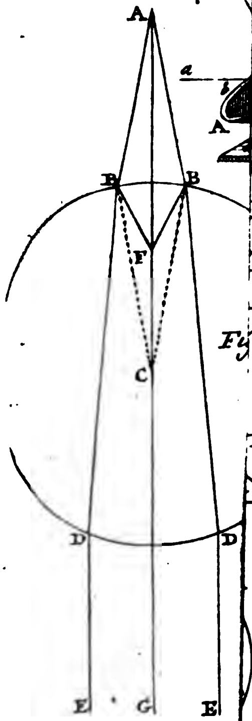
Fig. 138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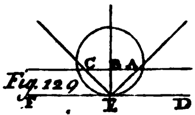
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Fig. 129
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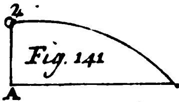
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Fig. 141
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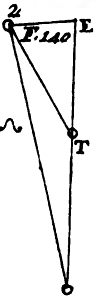
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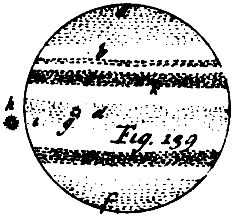
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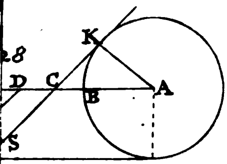
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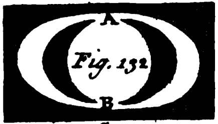
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Fig. 132
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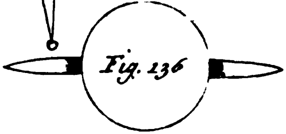
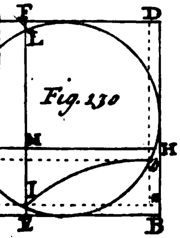


Fig. 136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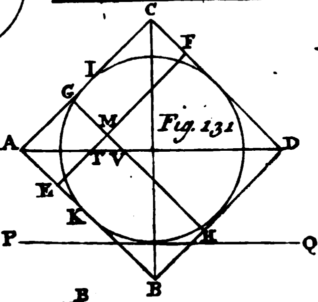
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Fig. 131
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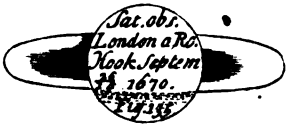
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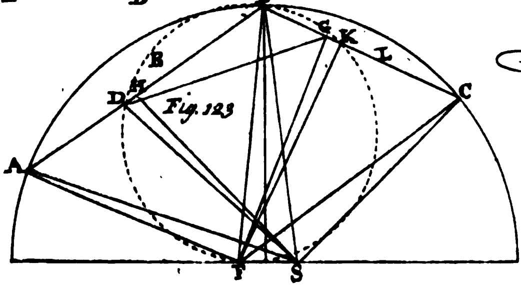
Fig. 135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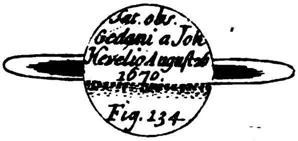
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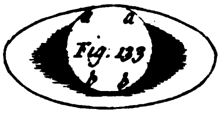
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Fig. 133
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But if we allow the Place of the fix'd Star to be as by the *Caroline* Tables, the Place of *Jupiter* will come out ♎ 13° 32′ 09″.

Having thus the Place and Latitude of the Planet, we ſhall thence endeavour to derive the Inclination of *Jupiter's* Orbit to the Plain of the Earth's Orbit.

For finding of which the mutual Diſtances of *Jupiter* and the Earth, together with the Place of the Sun, are required; which we may ſafely take from any approved Tables. I generally uſe the *Caroline* Tables, which I have found to be more accurate and more eaſy than any others. From theſe I have taken at 8ʰ 16′ after Noon,

The true Place of the Sun————	♈ 10° 40′ 18″
His Diſtance from the Earth ———	100084
The Diſtance of *Jupiter* from the Sun —	544921
————from the Earth ———	444952

Now in the annexed Figure let S be the Sun, T the Earth, ♃ the Planet, S E a Radius of the Ecliptick continued to the Orbit of *Jupiter*, and the Angle ♃ T S the Latitude of the Planet ſeen from the Earth 1° 35′ 40½″. *Fig.* 140.

In the Triangle ♃ S T, having given the Angle ♃ T S the Complement of the obſerved Latitude to a Circle, ♃ S and ♃ T the Diſtances of the Planet from the Sun and the Earth, being found above, the Angle ♃ S E will be had, the Latitude or Inclination of the Planet as ſeen from the Sun, 1° 18′ 7″.

The *Geocentrick* Place of *Jupiter* was ♎ 13° 35′ 33″ Therefore from the given Diſtances of *Jupiter* and the Earth from the Sun, the *Heliocentrick* Place of the Planet will be found ♎ 13° 3′ 33″; from whence ſeverally ſubtracting thoſe Places of the Node, which have been aſſumed by the Authors mention'd in the following Table, the Arguments of Latitude here annexed will come out. Whence it may be obſerved, that none of them makes *Jupiter* more advanced from his Limit than 6° 29′ 59″, or leſs than 3° 58′ 59″: Which how great ſoever the Difference may ſeem, in inveſtigating the greateſt Inclination of the Orbit, cannot produce a greater Error than 23 Seconds.

Authors.	*Places of* ☊				*Arguments of Latitude.*			
	s.	°	′	″	s.	°	′	″
Kepler.	3	6	33	37	3	6	29	56
Street.	3	6	33	47	3	6	29	46
Wing.	3	7	11	39	3	5	21	54
Ricciolus.	3	7	18	00	3	5	45	33
Caſſini.	3	8	45	00	3	4	18	33
Bullialdus.	3	9	4	34	3	3	58	59

That Place of the Node which Mr. *Caſſini* has made choice of, though it ſeems a little more advanced than it ought, yet for divers other Reaſons I like it beſt. Therefore in the Triangle ♃ A ☊, taking ☊ A the Argument of Latitude 94° 18′ 33″, and ♃ A the Inclination 1° 18′ 7″; the Angle of the In- *Fig.* 141.

clination of the Plain of *Jupiter*'s Orbit to the Ecliptick will be found 1 °1 20″, which *Kepler* makes 1° 19′ 0″, *Street* 1° 20′, *Bullialdus* and *Wing* 1° 21′ 48″; all of them ſomething greater than it ſhould be.

Almag. Nov. Par. 1. *p.* 710.

And that the Inclination is ſo much, at leaſt not greater than this, not only the Obſervations of laſt Night, but alſo thoſe of the Months *February*, *March*, and *May* of laſt Year, prevail with me to believe. Yet in the mean time I muſt acknowledge, that it may be proved to be greater, that is, 1° 20′ 20″, from that Tranſit of *Jupiter* near the 8th of ♍, *May* 29 and 30 *An.* 1649, obſerved at St. *Julian*, at *Bologna* and *Majorca*, by thoſe very learned Men *Ricciolus* and *Mutus*. Which ſeems to inſinuate to us, that ſince the Inclinations of the Orbits are eſteem'd as invariable by all, there muſt be ſome Error in thoſe Latitudes aſſigned by *Tycho* to ſome of theſe fix'd Stars. And this will hinder us from exactly determining the Quantity of this Inclination, till thoſe Latitudes are duly reſtored. Yet this I dare affirm, becauſe thoſe Latitudes of the fix'd Stars are alſo found to be immutable; that the Angle of the greateſt Inclination of the Plain of *Jupiter*'s Orbit to the Ecliptick, is leſs than 26′ 40″; or than the Latitude of the 9th Star of ♍ of the Fourth Magnitude, which by *Tycho* is call'd the laſt of the Four in the left Wing of *Virgo*. Whenever therefore this is given correct, the Inclination aforeſaid will be known alſo.

The Conjunctions of Saturn *and* Jupiter, An. 1682, *and* 1683. *at* Greenwich; *by Mr.* Flamſteed, n. 149. *p.* 241.

LXXXVI. Examining our antient *Ephemerides* I do not find that three *Conjunctions* of *Saturn* and *Jupiter* have ever happened in one Year's Space, ſince they were firſt in uſe to this preſent. Thoſe of *Moletius* calculated from the *Alphonſine Tables* indeed make three in the Space of eight Months betwixt *Auguſt* 1563, and *April* 1564, incluſive; but the *Ephemerides* of *Statius* calculated from the *Prutenick*, make only one, on the 26 of *Auguſt*, of which *Junctinus* gives us the following Obſervation in the Preface to his *Aſtronomical Tables*, *An.* 1563. *Aug.* 24. 14^{h} 36′ *p. m.* On the Northern Side *Jupiter* in a manner cover'd *Saturn*, who was on the Southern Side; now each of theſe Stars was found to be at the end of the 28th Degree of *Cancer*. *Riccioli* hence concludes, that the Planet ♃ covered ſome part of *Saturn* at this Time. But without Reaſon, for the Words *quaſi cooperiebat* intimate not that the one did corporally cover the other, but rather that there was ſome ſmall Interval betwixt them. The *Caroline Tables* make the viſible Latitude of *Saturn* now, 11′ 45″, of *Jupiter* 20′ 10″, both North; the Conjunction being ſome few Days paſt: but becauſe their Latitudes alter ſlowly, we may hence conclude the Difference 8′ 25″, to have been nearly their Diſtance at that Time. Theſe Tables being grounded on the *Tychonick* Obſervations, made within leſs than 40 Years after, and ſhewing the Latitudes of the *Planets* well at this Time near 100 Years later, we may conclude to have anſwer'd them as well then; and if we conſider how ſmall a Space the Diſtance of 8½ Minutes appears to the naked Eye in the Heavens, eſpecially betwixt two ſuch bright Planets as *Saturn* and *Jupiter* are, that the *Caroline* Diſtance agrees very well with the Words of *Junctinus*, and that *Riccioli* was miſtaken.

Their next *Conjunction* according to *Maginus*'s *Ephemerides* founded on the *Prutenick* Numbers, was *April* 29, 1583. in 21 *Deg.* of ♓, the *Sun* being then in 17 *Deg.* of ♉, ſo that that Planet's Riſing before him in *Signs* of ſhort Aſcenſion and with South Latitude, this *Congreſs* could not be obſerved by the Noble *Tycho* who was mindful of it, as appears by this Note in Page 55 of his *Hiſtoria Cœleſtis*. *May* 30. before Noon. As ſoon as ever we ſaw *Saturn* after the Conjunction, theſe Diſtances between *Jupiter* and *Saturn* were taken by a Radius ;

1ʰ	47′	3°	24′
1	50	3	24

The ſame *Ephemerides* ſhew the next *Conjunction* of *Saturn* and *Jupiter* 1603, *December* 14 at Noon in 9° 36′ of ♐ ; but the ingenious *Kepler* and our Sir *Chr. Heydon* found it by Obſervation 7 Days ſooner, or the Day of the ſame Month in the Morning, in near 8 *Degrees* of ♐, the Planets being but then newly emerged from the Rays of the *Sun*.

The *Ephemerides* of the Learned *Kepler*, calculated from his own *Rudolphine Tables*, make the next *Conjunction* 1623, betwixt the 7th and 8th of *July* in 6° 46′, of ♌, the Planet of *Saturn* being then only 4 Minutes to the North of *Jupiter* ; but this firſt *Conjunction* in the *Fiery Trigon* happening under the *Sun*'s Beams was not obſervable.

By the ſame *Tables*, and *Ephemerides* of *Eichſtade* calculated from them, theſe *Planets* met again in the 25° of ♓, betwixt the 15 and 16 of *February* 1643, with a Degree Difference of Latitude.

By the joint Conſent of *Eichſtade*'s and our *Wing*'s *Ephemerides*, the ſame Planets were in *Conjunction* again 1663, on the 10 of *October* at Noon in 13° 30′ of ♐, with one Degree of Difference of Latitude ; this *Conjunction* was obſervable after *Sun-ſet* in our Latitude, but I hear not that any one obſerved it.

In every one of theſe Years there happened only one *Conjunction* of the two *Superiors*, nor is it poſſible that there ſhould be more, except the *Heliocentrical Conjunction* fall near the *Oppoſition* of the *Sun* ; for then there may be three, two *Direct*, and one *Retrograde*, as has been within the Space of 7 Months, betwixt *October* and *May* laſt incluſive, of which the true Times are determined from the following Obſervations.

An. 1682.							
	d.				°	′	″
Oct.	5	17	51	Betwixt the Centers of *Sat.* and *Jup.*	0	34	54
		17	54	——————— rep.	0	34	48
	12	13	49	Betwixt their Centers———————	0	16	2
		13	54	——————— rep.	0	16	4
		14	3	Betwixt their next Limbs ———	0	15	22

Oct. 17.

	d.	h.	′		°	′	″
Oct.	17	14	10	Betwixt their Centers ———————	0	20	9
		14	17	——————— ——————— rep.	0	20	12
		14	21	——————— ——————— again	0	20	14
		14	25	Betwixt their next Limbs ———————	0	19	44
		14	33	Betwixt their remoter Limbs ———————	0	20	37
		15	9	*Saturn* from the *Heel* of *Caſtor* ———————	48	32	25
		15	14	*Jupiter* from the ſame *Star*——— ———	48	45	5
		15	17	——————— ———————	48	45	20
		15	20	——————— ——————— rep.	48	32	20
		15	50	*Saturn* from the ſame *Star* again———	0	20	30
———	———	———	———	Betwixt their Centers again———————	———	———	———
	19	15	41	Betwixt their remote Limbs———————	0	26	2
		15	45	Their Center ———————	0	25	37
		15	47	Their next Limbs ———————	0	25	11
	22	18	25	Betwixt their Centers———————	0	33	19
		18	29	——————— ——————— rep.	0	33	26

The *Diſtances* betwixt the *Planets* were meaſured with the *Micrometer* and 16 Foot Glaſs, from the *Fixt Stars* with the *Sextant* : thoſe of the 12th by my Aſſiſtant, the reſt myſelf.

On the 22 Day, the Planet *Jupiter* was in Conſequence of *Saturn* ſomething leſs diſtant from him than he had been obſerved on the 5th Day near the ſame Hour. Hence the middle Time, betwixt theſe Obſervations is pointed out for the Time of their true *Conjunction* ; but to determine it more accurately, I ſhall examine the Obſervations made with the Sextant on the ſeventeenth Day, which being neareſt the Time are moſt proper for this Purpoſe.

The correct Longitude of the *Heel* of *Caſtor* now ♋ 0° 50′ 42″, its Latitude 51′ 40″, South. The Latitude of *Saturn*, by the *Caroline Tables* 96′ 20″, of *Jupiter* 41′ 30″, both North.

By the aſſumed Latitude of *Saturn* 56′ 20″, and his Diſtance from the *Heel* of *Caſtor* obſerved and corrected 48° 32′ 30″, I found their Difference of Longitude 48° 30′ 37″; therefore *Saturn* in ♌ 19° 21′ 19″.

By the Latitude of *Jupiter* aſſumed 41′ 30″, and his Diſtance from the *Star* 48° 45′ 20″, their Difference of Longitude 48° 43′ 56″, and *Jupiter*'s Place in ♌, 19° 34′ 39″.

Hence *Jupiter*'s Place in conſequence of *Saturn*'s 13′ 20″, with which and the Diſtance of their Centers obſerved the ſame Night 20′ 12″, I find the true Difference of their Latitudes 15′ 20″; but half a Minute different from what I aſſumed it on the Authority of the *Tables*.

The apparent Motion of *Jupiter* from the 14 to the 18 Day of *October* by an *Ephemeris* exactly calculated and made agreeable to theſe Obſervations, is 29′ 16″, of *Saturn* 15′ 01″, both direct: hence the Motion of *Jupiter* from *Saturn* in four Days is 14′ 15″. I ſay therefore as 4 Days Motion, or 14′ 15″, 15″, is to 4 Days, or 96 Hours; ſo is 13′ 20″ (which *Jupiter* is paſt the *Conjunction* of *Saturn*) to 90 Hours or 3 Days 18 Hours; the Time interlapſed ſince the *Conjunction*; which taken from the 17 Day 15 Hours, the Time of my Obſervation, gives the true Time of the *Conjunction* of the two Planets on the 13 Day, one and twenty Hours after Noon, or according to the common Account, the 14 Day, at 9 a Clock in the Morning.

At which Time *Saturn* is with *Jupiter* in ♌ 19° 07′$\frac{1}{5}$, with 15′$\frac{1}{3}$ more Northern Latitude.

The *Acta Eruditorum Lipſienſia*, p. 366. make this *Conjunction* to have happened the ſame Day in the ſame Longitude with the *Eleventh Star of Leo*; whoſe Place they ſtate in ♌ 19° 04′ Lat. 0′ 16″ N. with 14 Minutes Difference of Latitudes betwixt the two *Planets*. But their Obſervation ſeems to have been made only by the Judgment of the bare Eye without an Inſtrument, which conſider'd, I wonder not that it differs at all, but rather that the Difference is ſo ſmall from this Determination.

On the 19 of *January* following, viewing then the Planets both *Retrograde* with the 16 Foot Glaſs, I found them approached within a meaſurable Diſtance of each other.

An. 1683.						
d.	h.	′		°	′	″
Jan. 19	6	41	Betwixt their Centers —— ——	0	33	28
	6	45	—— —— —— —— —— rep.	0	33	24
	6	49	Betwixt their remote Limbs —— —	0	33	52
26	6	3	Betwixt their Centers — — — —	0	15	8
Both being	6	7	— — — — — — — rep.	0	15	6
in ☍ *to the*	7	00	Betwixt their remote Limbs — ——	0	15	31
Sun.	7	8	By *T. Smith* — — — — rep.	0	15	9
	7	12	Betwixt their Centers —— ——	0	15	5
	7	14	——————————	0	15	2
	7	17	Betwixt their next Limbs — — —	0	14	29
	7	20	— — — — — — — rep.	0	14	31
	7	21	—— —— —— again	0	14	26
	9	24	*Jupiter* from the *Heel* of *Caſtor* ——	46	18	10
	9	26	—— —— —— rep.	46	18	5
	9	28	*Saturn* from the ſaid *Heel* — ——	46	8	50
	9	30	—— —— —— rep.	46	9	55
	9	37	*Jupiter* from the bright Star of the *Lion's Head* E. — —— ——	8	42	5
	9	39	—— —— —— rep.	8	42	5
	9	40½	*Saturn* from the ſame Star —— ——	8	29	35
	9	42½	—— —— —— rep.	8	29	40

	h. ′		° ′ ″
	9 48	*Jupiter* from the *Lion's Heart* ———	8 18 00
	9 50	——— ——— rep.	8 17 55
	9 52	*Saturn* from the ſame ——— ———	8 29 35
	9 54	——— ——— ——— rep.	8 29 35
	9 59	The *Lion's Heart* from *E* in the *Head* ——— ——— ———	12 58 50
	10 3	The *Heel* of *Caſtor* from the *Lion's Heart* ——— ——— ———	54 34 20
	10 8½	The *Heel* of *Caſtor* from *E* ♌ ———	46 24 45
Jan. 30.	5 28	Betwixt their Centers — ———	0 11 36
	5 30	——— ——— — rep.	0 11 33
	5 34	Betwixt their remote Limbs ———	0 11 58
	5 36	——— ——— ——— rep.	0 12 1
	5 38	Betwixt their next Limbs ———	0 11 1
	5 41	——— ——— — rep.	0 11 00
Feb. 7	7 37	Between their Centers ——— ———	0 28 35
	7 40	——— ——— ——— rep.	0 28 34

From Obſervations formerly made, I have determined the true Places and Latitudes, to this preſent Time, of,

	s ° ′ ″		° ′ ″	
The Heel of Caſtor ———	♋ 0 51 10	*Lat.*	0 51 40	*South.*
Bright * in the *Lion's Head*, E —	♌ 16 15 27		9 41 07	*North.*
Lion's Heart ——— ———	♌ 25 24 45		0 26 20	*North.*

And from the above recited Meaſures, the true Diſtances of the *Planets* from theſe Stars, *January* the 26th at 9ʰ 40′ *p. m.* as follows,

	°	′	″
Saturn from the *Heel* of *Caſtor* ———	46	09	00
Jupiter from the ſame ———	46	18	10
Saturn from the *Lion's Heart* ———	8	29	40
Jupiter from the ſame ——— — —	8	18	00
Saturn from the bright * in the *Lion's Head* E —	8	29	40
Jupiter from the ſame ——— ———	8	42	10

Whence I collect the true Places at this Time.

	s	°	′	″		°	′	″
Of *Saturn*	♌	16	57	10	*Lat.*	1	13	10
Of *Jupiter*	♌	17	07	10	*Lat.*	1	01	30
Difference of *Long.* ———		10	00		of *Lat.*		11	40

The *Retrograde* Motion of *Jupiter* from *Saturn* in 4 Days, betwixt the Twenty-ſixth and thirtieth of this Month, by my correct *Ephemeris* is 12′ 15″. I ſay therefore as 12′ 15″, is to 4 Days or 96 Hours; ſo is 10′ 00″ (the Difference of the *Planets* preſent Longitudes) to 78 Hours, or three Days ſix Hours; which therefore added to the Time of that Obſervation *Jan.* 26ᵈ $9^h\frac{2}{3}$, gives the true Time of the *Conjunction* Jan. 29ᵈ 16ʰ, or according to the common Account *Jan.* 30 at 4ʰ in the Morning. At which Time both the *Planets* are in ♌ 16° $51'\frac{1}{3}$, with $11'\frac{1}{2}$ Difference of Latitude or Diſtance from each other; which is further confirmed by the meaſured Diſtances of the *Planets* on the 30*th* at Night before recited.

On the 26*th* Day at 9ʰ 40′ the *Sun*'s true Place was by my Tables in ♒ 17° $21'\frac{1}{2}$, ſo that he was now about $\frac{1}{3}$ of a Degree paſt their Oppoſition.

Towards the latter end of the following *April*, the Planet *Jupiter* began to approach *Saturn* again, both being now direct; the 28. at Night with the 16 Foot Glaſs and *Micrometer* I meaſured the Diſtances.

An. 1683.							
	d.	h.	′		°	′	″
Apr.	28.	10	21	Betwixt their Centers ——	0	32	35
		10	23	—— —— rep.	0	32	33
		10	24	Betwixt their next Limbs ——	0	32	4
		10	26	—— —— rep.	0	32	2
				Betwixt their remote Limbs but not accurately, Clouds interpoſing.			
		10	29	—— —— ——	0	33	22
May	7.	8	59	*Jupiter* from the *Lion's Heart* ——	10	59	00
		9	1	—— —— rep.	10	59	00
		9	$3\frac{1}{2}$	*Saturn* from the *Lion's Heart* ——	10	58	50
		9	5	—— —— — rep.	10	58	50
		9	11	*Jupiter* from *E* in the *Lion's Head*	8	55	35
		9	15	—— —— — rep.	8	55	40
		9	17	*Saturn* from the ſame Star —	8	39	40
		9	18	— — — rep.	8	39	40
		9	30	Betwixt their Centers —	0	15	38
		9	33	— — — — rep	0	15	37
		9	35	Betwixt their next Limbs	0	15	3
		9	36	—— — rep.	0	15	00
		9	40	Betwixt their remote Limbs	0	16	2
		9	42	—— —— rep.	0	15	58
		10	20	*Jupiter* from *B* in ♍ —	38	11	45
		10	23	— — rep.	38	11	45
		10	26	*Saturn* from the ſame Star —	38	10	55
		10	28	— — rep.	38	10	45
May	11.	9	28	Betwixt their Centers —	00	20	2
			31	— — rep.	00	20	2
	16.	9	22	Betwixt their Centers ——	00	34	4

From theſe Obſervations I ſtate the Diſtances of the *Planets* from the fixed Stars *May* 7th at 9h 5′ *p. m.* as follows.

	°	′	″
Saturn from the *Lion's Heart* ——— —	10	58	50
Jupiter from the ſame ——— ———	10	59	00
Saturn from E in the *Lion's Head* ———	8	39	40
Jupiter from the ſame ——— ———	8	56	35

		°	′	″		°	′	″	
Hence the true Longitude of *Sat.*	♌	14	27	42.	*Lat.*	1	12	46.	*North.*
of *Jupiter*	♌	14	26	37.	*Lat.*	0	56	43.	*North.*
Difference of Longitude			1	4.	*Lat.*	0	16	3.	

The Difference of Latitudes ſomething exceeds the Diſtance meaſured with the *Micrometer*, by reaſon that the Wind then ſhaking the *Sextant* permitted us not to be ſo exact as uſually; but the Difference, being leſs than half a Minute, I eſteem inconſiderable.

The Diurnal Motion of *Jupiter* from *Saturn* was now 3′ 15″. It holds therefore, as 3′ 15″ (one day's Motion,) is to one Day or 24 Hours; ſo 1′ 4″ (the Diſtance of *Jupiter* from the ☌ with *Saturn*) to 8 Hours, the Interval betwixt the Obſervation and following *Conjunction*, which was therefore 17h after Noon, or according to the vulgar reckoning, *May* 8. at 5 a Clock in the Morning: At which Time the true Place of the *Planet* is ♌ 14° 28′¼, the Difference of their Latitudes, 15′ 40″. *Saturn* being ſo much more Northerly than *Jupiter*.

In all beſt eſteemed *Aſtronomical Tables* extant, the Mean Motions of the *Planet Saturn* are too ſwift, of *Jupiter* too ſlow conſiderably: Hence it came to paſs that they made the *Direct Conjunctions* ſome Days later, the *Retrograde* earlier than they were found by Obſervation.

At Dantzick; *by M.* Hevelius. *n.* 143. *p.* 18. & n. 151. *p.* 326.

2. *Octob.* 26. New Style, at 1h 40′ in the Morning, I happen'd to take the Situation of *Jupiter* and *Saturn* with my Tube and Micrometer, as well as I could wiſh. At the ſame time a certain conſpicuous fix'd Star, (which is worth obſerving) was very near the ſaid Planets. *Jupiter* then offer'd himſelf to me with three of his Companions; and perhaps the fourth was preſent, but becauſe of the little Clouds was not to be ſeen. *Saturn* was diſtant from *Jupiter* 16′ 44″; *Jupiter* was diſtant from the Star (in the right Shoulder of ♌, if I am not miſtaken,) 27′ 55″; and again *Saturn* was diſtant from the Star 38′ 1″. This Star was now according to our Catalogue in 19° 2′ 9″ of ♌, with Northern Latitude 0° 20′ 45″. On *Octob.* 30. at 5h in the Morning, the Diſtance of *Saturn* and *Jupiter* was 25′ 5″; whence we may ſurely conclude, that the Conjunction was celebrated ſeveral Days ſooner than *Nov.* 3. when the Ephemerides and the Calculation exhibit it.

This the following Obſervations will ſtill farther evince. For if the Conjunction was near, the Diſtance of *Jupiter* and *Saturn* would daily decreaſe; whereas it continually increaſed. On the firſt Day of *November* at two in the Morning, by help of our Micrometer the aforeſaid Diſtance was found to be 31′ 35″. And *Nov.* 2. I found the ſame Diſtance to be 35′ 21″. On *Nov.* 3. at One in the Morning, I found it now 39′ 9″. And *Nov.* 4. the Sky being very clear, the ſame Diſtance was ſtill greater.

As to the other Conjunction, which according to the Writers of Ephemerides ought to happen by the Retrogreſſions of theſe Planets on the 26 Day of *January* of the current Year 1683; I ſhall here add ſome of the principal Obſervations.

An. 1683.	*Time correct. by Alt. in Evening.*			*Obſervations.*		*Diſtances.*		
	h.	′	″			°	′	″
Feb. 1	6	40	0	Diſt. of *Sat.* and *Jup.* was found to be	3300	0	35	5
Feb. 2	9	30	0	Again, &c. ———	2900	0	22	3
Feb. 3	9	00	0	Again, &c. ———	2500	0	19	0
Feb. 4	10	00	0	Again, &c. ——— ———	2300	0	17	29
Feb. 5	8	30	0	Again, &c. ———	2100	0	15	59
Feb. 6	7	51	0	Again, &c. —— ——	1850	0	14	6
Feb. 7	8	17	19	Again, &c. ———	1700	0	12	55
Feb. 8	6	10	0	Again, &c. ——— ——	1600	0	12	10

Feb. 9. in the Evening, at $9^h\ 0'\ 8''$, I perceived the Planets in my Tube through the thick opening Clouds, and purſuing them with my Eye, I perceived the Conjunction had been celebrated the Night before between the 8 and 9 of *February*. For the ſaid Diſtance now appear'd ſomething larger: As alſo happen'd *Feb.* 11 in the Evening at $9^h\ 0'\ 0''$. For the Diſtance between *Saturn* and *Jupiter* was 2000, that is, $0°\ 15'\ 12''$ in the Micrometer, which on the 8 Day of *February* was found only $0°\ 12'\ 0''$.

Beſides, that the Conjunction was now paſt moſt ſurely appeared from hence, that each Planet was no longer in a right Line with the Belly of *Urſa Major*, as it was on the 8th of *February*, and likewiſe that *Saturn* was no longer in a right Angle to the Orbit of *Jupiter*'s Satellites.

Now that this may become ſtill more evident, I ſhall here add ſome Obſervations, which were continued on the following Days.

	h.	″			′	″
Feb. 12	9	0	Diſt. of *Sat.* and *Jup.* was taken —	2200	17	6
13	7	15	——— ———	2550	19	24
14	9	0	—— ——	2900	22	3
17	6	0	—— ——	3750	28	30
20	9	0	——— ——	5250	30	12

From the Continuation of theſe Obſervations it is abundantly manifeſt, that ſince the Planets from day to day recede farther from one another, the Conjunction itſelf happen'd really between the 8 and 9 of *February*.

Laſtly, for the Sake of the Lovers of Aſtronomy, I ſhall here add what I have obſerved concerning their third Conjunction in the Month of *May*.

An. 1683,

An. 1683.		*Time by the Watch in Evening.*		*Obſervations.*		*Diſtances.*	
		h.	'			'	"
May	8	9	6	The Diſtance of *Saturn* and *Jupiter* was found by the Micrometer.	4300	32	41
	10	9	14	—— ——	3750	28	30
	11	9	10	—— ——	3450	26	13
	12	8	45	—— ——	3050	23	10
	13	9	15	—— ——	2800	21	17
	14	9	45	—— ——	2550	19	23
	15	9	30	—— ——	2400	18	15
	16	9	30	—— ——	2250	17	6
	17	9	40	—— ——	2150	16	10
	18	10	00	—— ——	2100	15	58
				On this day happen'd the true Conjunct.			
	20	11	45	—— ——	2450	18	37
	21	9	15	—— ——	2650	20	9
	22	9	20	—— ——	2900	22	3
	23	9	5	—— ——	3250	24	43
	24	10	6	—— ——	3600	27	22
	25	9	30	—— ——	4000	30	25
	26	11	0	—— ——	4453	33	50
	27	9	25	—— ——	4900	37	15
	28	9	56	—— ——	5325	40	29

From which Obſervations it now appears to every one, that ſince the Diſtance of *Saturn* and *Jupiter* continually decreaſed from day to day to *May* the 18th, and from this Day again increaſed, the Conjunction of theſe Planets happen'd on that ſame Day; and indeed, (as appears from the Obſervations of the 15 and 20 of *May*,) at the Hour of 10 in the Forenoon: Which according to the Compilers of the Ephemerides, ought to have come on not before the 26. So that this third and laſt great Conjunction of this Year has alſo much eluded the Tables, as having happen'd above 8 whole Days ſooner than they predicted

Beſides, it appears from hence, that this laſt Conjunction was celebrated on *May* 18 at 10 in the Evening, becauſe *Saturn* at that time was no longer at right Angles to the Orbit of *Jupiter*'s Satellites; and further, (as may be ſeen from the Obſervations of the following days,) that from the 18 of *May* to the 28, the Diſtance of *Saturn* and *Jupiter* went on continually increaſing, as far as I could determine that Diſtance by my Micrometer.

Laſtly, it is alſo to be noted, that on the 21 of *May* in the Evening, when among other Obſervations I alſo obtain'd with my Sextant the Diſtance of each of the Planets from the upper Star in the anterior right Foot of the great Bear, and the aforeſaid Star was then almoſt in the ſame right Line with each of the Planets; ſo that it is eaſy for an Aſtronomer to judge, whether the Obſervations obtain'd with my new Sextant agree with thoſe taken by the Micrometer. By the Sextant the Diſtance of *Jupiter* from the ſaid Star was 32° 38′ 40″, and of *Saturn* 32° 19′ 45″. So that the true Diſtance of each Planet was 18′ 55″. But the ſame Diſtance found by the Micrometer the ſame Day, that is, *May* 21, was 20′ 9″; ſo that it was 1′ 14″ leſs than that obtain'd by the Sextant; yet I would not have you think, my Friend, that I err'd either with this or that Inſtrument; by no means. For *Saturn* and *Jupiter* were not exactly in the ſame Line with the ſaid Star, as may appear to any one either from the Globe, or by Calculation. Hence it neceſſarily follows, that the Diſtance derived by the Sextant ſhould be ſomething leſs.

The mean Conjunctions of Saturn and Jupiter; by Mr. Flamſteed. *n.* 149. *p.* 254. *July* An. 1683.

LXXXVII. *Riccioli* in the Second Part of the firſt Tome of his *Almageſt*, has given us a Table of all the mean *Conjunctions* of the two Superiors from the Creation till the *Year* of *Chriſt* 1358, but very coarſe and incorrect. I have therefore made a new one for 43 *Revolutions*, which are completed in 853 *Julian Years*, and 235 Days, from their mean Motions, which I have corrected by very late Obſervations. This being the *Period* of the *greateſt Conjunctions*, after which Space of Time they return to the ſame Place of the *Zodiack* within $\frac{3}{5}$ of a Degree.

The *ordinary Conjunctions* happen once in twenty Years, or more preciſely in 19 *Julian* Years, and 312 Days; in which Time *Saturn*'s mean Motion is $8^{s}\ 2^{\circ}\ 48'\frac{1}{20}$, *Jupiter*'s the ſame above one *Revolution*.

Theſe are commonly term'd the *Leſſer* of the *Great Conjunctions*, which continue in *Signs* of the ſame *Triplicity* for 10 *Revolutions* to each other, or 198 Years: each *Conjunction* according to the mean Motions being $8^{s}\ 2^{\circ}\ 48'\frac{1}{3}$ removed from the preceding; ſo that if any *Conjunction* was made upon the firſt of ♈, the next following ſhall be in 2°, 48′ of ♐, and all the following for 198 Years ſhall fall in ♈, ♌ and ♐, *Signs* of the ſame *Triplicity*.

But the 11 *Conjunctions* after, ſhall happen in the firſt Degree of ♍, and the following ten *Conjunctions* in ♉, ♍, and ♑, *Signs* of the ſame *Triplicity*. Of theſe the firſt are called by our Aſtrologers the *Greater Conjunctions*.

But the *Greateſt* is, when after 43 *Conjunctions* completed in 853 Years, 235 Days, the *mean Conjunctions*, having been made in all the *Signs*, return to that Point of the *Ecliptick* from whence they began: though I muſt confeſs, had I been to name them, I would have called thoſe the *greateſt* which happen in the *Signs*, ♋ and ♌, becauſe then the *Planets* riſe higheſt, and are longeſt viſible in our *Horizon*; as alſo being near their North *Nodes*, they approach neareſt, and if they have any extraordinary Influence (which *Naboyd* thinks either they have not, or if they have, we underſtand not) it muſt according to their Axioms be ſtrongeſt.

Thoſe which happen in ♑ and ♒ I ſhould call the *Greater* or *Middle*, becauſe the *Planets* being then near their South *Nodes*, may approach each other again very nearly, tho' they riſe not high in our *Horizon*, being in Southern *Signs*; the reſt might be accounted the *Leſſer* or *Ordinary*.

The *Mean Conjunction* of *Saturn* and *Jupiter* this Year 1683, was on the 14th Day of *January* old Stile, at 12 Hours after Noon, in the Meridian of *London*; at which Time the mean Motions of both the *Planets* were $4^s\ 11^\circ\ 45'$. This may be the *Radix* of the following *Table*.

A Table of the *mean Conjunctions* of *Saturn* and *Jupiter*.

	Intervals.						*Intervals.*				
Revolutions complete	*Time.*		*Motion.*			*Revolutions complete*	*Time.*		*Motion.*		
	y.	d.	s	°	′		y.	d.	s	°	′
1	19	312	8	2	48	23	456	219	6	4	31
2	39	258	4	5	37	24	476	165	2	7	19
3	59	204	0	8	25	25	496	111	10	10	8
4	79	150	8	11	13	26	516	57	6	12	56
5	99	96	4	14	1	27	536	3	2	15	44
6	119	42	0	16	50	28	555	315	10	18	32
7	138	353	8	19	38	29	575	261	6	21	21
8	158	299	4	22	26	30	595	207	2	24	9
9	178	245	0	25	15	31	615	153	10	26	57
10	198	191	8	28	3	32	635	99	6	29	45
11	218	137	5	00	51	33	655	45	3	2	34
12	238	83	1	3	40	34	674	356	11	5	22
13	258	29	9	6	28	35	694	302	7	8	10
14	277	340	5	9	16	36	714	248	3	10	59
15	297	286	1	12	4	37	734	194	11	13	47
16	317	232	9	14	53	38	754	140	7	16	35
17	337	178	5	17	41	39	774	86	3	19	24
18	357	124	1	20	29	40	794	32	11	22	12
19	377	70	9	23	18	41	813	343	7	25	00
20	397	16	5	26	6	42	833	289	3	27	49
21	416	327	1	28	54	43	853	235	0	00	37
22	436	273	10	1	43						

By this *Table* to find the Time of any *mean Conjunction*, paſt or future, neareſt to any Place of the *Zodiack*; for Times paſt, ſubtract the Longitude of the given Place from the Longitude of the *Radix* $4^{s}\ 11^{\circ}\ 45'$, the Reſidue ſeek in the laſt Column of the *Table*; if you find not the preciſe Number take the next to it, againſt this you have in the ſecond Column the Years and Days, in the firſt the Number of *Conjunctions* paſt ſince any was made in that Place; ſubtract the Years and Days from 1683. *Jan.* 14. and the Motion from $4^{s}\ 11^{\circ}\ 15'$, ſo have you the true Time of the *mean Conjunction*, and Longitudes of the *Planets* then.

But for Times to come ſubtract the *Radix* from the given Place, ſeek the Reſidue as before in the laſt Column; if you find it not, take that you find neareſt it; againſt which, as before, you have in the ſecond Column, the Years and Days; in the firſt, the Revolutions future: for Example,

If it were required to know when the laſt *Conjunction* happen'd in the firſt Deg. of ♒, Subtracting ♒ or Ten *Signs* from $4^{s}\ 11^{\circ}\ 15'$, the Reſidue is $6^{s}\ 11^{\circ}\ 15'$, which ſeeking I cannot find in the third Column of the *Table*, but I find $6^{s}\ 12^{\circ}\ 56'$, which is not two Degrees more, and againſt them 516 Years 57 Days, and in the firſt Column 26, for the Number of *Conjunctions* interlapſed. Subtracting 516 Years 57 Days from 1683. *Jan.* 14. there remains 1166 Years, 322 Days; which ſhews me that the *Conjunction* was in the *Year* 1166. *Nov.* 18. and ſubtracting the Motion $6^{s}\ 12^{\circ}\ 56'$, from $4^{s}\ 11^{\circ}\ 45'$, it points me to the Place in $9^{s}\ 28^{\circ}\ 49'$.

Or if the Time of the firſt *Conjunction* in ♎ to come were demanded, I ſubtract the *Radix* $4^{s}\ 11^{\circ}\ 45'$, from 6 *Signs*, the Reſidue $1^{s}\ 17^{\circ}\ 15'$, I ſeek in the *Table* but find it not, I take therefore the next to it $1^{s}\ 20^{\circ}\ 29'$, againſt which ſtands 357 Years, 124 Days, theſe added to 1683. *Jan.* 14, give me the Year 2040, and 138 Days, *May* 18, for the Time of this *Conjunction*, and adding the $1^{s}\ 20^{\circ}\ 29'$, to the *Radix* $4^{s}\ 11^{\circ}\ 45'$, it makes $6^{s}\ 2^{\circ}\ 14'$, for the true mean Longitude of this *Conjunction*.

From the *mean Conjunction* the apparent may be found by the Help of a Planetary Inſtrument, or the uſual Aſtronomical Tables.

LXXXVIII. 1. S. *Campani* affirms, that he hath remarked in the *Belts* of *Jupiter*, the *Shadows* of his *Satellites*, and followed them, and at length ſeen them emerge out of his *Diſk*.

The Shadows of Jupiter's Satellites *obſerved; by* S. Campani. *n.* 1. p. 3. *By M,* Caſſini. *and others.* n. 8. *p.* 143. *n.* 10. *p.* 171. n. 82. *p.* 4039.

2. M. *Caſſini*, after he had diſcover'd (by the Means of thoſe excellent *Glaſſes* of 35 Foot made by *S. Campani*) the *Shadows* caſt by the *Satellites* of *Jupiter* upon his *Diſk* when they happen to be between the *Sun* and *Him*; and after he had alſo diſtinguiſh'd their Bodies upon the *Diſk* of *Jupiter*, made ſome Predictions when they ſhould appear, to the End that the Curious might be convinc'd of this Matter by their own Obſervations.

Some of theſe Predictions have been verified, not only at *Rome*, and in other Places of *Italy*, but alſo at *Paris* by M. *Auzout*, and in *Holland* by M. *Huygens*; particularly *Sep.* 26, 1665. at half an Hour after Seven a Clock, one of theſe *Shadows* was ſeen both in *France* and in *Holland*.

Theſe *Spots* have this Peculiar, which diſtinguiſheth them from all others, that they are found preciſely in that Place of *Jupiter*, where ſome *Satellite* is ſeen by the *Sun*; that they go from the Oriental Limb to the Occidental of the Diſk of *Jupiter*, with a Motion always equal to that of the *Satellite*; that in reſpect to us they precede the *Satellite*, before the Oppoſition of *Jupiter* to the *Sun*, and follow him after the Oppoſition; that the further *Jupiter* is diſtant from the Oppoſition, the greater is the apparent Diſtance of the ſame *Satellite*; that at divers Times of the Year, this Diſtance changeth in proportion of the annual *Parallax* of the *Satellite*, according as he is differently ſeen by the *Sun*, and by the Earth; and that at one and the ſame Time of the Year, when divers *Satellites* happen to be between *Jupiter* and the *Sun*, the Spots correſpondent to them are diſtant from them in proportion of the Semidiameters of the Circles of the ſame *Satellites*.

By *Dr*. Hook. n, 14 *p*. 245. *Fig*. 139.

3. *An*. 1666. *Jan*. 26. about $3^h\ 15'$ in the Morning, I perceived (with a 60 Foot Glaſs) near the Middle of the Zone *d*, a very round Spot, like that repreſented at *g*, which was not to be perceived about half an Hour before; and I obſerv'd it in about 10′ Time to be gotten almoſt to *d*, keeping equal Diſtance from the *Satellite b*, which moved alſo weſtwardly, and was joined to the Diſk at *i*, at $3^h\ 25'$: So that it was ſufficiently evident that this black *Spot* was nothing elſe, ſave the Shadow of the *Satellite b*, eclipſing a Part of the Face of *Jupiter*. The other three *Satellites* in the Time of the *Eclipſe* were weſtwards of the Body of *Jupiter*.

The Elongations of Jupiter's *Satellites; by Mr.* Flamſteed. n. 82. *p*. 4036, 4337. n. 94. *p*. 6033. n. 96. *p*. 6094.

LXXXIX. 1. 'Tis now above two Years ſince the learned *R. Townley* Eſq; communicated to me the greateſt Digreſſions, which he had obſerved, of *Jupiter*'s Satellites from the Center of *Jupiter*, as alſo the mean Motion of every one, and the Epochs of thoſe Motions, deduced from his Obſervations made at his Seat at *Townley*. Afterwards I obtain'd from him your Ephemerides, O moſt learned *Caſſini*, of the *Medicean* Stars, for the Year 1668; from which having found, that not only the Motions, but the Epochs of Motion, as alſo the greateſt Elongations determin'd by you, were ſomething different from thoſe of Mr. *Townley*; I thought it might be worth while to make ſome Obſervations as Opportunity ſhould preſent, being earneſtly deſired by him ſo to do. Therefore in the Month of March of the Year 1672, *Julian* Style, with a Micrometer and a Tube of 14 Feet, with as much Care as I was able, I made the following firſt Experiments, repeating my Obſervations every Night for greater Certainty.

* $9'\ 37''$. n. 28. *p*. 4637.

An. 1672. *Mar*. $19^d\ 7^h\ 11'$. The remoter Limb of *Jupiter* was diſtant from the fourth Satellite 1601 *$=9'-34''$

27. 8. The remoter Limb from the ſame fourth Satellite 1591 $=9-30$

28. 8. The ſame Diſtance 1598 $=9-33$

By ſeveral Obſervations the Diameter of *Jupiter* was found 128. Therefore its Semidiameter was 64; by which the obſerved Diſtances being divided, there will ariſe the apparent Diſtances of the Satellite from the remoter Limb of *Jupiter* in his Semidiameters.

	d.	ſd.	′		ſd.	′
Mar.	19.	25	1	The Semidiameter being taken away, it will be from the Center ——	24	1
	27.	24	51		23	51
	28.	24	58		23	58

Then the Motion of which from *Jupiter*, and the Diſtances from the Center, according to your Numbers were thus.

	d.	h.	′	•	°	′	ſd.	′
Mart.	19	7	11	8	25	33	22	56
	27	8	..	2	19	35	22	37
	28	8	..	3	11	13	22	34

Therefore in the firſt Obſervation the Satellite was diſtant from his extream Elongation only 4′; in the ſecond 23′; in the third 26′; ſexagenary Scruples of the Semidiameter. Therefore if we add theſe to the obſerved Elongations in a due manner, the greateſt Digreſſions of this fourth Satellite from the Center of *Jupiter* will be by the firſt Obſervation 24 Semidiameters, 5′; by the ſecond 24 Semidiameters, 14′; by the third 24 Semidiameters, 24′; which you fix at only 23 Semidiameters, and Mr. *Townley* at 24 Semidiameters, 72′.

Of theſe three Elongations I judge the two latter to be the more accurate, becauſe for their Determination I had very commodious Obſervations, which I perform'd with all poſſible care. Afterwards I found the former among ſeveral Notes which I had taken on the 19th Night, which I cannot affirm was made with the ſame Exactneſs. However I admitted the Obſervations, as not ſo far diſagreeing with the reſt, but that it may ſerve to confirm them; as alſo to ſhew, very little if at all leſs, that the Elongation of this Satellite from *Jupiter* was to the Left Hand rather than to the Right.

Yet in my Obſervation I felt the Motion of the Air and Wind, which by ſhaking and agitating the Tube, (hanging in the open Air at an upright Pole, by means of a Rope and Pulley,) made the Obſervation difficult; and often was the Occaſion, that I might take the Diſtances too narrow. Therefore I reſolved to forbear from making many ſuch Obſervations, which require the utmoſt Care and Preciſion, till I have fitted up a more convenient Place for performing them, which at laſt I have provided. I have took care to have a Wooden Machine, like a ſhort Ladder, fitted into my Window, upon which my Tube being layed, may be turned about all ways, nor can be agitated this way or that by the Winds, except they are very violent, as it was in the open Air. Laying my Tube upon this, the 4th in the Evening, *An.* 1683. I apply'd my ſelf to making Obſervations with the greateſt Diligence, nor without Succeſs. For the Sky being exceeding clear, I ſaw all the four Satellites thro' a Tube conſiſting of convex Lens's, and meaſured their Diſtances from the remoter Limb of *Jupiter*, as follows.

3 985 Again—988
2 628 ——— 636
1 425 ——— 427
4 272 ——— 272. The Altitude of *Jupiter* taken with a Quadrant near two Foot long 24°—00′ Therefore the apparent Hour at *Derby* was 8ʰ 26′ after Noon. And then the fourth Satellite appear'd below the Line drawn on both Sides through the outmoſt Satellites; but, if I miſtake not, hardly a whole Semidiameter.

The Diameter of *Jupiter* was found 133, by Obſervations ſeveral Times repeated. Therefore the Semidiameter was 66½, which being taken from the obſerved Diſtances, the Intervals between the Centers of *Jupiter* and of the Satellites will become, that of the firſt 360, of the ſecond 569, of the third 921, of the fourth 205; which being divided by 66½, there will ariſe the viſible Elongations from the Center of *Jupiter* in his Semidiameters.

	ſd.	′			s	°	′	ſd.	
1	5	25	The Motions of the Satellites from *Jupiter*, and the apparent Diſtances, according to your Tables accommodated to *Derby*, were	1	9	4	52	4	59
2	8	33		2	2	22	47	7	57
3	13	51		3	2	20	26	12	48
4	3	5		4	4	23	49	2	29

Therefore the firſt Satellite was deficient only 1′, the ſecond 3′, the third 12′, ſexagenary Scruples of the Semidiameter, from the utmoſt Elongation; if therefore we add theſe to what were obſerved, the extream Digreſſions will become

	ſd.	′		ſd.		ſd.	′
Of the Firſt,	5	26	Which you make,	5	But Mr. *Townley*, as I find in ſome Papers,	5	31
Second,	8	37		8		8	47
Third,	14	2		13		13	28

It being foreſeen that a convenient Opportunity would be offer'd *Apr.* 11. in the Evening, and as I concluded that theſe Diſtances were not to be deduced from an Experiment or two, I reſolved with my ſelf to make a farther Inquiry, by Obſervations that might then be had. When I firſt began theſe, the Sky near *Jupiter* was ſo cover'd with thin Clouds, that ſometimes I could perceive the Satellites but obſcurely. Yet I took their Diſtances from the remoter Limb of *Jupiter*, as the Air would give me Leave, which were as follows.

At the Hour 7½ after Noon 3 947 Again 932
2 628 ——— 614
1 405. Afterwards when the Heaven became as clear as I could wiſh, I obſerved more accurately

3 947
2 622
1 405
4 942. Again 957. The Altitude of *Jupiter* was 24° 0′ Therefore the apparent Hour was 7ʰ 56′.

The fourth Satellite appear'd a little above the Line drawn through the firſt and ſecond; the third below that Line, but ſometimes I thought it was in it. The Diameter of *Jupiter* was taken at 132, and therefore his Semidiameter is 66; which taken from the obſerved Diſtances, gives the Interval between the Center of *Jupiter* and of the firſt 339, of the ſecond 556, of the third 881, of the fourth 891; which being ſeverally divided by 66, the apparent Elongations of the Center of *Jupiter* come out in the ſame Semidiameters, of the firſt 5 Semidiameters, 8′; of the ſecond 8 Semidiameters, 25′; of the third 13 Semidiameters, 21′; of the fourth 13 Semidiameters, 30′.

	s	°	′	ſd.	′
1	8	15	35	4	50
2	2	10	59	7	34
3	2	12	2	12	22
4	10	25	8	13	15

The Mean Motions of the Satellites from the *Pleni-mediceans*, with their Diſtances from the Center of *Jupiter*, according to your Numbers were as is ſet down in this Table. Whence it is to be ſeen, that the firſt was diſtant from its greateſt Elongation 10′ Scruples of the Semidiameter, the ſecond 26′, the third 38′. Which therefore if we add to the obſerved Elongations, the greateſt Digreſſions to be deduced from hence will become

Of the firſt	5	18
the ſecond	8	51
the third	13	59

very little differing from thoſe which we have deduced from the Obſervations of the fourth Night.

Now at each Time the inmoſt Satellite appear'd to the Left Hand from *Jupiter*, and the ſecond and third to the Right Hand. But *Apr.* 15. in the Evening I foreſaw that the third would appear on the Left Hand in its greateſt Elongation, on which Phænomenon therefore I thought it worth while to attend, that I might be ſatisfied, whether the greateſt Elongation from the Center of *Jupiter*, of the ſame Satellite, were the ſame on either Hand. The Sky was clear for Obſervation that Night, ſo that about the Hour of 7½ I obſerved according to my Wiſh, that the Diſtance of the third was 955, and the Diameter of *Jupiter* 131. Therefore his Semidiameter was 56½, which being ſubducted from the obſerved Diſtance, made the Interval between the Center of *Jupiter* and the Satellite to be 889; which being divided by the ſame viſible Semidiameter, gives the Elongation of the Satellite from the Center of *Jupiter* in his Semidiameters 13 Semidiameters, 35′. The Mean Motion of the Satellite was $3^{s}\ 4^{\circ}\ 9'$. The true Place of *Jupiter* ♎ 10° 27′. Therefore the Planet was diſtant from the *Pleni-medicean* $9^{s}\ 3^{\circ}\ 42'$, and from the greateſt Elongation only 3 Scruples. Now if we add theſe to the obſerved Digreſſion 13 Semidiameters, 35′; we ſhall have the greateſt Digreſſion this Time, to the Left Hand 13 Semidiameters, 38′. This is leſs by the third part of a Semidiameter than that on the Right Hand, as we have found by Obſervations twice repeated and agreeing together. This ſeems to inſinuate, that there is ſome Excentricity of the Center of the Orbit of this Planet from the Center of *Jupiter*.

2. The Little Circle in the Middle repreſents the Planet *Jupiter*, the four Concentrick Circles, the proper Orbits of his four *Satellites*, duly proportion-

An Inſtrument for finding the Diſtances of Jupiter'*s* Satellites *from his Axis; by Mr.* Flamſteed. *n.* 178. *p.* 1262. Dec. An. 1685. *Fig.* 142.

ed to the Breadth of his Body; the Diſtances betwixt the parallel Lines interſecting them, being each equal to one of his Semidiameters.

The 4 divided Circles next without theſe, are diſtinguiſhed into ſo many Parts as there are Days and Hours in each *Satellite*'s Revolution; the innermoſt of them ſerving for the *firſt* or *innermoſt Satellite*; that next it, for the 2ᵈ; that next without this for the 3ᵈ; and outermoſt for the 4ᵗʰ; above which is a ſmall divided Arch of 15 Degrees.

By this to find the Diſtances of the *Satellites* from ♃'s *Axis* to a propoſed Time.

1. Find the *Parallex* of *Jupiter*'s Orb to the Time propoſed, and note whether it be to be added or ſubtracted.

2. Extend the Thread from the Center of the *Inſtrument* over the *Parallax* numbred in the ſmall Arch; it cuts off in the four divided Circles, ſo many Hours as each *Satellite* ſpends in paſſing from the *Axis* of the Shadow to the *Axis* of ♃ viewed from our Earth; theſe I call the ſimple *Sarallactick Intervals*, which if the Parallax was to be added, are alſo Additional, if to be ſubtracted, Subductive.

3. To theſe *parallactick Intervals* add the Times of half the Duration of the *Eclipſe* of each *Satellite*, which for the *1ſt* may be aſſumed 1ʰ. 10′; for the *2d.* 1ʰ 30′, (greater Exactneſs being needleſs;) but for the *3d* and *4th* when *Eclipſed*, (their *Immerſion* into the Shadow and *Emerſion* from it being commonly given in the *Catalogues*) take half the Difference of theſe Times at the next *Eclipſe* to the Time propoſed for half the Duration, and add them to the ſimple *parallactick Intervals*, ſo have you them augmented. But as often as the 4th *Satellite* is not *Eclipſed*, (which is two Years in every ſix) its Interval needs no Augmentation.

4. Find in the *Tables* the Times of the *Eclipſes* of each *Satellite* next preceding the Time propoſed, and when the 4th is not *eclipſed*, of its paſſing the *Axis* of the Shadow; to which if the *parallactick Intervals* augmented were Additional, add them to, if Subductive, ſubtract them from, each the Time of its proper *Satellite*'s *Eclipſe*; ſo have you very near the apparent Times when each *Satellite* laſt paſt over the *Axis* of ♃ viewed from our Earth.

5. Subtract each of the Times thus got from the Time propoſed, the Remainders are the Intervals of the Motion of each *Satellite* from ♃'s *Axis*.

6. Extend the Thread from the Center, over each of theſe Intervals of Motion numbred ſeverally in the divided Circles, belonging each to its proper *Satellite*; where it cuts the proper *Orbit* of that *Satellite*, whoſe Interval was numbred in its peculiar Circle, it ſhews amongſt the Parallels how many Semidiameters of ♃ that *Satellite* is diſtant from him, and on which ſide of him 'tis poſited.

Note further, that the Thread as it lay extended over the *Parallax* of the *Orb* numbred in the ſmall Arch, where it cut the ſeveral proper Orbits of each *Satellite*, ſhewed amongſt the Parallels, how many Semidiameters of ♃ the Center of the Shadow was diſtant from the Center of ♃, viewed from our

Earth: And that if the *Parallax* of the *Orb* were Additional, the Shadow lies on the Right-hand from ♃, if Subductive, on the Left.

To explain theſe Precepts, I ſhall give two brief Examples. Let it be then propoſed to know, how far each *Satellite* appears diſtant from ♃ on the 26th of *Dec.* this preſent Year 1685, at $6^h\ 52'$ *p. m.* when the *Third Satellite* falls into the Shadow; alſo on *Jul.* 16. 1686. at $10^h\ 00'$ *p. m.* when there is no *Eclipſe.*

An. 1685. *December* $26^d\ 16^h\ 52'$ *p. m.* the *Parallax* of the *Orb* is 9° 20′ Additional; Therefore Fig. 143.

	1			2			3			4		
	d.	h.	′	d.	h.	′	d.	h.	′	d.	h.	′
The *Simple Parallactick* Intervals *add.*	0	1	15	0	2	10	0	4	25	0	10	20
The half *Durations* of the *Eclipſes* to be added	0	1	10	0	1	30	0	1	18	0	0	0
The *Parallactick Intervals* augmented	0	2	15	0	3	40	0	5	43	0	10	20
Laſt *Immerſions*, and 6. *Dec.*	25	9	37	25	5	47	19	12	58	10	00	30
Times of laſt paſſing *Jupiter's Axis, Dec.*	25	11	52	25	9	27	12	18	41	10	10	50
Subtracted from the Time propoſed. *Dec.*	26	16	52	26	16	52	26	16	52	26	16	52
Leaves the *Intervals* of *Motion.*	1	5	00	1	7	25	6	22	11	16	6	2
Over which, numbred in their peculiar Circles, the Thread being ſeverally laid, cuts the proper *Orbit* of each at their viſible *Diſtances* from *Jupiter*	ſd. 5. *dext.*			ſd. 6½ *ſin.*			ſd. 3. *dext.*			ſd. 4½ *dext.*		

Again, *An.* 1686. *Jul.* 16, 10ʰ *p. m.* the *Parallax* of the *Orb* is 10° 46 Subductive. Hence

	1	2	3	4
	d. h. ′	d. h. ′	d. h. ′	d. h. ′
The *Simple Parallactick Intervals* Sub. ———	0 1 12	0 2 35	0 5 10	0 12 00
Half *Duration* of the *Eclipſes* add. ———	0 1 10	0 1 30	0 1 21	— — —
The *Parallactick Intervals* augmented. ———	0 2 22	0 4 5	0 6 31	0 12 00
The next laſt *Emerſions*, and paſſing the *Axis* of the *Shadow*, *Jul.* ——— ——— ———	15 5 35	15 22 2	15 9 19	15 17 52
The Time of laſt paſſing the viſible *Axis* of *Jupiter* ——— ———	15 3 33	15 17 57	15 2 48	15 5 52
The Time propoſed ——	16 10 00	16 10 00	16 10 00	16 10 00
The *Intervals* of *Motion* ——— ——— ———	1 6 27	0 16 3	1 7 12	1 4 8
Therefore *Diſtan.* from *Jupiter*'s *Axis* ———	ſd. 5½ *dext.*	ſd. 8½ *ſin.*	ſd. 13½ *ſin.*	ſd. 10½ *ſin.*

And the *Satellites* ſtand at the two propoſed Times as in the two Figures. In drawing of which, though I have conſidered their *Latitudes* from the Line of their utmoſt *Elongations* paſſing thro' *Jupiter*'s Center; yet I give no Rules for determining it, the Contrivances and Directions neceſſary on that Account, being too many and troubleſome to be inſerted here: my deſign is only to ſhew the ingenious Obſerver, how to find at what Diſtance from ♃ each *Satellite* appears, that ſo he may not miſtake one for another when he is to obſerve any of their *Eclipſes*.

Eclipſes and Places of the Satellites of Jupiter *obſerved at* Paris; *by* . . . . *n.* 44. *p.* 892.

XC. 1. *An.* 1688. The *French Aſtronomers* have made theſe Obſervations by a 14 Foot Teleſcope.

Octob. 7. 10ʰ 32′ *p. m.* The *Firſt Satellite* (called *Pallas*) entred upon the Face of *Jupiter*.

Octob. 8. 8^{h} 11′. The *Second Satellite* (called *Juno*) went out behind *Jupiter.*

Octob. 9. 8^{h} 54′. The *Second Satellite* went out from the Face of *Jupiter.*

Octob. 16. 10^{h} 4′. The *Second Satellite* entered upon the Face of *Jupiter.*

Octob. 22. 10^{h} 41′ 33″. The *Firſt Satellite* entred into the Shadow of *Jupiter.*

Octob. 23. 8^{h} 32′. The *Firſt Satellite* entred upon the Face of *Jupiter.*

Nov. 12. 10^{h} 40′. The *Second Satellite* entred into the Shadow of *Jupiter.*

Nov. 20. 2^{h} 38′ 30″. After Midnight, the *Third Satellite* (called *Themis*) entred into the Shadow of *Jupiter.*

At Dantzick; *by M.* Hevelius. *n.* 78. *p.* 3029.

2. *An.* 1671, *Sept.* 7. New Style, in the Morning. It had been agreed on by Mr. *Caſſini* at *Paris*, and Mr. *Picard* at *Uraniburg*, to obſerve the Occultation of the firſt *Satellite* of *Jupiter*; wherefore I alſo thought proper to attend diligently to the ſame Phænomenon. Therefore at 4^{h} 27′ as ſoon as *Jupiter* appeared, I found that all his *Satellites* were preſent alſo, three to the left Hand, and one to the Right. The two which were neareſt him to the left ſeemed to be not far from his Limb, as alſo that on the right, which was the leaſt of them all. All four appear'd diſtinctly almoſt to 5^{h} 7′, tho' now the Heavens began to grow blue. At 5^{h} 12′, beyond my Expectation that nearer *Satellite* on the left Hand (in reſpect of my Tube, which exhibits Objects in an inverted Order,) ſeem'd to me intirely to vaniſh, the three others remaining, though that on the right Hand alſo ſeem'd to approach nearer and nearer to *Jupiter.* Whether indeed that was the very Moment of the Immerſion of that *Satellite*, I dare not certainly affirm; however that Occultation did not happen later. But I can readily grant that it might happen perhaps about a Minute ſooner, or thereabouts.

By the Watch in the Morn.			*Obſervations.*	*Diſtances and Altitudes.*			*The Time corrected.*		
h.	′	″		°	′	″	h.	′	″
4	36	25	*Jupiter* was firſt ſeen————	—	—	—	4	32	0
5	7	25	The Altitude of *Procyon*————	34	43	0	5	2	7
5	16	35	The firſt of *Jupiter*'s *Satellites* diſappeared —	—	—	—	5	12	0
5	26	5	The Altitude of *Procyon* ————	36	39	0	5	23	27

At Derby; *by Mr.* Flamſteed. *n.* 82. *p.* 4036.

3. *An.* $1671\frac{1}{2}$. *Feb.* 17. 7^{h} 25′ Afternoon, the Altitude of *Jupiter* was 15° 54′. But at 8^{h} 59′, Afternoon, or perhaps one Minute ſooner, the firſt *Satellite* on the right Hand of *Jupiter* fell into his Shadow, and as he vaniſh'd his Diſtance from the Limb was ſo ſmall, that I could not form a Judgment how much it was.

ib. 4037.

March 19^{d} 6^{h} 45′. The Altitude of *Jupiter* was 29° 35′. The firſt *Satellite* approach'd to the Limb of *Jupiter*, to which it was joyn'd at 7^{h} 51′.

n. 96. *p.* 6099.

An. 1672. *Apr.* 15. 7^{h} 43′ in the Evening. The Altitude of *Jupiter* was 25° 0′. The firſt *Satellite* appeared to be about $\frac{1}{6}$ of his Diameter from his Limb, being about to diſappear behind him.

At 8^h 6′ The Altitude of *Jupiter* was 27° 20′. He enter'd *Jupiter*, his height being 27° 26′; for certainly he could not be ſeen.

At Paris; *by M.* Caſſini. n. 117. *p.* 389.

4. *July* 6. New Style, 1675. before Midnight, the Hour being 11^h and 16″ preciſely. *Jupiter*'s ſecond *Satellite* began to emerge out of the Shadow of this Planet, which had obſcur'd him.

Ph. Col. n. 1. p. 33. §. LXIV. 2.

5. An. 1679. *Jun.* 5. ſt. n. 3^h *m.* I diſcovered 3 *Satellites* of *Jupiter*: The *Firſt* was diſtant Weſtward of the Limb of *Jupiter*, a little leſs than a Diameter; the *Second* was diſtant, on the Eaſt-ſide, a little more than a Diameter. The *Third* was more Eaſtward than the *Second*, by ſomewhat leſs than a Diameter of *Jupiter*.

The Equation of Light; by M. Romer. n. 136. *p.* 893. *Fig.* 145. June, An. 1677.

XCI. 1. Let A be the *Sun*, B *Jupiter*, C the *Firſt Satellite* of *Jupiter*, which enters into the *Shadow* of *Jupiter*, to come out of it at D; and let EFGHLK be the *Earth* placed at divers Diſtances from *Jupiter*.

Now ſuppoſe the *Earth*, being in L towards the 2^d Quadrature of *Jupiter*, hath ſeen the *Firſt Satellite* at the Time of its *Emerſion*, or iſſuing out of the Shadow, in D; and that about 42½ Hours after (*viz.* after one Revolution of this *Satellite*) the *Earth* being in K, doth ſee it returned in D; it is manifeſt, that if the Light require time to traverſe the Internal LK, the *Satellite* will be ſeen returned later in D, than it would have been if the *Earth* had remained in L; ſo that the Revolution of this *Satellite* being thus obſerved by the *Emerſions*, will be retarded by ſo much time as the Light ſhall have taken in paſſing from L to K; and that on the contrary, in the other Quadrature FG, where the *Earth* by approaching goes to meet the light, the Revolutions of the *Emerſions* will appear to be ſhortened by ſo much, as thoſe of the *Emerſions* had appeared to be lengthened.

The Theory of Jupiter's Satellites; *by M.* Caſſini. n. 128. *p.* 681. Sept. An. 1676.

This new *Equation* of the Motion of Light, which hath been eſtabliſhed by the *Royal Academy*, and in the *Oſervatory* for the ſpace of 8 Years, was confirmed by the *Emerſion* of the *Firſt Satellite* obſerv'd at *Paris* 1676. *Nov.* 9. 5^h 35′ 45″, at Night, 10′ later than it was expected by deducing it from thoſe that had been obſerved in the Month of *Auguſt*, when the Earth was much nearer to *Jupiter*:

M. Caſſini's *Tables, for the Eclipſes of the* Firſt Satellite of Jupiter, *Abridged, and Reduced to the Meridian of* London; *by Mr.* Edm. Halley. n. 21. *p.* 238. Nov. An. 1694.

XCII. 1. *M. Caſſini*, having formed a new Hypotheſis for the *Satellites* of *Jupiter*, different from that of *Galileo*, thinks that the Plane of their *Orbs* is inclined to the Plane of the *Ecliptick*: He ſettles their *Nodes* with the *Orbs* of *Jupiter* towards the 13° of *Leo* and *Aquarius*: and finds that the *Obliquity* of their Circles to the Orbit of *Jupiter* is almoſt double to the *Obliquity* of this Orbit to the *Ecliptick*.

2. *M. Caſſini*, in the laſt Treatiſe of a Book, entitul'd, *Recuil d'Obſervations faites en Pluſieurs Voyages*, &c. has employed his Skill, to make eaſy the Calculation of the *Eclipſes* of the *Firſt Satellite* of *Jupiter*, which is otherwiſe operoſe even to the ſkilful. The Tables have for Principles, that this *Satellite* revolves to the *Sun* in $1^d\ 18^h\ 28'\ 36''$, ſo preciſely, that in 100 Years the Difference is not ſenſible: That in the Time of the Revolution of *Jupiter* to his *Aphelion*, which he ſuppoſes in $4332^d\ 14^h\ 52'\ 48''$, this *Satel-*

lite makes exactly 2448 Months or Revolutions to the *Sun*; and dividing the Orbit of *Jupiter* into 2448 parts, he has in a large Table of *Equation* ſhewn what is the Inequality of the Motion of *Jupiter* in each Revolution reduced to time; aſſuming, *Thirdly*, the greateſt *Equation* of *Jupiter* 5° 32′ 40″, whence the hourly Motion of the *Satellite* from *Jupiter* being 8° 28½′, it follows that the greateſt Inequality (*Jupiter* paſſing the Signs of *Cancer* and *Capricorn*) amounts to 39′ 8″ of Time, to be added in *Cancer*, ſubtracted in *Capricorn*. *Laſtly*, as to the *Epocha*, or beginning of this Series of Revolutions, he has determined the *Aphelion* of *Jupiter* about 1½ Degree forwarder than *Aſtronomia Carolina*, and above two Degrees more than the *Rudolphine* Tables, *viz.* preciſely in 9° of *Libra*, in the beginning of this Century; which perhaps he finds the proper Motion of *Jupiter* about the *Sun* at this Time to require: and the Number of Revolutions, ſince *Jupiter* was laſt *in Perihelio*, is here ſtiled *Num.* I.

A ſecond Inequality is that which depends on the Diſtance of the Sun from *Jupiter*, which he ſays Mr. *Romer* did moſt ingeniouſly explain by the Hypotheſis of the Motion of Light; to which yet *Caſſini* by his manner of *Calculus* ſeems not to aſſent, tho' it be hard to imagine how the *Earth*'s Poſition in reſpect of *Jupiter* ſhould any way affect the Motion of the *Satellites*. This Inequality he makes to amount to two Degrees in the *Satellite*'s Motion, or 14′ 10″ of Time; wherein he ſuppoſes the *Eclipſes* to happen ſo much ſooner when *Jupiter* oppoſes the *Sun*, than when he is in Conjunction with him. The Diſtribution of this Inequality he makes wholly to depend on the Angle at the *Sun* between the *Earth* and *Jupiter*, without any Regard to the *Excentricity* of *Jupiter*, (who is ſometimes ½ a Semidiameter of the *Earth*'s Orb farther from the *Sun* than at other times) which would occaſion a much greater Difference than the Inequality of *Jupiter* and the *Earth*'s Motion; both of which are accounted for in theſe Tables with great Skill and Addreſs. But what is moſt ſtrange, he affirms that the ſame Inequality of two Degrees in the Motion, is likewiſe found in the other *Satellites*, requiring a much greater Time; as above two Hours in the 4th *Satellite*: which if it appeared by Obſervation would overthrow M. *Romer*'s Hypotheſis entirely. Yet I doubt not herein to make it demonſtratively plain, that the Hypotheſis of the Progreſſive Motion of Light is *Vid. Sup.* §.
found in all the other *Satellites* of *Jupiter* to be neceſſary, and that it is the ſame XCI.
in all; there being nothing near ſo great an Annual Inequality as M. *Caſſini* ſuppoſes in their Motions, by his Table, *p.* 9. and his *Præcepta Calculi*. The Method however uſed to compute this, is very curious; for having found that whilſt the *Sun* revolves to *Jupiter* there paſs $398^d\ 29^h\ 13'$, wherein are made 225⅜ Revolutions of the *Satellite* of *Jupiter*, the Number of Revolutions ſince *Jupiter* was laſt in Oppoſition to the *Sun*, is what he calls *Num.* II. in which the Inequality of the *Earth*'s Motion is allowed for in the Months, and that of *Jupiter*'s Orb by a Table of the *Equation* of *Num.* II. amounting in all to 3½ Revolutions of the *Satellite* to *Jupiter*. This in the Tables following I have thought fit to leave out, ſhewing how to find it by the help of the former Equation of *Num.* I. The Numbers are in effect the ſame with M. *Caſſini*'s, only reduced to our Style and Meridian, and the form of them abridged, and, it's hoped, amended.

Epochs of the Revolutions of the firſt Satellite *to the Shadow of* Jupiter, *under the Meridian of* London.

In the Julian *Year current.*					Num. I.	Num. II.
	d.	h.	′	″		
1660	0	11	5	48	968	200 6
1661	0	1	17	24	1174	181 2
1662	1	9	57	36	1381	162 2
1663	1	00	9	12	1587	143 5
1664	1	8	49	24	1794	125 1
1665	0	23	1	00	2000	105 7
1666	0	13	12	36	2206	86 4
1667	0	3	24	12	2412	67 0
1668	0	12	4	24	171	48 6
1669	0	2	16	00	377	29 2
1670	1	10	56	12	584	10 9
1671	1	1	7	48	790	216 9
1672	1	9	48	00	997	198 5
1673	0	23	59	36	1203	179 1
1674	0	14	11	12	1409	159 7
1675	0	4	22	48	1615	140 3
1676	0	13	3	00	1822	121 9
1677	0	3	14	36	2028	102 5
1678	1	11	54	48	2235	84 1
1679	1	2	6	24	2441	64 7
1680	1	10	46	36	200	46 4
1681	1	00	58	12	406	27 0
1682	0	15	9	48	612	7 6
1683	0	5	21	24	818	213 6
1684	0	14	1	36	1025	195 3
1685	0	4	13	12	1231	175 9
1686	1	12	53	24	1438	157 5
1687	1	3	5	00	1644	138 1
1688	1	11	45	12	1851	119 7
1689	1	1	56	48	2057	100 4

Epochs of the Revolutions of the first Satellite *to the Shadow of* Jupiter, *under the Meridian of* London.

In the Julian *Year Cur.*	d.	h.	'	"	*Num.* I.	*Num.* II.
1690	0	16	8	24	2263	81 0
1691	0	6	20	00	31	61 6
1692	0	15	00	12	228	43 3
1693	0	5	11	48	434	23 9
1694	1	13	52	00	641	5 5
1695	1	4	3	36	847	211 5
1696	1	12	43	48	1054	193 1
1697	1	2	55	24	1260	173 7
1698	0	17	7	00	1466	154 4
1699	0	7	18	36	1672	135 0
1700	0	15	58	48	1879	116 6
1701	0	6	10	24	2085	97 3
1702	1	14	50	36	2292	78 9
1703	1	5	2	12	50	59 5
1704	1	13	42	24	257	41 1
1705	1	3	54	00	463	21 8
1706	0	18	5	36	669	2 4
1707	0	8	17	12	875	208 4
1708	0	16	57	24	1082	190 0
1709	0	7	9	00	1288	170 6
1710	1	15	49	12	1495	152 3
1711	1	6	00	48	1701	132 9
1712	1	14	41	00	1908	114 5
1713	1	4	52	36	2114	95 1
1714	0	19	4	12	2320	75 8
1715	0	9	15	48	78	56 4
1716	0	17	56	00	285	38 0
1717	0	8	7	36	491	18 6
1718	1	16	47	48	698	0 3
1719	1	6	59	24	904	206 3
1720	1	15	39	36	1111	187 9

A Table

A Table of the Revolutions of the first Satellite *of* Jupiter *in a Year.*

January.				*N.* I.	*N.*II	*March.*				*N.* I.	*N.*II.
d.	h.	′	″			d.	h.	′	″		
0	0	0	0	0	0 0	1	4	12	24	34	34 8
1	18	28	36	1	1 0	2	22	41	00	35	35 8
3	12	57	12	2	2 1	4	17	9.	36	36	36 8
5	7	25	48	3	3 1	6	11	38	12	37	37 9
7	1	54	24	4	4 1	8	6	6	48	38	38 9
8	20	23	00	5	5 2	10	00	35	24	39	39 9
10	14	51	36	6	6 2	11	19	4	00	40	40 9
12	9	20	12	7	7 2	13	13	32	36	41	41 9
14	3	48	48	8	8 2	15	8	1	12	42	42 9
15	22	17	24	9	9 3	17	2	29	48	43	43 9
17	16	46	00	10	10 3	18	20	58	24	44	44 9
19	11	14	36	11	11 3	20	15	27	00	45	45 9
21	5	43	12	12	12 3	22	9	55	36	46	46 9
23	00	11	48	13	13 4	24	4	24	12	47	47 9
24	18	40	24	14	14 4	25	22	52	48	48	48 9
26	13	9	00	15	15 4	27	17	21	24	49	49 9
28	7	37	36	16	16 5	29	11	50	00	50	50 9
30	2	6	12	17	17 5	31	6	18	36	51	51 9
31	20	34	48	18	18 5						
February.						*April.*					
0	20	34	48	18	18 5	0	6	18	36	51	51 9
2	15	3	24	19	19 6	2	00	47	12	52	52 9
4	19	32	00	20	20 6	3	19	15	48	53	53 9
6	4	00	36	21	21 6	5	13	44	24	54	54 9
7	22	29	12	22	22 6	7	8	13	00	55	55 9
9	16	57	48	23	23 7	9	2	41	36	56	56 9
11	11	26	24	24	24 7	10	21	10	12	57	57 9
13	5	55	00	25	25 7	12	15	38	48	58	58 9
15	00	23	36	26	26 7	14	10	7	24	59	59 9
16	18	52	12	27	27 7	16	4	36	00	60	60 8
18	13	20	48	28	28 7	17	23	4	36	61	61 8
20	7	49	24	29	29 7	19	17	33	12	62	62 8
22	2	18	00	30	30 8	21	12	1	48	63	63 8
23	20	46	36	31	31 8	23	6	30	24	64	64 8
25	15	15	12	32	32 8	25	00	59	00	65	65 7
27	9	43	48	33	33 8	26	19	27	36	66	66 7
						28	13	56	12	67	67 7
						30	8	24	48	68	68 6

A Table of the Revolutions of the first Satellite *of Jupiter in a Year.*

May.				*N.* I.	*N.*II.
d.	h.	′	″		
0	8	24	48	68	68 6
2	2	53	24	69	69 6
3	21	22	00	70	70 6
5	15	50	36	71	71 6
7	10	19	12	72	72 5
9	4	47	48	73	73 5
10	23	16	24	74	74 5
12	17	45	00	75	75 5
14	12	13	36	76	76 4
16	6	42	12	77	77 4
18	1	10	48	78	78 4
19	19	39	24	79	79 3
21	14	8	00	80	80 3
23	08	36	36	81	81 3
25	3	5	12	82	82 3
26	21	33	48	83	83 3
28	16	2	24	84	84 2
30	10	31	00	85	85 2
June.					
1	4	59	36	86	86 1
2	23	28	12	87	87 1
4	17	56	48	88	88 0
6	12	25	24	89	89 0
8	6	54	00	90	90 0
10	1	22	36	91	90 9
11	19	51	12	92	91 9
13	18	19	48	93	92 9
15	8	48	24	94	93 8
17	3	17	00	95	94 8
18	21	45	36	96	95 7
20	16	14	12	97	96 7
22	10	42	48	98	97 7
24	5	11	24	99	98 6
25	23	40	00	100	99 6
27	18	8	36	101	100 6
29	12	37	12	102	101 5

July.				*N.* I.	*N.* II.
h.	d.	′	″		
1	7	5	48	103	102 5
3	1	34	24	104	103 5
4	20	3	00	105	104 4
6	14	31	36	106	105 4
8	9	00	12	107	106 5
10	3	28	48	108	107 3
11	31	57	24	109	108 3
13	16	26	00	110	109 3
15	10	54	36	111	110 2
17	5	23	12	112	111 2
18	23	51	48	113	112 2
20	18	20	24	114	113 1
22	12	49	00	115	114 1
24	7	17	36	116	115 1
26	1	46	12	117	116 0
27	20	14	48	118	117 0
29	14	43	24	119	118 0
31	9	12	00	120	119 0
August.					
0	9	12	00	120	119 0
2	3	40	36	121	119 9
3	22	9	12	122	120 9
5	16	37	48	123	121 9
7	11	6	24	124	122 9
9	5	35	00	125	123 8
11	00	3	36	126	124 8
12	18	32	12	127	125 8
14	13	00	48	128	126 8
16	7	29	24	129	127 7
18	1	58	00	130	128 7
19	20	26	36	131	129 7
21	14	55	12	132	130 7
23	9	23	48	133	131 7
25	3	52	24	134	132 7
26	22	21	00	135	133 6
28	16	49	36	136	134 6
30	11	18	12	137	135 6

A Table of the Revolutions of the firſt Satellite of Jupiter in a Year.

September.				N.I.	N. II.		*November.*				N.I.	N. II.	
d.	h.	′	″				d.	h.	′	″			
1	5	46	48	138	136	6	0	9	59	12	172	170	7
3	00	15	24	139	137	6	2	4	27	48	173	171	8
4	18	44	00	140	138	6	3	22	56	24	174	172	8
6	13	12	36	141	139	6	5	17	25	00	175	173	8
8	7	41	12	142	140	6	7	11	53	36	176	174	8
10	2	9	48	143	141	5	9	6	22	12	177	175	9
11	20	38	24	144	142	5	11	00	50	48	178	176	9
13	15	7	00	145	143	5	12	19	19	24	179	177	9
15	9	35	36	146	144	5	14	13	48	00	180	178	9
							16	8	16	36	181	180	0
17	4	4	12	147	145	5	18	2	45	12	182	181	0
18	22	32	48	148	146	5	19	21	13	48	183	182	0
20	17	1	24	149	147	5	21	15	42	24	184	183	0
22	11	30	00	150	148	5	23	10	11	00	185	184	0
24	5	58	36	151	149	5	25	4	39	36	186	185	1
26	00	27	12	152	150	5	26	23	8	12	187	186	1
27	18	55	48	153	151	5	28	17	36	48	188	187	2
29	13	24	24	154	152	5	30	12	5	24	189	188	2
October.													
1	7	53	00	155	153	5	0	12	5	24	189	188	2
3	2	21	36	156	154	5	2	6	34	00	190	189	2
4	20	50	12	157	155	5	4	1	2	36	191	190	3
6	15	18	48	158	156	5	5	19	31	12	192	191	3
8	9	47	27	159	157	5	7	13	59	48	193	192	3
10	4	16	00	160	158	5	9	8	28	24	194	093	4
11	22	44	36	161	159	5	11	2	57	00	195	194	4
13	17	13	12	162	160	5	12	21	25	36	196	195	5
15	11	41	48	163	161	6	14	15	54	12	197	196	5
17	6	10	24	164	162	6	16	10	22	48	198	197	6
19	00	39	90	165	163	6	18	4	51	24	199	198	6
20	19	7	36	166	164	6	19	23	20	00	200	199	7
22	13	36	12	167	165	6	21	17	48	36	201	200	7
24	8	4	48	168	166	6	23	12	17	12	202	201	8
26	2	33	24	169	167	7	25	6	45	48	203	202	8
27	21	2	00	170	168	7	27	1	14	24	204	203	9
29	15	30	36	171	169	7	28	19	43	00	205	204	9
31	9	59	12	172	170	7	30	14	11	36	206	206	0

A Table of the first Equation of the Conjunctions of the first Satellite *with* Jupiter.

N. I.	*Æquat.*		*N. I.*	*Æquat.*		*N. I.*	*Æquat.*	
	′	″		′	″		′	″
0	0	0	400	34	20	810	33	21
10	1	3	410	34	51	820	32	50
20	2	5	420	35	21	830	32	17
30	3	8	430	35	47	840	31	44
40	4	12	440	36	6	850	31	10
50	5	15	450	36	26	860	30	32
60	6	16	460	36	47	870	29	56
70	7	19	470	37	8	880	29	19
80	8	20	480	37	29	890	28	40
90	9	23	490	37	44	900	27	59
100	10	25	500	37	59	910	27	19
110	11	25	510	38	16	920	26	37
120	12	25	520	38	29	930	25	53
130	13	25	530	38	39	940	25	8
140	14	25	540	38	49	950	24	23
150	15	22	550	38	55	960	23	37
						970	22	50
160	16	18	560	38	59	980	22	3
170	17	17	570	39	3	990	21	15
180	18	11	580	39	6	1000	20	26
190	19	9	590	39	8	1010	19	37
200	20	5	600	39	7	1020	18	47
210	20	56	610	39	5	1030	17	56
220	21	49	620	39	03	1040	17	5
230	22	41	630	38	58	1050	16	13
240	23	32	640	38	51	1060	15	19
250	24	20	650	38	44	1070	14	25
260	25	7	660	38	34	1080	13	32
270	25	57	670	38	24	1090	12	37
280	26	43	680	38	10	1100	11	42
290	27	27	690	37	56	1110	10	47
300	28	9	700	37	40	1120	9	52
310	28	54	710	37	24	1130	8	57
320	29	35	720	37	5	1140	8	00
330	30	11	730	36	45	1150	7	3
340	30	45	740	36	25	1160	6	7
350	31	28	750	36	4	1170	5	1
360	32	10	760	35	40	1180	4	13
370	32	44	770	35	15	1190	3	15
380	33	15	780	34	49	1200	2	19
390	33	49	790	34	19	1210	1	21
400	34	20	800	34	49	1220	0	20
			810	33	21	1224	0	0

A Table of the ſecond Equation of the Conjunctions of the firſt Satellite with Jupiter.

N. II	*Æquat. add.*		N. II	*Æquat. add.*		N. II	*Æquat. add.*		N. II	*Æquat. add.*	
	′	″		′	″		′	″		′	″
0	0	0	28	2	4	56	7	0	84	12	0
1	0	0	29	2	13	57	7	12	85	12	9
2	0	1	30	2	21	58	7	24	86	12	16
3	0	2	31	2	30	59	7	36	87	12	24
4	0	3	32	2	39	60	7	47	88	12	32
5	0	4	33	2	48	61	7	59	89	12	40
6	0	6	34	2	58	62	8	11	90	12	47
7	0	8	35	3	8	63	8	22	91	12	53
8	0	10	36	3	17	64	8	34	92	13	00
9	0	14	37	3	27	65	8	46	93	13	6
10	0	17	38	3	37	66	8	57	94	13	13
11	0	20	39	3	48	67	9	8	95	13	19
12	0	23	40	3	59	68	9	20	96	13	24
13	0	27	41	4	9	69	9	32	97	13	30
14	0	32	42	4	20	70	9	44	98	13	35
15	0	37	43	4	31	71	9	54	99	13	39
16	0	42	44	4	41	72	10	3	100	13	45
17	0	47	45	4	53	73	10	14	101	13	48
18	0	53	46	5	4	74	10	25	102	13	51
19	0	58	47	5	15	75	10	35	103	13	54
20	1	4	48	5	27	76	10	45	104	13	57
21	1	11	49	5	39	77	10	55	105	14	00
22	1	18	50	5	50	78	11	5	106	14	3
23	1	25	51	6	2	79	11	15	107	14	5
24	1	32	52	6	14	80	11	25	108	14	7
25	1	40	53	6	25	81	11	34	109	14	8
26	1	47	54	6	37	82	11	43	110	14	9
27	1	56	55	6	49	83	11	52	111	14	10
28	2	4	56	7	00	84	12	00	112	14	10

A Table

A Table of half the Moræ *of the firſt* Satellite *in the Shadow of* Jupiter.

N. I.	h.	′	″	N. I.	h.	′	″
0	1	4	56	1200	1	5	6
40	1	4	33	1240	1	4	48
80	1	4	12	1280	1	4	26
120	1	3	59	1320	1	4	7
160	1	3	48	1360	1	3	54
200	1	3	39	1400	1	3	38
240	1	3	38	1440	1	3	38
280	1	3	48	1480	1	3	44
320	1	4	1	1520	1	3	52
360	1	4	16	1560	1	4	7
400	1	4	36	1600	1	4	24
440	1	4	56	1640	1	4	42
480	1	5	18	1680	1	5	00
520	1	5	41	1720	1	5	22
560	1	6	1	1760	1	5	46
600	1	6	21	1800	1	6	10
640	1	6	39	1840	1	6	28
680	1	6	53	1880	1	6	45
720	1	7	3	1920	1	6	57
760	1	7	11	1960	1	7	7
800	1	7	15	2000	1	7	13
840	1	7	13	2040	1	7	14
880	1	7	9	2080	1	7	15
920	1	7	2	2120	1	7	15
960	1	6	54	2160	1	7	10
1000	1	6	39	2200	1	6	49
1040	1	6	22	2240	1	6	32
1080	1	6	5	2280	1	6	15
1120	1	5	45	2320	1	5	58
1160	1	5	26	2360	1	5	38
1200	1	5	6	2400	1	5	18
				2440	1	5	2

From theſe Tables, to any given Year, Month and Day, to find the next Eclipſe of the Firſt Satellite *of* Jupiter, *proceed thus.*

1. In the Table of the *Epochæ* find the Year of our Lord, and ſet down the Day, Hours, Minutes and Seconds, with *Num.* I. and *Num.* II. thereto annex'd; and in the Table of Revolutions, ſeek the Month, and Day of the Month, with the Hours and Minutes, and *Num.* I. and *Num.* II. affix'd, and add them together; and the reſpective Sums ſhall ſhew the mean Time of the middle of the *Eclipſe* ſought, with *Num.* I. and *Num.* II. required. But it muſt be obſerved, that in *Jan.* and *Feb.* in the *Leap Year*, one Day is to be added to the Day thus found.

2. If *Num.* I. be found leſs than 1224, with *Num.* I. or if greater than 2448, ſubtracting 2448, therefrom, with the Reſidue enter the Table, and you will have the firſt Æquation to be added to the mean Time before found. But if *Num.* I. be leſs than 2448, but greater than 1224, ſubtract it from 2448, and entring the ſame Table with the Remainder, you ſhall have the firſt Æquation to be ſubtracted from the mean Time. Then divide the Minutes of the ſaid firſt Æquation by 11, or rather $^{3}{}_{3}{}^{4}$, and the *Quote* ſhall be the Æquation of *Num.* II. (anſwering to the Eccentrick Motion of *Jupiter*) to be added thereto when the firſt Æquation ſubtracts, and *è contra* ſubtracted when that adds.

3. If *Num.* II. thus æquated exceed 225,4 ſubtract 225,4 therefrom; and if the Remainder or *Num.* II. be leſs than 113, with the ſaid Remainder or Number; or if greater than 113, with the Complement thereof to 225,4, ſeek in the Table the ſecond Æquation; which being added to the Time before found, gives the true Time of the middle of the *Eclipſe*.

4. With *Num.* I. ſeek the half Continuance of the Total *Eclipſe*, which is to be added for the *Emerſion*, when the æquated *Num.* II. is leſs than 113, or if more than 225,4, it be leſs than 338. But if it exceed 113, or 338, then is the *Semimora* to be ſubtracted for the *Immerſion*.

Vid. Sup. § XXII.

5. Laſtly, With the *Sun*'s true *Place* take out the Æquation of Natural Days, which added or ſubtracted according to the Title, gives the Time of the *Immerſion* or *Emerſion* ſought.

Now how few Figures ſerve for this Computation, will beſt appear by an Example.

An. 1677. 17. $8^h\ 9'\ 40''$ at *Greenwich*, Mr. *Flamſteed* obſerved the *firſt Satellite* to begin to *emerge*; that is $8^h\ 9'\ 26''$, at *London*.

	d.	h.	′	″	N. I.	N. II.
1677.	0	3	14	36	2028	102, 5
Sept.	17	4	4	12	147	145, 5
Sept	17	7	18	48	2175	248, 0
Æquat. 1.	—		26	11	2448	2, 3
Æquat. 2.	17	6	52	37	273	250, 3
	+	0	1	39		225, 4
Semimora	+	1	7	00		
						24, 9
Equal Time	17	8	1	16	11) 26,	2 (2, 3
Equation	+	0	9	25	☉ in ♎	5°, 00′
Apparent. T.	17	8	10	41		
Obſerv.	00	8	9	20		
Error.			1	21		

An *Immerſion* of this *Satellite* being computed after the ſame manner according to theſe Tables, ought to have happened, *An.* 1683. *Nov.* 30. $16^h\ 52'\ 7''$; but I obſerv'd it at $16^h\ 48'\ 40''$, ſo that the Error was, $-3'\ 27''$.

Again, M. *Caſſini* obſerved an *Emerſion* at *Paris*, *An.* 1693, *Jan.* $14^d\ 10^h\ 40'\ 28''$; that is, at *London* $10^h\ 30'\ 48''$, which theſe Tables give at $10^h\ 30'\ 39''$; and therefore the Error was no more than $+9''$.

After this manner I have compared theſe Tables with many good and certain Obſervations, and ſcarce ever find them err above 3 or 4 Minutes of Time; which Errors are exceeding ſmall in compariſon of the ſhort Time that the *Satellites* have been diſcover'd.

In the Conſtruction of the Table, which ſhews the half Continuance of theſe *Eclipſes*, the Semidiameter of the Shadow of *Jupiter* is made by *Caſſini* juſt 10 Deg. and that of the *Satellite* 30′; and the *Satellite's aſcending Node* being ſuppoſed in 15° of *Aquarius*, at the end of this Century (that is 55° 20′ before the *Perihelion* of *Jupiter*) it will thence follow, that *Num.* I. being 116, or 2102, *Jupiter* paſſes the *Nodes* of the *Satellite*'s Orb, and conſequently theſe *Eclipſes* are Central, and of the greateſt Duration. But *Num.* I. being 215, or 1481, the *Satellite* paſſes the Shadow with the greateſt Obliquity, *viz.* 2° 55′ from the Center; whence the *Semimora* becomes of all the ſhorteſt.

3. The Tables of the other three *Satellites* not being ſo perfect or exact as thoſe of the firſt, are here given in another Form. The Periods of their Revolutions to *Jupiter*'s Shade are as follow. *Of the other 3 Satellites Ib. p.* 253.

	d.	h.	′	″	‴		
Period. Secundi	3	13	17	54	3	*ſive* $2\frac{1}{123}$	*Rev.* Primi.
Period. Tertii	7	3	59	39	22	*ſive* $4\frac{3}{61}$	*Rev.* Primi.
Period. Quarti	16	18	5	6	50	*ſive* $9\frac{7}{15}$	*Rev.* Primi.

Whence the Table of the Firſt Æquation of the *Firſt Satellite*, or M. *Caſſini*'s larger Table, may by an eaſy Reduction ſerve the other three; the Æquation of the *2d* being $2\frac{1}{123}$, or twice the Minutes with half ſo many Seconds as there are Minutes in the Æquation of the *Firſt*, and the greateſt Æquation thereof $1^{h}\ 18'\ 35''$. Æquation of the *3d* is $4\frac{1}{26}$ times greater than that of the *Firſt*, and when greateſt amounteth to $2^{h}\ 29'$. And the Æquation of the *4th* being $9\frac{7}{15}$ times that of the *Firſt*, is had by ſubtracting $\frac{1}{2}$ and $\frac{1}{3}$ from 10 times the Æquation of the *Firſt*, whence the greateſt becomes $6^{h}\ 10'\ 28''$: ſo that *Num.* I. and *Num.* II. as here collected for the *Firſt*, may indifferently ſerve all the reſt.

M Romer's *Æquation of Light Defended. Ibid. p.* 254. *Vid. Sup.*

4. As to the Second Æquation of the other *Satellites*, M. *Caſſini* has, by his *Præcepta Calculi* (as is before mentioned) ſuppoſed the Minutes thereof to be increaſed in the ſame Proportion, as inſtead of $14'\ 10''$ in the *Firſt* to be $28'\ 27''$ in the *Second*, $57'\ 22''$ in the *Third*, and no leſs than $2^{h}\ 14'\ 7''$ in the *Fourth*; whereas if this Second Inequality did proceed from the ſucceſſive Propagation of Light, this Æquation ought to be the ſame in all of them, which M. *Caſſini* ſays, was wanting to be ſhewn, to perfect M. *Romer*'s Demonſtration: wherefore he has rejected it as ill founded. But there is a good Cauſe to believe, that his Motive thereto is what he has thought not proper to diſcover. And the following Obſervations do ſufficiently ſupply the Defect complained of in the making out of that Hypotheſis.

An. 1676, Oct. ſt. n. $6^{h}\ 10'\ 37''$, *App.* but $5^{h}\ 59'\ 37''$, *Equal Time*; M. *Caſſini* at *Paris* obſerved the *Emerſion* of the *3d Satellite* from *Jupiter*'s Shadow. And again *Nov.* 14. following $6^{h}\ 20'\ 55''$. *App.* Time, but $6^{h}\ 5'\ 55''$, *Eq. Time*, he obſerved the like Emerſion of the ſame *Satellite*. The obſerved Interval of Time between theſe *Emerſions* was $43^{d}\ 0^{h}\ 6'\ 18''$, which is $8'\ 22''$ more than 6 Mean Revolutions of this *Satellite*, of which $4'\ 27''$ ariſe from the Difference of the Firſt Equation, and the greater Continuance of the Latter *Eclipſe*; ſo that the other 4 Minutes is all that is left to anſwer for the Difference of the *2d* Æquations and *Num.* II. in that Time increaſing from 48 to 72, gives $4'\ 36''$ for the Difference of the *2d* Æquations of the *Firſt Satellite*. So that here the *2d* Æquation of the *Third* is found rather leſs than that of the *Firſt*; but the Difference is ſo ſmall that it may rather be attributed to the Uncertainty of Obſervation: Whereas according to M. *Caſſini*'s Method of Calculating, inſtead of four Minutes it ought to be $18'\ 38''$, and the Interval of theſe two *Emerſions* $43^{d}\ 0^{h}\ 21'$, exceeding the Time obſerved by a whole Quarter of an Hour; which that curious Obſerver could not be deceived in.

The like appears yet more evidently in the *Fourth Satellite*. By the Obſervations of Mr. *Flamſteed* at *Greenwich*, *An.* 1682. *Sept.* 24. $17^{h}\ 45'$, *T. App.* but $17^{h}\ 32'$, *T. Eq.* the *Fourth Satellite* was ſeen newly come out of the

Shadow; ſo that about $17^h\ 30'$, *T. Eq.* the firſt beginning of *Emerſion* was conjectured; and after 5 Revolutions, *viz. Decemb.* $17^d\ 11^h\ 16'$ or $11^h\ 18'$, *T. Eq.* he again obſerved the firſt Appearance of the *Satellite* beginning to *Emerge*, that is, after an Interval of 83^d, $17^h\ 48'$; whereas this *Satellite* makes five mean Revolutions in $83^d\ 18^h\ 25'\frac{1}{2}$. Here we have $37'\frac{1}{2}$ to be accounted for by the ſeveral Inequalities. Of this $21'$ is due to the firſt Æquations, which is reduced to $19'$ by the Greater Continuance of the latter *Eclipſe*, *Jupiter* then approaching to his *Deſcending Node*: So that there remains only $18'\frac{1}{2}$ for the Difference of the 2^d Æquations, whilſt the Earth approached *Jupiter*, by more than the *Radius* of its own Orb; and the Difference of the 2^d Æquations of the *Firſt Satellite*, being according to *Caſſini* $8'\ 30''$, the ſaid Difference in the *Fourth* ought to be $1^h\ 20'\frac{1}{2}$, inſtead of $18'\frac{1}{2}$; whence the Interval of theſe two *Emerſions* would be, according to his Precepts, but 83^d, $16^h\ 46'$, inſtead of $83^d\ 17^h\ 48'$ obſerved. And whereas $18'\frac{1}{2}$, may ſeem too great a Difference, it muſt be noted, firſt, that M. *Romer* had ſtated the whole 2^d Æquation $22'\ 00''$, which M. *Caſſini* has diminiſhed to $14'\ 10''$; ſo that inſtead of $8'\frac{1}{2}$. M. *Romer* allows $13'$; and ſecondly, that in the firſt of theſe Obſervations, being about half an Hour before Sun-riſe, the Brightneſs of the Morning might well hinder the ſeeing of this ſmalleſt and ſloweſt *Satellite*, till ſuch time as a good part thereof was *Emerged*.

The Phaſes and Revolutions of Mars about his Axis; by Dr. Hook.
n. 11. p. 198.
n. 14. p. 239.
Mar. An. 1666.

XCIII. Having a great deſire to obſerve the Body of *Mars*, whilſt *Acronycal* and *Retrograde* (having formerly with a Glaſs about 12 Foot long, obſerv'd ſome kind of Spots in the Phaſe of it) tho' it was not in the *Perihelium* of its Orb, but nearer its *Aphelium*; yet I found that the Face of it, when near its oppoſition to the Sun (with a Charge, the 36 Foot Glaſs, I made uſe of, would well bear) appeared very near as big, as that of the Moon to the naked Eye.

But ſuch had been the ill Diſpoſition of the Air for ſeveral Nights, that from more than 20 Obſervations of it, which I had made ſince its being *Retrograde*, I could find nothing of Satisfaction, tho' I often imagined I ſaw *Spots*; yet the inflective Veins of the Air (if I may ſo call thoſe parts, which being interſpers'd up and down in it, have a greater or leſs Refractive Power than the Air next adjoining, with which they are mix'd) did make it ſo confuſed and glaring, that I could not conclude upon any thing.

On the 3^d of *Mar.* $0^h\ 30'$, $166\frac{5}{6}$, in the Morning, tho' the Air was ſtill bad enough, yet I could ſee now and then the Body of *Mars*, which I deſcribed by the Scheme B, as exactly repreſenting what I ſaw thro' the Glaſs, as I could. *Fig.* 145.

An. $166\frac{5}{6}$, *Mar.* 10. $0^h\ 20'$, in the Morning, finding the Air very bad, I made uſe of a very ſhallow Eye-Glaſs, as finding nothing diſtinct with the greater Charge; and ſaw the Appearance of it as in C, which I imagined *Fig.* 147. might be the Repreſentation of the former *Spots* by a leſſer Charge. About 3 of the Clock the ſame Morning, the Air being very bad (tho' to appearance exceeding clear, and cauſing all the Stars to twinkle, and the minute Stars to appear

Fig. 148. appear very thick) the Body ſeemed like D; which I ſtill ſuppoſed to be the Repreſentation of the ſame *Spots* thro' a more confuſed and glaring Air.

But obſerving *Mar.* 21. I was ſurprized to find the Air (tho' not ſo clear, as to the appearance of ſmall Stars) ſo exceeding tranſparent, and the Face of *Mars* ſo very well defined, and round and diſtinct, that I could manifeſtly ſee Fig. 149. it of the Shape in E, about half an Hour after 9 at Night. The Triangular *Spot* on the Right-ſide (as it was inverted by the Teleſcope) according to the appearances, thro' which all the preceding Figures are drawn, appeared very black and diſtinct, and the other towards the left more dim; but both of them ſufficiently plain and defined. About a Quarter before 12 of the Clock the ſame Night, I obſerved it again with the ſame Glaſs, and found the Ap- Fig. 150. pearance exactly, as in F; which I imagined to ſhew me a Motion of the former Triangular *Spot.*

Mar. 22. about half an Hour after 8 at Night, finding the ſame *Spots* in Fig. 151. the ſame poſture (as in G) I concluded that the preceding Obſervation was only the Appearance of the ſame *Spots* at another Height and Thickneſs of the Air; and thought myſelf confirmed in this Opinion, by finding them in much the ſame Poſture, *Mar.* 23. about half an Hour after 9 (as in H) tho' the Air was Fig. 152. nothing ſo good as before.

Mar. 28. about 3 of the Clock, the Air being light (in Weight) tho' moiſt Fig. 153. and a little hazy, I plainly ſaw it to have the Form repreſented in I; which is not reconcileable with the other Appearances, unleſs we allow a Turbinated Motion of *Mars* upon its Center: Which, if ſuch there be, from the Obſervations made *Mar.* 21, 22, and 23. we may gueſs it to be once or twice in about 24 Hours, unleſs it may have ſome kind of Librating Motion; which ſeems not ſo likely.

The Parallax of Mars; by Mr. Flamſteed. n. 89. p. 5118. n. 96. p. 6100.

XCIV. *An.* 1672. *Sept.* I meaſured with a Micrometer and a Tube of 14 feet, the Diſtance of *Mars* from two fixt Stars the ſame Night. Whence I learn'd that his Acronic and Perigean Parallax is never greater than 25 ſecond Scruples. Whence it follows, that at moſt the *Sun*'s is 10″, and his diſtance is 21000 of the *Earth*'s Semidiameters.

Places of Mars *Obſerved at* Darby; *by Mr.* Flamſteed. n. 86. p. 5039.

XCV. 1. *An.* 1672. *May* 14 in the Morning. The Planet *Mars* paſſed near the *Star* called that at the Buttocks of *Aquarius*, whoſe Latitude is 2° 0′ 0″. Its Place at that Time according to me was 24° 12′ 9″ ♒; according to *Street* 24° 9′ 0″. From whence I obſerved

	° ′		° ′ ″
The true Altitude of the fixt Star being	9 40	At 2^h 29′ in the Morning, the diſtance of *Mars* exactly in the ſame *Azimuth*	0 24 17
	11 12	The ſame diſtance again — —	0 24 24
	12 00	At 2^h 51′. The *Planet* had departed from the ſame *Azimuth*, and was to the Eaſt of the *Azimuth* Line; the Diff. being	0 00 55

	°	′	″
At 2^{h} 29′. in the Morning the *Parallactical Angle* was	82	26	00
Therefore *Mars* is in Conſequence of the fixt *Star*	00	3	11
With greater Southern Latitude	00	24	4
Therefore the Latitude of *Mars*	2	24	4
His true Place according to *Street* ♒.	24	12	17
His true Place according to Me— ♒.	24	15	04

2. *An.* 1683. *May* 3^{d}. 9^{h} 12′. *Mars* celebrated a Conjunction with the laſt Star but one of the Southern Wing of *Virgo*, which now is in 13° 49′ 14″. of ♎, and its Latitude 1° 48′ 33″ North; ſo that at the Time of the Conjunction *Mars* paſs'd hardly more than 40″ below the ſaid Star.

At Dantzick; *by M.* Hevelius *n.* 151. *p.* 332.

XCVI. 1. *M. Burattini* hath ſignified from *Poland*, that he hath obſerved Inequalities in *Venus* as in the *Moon*.

Spots in Venus; *by M.* Burattini. *n.* 10. *p.* 173.

2. I have at laſt diſcovered towards the Middle of the Body of *Venus* a part clearer than the reſt, by which one may judge of the Motion, or the Reſt of this Planet.

The Rotation of Venus; *by M.* Caſſini. *n.* 32. *p.* 615. *n.* 35. *p.* 687.

The firſt time I ſaw it, was *Octob.* 14. 1666. 5^{h} 45′, *p. m.* and then this bright part was very near the Center, on the North-ſide. And at the ſame time I obſerv'd Weſtward two obſcure *Spots*, ſomewhat oblong; but I could not then ſee that reſplendent part long enough to conclude any thing from thence, nor was I able to ſee any thing well of thoſe parts till *April* 28. 1667. on which Day a quarter of an Hour before Sun-riſing, I ſaw again a bright part, ſituated near the Section, and diſtant from the Southern Horn a little more than $\frac{1}{4}$ of its Diameter. And near the Eaſtern Ring I ſaw a dark and ſomewhat oblong *Spot*, which was nearer to the Northern than the Southern Horn. At the Riſing of the *Sun*, I perceived, that this bright part was then no more ſo near the Southern Horn, but diſtant from it $\frac{1}{3}$ of its Diameter. This gave me great Satisfaction. But I was ſurprized at the ſame time to find, that the ſame Motion, which was made from South to North in the inferior part, of the Disk, was on the contrary made from North to South in the Superior part whence the Determination of the Motion may be better taken: For we have no Example of the like Motion, except it be that of the Libration of the *Moon*.

The next Day, at the Riſing of the *Sun*, the ſaid bright part was not far from the Section, and diſtant from the Southern Horn $\frac{1}{4}$ of the Diameter. When the *Sun* was 4 Degrees high, the ſame was ſituated near the Section, remote from the Southern Horn $\frac{2}{5}$ of the Diameter. The *Sun* being high 6° 10′, it ſeemed to have paſſed the Center, and that the Section of the Diſk did cut the ſame. The *Sun* being 7° high, it appeared yet more advanced Northward, together with two obſcure *Spots* ſeated between the Section and the Circumference, and equally diſtant from one another, and from each Horn on both Sides, And the Sky being very clear, I obſerved the Motion of the bright part for $1\frac{1}{8}^{h}$. which then ſeemed to be exactly made from South to North without auy ſenſible Inclination Eaſtward or Weſtward. Mean time I perceived in the Motion of the dark *Spots* ſo great a Variation, that it cannot be aſcribed to any Reaſon in *Opticks*.

May 10 and 13. Before *Sun-Riſing*, I ſaw ſtill the bright part near the Center Northward.

Laſtly, *June* 5 and 6. Before the Riſing of the *Sun*, I ſaw the ſame between the Northern Horn and the Center of this Planet, and I noted the ſame irregular Variation in the obſcure *Spots*. But when *Venus* began to be farther removed from the *Earth*, it was more difficult to obſerve the *Phænomena*.

It is very difficult to determine any thing of the Motion of theſe *Spots*: Yet this I can ſay (ſuppoſing that this bright part of *Venus*, which I have obſerv'd, eſpecially this Year 1667, hath always been the ſame) that in leſs than one Day it abſolves its Motion, whether of Revolution or Libration, ſo as in near 23 Hours it returns to the ſame Situation in this *Planet*, which yet happens not without ſome Irregularity.

A Place of Venus, obſerv'd at Dantzick; *by M.* Hevelius. *n.* 154. *p.* 419.

XCVII. *An.* 1683. *Aug.* 4. at almoſt 2^h in the Afternoon. *Venus* was diſtant towards the North only 16′ from a fixt Star of the third Magnitude, called the *Belly of Pollux*; which I found exactly by the Micrometer.

XCVIII. Mercury *obſerved in the Sun*, 1690. *at* Nuremberg; *by M.* Jo. Phil. Wurtzelbaur. *n.* 192. *p.* 483.

Time by the Clock.	*Obſervations.*			*Time by Calculation*
h. ′ ″	*Octob.* 30. Afternoon, *An.* 1690.			h. ′ ″
6 32 00	*Os Pegaſi* culminates —— —— —— ——			6 28 45
9 00 00	*Andromeda*'s Head culminates —— —— ——			8 52 17
9 4 30	*Jupiter* culminates —— —— —— ——			
	Octob. 31. Morning.			
8 30 00	The *Sun* emerged out of the Clouds; above his Diſk, in the Table of Obſervation, at the Diſtance of more than half a Digit from the Vertical to the right Hand, (tho' really to the left,) *Mercury* appear'd going out of his Limb.			
8 36 00	After *Mercury* had adhered a Minute of Time to the undulating Limb of the *Sun*, he went out at 14° from the *Zenith* towards the North.	°	′	h. ′ ″
8 49 00	The Altitude of the *Sun* —— —— ——	10	5	8 38 38
8 59 45	—— —— —— ——	11	10	8 47 48
9 7 10	—— —— —— ——	12	10	8 56 24
9 50 00	—— —— —— ——	16	28	9 38 7
11 1 30	—— —— —— ——	21	31	10 56 32

The Ratio of the Diameters of the *Sun*, and of the *Nucleus* of *Mercury*, while he continued in the lucid Diſk of the *Sun*, as far as could be gueſs'd through the Air that was pretty thick, was as 1000 to 8½. After he had arrived at the Limb of the *Sun*, and had adhered almoſt a Minute to the undulating Limb, and had recovered his genuine Roundneſs; (for he had aſſumed as it were an Elliptical Form before, from the Light of the *Sun*'s Diſk,) the proportion was as 1000 to $12\frac{1}{3}$.

Mercury obſerved in the Sun. Nov. 3. *ſ. n.* 1697; *by M.* Caſini. *n* 245. *p.* 372.

XCIX. *An.* 1697. *Nov.* 3. New Style; at 7^{h} 25′. After the *Sun* had emerged out of the Clouds, directing my Teleſcope to him, the Difference of the right Aſcenſion of the Center of *Mercury* to the Weſt, and the Center of the *Sun*, was obſerved by the Clock — — — 0° 11′ 52″

The Difference of the Declination of *Mercury* to the South was 0 6 20

At 8^{h} 3′. The Difference of Right Aſcenſion of the Centers of *Mercury* to the Weſt and of the *Sun* was, Hours — } 0 15 30

The Difference of Declination was, Degrees — — 0 4 42

At 8^{h} 8′ 38″. The preceding Limb of *Mercury* arrived at the preceding Limb of the *Sun*.

At 8^{h} 10′ 24″. *Mercury* wholly emerged out of the *Sun*'s Diſk, being obſerved with a Teleſcope of 18 Feet.

From theſe Obſervations compared together, as far as can be collected from ſo ſhort an Interval, I found by Trigonometry that *Mercury* came to the Middle of his Path in the *Sun*'s Diſk at 6^{h} 11′ 18″ in the Afternoon.

And that the Aſcending Node of *Mercury* was in ♉ 14° 42′. ſtill more advanced than by the Obſervations of the Year 1677.

Alſo I found by compariſon of the laſt Obſervations, that the Inclination of *Mercury*'s Orbit to the Ecliptick was 6° 23′; which however, becauſe of the ſhort Interval of theſe Obſervations, I dare not prefer to that which I derived from the *Chineſe* Obſervations of the Reverend Father *Fontenay*, which were diſtant by a much greater Interval. This was 6° 40′. which approaches nearer to the Rudolphine Tables.

The Viſible Conjunctions of the Inferior Planets *with the* Sun; *by Mr.* Halley. *n.* 193. *p.* 511. Mar. An. 1691

C. It has been long ago diſcover'd, not only from the Principles of true Aſtronomy, but upon the Credit of the beſt Obſervers, that the Planets *Mercury* and *Venus* ſometimes enter the Diſk of the *Sun*, and are ſeen within his bright Orb to paſs along like blackiſh Spots. But by what Law and on what Conditions, alſo in what Intervals of Years, theſe Phenomena are preſented to our View, has not yet (I think) been duly determin'd by our modern Aſtronomers. Sure I am, that about this Matter I have ſeen nothing yet in Print. Therefore I thought it might not be unacceptable, if I ſeriouſly apply'd myſelf to this Inquiry; and I truſt that in this Diſſertation I ſhall fully unfold the whole Affair, which is perplexed enough, and underſtood but by few.

It is ſufficiently manifeſt, that theſe Phaſes of theſe Planets always happen when they are retrograde and in their Conjunctions with the *Sun*; that is, when the *Sun* is ſo near their Nodes, that the Latitude of the Planet in Conjunction with the *Sun*, may not exceed the *Sun*'s Semidiameter. Now that I may the more eaſily inveſtigate the Limits and Conditions of theſe Conjunctions, and as the Elements of Calculation for each of theſe Planets are very different, I ſhall manage them ſeparately; and firſt I will begin with *Mercury*.

We are well aſſured, that the Aſcending Node of this Planet, according to the lateſt and beſt Obſervations, is about 15 Degrees in *Taurus*, or rather at $0^{s}\ 15^{\circ}\ 44'$ from the firſt Star in *Aries*, in the preſent Age. And that the Deſcending or Oppoſite Node is at $6^{s}\ 15^{\circ}\ 44'$ from the firſt Star of *Aries*. And the Angle in which the Plain of *Mercury*'s Orbit is inclined to the *Ecliptic*, is pretty well determin'd by *Kepler* to be $6^{\circ}\ 54'$. Now by the moſt approved Hypotheſes it appears, that when *Mercury* is in his Aſcending Node his diſtance from the *Sun* is 31365 of ſuch Parts, as the mean Diſtance of the *Sun* from the Earth is 100000. But when he is in his other Node, his Diſtance then meaſured in the ſame Parts is 45308. But the *Sun* when oppoſite to the Aſcending Node, is diſtant from the Earth in Conjunction to the ſame 98955 of the ſame Parts; but at the other Node the ſame Diſtance is 101007. And therefore *Mercury* in Conjunction with the *Sun* at the Aſcending Node is diſtant from the *Earth* 67591 Parts, and at the Deſcending Node 55699 Parts. As theſe differ very widely, the Conjunctions made at different Nodes muſt be conſidered ſeparately; and for brevity ſake we ſhall give the Elements of Calculation in a Synoptical Manner.

Mercury *being Retrograde is in central Conjunction with the* Sun, *at the Aſcending Node, in the Month of* October; *and from the aforeſaid Hypotheſes we ſhall have,*

		s	°	'	"
The *Sun*'s Longitude from the firſt Star of *Aries*		6	15	44	00
The Longitude of *Mercury* ſeen from the *Sun*		0	15	44	00
Mercury's Diſtance from the *Sun*	31365				
Mercury's Diſtance from the *Earth*	67591				
The Angle of the Inclination of *Mercury*'s Orbit		0	6	54	00
Six Hours Motions of *Mercury* ſeen from the *Sun*		0	1	30	58
The *Sun*'s Motion in the ſame ſix Hours		0	0	15	5
Hence *Mercury*'s Motion from the Sun in ſix Hours		0	1	15	53
And his Motion in 6 Hours from the *Sun* ſeen from the *Earth*		0	0	35	12
And the Angle of *Mercury*'s Path ſeen within the *Sun* with the *Ecliptic*		0	8	15	00
Hence *Mercury*'s Motion in 6 Hours in his viſible Orbit		0	0	35	40
Then *Mercury*'s Motion in a *Sidereal* Year beyond 4 Revolutions		1	24	45	8
Therefore in 13 Years		11	21	46	44
Therefore wanting to 54 intire Revolutions		0	8	13	16
			d.	h.	'
Which Space *Mercury* paſſes over in			2	00	11
			°	'	"
In which time the *Sun*'s Place is advanced, and the Situation of the Conjunction in the Node is as much diſtant from the Conjunction of the *Earth*			2	1	00
Now that Arch view'd from the *Earth* is			0	56	10
Whence from the given Angle of the Path ſeen 8° 15'. the Baſe may be found, or the Diſtance from the viſible Conjunction			0	55	34

			h.	″
Which Arch is paſſed over by *Mercury* according to the given horary Motion in			9	21
But 13 ſidereal Years exceed ſo many *Julian* with three Intercalations by			8	0
		d.	h.	′
Therefore *Mercury* returns to the *Sun* after 13 *Julian* Years, and moreover		2	17	34
Or with four Intercalations, if the foregoing Year be the third from the *Biſſextile*		1	17	34
			′	″
But from the Arch 56′ 10″. and the given Angle, the Perpendicular, or the neareſt Diſtance of the Conjunction from the *Sun*, is		0	8	3
Therefore the Conjunction conſpicuous within the *Sun* after 13 Years proceeds more Northerly 8′ 3″.				
	s.	°	′	″
By a like Reaſoning in 46 *Sidereal* Years the Conjunction moves	11	28	36	8
Therefore to 191 intire Revolutions are wanting	0	1	23	52
		h.	′	
That is in Time		8	12	
		°	′	″
In which the *Sun* advances		0	20	41
This Arch view'd from the *Earth* becomes		0	9	36
And the Baſe belonging to it is		0	9	30
		h.	′	″
But the Time in which *Mercury* deſcribes the Baſe is		1	30	00
But 46 *Sidereal* Years exceed ſo many *Julian* Years, with 11 Intercalations, by		19	3	00
		d.	h.	′
And *Mercury* returns to the *Sun* after 46 *Julian* Years, and moreover		1	4	51
Or with 12 Intercalations, as it happens when the foregoing Year in the ſecond or third from the *Biſſextile*		0	4	51
		°	′	″
But the Perpendicular, by which *Mercury* advances Northerly, is		0	1	22
		h.	′	″
Now the moſt accurate Period of *Mercury* to the *Sun* is finiſh'd in 263 *Sidereal* Years, and moreover		1	11	30
But theſe *Sidereal* Years exceed ſo many *Julian*, with 66 Intercalations, by		10	20	0
Whence after 263 *Julian* Years *Mercury* returns to the *Sun*, but later by		11	33	30
	d.			
Now if the foregoing Year is *Biſſextile*, let there be added	1	11	31	30
Then after this Interval he goes more to the North by	0	0	0	10

Now the other more extenſive Periods are very eaſily derived from theſe already found, and are of 6 or 7 Years.

That which is compleated in 7 Years depreſſes *Mercury* 22′ 47″, towards the South, and comes ſooner by 7 whole Days, excepting 9 Minutes, if there are two Intercalations. But with one Intercalation, that is when the foregoing Year is *Biſſextile*, 6 Days are to be ſubtracted, adding only 9 Minutes as before.

But that Vagabond Planet is ſeldom again to be found in the *Sun*'s Diſk after 6 Years; for having this Period, he paſſes 30′ 50″ more to the North: And this later by $9^d\ 17^h\ 25'$, if the foregoing Year be the ſecond or third from the *Biſſextile*, otherwiſe $8^d\ 17^h\ 25'$. are to be added.

In like manner if the Conjunction *happen at the* Deſcending Node, *in the Month of* April.

		s.	°	′	″
The *Sun*'s Longitude from the firſt Star of *Aries*		0	15	44	00
The Longitude of *Mercury* ſeen from the *Sun*		6	15	44	00
The Diſtance of the Planet from the *Sun* as before	45308				
His Diſtance from the *Earth*	55699				
The Motion of *Mercury* ſeen from the *Sun*, in ſix Hours		0	0	14	29
The *Sun*'s Motion in the ſame Time		0	0	43	21
The Motion of *Mercury* from the *Sun*		0	0	28	52
Hence the Angle of the Path of *Mercury* ſeen within the *Sun*'s Diſk made with the *Ecliptic* becomes		0	10	18	00
But his Motion ſeen from the *Earth*, in ſix Hours is		0	0	23	52

Whence if we follow the Method of the foregoing Calculation, it is found that after 13 Years, and moreover $3^d\ 7^h\ 37'$. *Mercury* will return to his Conjunction with the *Sun*. And if the foregoing Year be the third from the *Biſſextile*, then only $2^d\ 7^h\ 37'$. are to be added, then *Mercury* will be found to move 16′ 55″ more to the South.

But after 46 Years, with 12 Intercalations muſt be added $0^d\ 7^h\ 14'$. and there will be had *Mercury* in Conjunction with the *Sun* in a more Southern Path by 2′ 23″. But if the foregoing Year be *Biſſextile*, or the firſt from it, there muſt be added $1^d\ 7^h\ 14'$, that the Conjunction may be had accurately. In like manner after 263 Years, in which *Mercury* will decline towards the South 0′ 22″, there muſt be added $1^d\ 11^h\ 49'$ or $11^h\ 49'$. according to the Rule preſcribed in the former Caſe.

But in 6 or 7 Years, becauſe of the Vicinity of the *Earth* and the Planet, and therefore becauſe of the Arches enlarged at this Node, he does not return to the *Sun* ſo as to appear within his Diſk. But after 33 Years he paſſes the *Sun* at a more northerly Path of 14′ 2″. And the Moment of Conjunction will be had by ſubtracting $3^d\ 0^h\ 25'$, from the time of the former, if it be in the third Year from the *Biſſextile*. Otherwiſe only $2^d\ 0^h\ 23'$ muſt be ſubtracted.

Theſe being found it will be eaſy to continue the Calculation for all theſe Conjunctions of *Mercury* with the *Sun*, and that with the greateſt certainty, and without any doubt whether or no all that are poſſible are included. By Addition alone the Moments of the Conjunctions will be obtain'd, and the Diſtances of the Planet from the Center of the *Sun*; whence alſo by help of the Table the Durations of theſe *Eclipſes*, as they may be called, may be taken out, ſo that nothing may be wanting to compleat this Affair.

As to the *Epochs*, they may be found with more ſafety by the Induſtry of Obſervers, than derived by any ſubtile Calculation. Therefore in the firſt Caſe we have made choice of that notable Tranſit of *Mercury*, which I myſelf made a moſt compleat Obſervation of, in the Iſland of St. *Helena*, *Octob.* 28. *An.* 1677, Old Style, and whoſe Middle I determined from the Beginning and End, to be in the ſaid Iſland at 0^h 4′ Afternoon, but at *London* 0^h 28′ Afternoon. Now the Path which the Planet ſeem'd to obſerve, was 4′ 40″ more northerly than the *Sun*'s Center. In the other Caſe, that is, when *Mercury* is in Conjunction with the *Sun* in the Month of *April*, I have thought fit to take the *Epoch* from the learned *Hevelius's Treatiſe of* Mercury *ſeen in the* Sun, *p.* 72, 75, which is, that *April* 23. *An.* 1662, Old Style, at 6^h 8′ Afternoon at *Dantzick*, that is, at 4^h 52′ at *London*, *Mercury* appeared very near the Center of the *Sun*, as being in the Middle of his Tranſit, and at the ſame Time was diſtant from that Center 4′ 27″ to the North. Hence by the foregoing Precepts it will be a Work of but little Trouble, to exhibit in order all the viſible Conjunctions of *Mercury* with the *Sun* at the ſame Time. And for an Example that any one may imitate hereafter, here are all the Phenomena of this kind, that have ever appeared this preſent Age ſince the Invention of the Teleſcope, or that will appear to Poſterity in the Age following.

A Series of the Moments at which Mercury *is ſeen in Conjunction with the* Sun, *and within his Diſk, for the preſent and the next Age, with the Diſtances of the ſame Planet from the* Sun's *Center.*

In *APRIL.*

Years.	*Times of the Conjunction.*				*Diſtances from the* Sun's *Center.*		
	d.	h.	′		′	″	
1615	22	21	38	*	7	20	N.
1628	25	5	15	*	9	35	S.
1661	23	4	52	*	4	27	N.
1674	26	12	29		12	28	S.
1707	24	12	6		1	34	N.
1720	26	19	43	*	15	21	S.
1740	21	11	43		15	36	N.
1753	24	19	20	*	1	19	S.
1786	22	18	57	*	12	43	N.
1799	26	2	34	*	4	12	S.

In OCTOBER.

Years	*Times of the Conjunction.*				*Diſtances from the* Sun's *Center.*		
	d.	h.	′		′	″	
1605	22	8	29		12	48	South
1618	25	2	4	*	4	45	S.
1631	27	19	37	*	3	18	North
1644	30	13	11		11	21	N.
1651	23	13	20		11	26	S.
1664	25	6	54	*	3	23	S.
1677	28	0	28	* *	4	40	N.
1690	30	18	2	*	12	43	N.
1697	23	18	11	*	10	4	S.
1710	26	11	45		2	1	S.
1723	29	5	19	*	6	2	N.
1730	22	5	28		16	45	S.
1736	30	22	53	* *	13	5	N.
1743	24	23	2	* *	8	42	S.
1756	26	16	36		0	39	S.
1769	29	10	10		7	24	N.
1776	22	10	19		15	23	S.
		Nov.					
1782	1	3	44	*	15	27	N.
		Oct.					
1789	25	3	53	*	7	20	S.

Thoſe Tranſits which have the Mark *, are but partly viſible at *London*; but thoſe which are marked * *, are totally viſible.

Now it is to be obſerved, that at the Aſcending Node of *Mercury* in the Month of *October*, the Diameter of the *Sun* takes up 32′ 34″, and therefore the greateſt Duration of a Central Tranſit is 5ʰ 29′. But in the Month of *April* the Diameter of the *Sun* is 31′ 54″, whence by reaſon of the ſlower Motion of the Planet, there ariſes the greateſt Duration 8ʰ 1′. Now if *Mercury* approaches obliquely, theſe Durations become ſhorter on account of the Diſtance from the Center of the *Sun*. And that the Calculation may be made more perfect, I have added the following Tables, in which are exhibited the half Durations of theſe *Eclipſes*, to every Minute of the Diſtance ſeen from the Center of the *Sun*. Theſe added to or ſubtracted from the Moment of Conjunction found by the foregoing Table mark out the Beginning and End of the whole Phenomenon.

In

OCTOBER.

The Diff. in Min.	*The half duration*
′	h. ′
0	2 44½
1	2 44
2	2 43
3	2 41½
4	2 39½
5	2 36½
6	2 33
7	2 28½
8	2 23
9	2 17
10	2 10
11	2 1
12	1 51
13	1 39
14	1 24
15	1 4
15½	0 50
16	0 30

APRIL.

The Diff. in Min.	*The half duration*
′	h′ ′
0	4 0½
1	4 0
2	3 58½
3	3 56
4	3 53
5	3 48½
6	3 43
7	3 36
8	3 28
9	3 18½
10	3 7
11	2 54
12	2 38
13	2 19
14	1 55
15	1 21½
15½	0 56

Theſe Numbers truly repreſent all the Obſervations hitherto made, nor have I any reaſon to doubt of the future, ſince of all the Planets *Mercury* being neareſt the *Sun*, approaches ſo near to his Center, that it is leaſt diſturb'd by the Intervention of other Centers; nor is it ſenſibly affected by thoſe Deviations which ariſe from the Syſtem of the reſt, to which the ſuperior Planets, and eſpecially *Saturn*, is much expoſed.

I have purpoſely omitted the Parallaxes as very ſmall, and which being different in different Places, ought not to be admitted in a general Calculation. And likewiſe becauſe it is not yet certain how much they are, but rather may be very ſafely derived from ſuch Obſervations. Neither have I taken any Account of *Mercury*'s Diameter, becauſe being incredibly ſmall, he ſeems to adhere to the *Sun*'s Limb only a very few Minutes. From a moſt accurate Obſervation I have found, that hardly two Minutes were over whilſt he wholly came out of the *Sun*, *Octob.* 28. 1677. whence I concluded his Diameter to be

$0' \, 11''$; and according to the ratio of the Diſtances from the *Earth*, at the other *Node* it was almoſt $0' \, 13''\frac{1}{2}$. Therefore then $3\frac{1}{2}$ Minutes of Time were ſpent, whilſt the whole Planet directly paſs'd through the Limb of the *Sun*. But when he paſſes obliquely, he continues ſomething longer, according as the Secants of the Angles of Incidence are encreaſed. Alſo there is hardly any need that we ſhould make any Eſtimate of the Equations of Time; becauſe a great many Days they continue conſtant, and as it were invariable on both ſides in each Month.

Of the viſible Conjunction of Venus *with the* Sun.

Tho' *Venus* is the moſt beautiful of all the Stars, yet like the reſt of her Sex, ſhe does not care to appear in ſight without her borrowed Ornaments, and her aſſumed Splendor. For the confined Laws of Motion envy this Spectacle to the Mortals of a whole Age, like the Secular Games of the Ancients; tho' it be far the moſt noble among all thoſe that Aſtronomy can pretend to ſhow. Now it ſhall be declared hereafter, that by this one Obſervation alone, the Diſtance of the *Sun* from the *Earth* may be determined with the greateſt Certainty, which hitherto has been included within wide Limits, becauſe of the Parallax which is otherwiſe inſenſible. But as to the Periods, they cannot be deſcribed ſo accurately as thoſe of *Mercury*, ſince *Venus* has been obſerved within the *Sun*'s Diſk but once ſince the Beginning of the World, and that by our *Horrox*. Take here the Sum of the Calculation, the Motions being corrected, as far as was poſſible by the rude Obſervations of the Ancients.

	s.	°	′	″
The Longitude of the *Aſcending Node* of *Venus* from the firſt Star of *Aries* ——	1	15	16	00
Therefore the *Sun* is in Conjunction with it in the oppoſite Point, that is, for ſome Ages about the End of *November*	7	15	16	00
The Diſtance of *Venus* from the *Sun* in Parts —— 71997				
The Diſtance of *Venus* from the *Earth* —— 26438				
The Inclination of the Orbit of *Venus* to the *Ecliptic* ——	0	3	23	0
The Motion of *Venus* in 8 *Sidereal* Years, above 13 Revolutions	0	1	30	$28\frac{1}{2}$
The Motion of *Venus* in 235 *Sidereal* Years, above 381 Revolutions ——	11	29	17	39
The Motion of *Venus* in 243 *Sidereal* Years, above 395 Revolutions ——	0	0	48	8

From theſe Principles a Calculation being made, according to the Method explain'd in *Mercury*, the Intervals of Times and Diſtances will come out as follows.

After 18 Years *Venus* returns to the *Sun*, taking away $2^{d} \, 10^{h} \, 52'\frac{1}{2}$; from the Moment of the foregoing Tranſit; and the Planet proceeds in a Path which is $24' \, 41''$. more to the South than the former.

After 235 Years adding $2^{d} \, 10^{h} \, 9'$, *Venus* may again enter the *Sun*, but in a more northern Path by $11'\ 33''$. But if the foregoing Year is *Biſſextile*, $3^d\ 10^h\ 9'$ muſt be added.

After 243 Years *Venus* may alſo paſs the *Sun*, only taking away $0^h\ 43'$ from the Time of the former ; but procceeds more ſoutherly by $13'\ 8''$. Now if the foregoing Year be *Biſſextile*, add $23^h\ 17'$.

And in all theſe Appulſes of *Venus* to the *Sun*, in the Month of *November*, the Angle of her Path with the *Ecliptic* is 9° 5′, and her Horary Motion within the *Sun* is $4'\ 7''$. And ſince the Semidiameter of the *Sun* is $16'\ 21''$, the greateſt Duration of the Tranſit of the Center of *Venus* comes out $7^h\ 56'$.

Then let the *Sun* and *Venus* be in Conjunction at the *Deſcending Node* in the Month of *May*; and by the ſame Numbers the ſame Intervals may be computed. After 8 Years let there be taken away $2^d\ 6^h\ 55'$. And *Venus* will make her Tranſit in a more northern Path by $19'\ 58''$.

After 235 Years add $2^d\ 8^h\ 18'$, or if the foregoing Year be *Biſſextile* $3^d\ 8^h\ 18'$, and you will have *Venus* more to the South by $9'\ 21''$.

Laſtly after 243 Years add $0^d\ 1^h\ 23'$, or if the foregoing Year be *Biſſextile*, $1^d\ 1^h\ 23'$, and *Venus* will be found again in Conjunction with the *Sun*, but in a more northerly Path by $10'\ 37''$.

In every Tranſit within the *Sun* at this *Node*, the Angle of *Venus*'s Path with the *Ecliptic* is 8° 28′, and her Horary Motion is $4'\ 0''$ and the Semidiameter of the *Sun* ſubtending $15'\ 51''$, the greateſt Duration of the central Tranſit comes out alſo $7^h\ 56'$ exactly the ſame as at the other *Node*.

As to the *Epochs*, from that only Ingreſs which *Horrox* obſerved, the *Sun* being then juſt ready to ſet, it is concluded, that *Venus* was in Conjunction with the *Sun* at *London* in the Year 1639, *Nov.* $24^d\ 6^h\ 37'$, and that ſhe declined towards the South $8'\ 30''$. But in the Month of *May* no Mortal has ſeen her as yet within the *Sun*. But from my Numbers, which I judge to be not very different from the Heavens, it appears that *Venus* for the next Time will enter the *Sun* *An.* 1761. *May* $25^d\ 17^h\ 55'$, that being the Middle of the *Eclipſe*, and then will be diſtant from his Center $4'\ 15''$, towards the South. Hence and from the foregoing Revolutions all the Phenomena of this kind will be eaſily exhibited for a whole Millennium, as I have computed them in the following Table.

In *NOVEMBER*.

Years.	*Times of the Conjunction.*			*Dist. from the Cent. of the* Sun.		
	d.	h.	′	′	″	
918	20	21	53	6	12	N.
1161	20	21	10	6	55	S.
1396	23	7	20	4	38	N.
1631	26	17	29	16	11	N.
1639	24	6	37	8	30	S.
1874	26	16	46	3	3	N
2109	29	2	56	14	36	N.
2117	26	16	3	10	5	S.

In *MAY.*

Years.	*Times of the Conjunction.*			*Diſt. from the Cent. of the* Sun.		
	d.	h.	′	′	″	
1048	24	13	45	3	50	N.
1283	23	8	14	5	31	S.
1291	25	15	9	14	27	N.
1518	25	16	32	14	52	S.
1526	23	9	37	5	6	N.
1761	25	17	55	4	15	S.
1769	23	11	00	15	43	N.
1996	28	2	13	13	36	S.
2004	25	19	18	6	22	N.

As for the Durations of theſe *Eclipſes* of *Venus*, they may be computed after the ſame Manner as thoſe of *Mercury* in reſpect of the Center. But ſince *Venus*'s Diameter is pretty large, and ſince the Parallaxes alſo may bring a very notable Difference as to Time, a particular Calculation muſt neceſſarily be made for every Place.

Now the Diameter of *Venus* is ſo great, that while ſhe adheres to the *Sun*'s Limb almoſt 20 Minutes of Time will be elapſed, that is, when ſhe applies directly to the *Sun*. But when ſhe is incident obliquely, ſhe continues longer in the Limb. Now that Diameter, according to *Horrox*'s Obſervation takes up 1′ 18″, when ſhe is in Conjunction with the *Sun* at the *Aſcending Node*, and 1′ 12″ at the other *Node*. Now the chief uſe of theſe Conjunctions is accurately to determine the *Sun*'s Diſtance from the *Earth*, or his Parallax, which *Aſtronomers* have in vain attempted to find by various other Methods; for the Minuteneſs of the Angles required eaſily elude the niceſt Inſtruments. But in obſerving the Ingreſs of *Venus* into the *Sun*, and her Egreſs from the ſame, the Space of Time between the Moments of the internal Contacts, obſerved to a Second of Time, that is, to $\frac{1}{10}$ of a Second or 4‴ of an Arch, may be obtain'd by the Aſſiſtance of a moderate Teleſcope and a Pendulum Clock, that is conſiſtent with itſelf exactly for 6 or 8 Hours. Now from two ſuch Obſervations rightly made in proper Places, the Diſtance of the *Sun* within a five hundredth Part may be certainly concluded, as I ſhall ſhew at another Opportunity.

Fig. 154, 155. Leſt any thing ſhould ſeem obſcure to a Reader not much verſed in *Aſtronomical* Matters, I have delineated Schemes for the Tranſit of each Planet, by which I have endeavour'd to repreſent every thing to view.

The Motion of the Comet, A. 1664. *Predicted; by M.* Auzout. n. 1. p. 3. n. 2. p. 18.

CI. 1. M. *Auzout*, after he had ſeen the *Comet* (which was firſt obſerved in *Holland Decemb.* 2 1664.) 4 or 5 times, made the *Ephemerides* of its Motion upon an *Hypotheſis* that it moveth juſtly enough in the Plane of a great Circle, which inclineth to the *Equinoctical* about 30°, and to the *Ecliptick* 49° or 49°½, cutting the *Æquator* at about 45°½, and the *Ecliptick* at 28° of *Aries* or a little more.

n. 2. p. 19.

He takes notice, that more *Comets* enter into our Syſtem by the Sign of *Libra* and about *Spica Virginis* than by all the other parts of the Heavens: For, both the preſent *Comet* and many others regiſtred in *Hiſtory* have entered that way, and conſequently paſs'd out of it by the Sign of *Aries*; by which alſo many have entered.

Obſerved; by M. Auzout. ib.

2. Till the 6*th* of *Feb.* this *Comet* always advanced: But after that Day, I found that it returned in augmenting always its Latitude. I left it *Mar.* 8. at the 18. of the *Horn* of *Aries*, almoſt in the ſame Latitude; and I am apt to believe it will be *Eclipſed* this Evening,

I ſhall only add, that on *Feb.* 3. we were ſurprized, to ſee the *Comet* again much brighter than ordinary, and with a conſiderable *Train*. Some did believe that it approached again to us. But having beheld it with a Teleſcope, I ſoon ſaid, that it was joined with two ſmall Stars, whereof one was pretty bright, and that this *Conjunction* gave the *Comet* that Brightneſs. Hence it was, that I aſſur'd my Friends here, that we ſhould no more ſee it ſo bright.

n. 6. p. 107.

M. *Auzout* alſo ſtrongly conceives, that this *Comet* could not be *Feb.* 18 *ſt. n.* where M. *Hevelius*, in his *Prodromus Cometicus*, hath placed it, viz. *in prima Arietis*; unleſs it be ſaid that it viſited that Star of *Aries* on the 18th, and returned thence the 19th, into its ordinary Courſe: For, according to his, and his ſeveral Correſpondents Obſervations, the *Comet* on *Feb.* 17. was diſtant from that *Firſt Star of Aries* at leaſt 1° 17; and on *Feb.* 19. (He having miſſed, as well as his other Friends, the Obſervation on *Feb.* 18.) was advanced in its way 12′ or 13′, but yet diſtant from the ſaid *Star* ſome Minutes above a whole Degree, and conſequently far from having then paſſed it. After which time M. *Auzout* affirms to have ſeen it as well as ſeveral others, for many Days, and that until *Mar.* 17. obſerving, that about *Feb.* 26 or 27, when the *Comet* was neareſt to the often mentioned *Firſt of Aries*, it approached not nearer than 50′.

By ſome Engliſh Aſtronomers.
p. 108. n. 9.
p. 150.

3. Some Eminent *Engliſh Aſtronomers*, who have attentively obſerved the Poſition of this *Comet*, do jointly conclude, that whatever that Appearance was, which was ſeen near the *Firſt Star of Aries* by M. *Hevelius* (the Truth of whoſe Relation concerning the ſame they do in no wiſe queſtion) the ſaid *Comet* did not come near that Star in the *left Ear of Aries*, where the ſaid M. *Hevelius* ſuppoſes it to have paſſed, but took its Courſe near the *Bright Star* in its *Left Horn*, according to *Bayer*'s Tables.

The Principle of M. Auzout's *Hypotheſis; and the Motion of that* Comet *obſerved by M.* Caſſini.
n. 2. p. 17.

4. I have eaſily found the Principle of M. *Auzout's Ephemerides:* and 'tis this, that this *Comet* moves about a Centre, in a ſtraight Line drawn from the Earth thro' the *Great Dog*, in ſo great a Circle, that that Portion which is deſcribed, is exceeding ſmall in reſpect of the whole Circumference thereof, and hardly diſtinguiſhable by us from a ſtraight Line.

Concerning the *New Comet* you mention, I obſerv'd it *Feb.* 11, about the 24° of *Aries*, with a Northern Latitude of 24° 40′.

The Motion of the Comet A. 1665. predicted; by M. Auzout, n. 3. p. 36.

CII. 1. M. *Auzout*, after 3 or 4 Obſervations, hath publiſhed another *Ephemerides* concerning the Motion of the *Comet*, which he firſt began to obſerve *Apr*. 2. 1665. He affirms that the Line deſcribed by this *Star* reſembles hitherto a great Circle, as it is found in all other *Comets* in the midſt of their Courſe. He finds the ſaid Circle inclined to the *Ecliptick* about 26°. 30'. and the *Nodes* where it cuts it, towards the beginning of *Gemini* and *Sagittary*: that it declines from the *Æquator* about 26°, and cuts it towards the 11°; and conſequently that its greateſt Latitude hath been towards *Piſces*, and its greateſt Declination towards the 25° of the *Æquator*. He puts it in its *Perigee* about 15° of *Piſces*, a little more Weſterly than *Marchab*, or the *Wing of Pegaſus*.

Obſerved; by M. Auzout. *ib.* p. 37.

2. He obſerves in General, that this *ſecond Comet* is contrary to the precedent almoſt in all Particulars: Seeing that the former moved very ſwift, this pretty ſlow; that againſt the Order of the *Signs* from Eaſt to Weſt, this, following them, from Weſt to Eaſt; that, from South to North, this, from North to South, as far as it hath been hitherto, that we hear of, obſerved: that, on the ſide oppoſite to the *Sun*, this on the ſame ſide; that having been in its *Perigee* at the Time of its Oppoſition, this having been there, out of the time of its Conjunction. He taketh alſo notice, that this *Comet* differs in Brightneſs from the other, as well in its *Body*, which is far more vivid and diſtinct, as in its *Train*, whoſe Splendor is much greater, ſince it may be ſeen even with great Teleſcopes, which were uſeleſs in the former by reaſon of its Dimneſs.

A Comet. An. 1668. *at* Bononia; *by M.* Caſſini. n. 35. p. 683.

CIII. 1. *Ann*. 1668. *Mar*. 10^d 1^h of the following night (after the *Italian* way of counting) I obſerved a Path of Light extended from the *Whale* thro' *Eridanus*; which I judged to be the *Train* of a *Comet* both by the Figure and Colour, as alſo becauſe that the Direction of it was to the part oppoſite to the *Sun*, like other *Comets*. By its extreme Point it reached to that Star in *Eridanus*, which is called the 14 by *Bayerus*: But it iſſued out of the Horizontal Clouds, ſo that I thought the *Head* of the *Comet* was either veiled by them, or hid under the *Horizon*. *Mar* 11. there was ſeen a Brightneſs in the *Whale*, amongſt the thin Clouds, at leaſt for half an Hour, which was very like the Splendor of *Venus*, likewiſe veiled with thin Clouds.

Mar. 12. When the *Great Dog* was in the *Mid-heaven*, the ſame *Tail* appeared again. It paſſed thro' the Star in *Eridanus*, which *Bayerus* calls the 15, and left to the Southward the 14, where it did terminate *Mar*. 10. Being by the Imagination drawn out to about 3°, and further, it tended to that Southern Star which precedes the *Ear* of *Lepus*. It was therefore more Northerly than the Day before Yeſterday, and more Eaſterly. We were doubtful whether its Head was hid by the Clouds or under the Horizon. But the Line from *Jupiter* to the Extremity of the ſaid *Tail* in the Clouds was perpendicular to that *Tail*; ſo that it was in the *Whale*, and the apparent part of the *Train* reached out in Length about 32°.

At Lisbon; *by* ... p. 684.

2. *Mar* 5. *ſt. n.* The *Comet* was firſt diſcovered: but for as much as it ſet few hours after the *Sun*, there could hitherto be taken no conſiderable Obſervations of it. The *Body* thereof is not ſeen, becauſe it remains hid in

the Horizon. Its *Train* is of a ſtupendious Length, extended in Appearance over almoſt the 4th part of the viſible Heaven, from Weſt to Eaſt; its apparent Breadth is of a good *Palm*, and its Splendour very great, but it laſts but a few Hours.

At St. *Salvador Mar*. 5. (*ſt. n.*) at 7 a Clock at Night, *F. Eſtancel* began to ſee this *Comet* a little above the Horizon from Weſt to E.S.E. The beginning of its *Tail* was a little under the two lucid Stars, the 15 and 16th of the *Whale's Back*, over which it then paſſed, its Point being as 'twere at the 8 and 9th which are at the bottom of the *Whale's Belly*; and thus the whole Length thereof was about 23°. The *Globe* or *Head* of it was ſo ſmall and thin, that very few could diſcern it with the naked Eye. *In Braſil; by P. Valentine Eſtancel. n. 150 p. 91.*

Mar. 7. The former Brightneſs was ſomewhat leſs, and become ſo thin, that the Eye could eaſily ſee the Stars that were behind it, which by Conjecture were the 14 and 20th.

The *Tail* was always directly oppoſite to the *Sun*; and when it appear'd the firſt Time almoſt Horizontal, it was ſeen in the form of a Pillar, the *Head* ſtanding a little under, and on the ſide of the Star of the *Whale*, which is in the Lat. of 15° 46′. and the Long. of 12° 42′. of *Aries*. And the *Point* did ſhave the 14th North of the three that are in the *Belly*, in the Lat. of 20°. 30′. and Long. of 15° 57′. of *Aries*.

This *Comet* was at firſt very ſplendid, and caſt itſelf with that Vividneſs upon the Sea, that the Rays thereof were reverberated unto the Shoar, where the Obſervers ſtood. But this Brightneſs laſted only for three Days, after which it did conſiderably decay. But that which ſeemed ſomewhat ſtrange was, that having loſt ſo much of its Light, yet its Bulk was not diminiſhed, but continued rather increaſing until the *Comet* diſappear'd. It paſs'd more ſwiftly than *Venus*, whence he infers that it was under *Venus*: yet the Anticipation was not ſo great, that it could be believ'd to be under the *Moon*, as he would have it.

4. *P. Pietro Suſarte*, Rector of *Macao*, in the *Eaſt-Indies*, well verſed in Matters *Aſtronomical*, writes to have ſeen the ſame all along the Coaſt of *Bona Speranza*. *In Africa; by P. Pietro Suſarte. ib.*

CIV. There hath been ſeen here a New *Comet* from the 2d of *Mar. ſt. n.* 1671. It is but little, having a *Train* not above a Degree or a Degree and an half long. It is now (*Mar*. 9.) about the Stars in the *Right Arm* of *Andromeda* on her Shoulder Blade. As far as I can collect from one or two Obſervations, it tends towards the *Lucida* of *Andromeda*'s *Girdle*, and that with a direct diurnal Motion of about 2 Degrees in its Courſe. *A Comet. An. 167$\frac{1}{2}$. at Dantzick; by M.* Hevelius. *n.* 81. p. 40 17.

The 6th of *March* in the Evening 7h 40′ it was in 7° ♈. and in 35° of Northern Latitude; as I gueſs'd by the haſty Inſpection of a Globe.

Mar. 7. in the Morning 3h 30′, its Longitude was about 8° ♈. with a ſomewhat leſſer Latitude than before: in the Evening of the ſame Day it's Longitude was 10°. ♈. and Lat. 34° *ferè*.

Mar. 8. In the Morning 4h, the Long. was 12° ♈. and the Lat. 33° Which yet I would not have taken preciſely, becauſe I cannot yet reduce my Obſervations to a *Calculus*.

2. Mr.

At . . . . . . by Mr. Newton. *ib.* p. 4018.

2. Mr. *Iſaac Newton* about the 16 of *March. ſt. V.* ſaw a dull Star South-Weſt of *Perſeus*, which he now takes to have been that *Comet* It was very very ſmall, and had not any viſible *Tail*, which made him regard it no further.

At Paris *by M.* Caſſini. n. 81. p. 4018. n. 82. p. 4042.

3. The *Mathematicians* of *La Fleſche* perceived him from the 16 of *March, ſt. n.* and gave us here at *Paris* the firſt notice of it. Thoſe of the *College* of *Clermont* being advertiſed of it, ſaw him the 25 of the ſame Month.

Mar. 26. 7^{h} 30′ in the Evening, M. *Caſſini* ſaw him between the *Head* of *Meduſa* and the *Pleiades*; without a Teleſcope he appeared no otherwiſe than a Star of the 3^{d} Magnitude; his *Head* ſeen with a Teleſcope of 17 Foot, appear'd almoſt round; but it was well defined, and diſtinguiſh'd from the Miſtineſs, which formed a kind of *Chevelure*, wherewith it was encompaſſed; and even the middle was a little confuſed, and ſeemed to have Inequalities, as are ſeen in Clouds..

The *Tail* was almoſt imperceptible; yet by the Teleſcope it was ſeen turned oppoſite to the *Sun*, and it appeared of the Length of two Diameters of the *Head* or thereabout: For it was not eaſy to meaſure it preciſely, becauſe being thinner according as it was farther from the *Head*, its Extremity was inſenſibly loſt, And ſo the whole *Comet*, *Head*, *Tail*, and *Chevelure*, taken all together, took up no more than 3 or 4 Minutes of a Degree. At 7^{h} 48′, he was in a ſtraight Line with the *Lucida* in the *Head* of *Meduſa*, and with the moſt *Occidental* one of the *Pleiades*, and above the two cleareſt Stars of the *Southern Foot* of *Perſeus*; ſo that a ſtraight Line drawn thro' theſe two Stars, did almoſt touch the Southern Extremity of his *Chevelure*. This Place of the *Comet*, transferred upon the Map of the fix'd Stars, fell preciſely enough upon 23° 25′ of *Taurus*, in 15° Northern Latitude.

With a Teleſcope 3 Foot, we ſaw near the *Comet* two ſmall Stars diſtant one Diameter of the *Sun* from one another, which Stars are not in the Catalogues. The *Comet* was in a Straight Line, drawn from one of thoſe two Stars to the other perciſely at 9^{h} 15′. but a little nigher to that which was Weſtward: But 9^{h} 33′, he was equally diſtant from them both. It was taken notice of, that from 8^{h} 5′ till 10^{h} 26′, *He* made, in reſpect of theſe two Stars, an oblique Motion ſenſible enough, going from North to South in the ſame time that he advanced from Weſt to Eaſt.

Mar. 28. 7^{h} 42′. in the Evening, the *Comet* was diſtant from the leſs bright Star of the *Southern Foot* of *Perſeus*, no more than about 24′ Weſtward. He had almoſt the ſame Latitude with this Star; ſo that he was preciſely enough at 26° 8′ ♉, and in the Lat. 12° 8′. At 8^{h} 14′ we took, as well as we could, the Diſtance of the *Comet* to the Star in the Eye of *Taurus*, called *Aldebaran*, 19° 38′. and 8^{h} 29′. The Diſtance of the *Comet* to the Star called *Cappella*, was found to be of 22° 32′.

Mar. 30. 9^{h} 35′, at Night, the *Comet*, ſeen without a Teleſcope, appeared no otherwiſe than a Star of the fourth Magnitude: thro' the Teleſcope he exceeded even thoſe of the Firſt; but he was very dark, and in what manner ſoever we look'd upon him, we could obſerve almoſt no *Tail* at all of him. He had paſſed one Degree and an half beneath the *Lucida* of the *Southern Foot* of *Perſeus*; ſo that this Star was exactly in the midſt of the *Comet* and the little Star of the *Leg* of *Perſeus*, marked η by *Bayerus*, which then we ſaw not but by a Teleſcope. A ſtraight Line drawn from one of theſe Stars to the other, did almoſt touch the *Southern Limb* of the *Comet*, which being transferred upon the Map of the *Fix'd Stars*, fell upon 28° 45′ of *Taurus*, in the Northern Latit. of 9° 56′. At 9^{h} 45′, the Weſtern Limb of the *Comet* touched a ſtraight Line, drawn thro' this leſs bright Star of *Perſeus*'s *Southern Foot*, and thro' the moſt Northern of the Head of *Taurus*; but that he was already got ſomewhat nearer to the Latter.

Mar. 31. 8^{h} in the Evening, the *Comet* was in a direct Line with the *Lucida* in the *Foot* of *Perſeus*, and with the moſt Northern in the *Head* of *Taurus*; but he was more than twice as much remoter from the firſt than the other, and being transferred upon the Map of the *Fix'd Stars*, he was found at 15′ from *Gemini*, in the Latit. of 8° 49′. During the whole time that we could obſerve him this Night (which was till 10 a Clock) he quitted not this ſtraight Line, which was almoſt parallel to the Horizon: notwithſtanding that his own particular motion ſhould raiſe him a little above it; as the Parallax, on the contrary, ſhould ſink him beneath it in approaching to the Horizon. It may be, there was a compenſation made of theſe two contrary Motions: poſſibly alſo the Effect of both was not ſenſible.

April. 1. The *Comet* could not be ſeen without a Teleſcope, becauſe the *Moon*, being very near it, hid him from our ſight. But with a Teleſcope only of one Foot we diſcerned him eaſily enough, and found that he had paſſed 46′ beyond the moſt Northern Star of the *Head* of *Taurus*, and that he muſt have touch'd it by his Southern Limb; as alſo that he was diſtant 1° 43′, from the Star that was neareſt to that toward the South; which is equally bright, yet not marked by *Bayerus*. This place being transferred upon the Map of the *Fix'd Stars*, we found that he was at 1° 30′ of *Gemini*, in the Northern Latit. of 7° 44′.

April. 2. 8^{h} in Even. M. *Caſſini*, having obſerved the *Comet* with a Teleſcope of one Foot, which diſcovered 5°, found that he was two Deg. and half diſtant from the moſt Northern Star of *Taurus*; and one Deg. from the Star of the *Ear* marked φ by *Bayerus*, and by *Tycho* called *Sequentis Lateris Borei*.

Two Lines drawn from the moſt Northern Star of *Taurus*, one to the *Comet*, the other to the Star that is wanting in *Bayerus*, made a Right Angle; and the Diſtance of the *Comet* to this Angle, was double to that which is between theſe two Stars. This place transferred upon the Map of the *Fix'd Stars* fell on 2° 48′ of *Gemini*, in the Northern Latit. of 6° 40′.

April 3. 9^{h}, we ſaw him with the one Foot Teleſcope. He had paſſed over the upper Star of the *Ear* of *Taurus*, and he made with this Star the *Baſis* of an *Iſoſceles* Triangle, on the Top whereof was the inferior Star of the *Ear*. The two Sides of this Triangle were two times and an half bigger than the *Baſis*; ſo that the *Comet* was 4° of *Gemini*, in the Northern Latit. of 5° 38′.

Apr. 5. 8^{h} at Even, the *Comet* had paſſed the Northern *Ear* of *Taurus*, and was equally diſtant from the Upper Star of the Northern *Ear* and from that which was on the *Front* of *Taurus*. He was alſo as diſtant from the Inferior

Star of the *Ear* of *Taurus*, as this Star is from the next Weſtward, by *Tycho* called *Inferior præcedentis Lateris Quadrilateri*; and a ſtraight Line, drawn thro' the *Comet* and the upper Star of the *Ear*, made an almoſt Right Angle with another Line, drawn from the *Comet* to the Inferior of the two Stars, that are above the *Eye* of *Taurus*. This Place being carried over to the Map of the *Fix'd Stars*, the *Comet* was found at 6° 18′ of *Gemini*, in the Northern Latit. of 3° 41′. He was ſo confuſed this Night, that even with the 17 Foot Teleſcope we could not exactly diſtinguiſh the *Head* from the *Chevelure* which environed him. The whole appeared a little bigger than the Diſk of *Jupiter*, ſeen by the ſame Teleſcope.

Apr. 6. 8^h at Even, a ſtraight Line drawn from the *Comet* to the Star that is in the *Front* of *Taurus*, made a Right Angle with another ſtraight Line drawn from this ſame Star to the Inferior of the two that are above the *Eye*: and the Diſtance of this latter Star to that of the *Front* of *Taurus* was twice the Diſtance of the ſame Star of the *Front* of *Taurus* to the *Comet*. This Place being transferred upon the Map of the *Fix'd Stars*, the *Comet* was found at 7° 25′ of *Gemini*, in the Northern Latit. of 2° 45′. At 9^h 6′ we ſaw on the ſide of the *Comet* a Star ſufficiently clear, which was not farther diſtant from him than a little more than the Diameter of the *Comet*, and that was at the ſame Height of the Horizon.

Apr. 7. 9^h in the Evening, the *Comet* was equally diſtant from the Inferior Star of the *Northern Ear* of *Taurus*, and from the Superior of the Root of the *Northern Horn*. He was alſo as far diſtant from this latter Star, as this Star is from that of the *Front*. This Place being carried over to the Map of the *Fix'd Stars*, fell on 8° 30′ of *Gemini*, in the Northern Latit. of 1° 56′.

All the Places of the *Comet*, that we have obſerved till now, fall into a Line little differing from an Arch of a great Circle, which cuts the *Ecliptick* in 10° 45′ of *Gemini*, and which conſequently hath its greateſt Latitude in 10° 45′ of *Piſces*; which Latitude is between 39° and 40° Northward. The ſame Circle cuts the *Æquator* at 101 degrees of the *Vernal Section* Eaſtward; and its greateſt *Declination* from the *Æquator* Northward is of 38°½.

Having choſen two of our Firſt Obſervations (becauſe the latter are not ſo proper for this Purpoſe) and having taken a Mean between the firſt Obſervations of the *Mathematicians* of *La Fleſche*, we found, by our *Method* explained in the Theory of the *Comet* of 1665, that this *Comet* had been in his *Perigee* the 12 of *March* at 8 a Clock in the Morning: that in that time, which is that of his greateſt apparent Celerity, he made about 2° 32′ a day in the great Circle of his apparent Motion, and $\frac{444}{10000}$ of his *Perigee* Diſtance in the Line of his Equal Motion: that he was in his greateſt Declination the 11*th* and 12*th* of *March*; and that at that time, he paſſed thro' the Inferior Meridian at about two a Clock after Midnight.

If we have rightly determined his *Perigee*, and that the *Hypotheſis* of the Equality of his Motion be juſt for that time, he hath been viſible ſince the Middle of *February*, at which time he was as far diſtant from his *Perigee* by Approaching to the *Earth*, as he is at preſent by Receding from it. He muſt then have been at the extremity of the *Southern Wing* of the *Swan*, and arrived at the *Southern Foot* of *Pegaſus* on the 23 of *Febr.* of the ſame bigneſs that he was ſeen to be of, *Mar.* 28. He muſt have arrived at the Stars of the *Northern Arm* of *Andromeda*, *Mar.* 9. at thoſe of her *Girdle*, 12. when he was in his *Perigee*, and in his greateſt Declination; to her *Southern Leg*, *Mar.* 15. between her *Southern Leg* and the *Triangle*, *Mar.* 18. very near as he was obſerved at *La Fleſche*; and under the *Head* of *Meduſa*, *Mar.* 25. The Days enſuing he muſt have arrived at the Places marked in our Firſt Obſervations: But in the laſt he hath been ſwifter than this *Hypotheſis* will bear. To repreſent theſe latter Obſervations, the Line of the Motion ought to have been made Curve, as we did for the end of the Apparent Motion of the *Comet* 1665. with this difference, that inſtead of that Line's being Convex in regard of the *Earth*, becauſe the Motion was *Retrograde*, this was to be Concave towards the *Earth*, becauſe that the Motion of this *Comet* is *Direct*.

It's a Thing worth obſerving, that this *Comet* keeps his Courſe almoſt like that of the 2 *Comet* of 1665, and another of 1577, obſerved by *Tycho*. For they have paſſed thro' almoſt the ſame *Conſtellations*; tho' this be more inclined Northward, and cut the *Ecliptick* 5 or 6 Degrees more forward than that of 1665. So that it ſeems that in this Place of the Heavens there is, as 'twere, a *Zodiack* for *Comets*.

A Comet, An. 1677. *at* Paris; *by M.* Caſſini n. 135. p. 868.

CV. 1. Mr. *Romer* firſt took notice of the new *Comet*, *Apr.* 28. New Style, 1677. and I being preſently inform'd of this appearance, at 4^{h} 6′ 31″, after Midnight we took its Altitude 12° 22′ 10″. I judged it to be in a Vertical declining from the Eaſt towards the North about 33°. On the 29 in the Morning it was ſeen by Mr. *Picard* for a Moment through the Clouds at 3^{h} 9′ 31″ after Midnight in the Altitude of 4° 39′.

On *May* 2^{d} in the Morning, the Right Aſcenſion of the Midheaven by the fixt Stars being 267°. the Altitude of the Comet was 4° 5′. The Diſtance of the Vertical from the North towards the Eaſt was about 42° 8′. On the fourth Day in the Morning at 3^{h} 30′. after Midnight, the Altitude of the *Comet* was 5° 33′. The *Azimuth* Diſtance from the North to the Eaſt was about 42° 32′.

On the fifth Day at 3^{h} 32′. The Altitude of the *Comet* was 5° 10′. The *Azimuth* Diſtance from North to Eaſt was about 44° 10′.

By theſe Obſervations the *Comet* at firſt was in the Triangle, afterwards near the *Head* of *Meduſa*, and ſhew that it proceeded according to the Series of the Signs, by a Line that was very near and almoſt parallel to that, which was deſcribed by the *Comet* in the Year 1590, in the Month of *February*. The bigneſs of its *Head* ſeen through the Teleſcope was almoſt equal to the Diſk of *Jupiter*, or ſomething leſs. Nor did it appear perfectly round, but of an Oval Figure, the longer Diameter being parallel to the Horizon; which ſeem'd to be owing to the Horizontal Refraction.

Its *Capillitium* ſeen through the Teleſcope was ſomething wide, and nearly Parabolical; but to the naked Eye it ſeem'd narrow, and was a little inflected towards the Weſt.

At Dantzick; *by* M. Hevelius *ib* p. 869.

2. A *Comet* has appear'd lately, which was firſt obſerved here at *Dantzick*, *Apr.* 27. in the Morning. On the 29th it aroſe, or rather appear'd to ſight, at 1^{h} 52′. at North-Eaſt and by North. It had no large *Head*, yet it was

bright enough, conſiſting of one ſhining Nucleus, like that which was ſeen *An*. 1665. It ſtretch'd out a Tail pretty luminous, with Rays divaricating upwards, of almoſt two Degrees. The Line of Direction of the Tail being continued, proceeded between *Alamac* the bright foot of *Andromeda*, and her *Girdle*, and divided the Diſtance of theſe Stars as it were into two equal Parts. It was at that time above the *Head* of *Aries*, in the Triangle between the Apex and the Northern Star in its Baſe, that is, in 5° of *Taurus*, and in 12° North Latitude. At this time it was diſtant from the *Sun* only 5° according to Longitude, but in its own great Circle 20°. Since then this *Comet* was ſo near the *Sun*, it could not ſhew its Tail any longer, tho' in my Opinion it had one much longer, nay, I think in a few Days it will ſhew it much ſhorter.

Apr. 30. it was found in 9° of *Taurus*, with 18° of North Latitude, and almoſt as far from the *Sun*, being in 12° of ♉. It ſpread its Tail again two Degrees or more, to the Northern Star in the Baſe of the Triangle, which Star might plainly be ſeen by good Tubes at the Point of the Tail.

May 1. at 2^{h} 32′. in the Morning it was found in 11° of *Taurus* with North Latitude 18° almoſt in Conjunction with the *Sun*, and being diſtant almoſt as many Degrees from the *Sun*. Its Tail was ſtill bright, but ſomething ſhorter, tho' wider, which it ſtretched out to the bright Foot of *Andromeda*.

From *Apr*. 29. when firſt I obſerved it, to this Day *May* 1. it compleated nearly 5° 30′, by its own proper Motion.

As far as I can collect from theſe Obſervations, it moves with a direct Motion to the left Foot of *Perſeus*, above *Taurus*, to the Feet of *Gemini*, if it will continue long enough. The Deſcending *Node* is about 20° of *Gemini*, (but this is only a rude Conjecture,) and thus it will paſs the *Ecliptic* in that Place, and then will become Southward, with an Inclination of its Orbit of almoſt 27°.

May 2. in the Evening, at 8^{h} 45′ altho' no Stars ſhined out in that Part of the Heavens, and there was an intenſe Twilight, yet I preſently found the *Comet* with my Optical Tube. A little after I found him with 3° 30′ of Altitude. His Tail on account of the Twilight was very thin, which he ſtretched out between each *Knee* of *Caſſiopea*, but nearer to the left. It ſat that Evening at North-north-weſt.

On *May* 3. in the Morning, the *Comet* aroſe at N. N. E. at 1^{h} 23′, tho' the Tail was diſcovered by us ſomething ſooner at 1^{h} 18′. It was at 14° in ♉, almoſt in *Conjunction* with the *Sun*, having 17° of Latitude, and almoſt the ſame Diſtance from the *Sun*. This day it ſhew'd its Tail much longer and better defined, and very bright, of near 2 or 3 Degrees. Thus it was found by myſelf and other quick-ſighted Spectators with the naked Eye, at 3^{h} 34′, and with the Teleſcope at 3^{h} 40′, at the height of 11° 30′; ſo that the *Sun* at that time was depreſſed only 6° below the Horizon; nay, we had ſeen it longer, if ſome litte Clouds had not intervened. His daily Motion ſeem'd to decreaſe, as far as I could judge by Conjecture without any Calculation. For between 29 and 30 of *April* it was near 2° 45′, between *Apr*. 30 and *May* 1. it was 2° 15′, between 1 and 2 of *May* it was 1° 55′, between 2 and 3 *May* it was 1° 40′. But the Obſervations themſelves and the Calculation will ſhew this plainer.

On *May* 4. in the Evening, the Air being very pure, at 8^{h} 53′ the *Comet* was again diſcover'd, but was a little obſcurer than the foregoing Days, and its Tail was ſhorter. On *May* 5 in the Morning at 1^h 41′, projecting its Tail towards the right Knee of *Caſſiopea*, it was in 17° of ♉, with 16° of North Latitude, and at the ſame Diſtance from the *Sun*. Its proper Motion from the 3d to the 5th of *May* was almoſt 2° 40′, the Latitude decreaſing, that is, from the Beginning almoſt to 3°, ſo as from the 29 of *April*. The proper Motion of the *Comet* from *May* 5. was almoſt 12°. On *May* 6, in the Morning its Place was in 18° of ♉, with North Latitude 15° 30′. The *Sun* being then in 17° of ♉. The daily Motion was about 50′. As to its Head, it ſeem'd thinner and weaker than the Tail, becauſe the *Sun* was diſtant not above 16½ Degrees. On *May* 6, in the Evening it was ſeen with the Optic Tube at 8^h 35′. with its Tail ſtill ſhorter and more dilute; but as it was in a lower Situation, and an intenſe Twilight, it could not at all be perceived by the naked Eye.

On *May* 7. it was firſt perceived at 2^h 22′, at the Altitude of 3°, ſo that it ſeem'd to be very thin. It was at that time in 19° of ♉. Its proper Motion decreaſed more and more, as far as could be known without Calculation. On *May* 8. from one in the Morning it was carefully ſought for with the naked Eye, but did not appear. Yet it was found with a 12 foot Teleſcope, having a Tail ſtill, but a very ſhort one, extended a little from the Vertical Circle to the left Hand. As far as I could gueſs it was in 20° of ♉, at the Diſtance of 15° from the *Sun*, which was then in 19° of ♉. At this time it was nearly in a right Line with the right Shoulder of *Perſeus*, and *Algol* of *Meduſa*. The Diameter of the *Comet* compared with that of *Jupiter* hardly came up to half. As to the reſt, it was ſtill conſpicuous enough by help of the Tube, ſo that at 3^h 45′. I could ſee it diſtinctly, at the Altitude almoſt of 9°, whence we may collect, that the Arch of Viſion was then hardly 5°. For then the *Sun* was hardly 5° below the Horizon, at what time all the Stars except *Jupiter* were vaniſhed. On *May* 8 in the Evening, the *Comet* was no longer to be diſcovered, either with the naked Eye, or with any Teleſcope.

3. The firſt certain notice I had of this *Comet* was on *April* 21. The 22 of *Apr*. at about 2 a Clock after the Midnight following, I ſaw the *Tail* raiſed almoſt perpendicular to the Horizon; ſoon after the *Head* appeared thro' a thin Vapour, from which the *Tail* pointed, as near as I could gueſs, upon the * in the *Knee* of *Caſſiopeia*, its Length being about 6 *Deg*. and Breadth at the Top of about 7 or 8 *Min*. Viewing the *Head* with a Teleſcope of 16 Foot, I found it was not perfectly round, but indented, and not near one *Min*. Diameter. Afterwards I haſtened to meaſure its Diſtances from ſeveral fixed Stars; which were as follow.

At Greenwich; *by Mr.* Flamſteed, *ib. p.* 173.

h.	′	″		°	′	″
2	44	00	Its *Head* and the *Foot* of *Androm. Alamech.* —— —	11	26	0
2	47	15	That Diſtance repeated —— —— — ——	11	26	50
2	55	3	Its *Head* from *Capella* —— —— —— ——	31	1	15
2	59	10	—— —— —— —— ——repeated	31	1	24
3	12	2	Its *Head* from *Algol* in *Meduſa*'s —— —— ——	18	16	54
3	21	22	—— from *Mirach* —— —— —— ——	19	35	0
3	27	54	—— from *Alamech* again —— —— —— ——	11	33	30
3	36	20	—— from *Capella* again —— —— —— ——	30	59	45

At 4^h 21′½ *p. m.* the height of the *Comet* was about 5°½, therefore the Diſtance of the *Head* of the *Comet* from *Algol* corrected by Refraction, 8° 19′.
from *Mirach* —— 19 37

And admitting with *Hevelius* the Place of *Mirach* now in ♈. 21° 40′ 34″ with North Latitude 25° 57′, its Diſtance from *Algol* will be 23° 42′ 40″, and the Place of the *Head* of the *Comet* in ♉ 14° 48′⅙, with North Latitude 17° 8′.

At 3^h 28′, I ſtate the correct Diſtance of the *Comet's Head* from *Capella* 31° 00′; from *Alamech* 11° 40′; and therefore its true Place in ♉ 14° 05′½, with North Latitude 17° 6′ 25″: agreeing very well with the Place derived from the former Diſtances from two other and different Stars.

The *Tail* was not, it ſeems, directly oppoſite to the *Sun*: for the *Sun*'s Place was now ♉ 13° 7′; but the *Comet* being in 14° 47′ of the ſame *Sign*, that is 1° 40′ in the Conſequence of the *Sun*, the *Tail* ought, if it had been exactly oppoſite to the *Sun*, to have lain in Conſequence of the *Head*; but the *Knee* of *Caſſiopeia* is now in ♉, 13° 24′, in Antecedence of the *Comet*, whoſe *Tail* lay not therefore in Conſequence, but in Antecedence of the Line paſſing thro' its *Head* and the *Sun*, at about an Angle of 10°.

Next Night, being that following the 23 of *April*, about ¾ of an Hour after two, its *Tail* appeared much ſhorter than laſt Morning: At 2^h 51′, its *Head* was from *Mirach* 21° 9′. Hence, and from a Courſe of Obſervation of it ſent me by an ingenions Friend, I found its Motion was direct, and its Latitude decreaſing.

A Comet, An. 1682. *at* Dantzick; *by M.* Hevelius. *Ph. Col. n.* 3. *p.* 65.

CVI. I obſerved the late *Comet* firſt in the Morning before Sun-riſing, from the 2^d to the 4^{th} of *December*, *An.* 1680. Then in the Evening, from *December* 24. to the *Vernal Equinox*. In the Morning it was in ♎ and ♏, with Southern Latitude. In the Evening in ♑. ♒, ♓, ♈, and ♉, with Northern Latitude.

A Comet. An. 1682. *at* Dantzick; *by M.* Hevelius. *n.* 143. *p.* 16.

CVII. I have obtain'd many Diſtances from the fixt Stars, as alſo Meridian Altitudes, of the late *Comet*. It would be too long to mention them all, nor have I time to ſubmit them to a ſtrict Calculation. It may ſuffice to ſay at this time, that this *Comet* was firſt ſeen here at *Dantzick* on *Aug.* 25. New Style, 1682. And that from *Aug.* 26. to *Sept.* 7. it was duly obſerved by me. But what was its Path, what its Velocity, what was the Angle that its Orbit made with the *Ecliptic*, may appear from the following Table. But this I would have you know, that it was not compoſed by any accurate Calculation, but drawn from a conſideration of the Globe, by a looſer way of reaſoning.

Day of the Month.		*Longitude of the Comet.*	*Latitude of the Comet.*	*Motion in its own Orbit.*	*Daily Motion ſomething more exactly.*
	h. ′	° ′ s.	° ′	° ′	° ′
Aug. 26	3 0 *Morn.*	23 30 ♋	21 0 North.	——	——
Aug. 27	11 0 *Even.*	5 0 ♌	23 30 North.	10 0 nearly	5 28
Aug. 28	—— *Even.*	——	——	——	5 35
Aug. 29	—— *Even.*	——	——	——	5 41
Aug. 30	3 30 *Morn.*	18 0 ♌	25 20 North.	13 20 ——	——
Aug. 30	9 0 *Even.*	22 0 ♌	25 40 North	3 30 ——	5 46
Aug. 31	3 30 *Morn.*	24 30 ♌	26 0 North	2 20 ——	5 50
Sept. 1	3 30 *Morn.*	1 0 ♍	26 0 Nearly	5 45	
Sept. 1	9 0 *Even.*	6 0 nearly ♍	25 40 North.	4 45 ——	5 46
Sept. 2	——	——	——	——	5 43
Sept. 3	8 30 *Even.*	20 0 nearly ♍	24 30 North.	11 30 ——	5 40
Sept. 4	—— *Even.*	——	——	——	5 34
Sept. 5	—— *Even.*	——	——	——	5 24
Sept. 6	9 0 *Even.*	5 0 ♎	20 30 North.	15 0 ——	5 00
Sept. 7	—— *Even.*	——	——	——	4 30
Sept. 8	8 0 *Even.*	12 0 ♎	18 15 North.	8 0 nearly	4 00
Sept. 9	8 30 *Even.*	15 30 ♎	17 15 North.	3 30 ——	3 30
Sept. 10	8 0 *Even.*	18 30 ♎	15 45 North.	3 0 nearly	3 00
Sept. 11	—— *Even.*	——	——	——	2 40
Sept. 12	8 0 *Even.*	23 0 ♎	14 0 North.	5 0 ——	2 20
Sept. 13	7 30 *Even.*	25 0 ♎	13 30 North.	2 0 ——	2 00

So as by its own Motion in its own Orbit from *Aug.* 26. to *Sept.* 13. it has moved 83° 27′. And in the *Ecliptic* 91° 30′. But its Northern Latitude has increaſed to 26°, and again has decreaſed to 12° 30′.

Obſerve, that its Northern *Node* is in 24° ♉, and its Southern *Node* in 24° ♏. But its *Limits* were in 24° ♌ and ♒. The *Angle* of the *Orbit* and the *Ecliptic* was nearly 26°. But whether for its whole Duration it was intirely conſiſtent with its *Nodes*, or whether it varied and how much, will appear by the Calculation.

For its whole Continuance the Head was ſomething brighter and greater than that of the Year 1681. But on the contrary it had much a ſhorter Tail. In the Head, by means of a long Teleſcope, we could obſerve but one *Nucleus*, and that always of an *Oval* and *Gibbous Figure*; except that on *Sept.* 8. eſpecially, a very bright Ray proceeded from the *Nucleus*, which was partly crooked, ed, and paſſed into the Tail. This deſerves notice, for as I remember I have not ſeen the like appearance in any other *Comet.*

Beſides it may be obſerved, that ſometimes it directed its Tail pretty exactly in Oppoſition to the *Sun*, as *Aug.* 30. in the Morning. But often with a notable Deviation, as is common in moſt *Comets.* Alſo its *Coma* had not always the ſame length. At firſt the Tail ſeem'd to have a length of 12°, afterwards ſometimes ſhorter, and ſometimes longer as far as 15° or 16°. But towards the End it diminiſhed continually. *Fig.* 156.

CVIII. *An.* 1683. *Jul.* 30. at $11^h\ 30'$. In our new Conſtellation the *Tyger* or *Lynx*, I perceived a *Comet* here at *Dantzick*; ſtretching out its Tail, which was not very long, upwards between the *Polar Star* and *Caſſiopeia*, with ſome Inclination. It made a right Line with the uppermoſt Star of the *Head* of *Auriga* and the right *Shoulder* of *Perſeus*; as alſo with the *Belly* of the greater *Bear* and the right *Shoulder* of *Auriga*; as alſo with the middle Star of the *Tail* and of the *Side* of the great *Bear*. Then taking a Tube of 10 Feet, I conſidered this Phenomenon. The Head indeed was pretty large, but the Matter was not very denſe. So that in this there appear'd no bright *Nucleus*, nor diſtinct Corpuſcles, as are generally found in moſt others. At near 12^h its Altitude was 19° 57′.

A *Comet.* An. 1683. *at* Dantzick; *by M.* Hevelius. n. 154. *p.* 416.

July 31. in the Evening, that is, at $12^h\ 30'$, when its Altitude was 21° 28′ it made a right Line with the *Foot* of *Auriga* and *Capella*. The *Tail* was very dilute and thinner than Yeſterday, but ſomething longer.

Aug. 4. in the Morning, at that time it was ſo far removed from the right *Shoulder* of *Auriga*, as the ſaid *Shoulder* is diſtant from the *Head* of the *Kid.* But the left *Leg* of *Perſeus*, *Capella*, and the *Comet*, ſeem'd to be in a right Line.

Aug. 16. in the Evening, at near 11^h. the *Comet* was within four little Stars, one of which was at the upper Part of the *Comet* in Conjunction with it, at the Diſtance of only 1′, ſo near it was to the Limb. At which time I meaſured the Diameter of the *Comet* with my Micrometer, and found it 6′ 5″.

Aug. 20. in the Evening, it extended its very ſhort and thin *Coma* between *Capella* and the *Head* of the *Kid.*

On the 20. of *Aug.* in the Evening it was very near to each *Kid*, ſo that it made almoſt an Equilateral Triangle with them, the Sides of which almoſt equal'd the Diſtance of the *Kids*, which is about 47′. Beſides the *Comet* made an Equilateral Triangle with *Capella* and the Star at the bottom of the right *Foot* of *Perſeus*, the Baſe of which was the Diſtance of the ſame fixt Stars.

Aug. 24. in the Evening, it was between *Capella* and the *Pleiades*, ſo that it ſeem'd at the ſame Diſtance from each. Then *Capella*, the *Comet*, and the *Pleiades*; alſo *Almanac*, the *Head* of *Meduſa*, and the *Comet*; alſo the right *Shoulder* of *Auriga*, the *Comet*, and the Star that follows the left *Foot* of *Perſeus*; nearly made a right Line. Yet in this laſt Conſtitution, the *Comet* ſeem'd to go rather below the right Line.

Aug. 25. in the Evening, it almoſt made a right Line with *Capella* and the *Head* of the *Kid.*

Aug. 29. at 1^h 5′. in the Morning, it was accompany'd with many very minute and bright fixt Stars, that is, the *Subieſcian Stars*; and was diſtant from the Weſtern Point of the *Pleiades* not above 42′ 35″.

The ſame Day in the Evening, it was found much more advanced, contrary to the Series of the Signs; that is, almoſt 4° in the Space of 24 Hours.

Aug. 30. in the Evening, the *Comet* was diſtant not above 32′ 41″, from a certain very conſpicuous little Star, and conſtituted a right Line with *Muſca*, and that in the Baſe of the Triangle, and then alſo with the foremoſt in the *Foot*, and that in the *Knee* of *Perſeus*.

Sept. 2. in the Morning, the *Comet* was between the *Pleiades* and the Knot of the Line, making a right Line with *Muſca* and the bright Star in the *Whale's Jaw*, and with the loweſt in the *Haunch* of the *Bull* and the *Jaw* a Triangle almoſt equicrural, the Vertex of which is the aforeſaid *Jaw*. Alſo it made a right Line with the two in the Front of the *Whale*; alſo it was as far diſtant nearly from that towards the Weſt, as they were from one another.

This Day I again meaſured very diligently with a Micrometer the Diameter of the *Comet's Head*, being 9′ 7″. *Aug*. 16. with the ſame Micrometer I found it only 6′ 5″, ſo that it had notably increaſed in the Space of 17 Days. It might be ſaid the Reaſon is, becauſe in the laſt Obſervation it was much nearer the *Earth*. But therefore it ought to ſhew its Head much clearer and brighter, eſpecially if it were a permanent Body, as ſome aſſert, which finiſhing its Courſe in a certain Time comes again into our View. But on the contrary its Head is much more dull and rare at laſt, ſo that we might very diſtinctly perceive the Matter of its Head to be gradually diſſolved. And this agrees much better with our Hypotheſis.

Sept. 4. in the Morning the *Comet* ſeem'd to be in a right Line with the Star in the Weſtern Front of the *Whale*, and the bright Star of *Aries*. Alſo with that in the *Mouth* and *Jaw* of the *Whale*. Beſides it made almoſt an Equilateral Triangle with that in the *Mouth* and at the *Cheek* of the *Whale*. Laſtly, it fell out as I could wiſh, that to the South at 3^h 40′ in the Morning nearly, with a moſt exact Quadrant I obtain'd its Meridian Altitude, which was 38° 15′.

D. of the Month.		*Longitude.*			*Latitude.*			*Diurnal Motion.*		*Declination.*			*Right Ascen.*	
		°	′		°	′		°	′	°	′		°	′
July	30	7	0	♋	29	15	N			51	30	N	100	0
	31	6	25	♋	29	0	N	0	42					
Aug.	1	5	45	♋	28	45	N	0	44					
	2	5	0	♋	28	30	N	0	46					
	3	4	10	♋	28	15	N	0	48					
	4	3	20	♋	28	0	N	0	50	51	40	N	96	0
Aug.	5	2	20	♋	27	45	N	0	52					
	6	1	20	♋	27	30	N	0	54					
	7	0	20	♋	27	15	N	0	56					
	8	29	20	♊	27	0	N	0	58					
Aug.	9	28	20	♊	26	40	N	1	0					
	10	27	20	♊	26	20	N	1	2					
	11	26	20	♊	25	55	N	1	4					
	12	25	20	♊	25	30	N	1	6					
Aug.	13	24	20	♊	25	0	N	1	8					
	14	23	20	♊	24	30	N	1	10					
	15	22	20	♊	24	0	N	1	12					
	16	21	10	♊	23	20	N	1	14	46	0	N	77	0
Aug.	17	19	20	♊	22	30	N	1	19					
	18	17	40	♊	21	30	N	1	25	44	0	N	73	30
	19	16	0	♊	20	30	N	1	35					
	20	14	20	♊	19	15	N	1	45	41	0	N	69	30
Aug.	21	12	20	♊	18	0	N	1	55					
	22	10	20	♊	16	45	N	2	5					
	23	8	20	♊	15	30	N	2	15					
	24	6	20	♊	14	15	N	2	25	25	0		60	40
Aug.	25	3	50	♊	12	45	N	2	40					
	26	1	1	♊	11	0	N	2	55					
	27	28	15	♉	9	0	N	3	10					
	28	25	15	♉	6	30	N	3	25	24	30	N	51	0
Aug.	29	22	15	♉	4	0	N	3	40	21	30	N	48	30
	30	18	55	♉	1	30	N	3	55	18	00	N	45	40
	31	16	25	♉	1	0	S	4	5					
Sept.	1	12	15	♉	1	30	S	4	20					
Sept.	2	9	55	♉	6	0	S	4	40	10	30	N	40	
	3	6	25	♉	9	40	S	5	5					
	4	2	35	♉	11	20	S	5	30	1	30	N	34	

From hence it may very clearly be ſeen, that this *Comet* moved conſtantly againſt the Order of the Signs; ſo that in the *Eclиptick* it went 63° 55′, but in its own *Orbit* 74° 35′, under an Angle of its *Orbit* and the *Ecliptic* of almoſt 39°, but of its *Orbit* with the *Equator* of 56°. Its *Latitude* at firſt was 29° 15′ North, and at laſt 11° 20′ South. So that it varied its Latitude almoſt 41°.

This I ſhall take notice of as to its Head, that at firſt its Diameter was much leſs than afterwards; but on the contrary, that at firſt it was far brighter than towards the End. But it exhibited no diſtinct and fulgent *Nuclei*, as I have ſeen in moſt, but a confuſed Maſs of Matter, much thinner towards the End. This *Comet*, ſince it was ſeen moſtly without a Tail, may truly be reckoned among Hairy Stars, or thoſe that have Beards like Goats; for it extended upwards its very ſhort and dilute Briſtles only to *Aug.* 18. which afterwards quite diſappear'd.

CIX. A new *Comet* was lately ſeen in the Heavens, by the ſharp Eye of Ab. *Blanchini*, the Diſciple of *Geminiani Montanarius*. The *Comet* was indeed but ſmall but appear'd regular in its Orbit, of a thin Light, and as an obſcure Star. But it was more luminous through the Teleſcope.

A Comet, An. 1684. *at* Rome; *by* S. Ciampini. n. 169. p. 920.

June 30. New Style. *An.* 1684. the *Comet* was firſt ſeen by me in 9 Degrees of *Libra*, with ſome Minutes; the Latitude of the ſame was 8 Degrees North, with ſome Minutes.

Days of July.	*Longitude of the Comet.*	*Northern Latitude.*	*Days of July.*	*Longitude of the Comet.*	*Northern Latitude.*
	° ′	° ′		° ′	° ′
1	11 18 ♎	13 12	10	24 42 ♎	39 40
2	13 16 ♎	17 57	11	25 44 ♎	40 49
3	14 58 ♎	22 12	12	26 38 ♎	41 58
4	16 45 ♎	25 40	13	27 23 ♎	43 5
5	18 30 ♎	28 50	14	28 32 ♎	44 9
6	19 50 ♎	31 34	15	29 49 ♎	45 12
7	21 7 ♎	33 54	16	00 50 ♏	46 20
8	22 20 ♎	36 6	17	2 8 ♏	47 40
9	23 32 ♎	38 00			

The four laſt Obſervations require ſome better Calculation, for perhaps an ror of ſome Minutes may have crept in. The Obſervation of the firſt of y is very exact, for the *Comet* appear'd in the Teleſcope along with that Star *Virgo* which by *Bayer* is mark'd ω, and fell under the Girdle of the firſt rthern Part. The ſureſt Obſervation of all is that of the ſixth Day, when *urus* was ſeen through the Tube together with the *Comet*. Alſo on the 14th the *Comet* and the Star χ in *Bootes*'s Hunting Pole, under his *Shoulder*, were ſeen at one View.

A Comet. An. 1686. *at* Leipſick; *by M.* Kirk. n. 186. *p.* 256.

CX. *Sept* 8. *ſt. v. A.* 1686. 4^{h} *mane*, about Day-break, M. *Kirk* found this *Comet* in the Conſtellation of *Leo*, to the Right-hand of the *Lucida in Lumbis* ♌ (as it is conceived, for the *Latin* Copy is defective in this place) and reſembling that Star in Colour and Magnitude, with a thin and ſhort Tail extended upright. Over the *Comet* in the ſame Vertical was the Star of θ ♌ of *Bayer*, or 21 *Tychoni*, diſtant therefrom, by the Micrometer, exactly a Degree; and a Line drawn from the *Lucida in Lumbis* ♌ to the *Comet*, paſſed much about half a degree to the Right-hand of the θ *Leonis*. The Diſtance of the *Comet* from *Regulus* taken by a Radius was about 17°. The next Morning *Sept.* 9. at 3^{h} 58′, the Diſtance thereof from θ ♌ was found by the Micrometer 2°. 23′½, and at 4^{h} 40′, again 2° 25′¾. To verify the Times, the Altitude of the *Lucida in Lumbis* ♌ was obſerved 11° 10′, at 4^{h} 8′ *manè*. A Right Line drawn by the *Comet*, and the ſaid θ *Leonis* towards β *Leonis*, or the *Lucida Colli*, left that Star a little to the Right-Hand.

This *Comet* was ſeen by a Countryman, who firſt gave Notice thereof, from the *6th* to the 12*th* of *Sept.*

The Reſult of theſe Obſervations is, that the *Comet* was direct in Motion, that it mov'd about 1½ Degree *per Diem*, and that it ſeemed rather to decreaſe in Latitude. On the *7th* of *Sept.* it was about 24′ diſtant from θ *Leonis*, but its bearing therefrom is not ſet down.

This Star, θ ♌, was then in 9° 2′ of ♍, with North Lat. 9° 41′½. Whence at the time of the firſt Obſervation it may be concluded, that the *Comet* was in 9° 55′ of ♍, with North Lat. 9° 15′. And at the 2^{d} Obſervation, the Longitude of the *Comet* will be found about 11° 20′ of ♍, with much the ſame North Latitude as before.

A Comet. An. $169\frac{8}{9}$. *at* Paris; *by M.* Caſſini. n. 250. p. 79.

CXI. *Feb.* 19. New Style, *An.* 1699. in the *Royal Obſervatory* at *Paris*, a ſmall *Comet* began to be ſeen, which was like a nebulous Star of the third Magnitude. It reſembled that which was obſerved in *September* 1698.

Its Situation was among the unform'd Stars of the ſixth Magnitude, near the *Arctic Polar* Circle, above the *Head* of *Auriga*, almoſt at an equal Diſtance between the Weſtern *Elbow* of *Perſeus* and the *Head* of the great *Bear*. *Tycho* aſcribes them to the unform'd Stars about the leſſer *Bear*. As we continued our Obſervations, it ſeem'd by its own Motion to direct its way towards *Capella*, with a ſmall Deviation from its Circle of Declination. Its Velocity was ſuch, that in the Space of one Day it deſcribed about 7 Degrees of a great Circle, by which Motion in leſs than 4 Days it might come to the Pole, and be joyn'd to the *Polar Star*.

At 6^{h} after Midnight we compared the *Comet* with a Star of the 6th Ma
tude, which *Tycho* calls the ſecond of thoſe which are in a right Line with
Pole. In its Paſſage through the Horary Circle it preceded this Star by 53″, by which the Difference of right Aſcenſion will be given 4° 43′. But was more to the North than this Star by 8′. whence ſuppoſing *Tycho*'s Lon
tude and Latitude of this Star, to this Time, the *Comet* is referr'd to 15°
of *Gemini*, with North Latitude 37° 25′.

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This *Comet* moves to the oppoſite Parts of the Heavens in reſpect of thoſe to which the *Comet* of the foregoing Year tended; whereas it was almoſt at the ſame Diſtance from the *Pole*, as this *Comet* of ours when it was firſt ſeen; nor very diſtant from the ſame Place.

But the *Comet* of the Month of *September* proceeded in the ſame way as the *Comet* of the Year 1652. purſued among the Stars, as it was obſerved by us at *Bologna*. On which Occaſion we diſtinctly deſcribed that way by the ſame Stars as our *Comet* held, *An.* 1698. in Letters addreſs'd to the Moſt Serene *Fran. Eſt* Duke of *Modena*. That *Comet* paſs'd in the Month of *December* from the Southern Parts of the Heaven through the Conſtellations of the *Hare*, *Orion*, and the *Bull*, where it cut the *Ecliptic* with an Inclination of 76°, and through *Perſeus* and *Caſſiopeia*, where it ceas'd to be ſeen in the Month of *January*, *An.* 1653. This began to be ſeen in the beginning of the Month of *September*, in the ſame Part of *Caſſiopeia* where the other ceaſed to be ſeen, and thence proceeding through the *Shoulders* and the *Arms* of *Cepheus*, where it had the greateſt Latitude from the *Ecliptic* of 76° it paſs'd through the *Dragon* and the *Swan*, through the *Lion's Skin* in *Hercules*, through *Ophiucus*, to the Conſtellation of *Scorpio*, which it kept in our laſt Obſervations from the 24 to the 28 of *September*. Now from theſe Obſervations we collected, that this *Comet* had its *Perigee Sept.* 7. in the Evening, with the greateſt apparent Velocity of nearly 10° in the Space of one Day.

CXII. Papers *of leſs General Uſe* Omitted.

1. The Conſtellation of *Cygnus*, with the *New* Star *in Pectore* in it, by *Hevelius*; together with the Names of the Stars in that *Conſtellation* by *Tycho* and of thoſe added by *himſelf*. Cygnus. n. 21. p. 372. n. 65. p. 2088, 2090.

Mr. Horrox's *Lunar Syſtem.* n. 110. p. 219.

2. Mr. *Flamſteed* having peruſed Mr. *Street*'s Diſcourſe, and conſidered the Contrivance of his *Moon-Wiſer*, aſſures, that for the Motion of Longitude 'tis the very ſame, and for the Motion of Latitude not much better than Mr. *Horrox*'s. n. 116. p. 368.

But Mr. *Flamſteed* hath thought of another Contrivance, that will ſhew the *Moon*'s true Place to a Minute.

Caleſtial Phænomena calculated.

3. 1. The more Notable *Cæleſtial Appearances* calculated, by Mr. *Flamſteed*, for the *Year* 1670. n. 55. p. 1099.
2. The ſame for the *Year* 1671. n. 66. p. 2029.
3. The ſame for the *Year* 1672. n. 77. p. 2297.
4. The ſame for the *Year* 1673. n. 79. p. 3061. n. 86. p. 5040.
5. The ſame for the *Year* 1674. n. 89. p. 5118. n. 99. p. 6162.

Satellite Eclipſes.

4. 1. The *Eclipſes* of the *Satellites* of *Jupiter* viſible at *Uraniburg* the laſt four Months of the *Year* 1671. calculated by M. *Caſſini*. n. 77. p. 2238.
2. The *Eclipſes* of the *Satellites* of *Jupiter* viſible at *the Obſervatory* at *Greenwi* n the three laſt Months of the *Year* 1683. calculated by Mr. *Flamſteed*. n. 151. p. 322.
The *Satellite Eclipſes* calculated by Mr. *Flamſteed* for the *Year* 1684. n. 154. p. 404.
The ſame for the *Year* 1685. n. 165. p. 760.
The ſame together with the *Parallaxes* of *Jupiter*'s *Orb* and his *Geocentri* *Places*, for the *Year* 1686. n. 177. p. 1215.
The ſame for the *Year* 1687. n. 184. p. 196.
The *Satellites Eclipſes* calculated by Mr. *Halley*, for the *Year* 1688. n. 191. p. 435.

An Account of the *Ephemerides* of the *Comet*, *A.* 1665. calculated *Comets*, n. 1. p. 3.

n. 2. p. 17, 18. by M. *Auzout*; and the *Principle* of his *Hypotheſis* diſcovered by M. *Caſſini*.

n. 3. p. 36. 2. An Account of the *Ephemerides* of the *Comet A.* 1665. Calculated by M. *Auzout*.

CXIII. Accounts *of* Books *and* Emendations, Omitted.

n. 102. p. 40. 1. A new Size of *Globes* about 15 Inches Diameter rectified by *R. Morden* and *Will. Berry*.

Phil. Col. 2. A Repreſentation of the Heavens in two large *Hemiſpheres* of 30 Inches
n. 1. p. 44. Diameter, Stereographically projected upon the Plane of the *Æquinox*; by Mr. *Fr. Lamb*.

n. 90. p. 5150. 3. Deux Machines propres à faire les *Quadrans*, avec tres grande facilité, par le P. *Ignace Gaſton Pardies*, S. J. à *Paris*, 1673. in 12°.

n. 184. p. 213. 4. *Scioterícum Teleſcopicum*, or a new Contrivance of adapting a Teleſcope to an *Horizontal Dial*, for Obſerving the Moment of Time by Day or Night; by *Will. Molineux R. S. S. Dublin*, 1686. *in* 4*to*.

n. 241. p. 240. 5. The *Meridian Line* of the *Church* of St. *Petronio*, drawn and fitted for Aſtronomical Obſervations, in the *Year* 1655. reviſed and reſtored in the *Year* 1695; by *Jo. Dom. Caſſini*. At *Bononia* 1695. *Fol.*

n. 66. p. 2028. 6. *Joa. Hevelii* Machinæ cæleſtis Pars prior, *Organographiam Aſtronomicam*
n. 99. p. 6171. plurimis Iconibus illuſtratam & exornatam exhibens, *&c. Gedani* 1673 in *Fol.*

n. 109. p. 215. 7. *Animadverſions* on the Firſt part of the *Machina cæleſtis* of *Joa. Hevelius*, together with an Explication of ſome *Inſtruments* made by *R. Hook. P.* of
n. 111. p. 243. *Geometry* in *Greſh. Coll.* and *R. S. S. London* 1674. *in* 4*to*. Dr. *Wallis*'s Letter to M. *Hevelius*, concerning *Diviſions* by *Diagonals* there inſerted, but faultily, is here *Reprinted* more correctly.

n. 175. p. 1162. 8. *Joannis Hevelii* Conſulis *Dantiſcani* Annus Climactericus. *Gedani* 1685. *Fol.* Wherein (among other Things) M. *Hevelius* vindicates the Juſtneſs of his *Celeſtial Obſervations* againſt the Exceptions by ſome made to the *Accuracy*
Ib. p. 1164. of them. The *Controverſy* between *Him* and Dr. *Hook*, about the Uſe of *Te-*
n. 111. p. 244. *leſcopick* and *Plain Sights*, and Dr. *Wallis*'s Calculation, for *Dividing* the Limb
n. 175. p. 1176. of Inſtruments by *Diagonals*, are alſo here Abridged.

9. Excerpta ex Literis Ill. & Clariſſ. Virorum ad Nob. Ampliſſ. & Conſul-
n. 150. p. 308. tiſſ. D. *Jo. Hevelium* Conſ. *Gedanenſem* perſcriptis, Judicia de Rebus *Aſtronomicis*, ejuſdemque Scriptis, exhibentia; Studio ac Operâ *Joa. Erici Olhoffii* Secretarii *Gedani* 1683. in 4*to*.

10. A Deſcription of the *Helioſcop*, and ſome other *Inſtruments*, made by
n. 118. p. 440. *R. Hook*, R. S. S. *Lond.* 1675. in 4*to*.

11. The Sphere of M. *Manilius* made an *Engliſh* Poem, with Annotations, and an *Aſtronomical Appendix*; by *Ed. Sherburn*, Eſq; *Lond.* 1675. in *Fol.*

n. 101. p. 233. 12. *Albatenii* Obſervationes *Aſtronomicæ*, Quas ex *Arabico* in *Latinum* Tranſ-
n. 204. p. 913. tulit *Plato Tiburtinus*. *Noribergæ* 1537; & *Bononiæ* 1645. The *Arabick* opy of thoſe Obſervations does not appear, whereby that *Tranſlation* might *examined*: But Mr. *Halley*, by calculating Tables from the Principles there c ered, hath here *diſcovered* and *corrected* above 30 conſiderable *Faults* in a few P ges.

n. 43. p. 868. 13. *Hiſtoria cæleſtis*; ex Libris & Commentariis MS. *Obſervationum* V enalium *Tychonis Brahe*, *Dani*, *Auguſtæ Vindelic.* Ann. 1666. in *Fol.*

n. 102. p. 27. & 29. 14. All the *Manuſcripts* of the famous *Kepler*, (both *publiſhed* and *unpub* ed) which are purchaſed, and carefully preſerved by M. *Hevelius*.

Je-

15. *Jeremiæ Horroccii Angli* Opera Poſthuma: una cum *Guil. Crabtræi* Obſervationibus Cœleſtibus, nec non *Jo. Flamſteedii* de *Temporis Æquatione* Diatriba, numeriſque *Lunaribus* ad Novum *Lunæ* Syſtema *Horroccii. Lond.* 1672. in *4to.* n. 87. p. 5078.

16. *Aſtronomia Reformata.* Auctore *Joan. Bapt. Riccioli S. J. Stephano de Angelis*, conceiving the Arguments of this *Author*, againſt the *Motion* of the *Earth*, to be none of the ſtrongeſt, taketh Occaſion to let the world ſee that they are not more eſteem'd in *Italy*, than in other places: *Manfredi*, in behalf of *Riccioli*, endeavours to anſwer the Objections of *Angeli*, and this latter replies to *Manfredi*'s Anſwer. The Subſtance of which *Controverſy* is here given by Mr. *Ja. Gregory*; with ſome Remarks and Explications of his own upon it. n. 22. p. 394. n. 36. p. 693.

17. An Attempt to prove the *Motion* of the *Earth* from Obſervations, made by *R. Hook, F. R. S. Lond.* 1674. *in 4to.* The Method of this Undertaking is approved and commended by M. *Chr. Huygens*, and M. *Caſſini.* n. 101. p. 12. n. 105. p. 90.

18. *Nicolai Mercatoris* Holſati, è Soc. Regia, *Inſtitutionum Aſtronomicarum* Libri duo. *Lond.* 1676. in *Octavo.* n. 125. p. 611.

19. Annales Cœli & Temporum perpetui, ſive Myſteria *Aſtronomo-Chronologica* à Seculo abſcondita nunc per Dei Gratiam detecta & evidenter aſſerta, Libris tribus. *Kiloni.* This Book is Preparing by Dr. *Waſmuth:* n. 104. p. 74.

20. A *Catalogue* of *Fixed Stars* with their Longitudes, Latitudes, and Magnitudes, according to the *Obſervatiou* of *Uleg Beig. Oxford* 1666. n. 8. p. 145.

21. *Catalogus Stellarum Auſtralium*, ſive Supplementum Catalogi *Tychonici*; exhibens Longitudines & Latitudines *Stellarum Fixarum*, quæ prope *Polum Antarcticum* ſitæ, in Horizonte *Uraniburgico*, *Tychoni* inconſpicuæ fuere. Authore *Edm. Halleio*, è *Col. Reg. Oxon.* in *4to.* n. 141. p. 1032.

22. Congietture *Phyſico Aſtronomiche* della Natura del *Univerſo*; da *Pietro* M. *Cavina*, in *Faenza* 1669. in *4to.* n. 65. p. 2012.

23. Proſe de *Signori Academici* di *Bologna*, in *Bologna* 1672. in *4to. S. Montanari*'s Diſcourſe concerning the admirable *Changes* and other *Novelties* obſerved in the *Heavens.* n. 89. p. 5125.

24. *Iſmaelis Bullialdi* ad Aſtronomos Monita duo. Primum de *Stella nova*, quæ in *Collo* Ceti ante An. aliquot viſa eſt. Alterum de *Nebuloſa* in *Andromedæ Cinguli* parte Borea, ante Biennium iterum orta. Approv'd by M. *Hevelius.* n. 21. p. 381. n. 25. p. 460.

25. Three Letters of *Jo. Dominicus Caſſinus*, concerning his *Hypotheſis* of the *Sun's Motion*, and his Doctrine of *Refractions.* At *Bononia*, in *4to.* n. 84. p. 5001.

26. Refractio *Solis Inoccidui*, in Septentrionalibus Oris circa Solſtitium Æſtivum, *An.* 1695. aliquot Obſervationibus *Aſtronomicis* detecta. *Holmiæ*, in *4to. Tranſlated iuto Engliſh. Lond.* in *8vo.* n. 233. p. 731.

27. *Tabularum Aſtronomicarum* Paris prior; de Motibus *Solis* & *Lunæ* necnon de Poſitione *Fixarum*, ex ipſis Obſervationibus deductis: Authore *Ph. de la* H[illegible] *Paris* 1687. in *4to. Some Animadverſions on it are here inſerted.* n. 191. p. 443.

1. The *Royal Almanack* for the *Year* 1675: by *N. Stevenſon.* in 12°. n. 108. p. 192.

——— For the *Year* 1676. n. 120. p. 490.

——— For the *Year* 1677. n. 130. p. 774.

Ephemeris, ad *Annum* 1686, exactiſſime ſupputata. *Lond.* in *8vo.* n. 179. p. 35

The *Celeſtial World diſcovered*, or Conjectures concerning the Inhabitants, Plants, and Productions, of the *Worlds in the Planets.* Written in *Latin* by M. *Chr. Huygens.* in *8vo.* n. 256. p. 337.

31. Rag-

n. 1. *p.* 2. *n.* 45. *p.* 900. *n.* 4. *p.* 69. *Ib. p.* 74. 31. Ragguaglio de Nuove Obſervationi da *S. Gioſeppe Campani.* This Book was anſwer'd by M. *Auzout*, who gives his Opinion of *Campani*'s Glaſſes, and his new *Obſervations* of *Saturn* and *Jupiter* made with them: to this *S. Campani* publiſhes a *Reply*, and M. *Auzout* his *Animadverſions* thereon.

n. 35. *p.* 688. *n.* 44. *p.* 892. 32. 1. Ephemerides *Mediceorum Siderum*, ex Hypotheſibus & Tabulis *Jo. Dom. Caſſini*, *Bononiæ* 1668. in *Fol.* The like *Tables* have been formerly publiſh'd by the Learned *J. Bapt. Hodierna* at *Rome*, about 1656.

n. 214. *p.* 256. 2. The Table of the *Eclipſes* of the *Firſt Satellite* of *Jupiter* by M. *Caſſini*, publiſh'd at *Paris* in the *Recueil d' Obſervations faites en pluſieurs Voyages pour parfectioner l'*Aſtronomie *& la* Geographie, being not printed with the uſual Care of the *Imprimerie Royale*, Mr. *Halley* here amends ſome of the *Errata.*

n. 14. *p.* 242. *n.* 35. *p.* 687. 33. *Martis*, circa Axem proprium revolubilis, Obſervationes *Bononiæ* à *Joa. Dominico Caſſino* habitæ 1666. Here M. *Caſſini* judges it evident, that the Period of this *Planet*'s *Revolution* is not performed in the ſpace of 12^h 20′, but in about 24^h 40′; and that thoſe, who affirm the former, muſt have been deceived by not well diſtinguiſhing the two *Faces.*

n. 134. *p.* 853. 34. *Mercurius* in *Sole* viſus; à *Jo. Hevelio*, *Ged.* 1662.

n. 6. *p.* 104. 35. *Prodromus Cometicus*; by *Hevelius.*

n. 17. *p.* 301. 36. *Joannis Hevelii* Deſcriptio *Cometæ*, *An.* Æræ *Chriſtianæ* 1665. exorti: una cum Mantiſſa *Prodromi Cometici*, obſervationes omnes Prioris *Cometæ* 1654, ex iiſque genuinum Motum accuratè deductum, cum Notis & Animadverſionibus, exhibens.

n. 40. *p.* 805. 37. *Jo. Hevelii* Cometagraphia: *Dantzick*, in *Fol.*

n. 35. *p.* 691. 38. *Staniſlai* de *Lubienietz* Theatrum *Cometicum. Amſtelod.* 1668. in *Fol.*

n. 53. *p.* 1069. 39. Del Movimento della *Cometa*, apparſa il meſe di *Decembri* 1664. da *Pietro Maria Mutoli*, in Piſa, in 4*to.*

n. 53. *p.* 1071. 40. *Eraſmi Bartholini* de *Cometis*, *An.* 1664 *&.* 1665. Opuſculum; ex Obſervationibus *Hafniæ*, habitis adornatum. *Hafniæ*, in 4*to.*

n. 139. *p.* 980. 41. *Joh. Walliſii*, De *Cometarum* Diſtantiis inveſtigandis. *Lond.* 1678.

Ib. 986. 42. Lectures and Collections made by *R. Hook*, Sec. of the *R. S. Lond.* 1678 in 4*to.*

Ph. Col. n. 4. *p.* 106. 43. Obſervat. of the *Comet* of 1680 and 1681. made at the *Col.* of *Clermont*; by *P. J. de Fontenay* è *S. J. Profeſſ.* of *Mathematicks*, Paris 1681.

Ib. p. 114. 44. A Treatiſe concerning the late *Comet*, publiſhed at *Turin*, 1681. by *Donato Roſſetti*, S. T. D. Canon of *Legborn.* and *Tutor* in *Mathematicks* to the *Duke of* Savoy.

Ib. p. 116. 45. An Explication of the *Comet* which appeared at the End of 1680, and in the Beginning of 1681, upon the Obſervations of D. *Anthelme*, *Carthuſian* of *Dijon.* At *Dijon*, 1681. in one ſingle Sheet.

Ph. Col. n. 7. *p.* 196. 46. A ſmall Diſcourſe about *Comets* publiſhed in the *High Dutch* at
1681; by a *Lover of Aſtronomy.*

Ib. p. 199. 47. A new Introduction, ſhewing how the *Motions* of the *Comets*
reduced to ſome certain and Geometrical Rules, that their *Appearance* ma
predicted: in *High Dutch*; by *Ja. Bernouly*, at *Bazil. An.* 1681.

n. 149. *p.* 272. 48. *Joannis Jacobi Zimmermanni Cometo-ſcopia.* Or, three *Aſtronomical*
lations concerning the *Comets* that have been ſeen in the *Years* 1680, 16
1682. *Stutgard*, 1682. in 4*to.*

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CHAP. V.

MECHANICKS. ACOUSTICKS.

The General Laws of Motion by Dr. Wallis. n. 43. p. 864. Nov. An. 1668.

I. 1. IF an Agent as A produces an Effect as E; an Agent as 2 A, will produce the Effect 2 E, 3 A as 3 E, &c. all other things being like. And univerſally, m A will produce the Effect m E, whatever ratio m be the Exponent of.

2. Therefore if a Force as V move the Weight P, the Force mV will move m P, other things alike: Suppoſe the ſame Space in the ſame Time, that is, with the ſame Velocity.

3. Alſo if in the Time T it move it the Length L; in the Time nT it will move it the Length n L.

4. Therefore if the Force V, in the Time T, move the Weight P, the Length L; the Force m V, in the Time n T; will move the Weight m P, the Length n L. And therefore as VT, (the Product of the Forces and Time,) to PL, (the Product of the Weight and Length,) ſo is mnVT, to mnPL.

5. Becauſe the Degrees of Velocity are proportional to the Lengths perform'd in the ſame Time; or, (which comes to the ſame,) are reciprocally proportional to the Times ſpent in performing the ſame Length; it will be $\frac{L}{T} : C :: \frac{mL}{nT} . \frac{m}{n}C$. That is, the Degrees of Velocity are in a ratio compounded of the Direct Ratio of the Lengths, and a Reciprocal Ratio of the Times.

6. Therefore becauſe VT . PL :: mnVT . mnPL; it will be $V . \frac{PL}{T} :: mV . \frac{mnPL}{nT}$. That is, $V . PC :: mV : mPC = mP \times C = P \times mC$.

7. That is, if the Force V is able to move the Weight P with the Velocity C; the Force mV will move either the ſame Weight P with the Velocity mC; or with the ſame Velocity will move the Weight mP; or laſtly, any Weight with ſuch Velocity, that the Product of the Weight and Velocity may be mPC.

d on this depends the Reaſon of the Conſtruction of all Machines for facilitating Motions: So that in whatever ratio the Weight is increaſed, the Velocity muſt be diminiſh'd in the ſame. So that the Product of the Velocity and Weight, that is to be moved with the ſame Force, may always be the ſame. That is, $V : PC :: V : mP \times \frac{1}{m}C = PC$.

9. If the Weight P, moving with the Velocity C, by means of the Force V, ſtrikes directly againſt the Weight m P, which is at reſt, and is no ways hindred; each will be carry'd by the Velocity $\frac{1}{1+m}$ C. For becauſe of the ſame Force, which is apply'd to the moving of a greater Body; the Velocity of the increaſed Body muſt be diminiſhed in the ſame ratio. That is, $V : PC :: V : \frac{1+m}{1} P \times \frac{1}{1+m} C = PC$. Therefore the Shock of one, (by which is meant the Product of the Weight and Velocity,) will become $\frac{1}{1+m}$ PC, and of the other $\frac{1}{1+m} m$ PC.

10. If againſt the Body P, moving with the Velocity C, (by means of the Force V,) another Body follows it the ſame way with greater Velocity, and ſtrikes directly againſt it; ſuppoſe the Body m P, with the Velocity n C; (and therefore moved by the Force mn V;) both will be carry'd with the Velocity $\frac{1+mn}{1+m}$C. For it is $V : PC :: mnV : mnPC :: V+mnV (\frac{1+mn}{1}V) : \frac{1+mn}{1} PC = \frac{1+m}{1} P \times \frac{1+mn}{1+m} C$. Therefore the Shock of the preceding will be $\frac{1+mn}{1+m}$ PC, and that of the ſubſequent will be $\frac{1+mn}{1+m} m$ PC.

11. If Bodies moving contrary ways meet and ſtrike each other directly, ſuppoſe the Body P moves to the right Hand with the Velocity C by means of the Force V; and the Body mP moves to the left Hand with the Velocity n C, and therefore by the Force mn V; the Velocity, Shock, and Direction of each may be determined. The Body that moves to the right Hand would communicate to the other, if it were at reſt, the Velocity $\frac{1}{1+m}$C, and therefore the Impetus $\frac{1}{1+m} m$ PC to the right Hand, and would retain to itſelf this ſame Velocity, and conſequently the Impetus $\frac{1}{1+m} m$PC to the right Hand, by Sect. 9. And the Body moving to the left Hand (by a like reaſoning) would communicate to the other, if at reſt, the Velocity $\frac{mn}{1+m}$ C, and therefore the Impetus $\frac{mn}{1+m}$ PC to the left; and would retain to itſelf this ſame Velocity, and therefore the Impetus $\frac{mn}{1+m} m$ PC to the left. Now ſince the Motion is on each ſide, the Impetus of the Body that before moved to the right Hand will now be the Aggregate of $\frac{1}{1+m}$ PC to the right Hand, and $\frac{mn}{1+m}$ PC to the left Hand; and therefore will really be to the right Hand or left, according

ing as this or that is the biggeſt, with that Impetus which is the Difference of the two. That is, (making the Sign + to ſignifie to the right Hand, and — to the left Hand,) the Impetus will be $+\frac{1}{1+m}PC-\frac{mn}{1+m}PC=\frac{1-mn}{1+m}PC$; and the Velocity $\frac{1-mn}{1+m}C$; and therefore to the right Hand or left, according as 1 or mn ſhall be greater. And in like manner the Impetus of that Body that before moved to the left Hand will be $+\frac{1}{1+m}mPC-\frac{mn}{1+m}mPC=\frac{1-mn}{1+m}mPC$, and its Velocity $\frac{1-mn}{1+m}C$; and therefore to the right Hand or left, according as 1 or mn is the greater.

12. But if the Bodies do not move directly the ſame way, nor directly contrary ways, but ſtrike one another obliquely; the foregoing Calculation is to be accommodated to the Degree of Obliquity. Now the Impetus of a Body ſtriking obliquely, is to the Impetus that would be produced if they ſtruck directly, other things being alike, in the ratio of Radius to the Secant of the Angle of Obliquity. Which is alſo to be underſtood, when the the Body falls not perpendicularly but obliquely on the Surface of the Body that is ſtruck, as well as when the Ways of their Motion croſs one another obliquely. This Conſideration rightly apply'd to the foregoing Calculation, will determine what will be the Velocity, Impetus, and Direction of Bodies that impinge thus obliquely. That is, with what Impetus, with what Velocity, and towards what Parts thoſe Bodies will reflect from each other, which impinge in this manner. The Reaſon is the ſame of the Gravitation of heavy Bodies that deſcend obliquely, to the Gravitation of thoſe which deſcend perpendicularly.

13. If the Bodies ſo impinging are ſuppoſed to be not abſolutely hard, (which we have conceived hitherto,) but yielding to the Stroke, yet ſo as to be able to reſtore themſelves by their Elaſtic Force; it may hence happen that thoſe Bodies ſhall rebound from one another which otherwiſe would go together; (and indeed more or leſs, according as this Power of Reſtitution is greater or leſs;) that is, if the Impetus proceeding from the Reſtitutive Force is greater than the Progreſſive.

In Motions that are continually accelerated or retarded, that is to be eſteemed the Impetus for every Moment, which belongs to the Degree of Velocity then acquired. But when the Motion is along a Curve, that is to be reputed the Direction in every Point of the Motion, which is the Direction of the Tangent in that Place. And whenever the Motion is accelerated or retarded, and alſo along a Curve, as in the Vibrations of a Pendulum; the Impetus is to be eſtimated for every Point, both according to the Degree of Acceleration, and the Obliquity of the Tangent in that Place.

2. The Law of Nature in the Colliſion of Bodies.

The proper and moſt natural Velocities of Bodies are reciprocally proportional to thoſe Bodies.

By Sir Chriſt. Wren. Ibid. p. 867.

Therefore the Bodies R, S, having their proper Velocities, alſo after Colliſion retain thoſe proper Velocities.

And the Bodies R, S, having improper Velocities, by Impulſe are reſtored to Equilibrium. That is, as much as R exceeds, and S is deficient from their proper Velocity before Impulſe, ſo much by the Impulſe is taken from R, and is added to S: And contrarywiſe.

Wherefore the Colliſion of Bodies having their proper Velocities is equivalent to a Balance oſcillating upon two Centers, which on each ſide are equally diſtant from the Center of Gravity. Now the Beam of the Balance is produced as there is occaſion.

Therefore there are three Caſes of equal Bodies moving improperly. But of unequal Bodies moving improperly, (whether contrary ways or the ſame way) there are in all ten Caſes, five of which ariſe by Converſion.

Fig. 157. R, S, are equal Bodies, or R is the greater Body, and S the leſſer. *a* is the Center of Gravity, or the Handle of the Balance; Z is the Sum of the Velocities of both Bodies.

$$\left.\begin{array}{l}\left\{\begin{array}{l}R\,e\\ S\,e\end{array}\right\}\text{Veloc. of Body}\left\}\begin{array}{l}R\\ S\end{array}\right\{\text{given before Impulſe.}\\ \left\{\begin{array}{l}o\,R\\ o\,S\end{array}\right\}\text{Veloc. of Body}\left\}\begin{array}{l}R\\ S\end{array}\right\{\text{required after Impulſe.}\end{array}\right\}\text{or}\left\{\begin{array}{l}\left.\begin{array}{l}S\,o\\ R\,o\end{array}\right\}\text{Veloc. of Body}\left\{\begin{array}{l}S\\ R\end{array}\right\}\text{given before Impulſe.}\\ \left.\begin{array}{l}e\,S\\ e\,R\end{array}\right\}\text{Veloc. of Body}\left\{\begin{array}{l}S\\ R\end{array}\right\}\text{required after Impulſe.}\end{array}\right\}$$

The Rule. R *e*, S *e*, make *o* R, *o* S: R *o*, S *o*, make *e* S, *e* R.

[Read the Syllables (tho' disjoyn'd) R *e*, S *e*, *o* R, *o* S; or R *o*, S *o*, *e* S, *e* R, in the Line of every Caſe; and of theſe that which is written in the Scheme in the *Hebrew* manner, ſhews a Motion contrary to the Motion which the *Latin* way of writing of each Syllable denotes. A Syllable conjoyn'd ſignifies the Reſt of the Body.

$$\text{Calcul.}\ \begin{array}{l|l|l} R+S:S::Z:Ra & Re-2Ra=oR & So-2Sa=eS. \\ R+S:R::Z:Sa & 2Sa\pm Se=oS & 2Ra+Ro=eR. \end{array}$$

Nature obſerves the Rules of Specious Addition and Subtraction.

By M. Huygens. n. 46. p. 927. Apr. An. 1669. 3. The Rules concerning the Motion of Bodies, after their mutual Impulſe.

1. If a hard Body ſtrikes another equal hard Body at reſt, after the Contact that will be at reſt, and the Body at reſt will acquire the ſame Velocity as was in the ſtriking Body.

2. But if the other equal Body move alſo, and in the ſame right Line, after Contact they will both continue to move, but with Velocities mutually interchanged.

3. A Body ever ſo great will be moved by a Body ever ſo little, that ſtrikes againſt it with any Velocity.

4. The General Rule for determining the Motion which hard Bodies acquire by their direct meeting is this following.

Fig. 158. Let the Bodies be A and B, of which A is moved with the Velocity AD; and let B meet it, or let it move the ſame way with the Velocity BD, or laſtly let it be at reſt, that is, in this caſe let the point D fall in B. Let the Line AB be divided in C, (which is the Center of Gravity of the Bodies A, B,) and let C E be taken equal to C D. I ſay, E A will be the Velocity of the Body A after meeting, and E B of the Body B, and each will proceed in that way as is ſhewn by the order of the Points E A, E B. Now if the Point E falls in A or B, the Bodies A or B will be reduced to reſt.

5. The Quantity of Motion of two Bodies may be increaſed or diminiſhed by Conflict: But there always remains the ſame Quantity the ſame way, taking from thence the Quantity of contrary Motion.

6. The Sum of the Products made by the Bulk of every hard Body drawn into the Square of its Velocity, is the ſame both before and after the Conflict.

7. A hard Body at reſt receives more Motion from another hard Body, whether greater or leſs, by the Interpoſition of ſome third Body which is of an intermediate Quantity, than if it had been ſtruck by it immediately. And if that interpoſed Body ſhould be a mean Proportional between the two others, it will act the moſt forcibly of all againſt the Body at reſt.

In all theſe Concluſions the Author ſuppoſes the Bodies to be of the ſame Matter, as he acknowledges himſelf; or he intends, that their Bulk may be eſtimated by their Weight.

But he adds, that he has obſerved a certain wonderful Law of Nature, which he affirms that he can demonſtrate in Spherical Bodies, and which ſeems to him to be general in all other Bodies whether hard or ſoft, or whether they ſtrike directly or obliquely; that is, that the common Center of Gravity of two, three, or any Number of Bodies, is always equally promoted the ſame way, in the ſame right Line, both before and after the Percuſſion.

Some Hiſtorical Paſſages relating to theſe Papers; by Mr. Oldenburg. *Ibid. p.* 925.

4. For ſome Months laſt paſs'd, ſeveral Members of the *Royal Society*, at their publick Meetings, had inſiſted very earneſtly, that that important Subject of the Laws of Motion ſhould at laſt undergo a ſtrict Examination, having formerly been propoſed to the Society, but not yet diſcuſs'd as it deſerved to be. It then ſeem'd proper to that Illuſtrious Society to determine, that which ever of their Members ſeem'd fitteſt for this Inquiry, into the Nature of Motion, ſhould be deſired to produce their Thoughts and Diſcoveries about it, and likewiſe to collect what had been done in this matter by other excellent Men, as *Galilæus*, *Des Cartes*, *Honoratus Faber*, *Joachimus Jungius*, *Peter Borellus*, and others. Chiefly with this View, that thus conſulting and comparing the Opinions of all, a Theory might thence be eſtabliſhed in the Philoſophical World, which might agree as much as poſſible with Obſervations and Experiments, which ſhould be often repeated with due Care and Fidelity.

This their Deſire being made known, ſeveral of the Members of the ſaid Society comply'd with it, as *Chriſtian Huygens*, *John Wallis*, and *Chriſtopher Wren*; who undertook to compleat as ſoon as might be thoſe Hypotheſes and Rules of Motion, in digeſting of which they had been employed for ſome time. Hence it was that thoſe three great Mathematicians, in the Space of a few Weeks, comunicated their Theories neatly abridged, and as it were by ſtrife, deſiring the Sentiments of the *Royal Society* upon the ſame. Firſt of all Dr. *Wallis* tranſmitted his Principles concerning the Eſtimation of Motion, by a Letter dated *Nov.* 15. 1688, which was deliver'd and read the 26 of the ſame Month. He was ſoon ſucceeded by Sir *Chriſtopher Wren*, who on the 17th of the Month following exhibited to the Society the Laws of Nature concerning the Colliſion of Bodies. Theſe the Society ordered to be printed, having firſt obtained the Conſent of the Authors, for the more commodious Communication and ampler Diſcuſſion of this Subject.

Whilſt theſe Things were doing with us, on the 4th of *January* following, *Engliſh* Stile, the Poſt brought us Letters from Mr. *Huygens*, written on the 5th day of the ſame Month, but New Stile, containing the four firſt Rules concerning the Motion of Bodies ariſing from mutual Impulſe, together with their Demonſtrations. I had at hand a Copy of *Wren*'s Theory, which I ſent the ſame Day to Mr. *Huygens* by Way of Retaliation, the Poſt then favouring. I forbore to open Mr. *Huygens*'s Letters, ſuſpecting ſomething of this Nature to be included in them becauſe of their Bulk, and becauſe of his Promiſe, till I had an Opportunity of laying them before the moſt noble and worthy Preſident of the Royal Society, the Lord Viſcount *Brounker*. Which being done, and the Rules of each being compared by the ſaid Society, there preſently appeared a wonderful Agreement between them; which produced in us a great Deſire of committing both their Writings to the Preſs. On Mr. *Huygens*'s Side nothing was wanting but his Conſent, without which we did not think it fair to print his Diſcoveries, eſpecially as he had not given them then compleat. However we took care to regiſter his Paper in the publick Acts of the Royal Society, and to return our ſolemn Thanks to the Author on *Jan.* 11. for his agreeable Communication; afterwards adding (on *Feb.* 4.) our earneſt Wiſhes, that he would cauſe his Theory to be printed either at *Paris*, (which he might eaſily do in the Journal *des Scavans*, as it is called,) or ſuffer us to print it here at *London* in the Philoſophical Tranſactions. Which Letters being ſent to Mr. *Huygens*, we ſoon afterwards received an Anſwer from him, acknowledging the Receipt of Sir *Chriſtopher Wren*'s Paper on this Subject, but mentioning nothing about the Publication of his own Paper, either at *Paris* or *London*.

Hence I conclude it is very plain, that Mr. *Huygens* has been wanting to himſelf, in not haſtning the Publication; and by his Delay has given occaſion, that Mr. *Wren* having by his Sagacity diſcovered both the Theories, juſtly claims a right to the Glory due to this Diſcovery. Since it is beyond all doubt, that neither of the Gentlemen knew what was done by the other, before their Writings appeared together; but each by his own Ingenuity produced theſe beautiful Originals.

Indeed Mr. *Huygens*, when he was at *London* ſome Years ago, ſolved theſe Caſes of Motion which were then propoſed to him. A ſure Argument that he had even then found out the Rules, by the Evidence of which he performed this Matter. But he will not affirm, that he diſcloſed any Thing of his Theory at that Time to any of the *Engliſh*. Nay, he muſt confeſs, that though he was ſolicited by ſome of them to make this Communication, yet he could never be prevail'd on to do it till very lately.

II. Let all the Lines ab, bc, cd, de, &c. be all equal to one another, and $b\,1$, $c\,2$, $d\,3$, $e\,4$, $f\,5$, &c. increaſe equally as the Numbers 1, 3, 5, 7, 9, &c.

I ſay that any heavy Body falling in this Line from any Point of it, will reach the Bottom in the ſame Space of Time, as it would reach it if it ſhould fall from any other Point of the ſame.

*The Synchroniſm of the Vibrations made in a Cy*cloide; *Demonſtrated by a Perſon of* Quality. n. 94. p. 6032. *Fig.* 159. May An. 1673.

For if you ſuppoſe $a = ab = bc = cd$, &c. and $b = b\,1$, and $x =$ any Number of either; then if xa is put for af, then xxb muſt repreſent $f\delta$, and therefore the Time of Deſcent will neceſſarily be $\frac{xxb}{xxaa}$ or $\frac{b}{aa}$. And the ſame obtains in all Caſes. Therefore, &c.

I ſay moreover that this Curve is a Cycloid, which is eaſily demonſtrated from the Conſtruction, and from what is now ſhewn. For this Curve $abcdefz$ is equal to the double of the laſt of the right Lines, that is $2z\omega$, and $a\omega$ is equal to the Semicircumference of the Circle whoſe Diameter is $z\omega$; and in general the Triangle ♈♉♊ repreſents the right Line $z\omega$; and the Square ♈♊♉♋ repreſents the Curve $abcdefz$; and the Quadrant ♈♉♋ repreſents the right Line $a\omega$; and the Parts of one the Parts of the other reſpectively. As if ♈♎♍ repreſents $f\delta$, then will ♈♎♌♋ repreſent $a\delta$, and ♈♎♏♋ will repreſent af. But I have no time to purſue this farther.

Laſtly I ſay, that a Ball ſuſpended by a String of a due Length, and vibrating between two Cycloids, will move in a Cycloid. Wherefore ſuch Vibrations will be Synchronous. *Q. E. D.*

III. 1. Prob. To determine the Curve-line connecting two given Points which are at different Diſtances from the Horizon, and not in the ſame Vertical Line, upon which a Body moving by its own Gravity, and beginning to move from the upper Point, ſhall deſcend to the lower Point in the ſhorteſt time.

A Problem concerning the Line of quickeſt Deſcent between two Points given; propoſed by M. Jo. Bernoulli. n. 224. p. 384. Jan. An. 1697.

The Senſe of the Problem is this; of the infinite number of Lines that may be drawn between thoſe Points, from one to the other, to make choice of that, according to which if a Plate be bent having the form of a Tube or a Canal, ſo that a Ball being laid upon it, and ſuffer'd to deſcend freely, it may perform its Paſſage from one Point to the other in the ſhorteſt Time poſſible.

2. Yeſterday I received copies of two Problems, propoſed by that moſt acute Mathematician Mr. *John Bernoulli*, printed at *Groningen*, *Cal. Jan.* 1697. Of the firſt of which this is the Solution.

Solv'd; by . . . . *Ibid.*

From a given Point A let there be drawn an indefinite right Line APCZ parallel to the Horizon, and upon the ſame right Line let there be deſcribed any Cycloid AQB, meeting a right Line drawn through the other given Point B in the Point Q, as alſo another Cycloid ABC, whoſe Baſe and Altitude may be to the Baſe and Altitude of the former as AB to AQ reſpectively. Then this laſt Cycloid will paſs through the Point B, and will be that Curve-line, in which a Body falling by the Force of Gravity will arrive ſooneſt from the Point A to the Point B. Q. E. I.

Fig. 160.

3. Let AP be an Horizontal Line, P the Point from whence the heavy Body deſcends through the Curve-line required ADE, C and D two Points infinitely near, through which the Body will fall, CD a right Line connecting the two Points, DC and sC, DF and SG, FS and GC or sH, Moments of the Abſciſs of the Curve and of the Ordinate reſpectively. Take Dr = Ds, and tC = BC.

The Demonſtration; by Mr. R. Sault. n. 246. p. 425. Nov. An. 1698. *Fig.* 161.

Becauſe in the little naſcent Lines the Time is directly as the Way deſcribed and the Velocity inverſely, (that is, in this Caſe as the Square-root of the Altitude of the falling Body,) by Hypotheſis it will be $\frac{Ds}{\sqrt{QD}}+\frac{SC}{\sqrt{QF}}=$ the leaſt Time. And becauſe the Velocity in the Points of the ſame Altitude S and B along the Curve D s C and the right Line D B C is the ſame, the Time along D C, which evidently is the leaſt, will be as $\frac{BD}{\sqrt{QD}}+\frac{BC}{\sqrt{QF}}$. Therefore let theſe Times be equal, and then $\frac{Ds}{\sqrt{QD}}+\frac{sC}{\sqrt{QF}}=\frac{DB}{\sqrt{QD}}+\frac{BC}{\sqrt{QF}}$, that is, $\frac{DB-Ds}{\sqrt{QD}}=\frac{sC-BC}{\sqrt{QF}}$, or $\frac{Br}{\sqrt{QD}}=\frac{ts}{\sqrt{QF}}$.

But the Evaneſcent Triangles Brs, Bts, are equiangular to the Triangles DsF, HsC; therefore $\frac{Bs}{Ds}=\frac{Br}{sF}$, and $\frac{ts}{Hs}=\frac{Bs}{st}$. Let theſe two Ratios of Equality be compounded, and then $\frac{Br}{Ds\times Hs}=\frac{ts}{sF\times st}$. And *ex æquo* $\frac{\sqrt{QD}}{sF\times st}=\frac{\sqrt{QF}}{Ds\times Hs}$. Now becauſe any of the Elements may be ſuppoſed to flow equably, let us ſuppoſe $Ds=sC$, and the moſt ſimple Expreſſion of the Curve becomes $\frac{\sqrt{QD}}{sF}=\frac{\sqrt{QF}}{Ds}$ every where. That is, in the Point of Flexure the Curve will always be in a Ratio compounded of the Velocity directly, and of the Moment of the Ordinate reciprocally. Let $\dot x$, $\dot y$, and $\dot z$ be the Fluxions of the Abſciſs, the Ordinate, and of the Curve reſpectively, then $\frac{x^{\frac{1}{2}}}{\dot y}$ is conſtant as above. Therefore $\frac{x^{\frac{1}{2}}}{\dot y}=1$; but we ſuppoſed $\dot z$ $(=\sqrt{\dot x\dot x+\dot y\dot y})$ to be conſtant. Therefore that this may be conſtant Unity, and may obtain its due Dimenſions, will be $\frac{x^{\frac{1}{2}}}{\dot y}=\frac{a^{\frac{1}{2}}}{\sqrt{\dot x\dot x+\dot y\dot y}}$; and after Reduction $\dot y=\frac{x^{\frac{1}{2}}\dot x}{\sqrt{a-x}}$, which is a known Expreſſion of the Cycloid P D E. Q. E. D.

How much the Deſcent is Quicker in the Cycloid *than in a ſtraight Line by . . . . n.* 225. p. 424. Feb. An. 1697.

IV. *Theorem.* In the *Cycloid* AV D, whoſe Baſe A D is parallel to the Horizon, and Vertex V is downwards; if from A be any how drawn the right Line A B meeting the *Cycloid* in B, from whence let there be drawn the right Line B C perpendicular to the Curve of the *Cycloid* in B, to which from A let there be drawn the Perpendicular A C. I ſay that the Time in which a Body at reſt falling from A by the force of its Gravity deſcribes the right Line A B, is to the Time in which it paſſes along the Curve AV B, as the right Line A B to the right Line A C.

Fig. 162.

Through B draw B L parallel to the Axis of the *Cycloid* V E, and B K parallel to the Baſe A D, meeting the Axis in G, and a Circle deſcribed upon the

Diameter E V in F and H, and laſtly meeting the *Cycloid* in K. Draw the right Line E F, which from the Nature of the *Cycloid* will be parallel to the right Line B C. Whence B M is equal to E F, and E M to B F, which becauſe of the *Cycloid* is equal to the Arch V F. Therefore A M is equal to the Arch E H V F.

By *Prop.* 25. *Part* II. of *Huygens*'s *Horologium Oſcillatorium*, the Time in which a Body falling from reſt deſcribes AV, is to the Time of the Fall along E V, as the Semicircumference is to the Diameter; and by the laſt Propoſition of the ſame, the Time in which a Body deſcribes V B, after having deſcribed AV, (which is equal to the Time in which a Body deſcribes K V, after having deſcribed A K,) is to the Time of the Fall along AV, as the Arch V F is to the Semicircumference; and therefore to the Time of the Fall along E V, as F V to the Diameter. Wherefore the Time in which a Body deſcribes the Curve AV B is to the Time of the Fall along E V, as the Arch E H V F to the Diameter E V. But the Time of the Fall along E V is to the Time of the Fall along L B, or E G, as E V is to E F. Therefore *ex æquo*, the Time in which a Body deſcribes AV B is to the Time of the Fall along L B, as the Arch E H V F to the Subtenſe E F; that is, as the right Line A M to M B. Again, the Time of the Fall along L B is to the Time of the Fall along A B, as L B to A B. Therefore the Ratio of the Time in which the Body deſcribes AV B to the Time in which it deſcribes A B, is compoſed of the Ratio of A M to M B, and of the Ratio of L B to B A; and therefore is equal to the Ratio of A M × L B to M B × B A. But A M × L B is equal to M B × A C, becauſe each is equal to the double of the Triangle A B M. Therefore the Time in which a Body falling from reſt deſcribes the Curve of the *Cycloid* AV B, is to the Time in which it deſcribes the right Line A B, as M B × A C is to M B × B A, that is as A C to A B. *Q. E. D.* And the Demonſtration proceeds in the ſame Manner, if the Point B be between A and V.

Exact Portable Watches; by M. Huygens. n. 112. p. 272. *Fig.* 163. Mar. An. 1675.

V. 1. The upper Plate of the *Watch* is A B: The Circular *Balance-Wheel* C D, of which the *Arbor* is E F: The *Spring* turned *Spirally*, G H M, faſtned to the *Arbor* of the *Balance-Wheel* in M, and to the piece that is faſt to the Watch-Plate, in G, all the *Spires* or Windings of the *Spring* being free without touching any thing. N O P Q is the *Cock*, in which one of the *Pivots* of the *Balance-Wheel* turns; R S, is one of the *Indented-Wheels* of the *Watch* having a *Balancing* Motion, which the *Balance-Wheel* gives to it. And this *Wheel* R S, catches in the *Pinion* T, which holds on the *Arbor* of the *Balance*, of which by this means the Motion is entertained as much as is neceſſary. Theſe *Watches* are exact for the Pocket, and when made greater, will be uſeful to find the *Longitudes* both by Sea and Land.

By Dr. Goth. Guil. Leibnitz. n. 113. p. 285. Apr. An. 1675.

2. The Principle I thought upon ſome Years ago for making exact *Portable Watches*, is altogether different from that of M. *Huygens*: his depending upon a phyſical Obſervation, but mine upon a mere mechanical Reflection; which hath not been taken notice of for want of the *Art* of *Combination*, the uſe of which is far more general than that of *Algebra*. For, having conſidered

with my ſelf, that a *Spring* being bent to the ſame Degree, will always unbend itſelf in the ſame Time, provided it find the ſame freedom of unbending it ſelf ſuddenly; I inferred from thence, that there might be imployed two ſuch, one of which ſhould play, whilſt the *Firſt Mover* of the *Watch* did bend the other again.

Fig. 164. These Thoughts I have executed in the following manner: Let A B be one of the *Watch-Plates*, C and M, two indented Barrels wherein the ſmall *Springs* are incloſed. The Teeth of the Barrels catch thoſe of the Pinions *d d*, which carry the Balances *e e*; and other Teeth of the ſaid Barrels are catched by thoſe of the interrupted Wheel F G. Now let us imagine, that this Wheel F G, being moved towards H F, by the force of the *Firſt Mover* of the *Watch*, and turning the Barrel C, bends the *Spring* incloſed in it, and ſtops with the Barrel as ſoon as it hath bent this *Spring*. This piece which ſerves to ſtop, is eaſy, and hath not been thought neceſſary to be marked here, to avoid embarraſſing the Figure. But whilſt one indented part of the interrupted Wheel F G, *viz.* F, turns the Barrel C, the empty part, oppoſed thereunto, which is G, anſwers to the other Barrel M, and gives Liberty to the *Spring*, it incloſeth, to unbend itſelf. Thus whilſt the *Movement* of the *Watch* bends the ſmall *Spring* of the Barrel C, in the ſame Time the ſmall *Spring* of the other Barrel M, unbends of itſelf: I ſay, in the ſame Time, except the *Spring* C, ſhall have done bending a little ſooner than the *Spring* M ſhall have unbent itſelf: So that the *Spring* C, being bent, and the Wheel F G ſtopped; both of them ſtay in this Poſture, till the *Spring* M, when it ſhall be quite unbent, doth, at the end of its Motion, touch a Piece which delivers it. And then the *Spring* C unbends of itſelf in its turn; the Teeth of the interrupted Wheel, which continues its Motion the ſame way as before, ſince 'tis delivered, not being any more able to hinder it therefrom, becauſe the Barrel C, doth now meet with the empty part H, of the ſaid Wheel. But before it hath done with unbending it ſelf, the indented part L, being oppoſite to the empty part H, that turns the Barrel M, bends its *Spring* again, and having done ſo, ſtops with it; whilſt the *Spring* C, making an end of unbending itſelf, delivers them by a reciprocal good Office, and renders to the *Spring* M, the ſame Services which it had received from it, with an Expectation of receiving the like again.

Which being well conſidered, 'tis manifeſt, That the ſame *Alternative Motions* will continue always: That the Periods, taken from the very Moment of that one *Spring* begins to unbend, until the Moment it once unbends itſelf again, will always be of *equal Duration*, tho' the two ſmall *Springs* be not equally ſtrong: That the Balance of ſuch a *Watch* will be double, and may be charged more or leſs, and receive delay, by advancing or recoiling along the two Arms two equal Weights, Counter-balancing one another, that ſo the Change of the Situation may not at all prejudice the *Equality* of the *Watch*. For the reſt, we may in this kind of *Watches* ſpare the *Fuſee*, and conſequently the String or Chain. 'Tis alſo eaſy to judge, that ſuch *Watches* as theſe may be of a Size ſufficiently ſmall; that they will make no more Noiſe than ordinary *Watches*; that they will be as exact as *Pendulums*, and ceaſe not to go whilſt they are Winding up. And tho' the Motion of the Watch Wheels may be altered by many Accidents, yet the *Periods* of the *ſmall Springs* will not be concerned in all or any of them, provided the Motion of the Watch Wheels have always more Strength than it needs to bend them again; which is in our Power.

The Objections that have been made againſt this Contrivance, if employed for *finding Longitudes*, are theſe; that toſſing of Ships would ſhake the *Springs* as well as other pieces; that Ruſt would ſpoil them, ſince the ſaltiſh Humidity of the Sea in remote Voyages, ſpares not the very Needles of Compaſſes tho' incloſed in Boxes; that the Changes of Seaſons and Climates will ſenſibly alter the *Springs*, eſpecially the great Heats or Rains within the Tropicks, which at length will ſomewhat untemper the Steel; as is confirmed by the Experiments of the *illuſtrious Academy of Florence*, ſhewing how eaſily that Heat and Cold do change *ſlender Springs*: Beſides that, the Air more or leſs condenſed will alſo more or leſs reſiſt the Motion of the Balance. To which may be added, that *Springs* by working are weakened: and laſtly, That there will be always ſome little *Friction*, that will make the ſeveral pieces go more or leſs eaſily, and that even in length of time they will wear out.

But I anſwer, That all theſe Defects, that proceed from the Imperfection of the Matter, may be ſurmounted by a general Remedy, without Examining them here in particular: And that is, That for executing it in great, we make uſe of maſſy Springs, as are thoſe of Croſs-Bows, we being Maſters of them, not wanting Force or Place in a Ship, to govern a great Weight that may ſerve to bend them continually again. Now theſe *maſſy Springs* that may be ſo great, and their Reſtitution ſo ſpeedy, by augmenting their Number, that all the above-named Defects will have no conſiderable Proportion to this Strength, and the Aggregate of their Repetitions will not be ſenſible till after a very long time. And 'tis eaſy to *Demonſtrate*, That by augmenting the bigneſs of the Engine, and the force of the *maſſy Springs*, we may make the Error as ſmall as we will, provided we paſs not the bounds of Conveniency, and content our ſelves with Exactneſs ſufficient for their chief End, *viz.* For finding the *Longitudes*.

A Clock Aſcendent upon an inclined Plane; by M. de Gennes. n. 140. p. 1006. *Fig.* 165. July An. 1678.

VI. The Circle, F G H, being placed upon a *Plane inclined* A B, is divided into two unequal Parts by the Line G I. To reſtore to the leſs Secture its *Æquilibrium*, there is faſtened to the Extremity of the *Radius* D F, a Weight F, which is ſufficiently heavy to recover what the leſſer Secture loſes by its Situation. That a Wheel or Clock may thus ſtand not only in *Æquilibrium*, but alſo aſcend upward, there is placed in the middle of the *Clock* a *Drum*, which incloſes the *Spring* of the *Pendulum*; upon which *Drum* is faſtened the *Radius* D F. For thus the *Spring* being mounted, enforces the *Drum* to turn, and ſo to raiſe the Weight, which it cannot raiſe without its becoming more heavy, in regard that coming to the Point E, it is farther from the Center, than when it was in F, and thus all the *Wheel* turns on that Side as the *Spring* gives way.

A Clock Deſcendent *on a Plane Inclined; by Mr.* Maurice Wheeler. n. 161. *p.* 647. May An. 1684.

VII. Altho' the *Marquis* of *Worceſter* is ſaid to have contrived a *Watch* that ſhould *Move* upon a *Declivity*, and M. *de Gennes* has given ſome Account of a *Clock Aſcendent on a Plane inclined*; yet neither of them, nor any like them, was ever ſeen by me, and for ought I could ever learn, the *Reaſon* of their *Motions* remains to this Hour as great a Secret, as if they had never been. I ſhall therefore give an Account of a *Movement*, which I have deſign'd to meaſure Time after a peculiar Manner.

Fig. 166.

1. The exteriour Structure of it is a circular Body of 3½ Inches Diameter, conſiſting of two Plates meaſured by the ſame *Radius*, and fixed in a parallel Poſition to each other by the Hoop *b*, the breadth of which is about an Inch. This Hoop and the two Plates form the Caſe of the *Movement*; of which, that which appears in the Front, is towards the Verge thereof inſcribed with a *Horary Circle*, the Diviſions whereof anſwer the Hours of a natural Day. The deep Shades within this Circle are intended to repreſent a Concave, of near half a Inch deep; and the Prominence *g*, in the middle of this Concave, is a Hemiſphere of Braſs or Silver, riding looſly on a Pin, which lies hid, and is the *Axis* of the *Movement*. The upper half of this Hemiſphere is hollow, but the nether filled with Lead; and the ſmall Gentleman that ſits thereon, does with an erected Finger perform the Office of an Index. But this being only for Ornament, you may ſubſtitute in the room thereof any other Index, provided the *Axis* whereon it is ſupported, move freely in the Hole H, and the lower part thereof HL, ſo far preponderate to HP, as always to keep it Pendulous, with its Point to the Vertical Hour.

Fig. 169.

Fig. 166.

2. For the manner of its Motion, as far forth as it appears outwardly; it is thus: SE repreſents a Board or Shelf, of a ſtraight and even Surface, about 6 Foot long, and ſo thick as not to be apt to caſt with change of Weather; nor to grow camber under a ſmall Weight; on this is the *Movement* placed, and here to perform its Courſe; and therefore I call it the *Stage* of the *Movement*. This *Stage* is raiſed at the end S, about 10 *Deg.* above the Horizon or Line of Level HE; but this Angle of its Declivity DEH, is variable. The two Plates which form the *Caſe* of the *Movement*, are to be extant all round without the Hoop *b*, ¼ of an Inch, and the Edges of them lightly indented; that while the *Movement* deſcends upon the *Stage*, it may turn only, and not ſlide. The *Movement* being placed as high as it may, near the point S, ſhall move downward towards E, with that ſlowneſs, as to finiſh one entire Revolution in 24 hours; and while it does ſo, the Diviſions on the *Horary Circle* (or Dial-Plate) ſucceſſively Culminating over the Point of the Index (which is always to keep the ſame Poſition) will ſhew the Hours of the Day and Night. And when by ſeveral repeated Revolutions, it has meaſured the length of its *Stage*, it is to be replaced at S, as before; which may be done in leſs than half the time you are Winding up a Watch; and if the *Stage* be 6 Foot long, no oftener than once in a whole Week.

3. The way of Adjuſting the Motion to the exact Meaſure of an Hour, and Rectifying its Errors, is thus: *viz.* By the turning a Skrew inſerted at S, the *Stage* may be elevated or depreſs'd, and accordingly the *Movement* will go Faſter or Slower: Faſter, if raiſed up, and Slower, if let down;

and

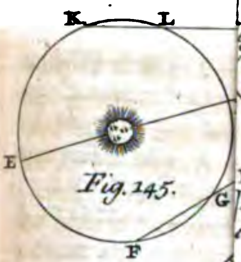

Fig. 145.

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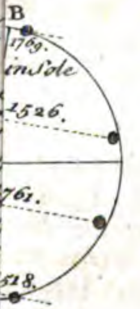

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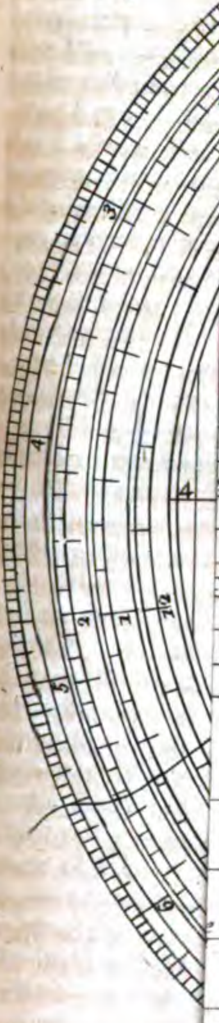

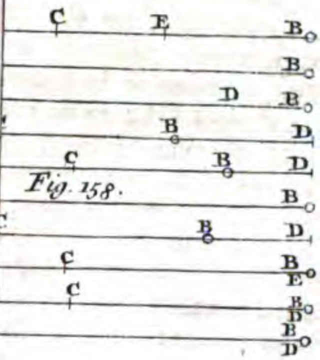

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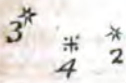

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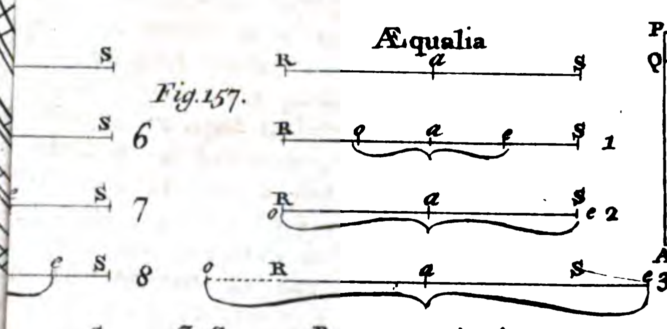
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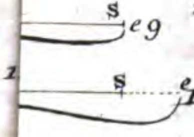
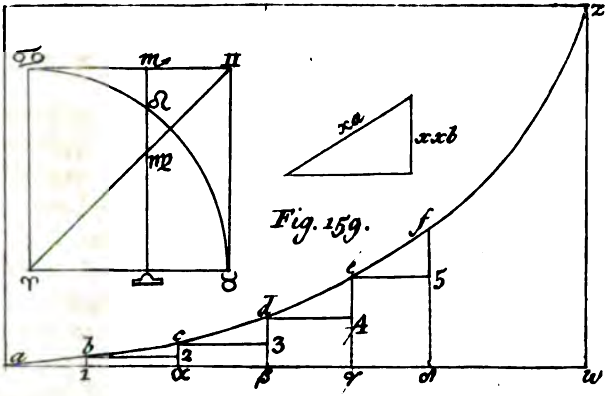
Fig. 157.





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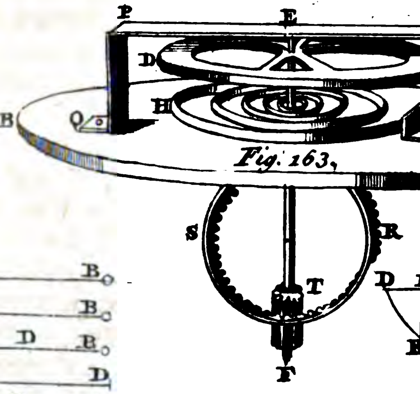

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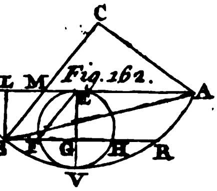

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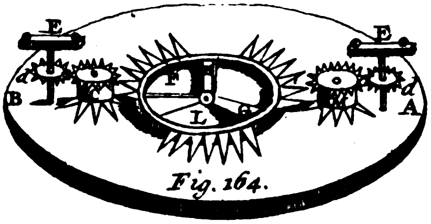

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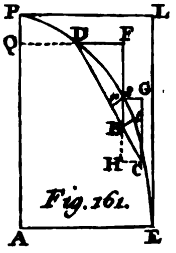

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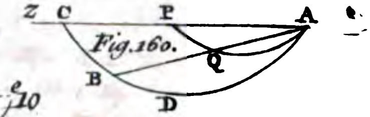

Fig. 160.

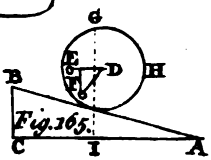

Fig. 165.

and by making the *Horary Circle* moveable, and Inſerting ſeveral ſmall Boſſes or Buttons, here and there upon the Verge thereof, it may with an eaſy touch of the Finger be moved to the right and left as there ſhall be Occaſion, till the juſt Time be brought to the Point of the *Suſpended Index*.

Fig. 167. The Reaſon of this *Movement* may be thus explained: 1. Let the Circle L O D N, repreſent any Circular Body, whoſe Centers both of Gravity and Magnitude are coincident at M. Let this Circular Body be placed upon ſome Level Plane G G; and then it is evident that the Angle of its Contact with that Plane at *a*, will alſo be the Point of its Libration, and conſequently it muſt reſt there: Becauſe the Moment and Impediment are equal.

2. Let D E repreſent a Deſcending Plane, making an Angle of Contact with this Circular Body at *b*: and here 'tis manifeſt it cannot reſt: becauſe the Line of Direction *r a*, which (while it inſiſted upon a Level) divided the Circular Body by the Centers of Magnitude and Gravity into parts Æquipонderate, is now removed to L D; which Line L D falling without, or beſide, the Center M, evidently deſtroys the Æquipoiſe of its Parts, and therefore muſt leave it to tumble down towards E. For here the Moment is greater than the Impediment. The Reaſon therefore of its Deſcent now being the Over-balance of the Parts L N D, to the remaining Section L D O, it muſt neceſſarily follow,

3. That if ſome Weight equal to the Exceſs of L N D, above L O D, were affix'd to the Limb of the Quadrant O *a*, as at P: then the Circular Body would reſt as quietly at *b*, as it did before at *a*. The Suppoſition cannot be denied, and the conſequence is unavoidable, becauſe $LDO + P = LND$, the Impediment is equal to the Moment.

Fig. 168. Let then the Numbers, 1, 2, 3, 4, repreſent a Train of Wheel-work, wherein there is no material difference from what is found in a common Watch; only the numbers of the Teeth on the Wheels and Pinions are to be ſo calculated, that the Motion of the Whole Train may correſpond to the aſſigned Revolution of the Body of the *Movement* which is to be once in 24 Hours: It would be expedient alſo, That a *Spiral Spring* were applied to its *Balance*, as in later *Movements*, is uſual; but of a *Fuſee* here's no need, for the Turns of the Body of the *Movement* as it deſcends upon the *Stage*, anſwer all the Intentions of a String or Chain; and the *Contranitence* of the Weight P, to the Exceſs of LED, above LQD, ſerves inſtead of a Perpetual Spring; and the *Movement* wants only a *Perpetual Deſcent*, to make its *Motion* ſo. And whereas the great Wheel in ordinary *Movements*, is placed as near the edge of the Framing Plate *f r*, as it may be; here it muſt (with its *Axis* or Arbor M) poſſeſs the Center of the *Movement*: Becauſe this Wheel is to carry the Weight of *Power* P, by the *Vectis* M P, and that Weight P muſt always keep an Equidiſtance from the Center of the *Movement*; that while the Body thereof (*i. e.* of the *Movement*) performs its Revolutions; the ſaid Weight P, and the great Wheel (to which it is affixed) may, without any conſiderable Variation, continue in, or near the ſame Poſition, wherein they now are. Now ſuppoſe this Weight P, with its *Vectis* M P, to be taken quite out of the *Movement*, and the Body of the *Movement* to be placed on a Horizontal Plane H H, its Point of Contact in that Plane is T; where it ſhould, but cannot, reſt;

becauſe the Weight of that part of the Train, marked with the Numbers, 2, 3, 4, removes the Center of Gravity from M; and therefore on the oppoſite Part of the *Movement* as about C Q, the Inſide of the Hoop, which forms the Caſe, is to be loaded with a thin Lining of Lead, which may be a Counterpoiſe to that part of the Train; that ſo, the whole Body of the *Movement*, together with all its Furniture, within and without (excepting only P, with its *Vectis*) may, on that Horizontal Plane, or while it rides upon its own *Axis*, reſt indifferently in any Point. This reducing of the *Movement* to an Equilibration of all its Parts in the Center M, muſt be performed *Tentando*, *i. e.* by Raſping the Lead at C Q, as much and in ſuch Places as is needful; which, to an Artificer of Ordinary Sagacity, will not be at all difficult.

Fig. 170.

Fig. 168.

The Center of Gravity being thus reduced to M, replace the Weight P, by the Hole H, on the Arbor of the Central-Wheel M. Then let the Body of the *Movement* be placed on the Declivity DE, and ſuppoſing $P + LQD = LDE$, then the Body muſt needs reſt there: But becauſe the Weight P, is not now fix'd to any part of the Quadrant Q D, but hangs upon the Train of Wheel-work 1, 2, 3, 4, it evidently follows, That if the Power thereof be ſuperior to the Reſiſtance of the Train, then the whole Body of the *Movement* muſt needs deſcend, towards E. By this you ſee there are two Offices aſſigned to the Weight or Power P. The Firſt is, to be a Counterpoiſe to the exceſs of the Weight of L E D, above L Q D.

The Second is, that it be of force ſufficient to put the Train into a Motion ſo adjuſted, as may exactly comport with the time aſſigned for the Revolution of the whole Body. So that if there be any Difficulty remaining, it conſiſts in ſuch an exact Stating of the Weight and Power of P, that it may adequately ſerve both theſe Intentions. Now how eaſy this is, will be manifeſt from theſe Propoſitions following.

1. That whatever the intrinſick Weight of P ſhall be, as ſuppoſe it 4 Ounces *Troy*; yet the Power of that Weight will be augmented or diminiſhed according to the different Degrees of its Elevation in the Quadrant TQ. Thus conſidering P M, as a *Vectis*, its *Hypomochlium* is M, the Point where it exerts its Power on the Train, is at V; I ſay then, whatever Power it has upon the Point V, in its preſent Elevation of 45 *Deg.* it will acquire a greater by being raiſed to 50, 55, *&c.* and the greateſt of all in 90 *Deg.* at Q: And on the contrary, let it ſink to 40°, 35°, *&c.* its Power upon the Point V, will ſtill be diminiſhed, inſomuch that in T, it will be utterly extinguiſh'd. And therefore if P be of a competent Weight (*i. e.* not utterly too light) to *move* the Train at all, it will certainly move it in ſome Degree of Elevation or other in the Quadrant Q T.

2. If the Weight P be conſidered as to its Office of being a Counterpoiſe to the Body of the *Movement*; as I need not to prove, that it will perform this no leſs, while it hangs by upon the *Vectis* M P, than if it were faſt riveted in the ſame place to the Caſe of the *Movement*: ſo, in what Point of the Quadrant ſoever it will move the Train, it may be alſo a Counterpoiſe to the Body of the *Movement*. For,

1. At what Point ſoever of the Circle L E T Q, the Line of Declivity D E, makes an Angle of Contact; on the ſame Point will the *Diameter* S D fall at Right Angles with D E.

2. The Line of Direction LD, will ever fall upon the Point of Contact D, making an Angle with the Diameter, as S D L.

3. The Angle S D L, will be always equal to D E H, *i. e.* As great as is the Elevation of the Line of Declivity D E, above the Horizontal E H; ſo great will the Angle of Diſtance be between the Diameter S D, and the Line of Direction L D. Fig. 166.

4. The greater the Angle of Declivity is, the leſs will be the Section L Q D; and ſo on the contrary, the leſs that Angle is, the greater the Section. And therefore,

5. The Exceſs of the Weight of L E D, above L Q D, muſt be alſo greater, by Raiſing up the *Stage* with the Skrew at S; and that Exceſs leſs by Skrewing it down.

6. The Lighter that part of the Body is, which is repreſented by the Section L Q D, the more Heavy ought the Counterpoiſe P, to be; and that either in its own Intrinſick Weight, (in Ounces and Parts of Ounces) or elſe in its Potential Weight, by being raiſed higher in the Quadrant Q T.

7. The Skrewing up the *Stage* of the *Movement* at S, will raiſe the Counterpoiſe higher in the Quadrant Q T, by *Prop.* 3. and therefore potentially heavier. And from hence appears (I take it moſt clearly) both the Reaſon of the due Adjuſtment of the Motion of the Train to the exact Meaſure of an Hour, and what Weight is to be aſſigned to P, that moves it; and that we are not confined to Scruples and Grains, but are allowed ſuch a conſiderable Latitude, as it is not eaſy to err therein.

Having therefore ſet the *Stage* (by the help of the arched Skrew) at the Elevation of above 10 *Deg.* place the *Movement* thereon, and try what Weight, hanging at the end of the *Vectis* M P, will ſtir the Train; mean while holding the *Movement* with the Hand in ſuch a Poſition, as the *Vectis* may make an Angle of about 30 *Deg.* with the Perpendicular M T: then let the *Movement* looſe, to undulate upon the *Stage*; and when the Vibration ceaſes, obſerve to what Degree of the Quadrant the *Vectis* Points, and at the ſame time mind the Pulſes of the Balance. If at this Obſervation, the Weight lies low, (as for inſtance, between 25 and 35 *Deg.* of the Quadrant) and the Beats of the Balance are gueſſed to be not much different from their due time, the Weight P, is well enough proportioned. But if it chance to be much heavier than is abſolutely needful, that Exceſs will be moderated by Skrewing down the *Stage*; and if it be not abſolutely too light, its Defect will be compenſated, by Skrewing the *Stage* higher. Therefore of theſe two Extremes, chooſe the former; for the fewer Degrees that P ariſes in the Quadrant, beyond what is abſolutely neceſſary, it will (for Reaſons very obvious) be ſo much the better.

The Effects of Gravity in the Deſcent of Heavy Bodies, and the Motion of Projects, by Mr. Halley. n. 179. p. 3. Jan. An. 1686.

VIII. *Des Cartes* his Notion, I muſt needs confeſs to be to me incomprehenſible, while he will have the Particles of his *Cœleſtial Matter*, by being reflected on the Surface of the Earth, and ſo aſcending therefrom, to drive down into their Places thoſe Terreſtrial Bodies they find above them: This is, as near as I can gather, the Scope of the 20, 21, 22, and 23 *Sections* of the laſt Book of his *Principia Philoſophiæ*; yet neither he nor any of his Followers can ſhew, how a Body ſuſpended *in Libero Æthere*, ſhall be carried downwards by a continual Impulſe tending upwards, and acting upon all its Parts equally: And beſides, the Obſcurity wherewith he expreſſes himſelf, particularly, *Sect.* 23, does ſufficiently argue, according to his own Rules, the *Confuſed Idea* he had of the thing he wrote.

Others, and among them, Dr. *Voſſius*, aſſerts the Cauſe of the Deſcent of Heavy Bodies, to be the *Diurnal Rotation* of the *Earth* upon its *Axis*, without conſidering, that according to the Doctrine of Motion fortified with Demonſtration, all Bodies moved in *Circulo*, would recede from the Center of their Motion; whereby the contrary to *Gravity* would follow, and all looſe Bodies would be caſt into the Air in a Tangent to the Parallel of Latitude, without the Intervention of ſome other Principle to keep them faſt, ſuch as is that of *Gravity*. Beſides the Effect of this Principle is throughout the whole Surface of the Globe found nearly equal, and certain Experiment ſeems to argue it rather leſs near the Equinoctial, than towards the Poles; which could not be by any means, if the *Diurnal Rotation* of the *Earth* upon its *Axis* were the Cauſe of *Gravity*; for where the Motion was ſwifteſt, the Effect would be moſt conſiderable.

Others aſſign the *Preſſure* of the *Atmoſphere* to be the Cauſe of this *Tendency* towards the *Center* of the *Earth*; but unhappily they have miſtaken the Cauſe for the Effect, it being from undoubted Principles plain, that the Atmoſphere has no other Preſſure, but what it derives from its *Gravity*; and that the Weight of the upper Parts of the Air, preſſing on the lower Parts thereof, do ſo far bend the Springs of that Elaſtick Body, as to give it a Force equal to the Weight that compreſſed it, having of itſelf no Force at all: And ſuppoſing it had, it will be very hard to explain the *Modus*, how that Preſſure ſhould occaſion the *Deſcent* of a Body circumſcribed by it, and preſſed equally above and below, without ſome other Force to *Draw* or *Thruſt* it downwards. But to demonſtrate the contrary of this Opinion, an Experiment was long ſince ſhewn before the *Royal Society*; whereby it appeared, that the Atmoſphere was ſo far from being the Cauſe of *Gravity*, that the Effects thereof were much more vigorous where the Preſſure of the Atmoſphere was taken off; for a long Glaſs-Receiver, having a light Down-Feather included, being *Evacuated* of Air, the Feather, which in the Air would hardly ſink, did *in Vacuo* deſcend with nearly the ſame Velocity as if it had been a Stone.

Some think to illuſtrate this Deſcent of *Heavy* Bodies, by comparing it with the Virtue of the *Loadſtone*; but ſetting aſide the Difference there is in the manner of their Attractions, the *Loadſtone* drawing only in and about its Poles, and the Earth near equally in all Parts of its Surface, this Compariſon avails no more than to explain *Ignotum per æquè Ignotum*.

Others aſſign a certain *Sympathetical Attraction* between the Earth and its Parts, whereby they have, as it were, a Deſire to be united, to be the Cauſe we enquire after: but this is ſo far from explaining the *Modus*, that it is little more, than to tell us in other terms, that *Heavy* Bodies deſcend, becauſe they deſcend.

But tho' the *Efficient Cauſe* of *Gravity* be ſo obſcure, yet the *Final Cauſe* thereof is clear enough; for it is by this ſingle Principle that the Earth and all the Celeſtial Bodies are kept from Diſſolution: the leaſt of their Particles not being ſuffered to recede far from their Surfaces, without being immediately brought down again by Virtue of this *Natural Tendency*, which, for their Preſervation, the Infinite Wiſdom of their *Creator* has ordained to be towards each of their Centers; nor can the Globes of the *Sun* and *Planets* otherwiſe be deſtroyed, but by taking from them this power of keeping their Parts united.

The Affections or *Properties* of *Gravity*, and its manner of acting upon Bodies Falling, have been in a great meaſure diſcovered, and moſt of them made out by Mathematical Demonſtration in this our Century, by the accurate Diligence of *Galilæus*, *Torricellius*, *Huygenius*, and others; and now lately, by our worthy Countryman Mr. *Iſ. Newton.* Which *Properties* I ſhall here enumerate.

The Properties of Gravity. Ibid. p. 6.

1. The firſt *Property* is, That by this *Principle* of *Gravitation*, all Bodies do *deſcend* towards a *Point*, which either is, or elſe is very near to the *Center* of *Magnitude* of the *Earth* and *Sea*, about which the Sea forms itſelf exactly into a Spherical Surface, and the Prominences of the Land, conſidering the Bulk of the whole, differ but inſenſibly therefrom.

2. That this *Point*, or *Center* of *Gravitation*, is *fix'd* within the Earth, or at leaſt has been ſo, ever ſince we have any Authentick Hiſtory: For a Conſequence of its Change, tho' never ſo little, would be the over-flowing of the Low-lands on that ſide of the Globe towards which it approached, and the leaving new Iſlands bare on the oppoſite ſide, from which it receded; but for this Two Thouſand Years it appears, that the Low Iſlands of the *Mediterranean* Sea (near to which the Antienteſt Writers lived) have continued much at the ſame Height above the Water, as they now are found; and no Inundations or Receſſes of the Sea arguing any ſuch Change, are recorded in Hiſtory, excepting the *Univerſal Deluge*, which can no better way be accounted for, than by ſuppoſing this *Center* of *Gravitation* removed for a time, towards the Middle of the then Inhabited Parts of the World; and a Change of its place, but the two thouſandth Part of the Radius of this Globe, were ſufficient to bury the Tops of the Higheſt Hills under Water.

3. That in all Parts of the Surface of the Earth, or rather in all Points equidiſtant from its Center, the *Force* of *Gravity* is nearly *Equal*; ſo that the length of the Pendulum vibrating Seconds of Time, is found in all Parts of the World to be very near the ſame. 'Tis true, at St. *Helena*, in the Latitude of 16 *Deg.* South, I found that the Pendulum of my Clock, which vibrated Seconds, needed to be made ſhorter than it had been in *England*, by a very ſenſible Space (but which at that time I neglected to obſerve accurately) before it would keep Time; and ſince, the like Obſervations have been made by the *French Obſervers* near the Equinoctial; yet I dare not affirm, that in mine it proceeded from any other Cauſe, than the great Height of my Place of

Obſervation above the Surface of the Sea, whereby the *Gravity* being diminiſhed, the Length of the Pendulum, vibrating Seconds, is proportionably ſhortned.

4. That *Gravity* does *equally affect* all *Bodies*, without regard either to their Matter, Bulk, or Figure; ſo that the Impediment of the Medium being removed, the moſt compact and moſt looſe, the greateſt and ſmalleſt Bodies, would deſcend the ſame Spaces in equal Times; the Truth whereof will appear from the Experiment I before cited. In theſe two laſt Particulars, is ſhewn, the great Difference between *Gravity* and *Magnetiſm*, the one affecting only Iron, and that towards its Poles, the other all Bodies alike in every part. As a *Corollary*, from hence it will follow, That there is no ſuch thing as *Poſitive Levity*, thoſe things that appear *Light*, being only comparatively ſo; and whereas ſeveral things riſe and ſwim in Fluids: 'tis becauſe, Bulk for Bulk, they are not ſo heavy as thoſe Fluids; nor is there any Reaſon why *Cork*, for inſtance, ſhould be ſaid to be *Light* becauſe it *ſwims* on *Water*, any more than *Iron*, becauſe it *ſwims* on *Mercury*.

5. That this *Power Increaſes* as you *Deſcend* to, and *Decreaſes* as you *Aſcend* from the *Center*, and that in the Proportion of the *Squares* of the *Diſtances* therefrom *Reciprocally*, ſo as at a double Diſtance to have but a quarter of the Force: This Property is the Principle on which Mr. *Newton* has made out all the Phænomena of the Celeſtial Motions, ſo eaſily and naturally, that its Truth is paſt diſpute. Beſides that, it is highly rational, that the *Attractive* or *Gravitating Power* ſhould exert itſelf more vigorouſly in a ſmall Sphere, and weaker in a greater, in proportion as it is contracted or expanded; and if ſo, ſeeing that the Surfaces of Spheres are as the Squares of their *Radii*, this Power at ſeveral Diſtances will be as the Squares of thoſe Diſtances *Reciprocally*; and then its whole Action upon each Spherical Surface, be it great or ſmall, will be always equal. And this is evidently the Rule of *Gravitation* towards the Centers of the *Sun*, *Jupiter*, *Saturn*, and the *Earth*, and thence reaſonably inferred, to be the general Principle obſerved by Nature in all the reſt of the *Celeſtial Bodies*.

Theſe are the principal Affections of *Gravity*, from which the Rules of the *Fall* of *Bodies*, and the *Motion* of *Projects*, are Mathematically deducible. Mr. *Iſ. Newton* hath ſhewed how to define the Spaces of the Deſcent of a Body, let fall from any given Height, down to the Center, ſuppoſing the *Gravitation* to increaſe, as in the Fifth Property; but conſidering the Smallneſs of Height, to which any *Project* can be made to aſcend, and over how little an Arch of the Globe it can be caſt by any of our Engines, we may well enough ſuppoſe the *Gravity Equal* throughout, and the *Deſcents* of *Projects* in *Parallel Lines*, which, in Truth, are towards the *Center*; the Difference being ſo ſmall as by no means to be diſcovered in Practice.

Propoſitions concerning the Deſcent of Heavy Bodies, and the Motion of Projects.
Ibid. p. 9.

Prop. I.] *The Velocities of falling Bodies, are Proportionate to the Times from the Beginning of their Falls.*

This follows, for that the Action of *Gravity* being continual, in every ſpace of Time, the falling Body receives a new Impulſe, equal to what it had before,

fore, in the ſame Space of Time, received from the ſame *Power*; For inſtance, in the Firſt Second of Time, the falling Body has acquired a *Velocity*, which in That Time would carry it to a certain Diſtance, ſuppoſe 32 Foot, and were there no new Force, would deſcend at that Rate with an Equable Motion; but in the next Second of Time, the ſame *Power* of *Gravity* continually Acting thereon, ſuperadds a New *Velocity* equal to the former; ſo that at the end of two Seconds, the *Velocity* is double to what it was at the end of the firſt; and after the ſame manner may it be proved to be Triple, at the end of the Third Second, and ſo on. Wherefore the *Velocities of Falling Bodies, are proportionate to the Times of their Falls.* Q. E. D.

Prop. II.] *The Spaces deſcribed by the Fall of a Body, are as the Squares of the Times from the Beginning of the Fall.*

Demonſtration.] Let AB repreſent the Time of the Fall of a Body, BC, Fig. 171. perpendicular to AB, the Velocity acquired at the End of the Fall, and draw the Line AC; then divide the Line AB, repreſenting the Time, into as many equal Parts as you pleaſe, as *b*, *b*, *b*, *b*, *&c.* and through theſe Points draw the Lines, *bc*, *bc*, *bc*, *bc*, *&c.* parallel to BC; 'tis manifeſt that the ſeveral Lines, *bc*, repreſent the ſeveral *Velocities* of the Falling Body, in ſuch parts of the Time, as A*b*, is of AB, by the *Former Propoſition*. It is evident likewiſe that the Area, ABC, is the Sum of all the Lines *bc*, being taken according to the *Method* of *Indiviſibles* infinitely many; ſo that the Area ABC, repreſents the Sum of all the *Velocities* between none and BC, ſuppoſed infinitely many; which Sum is the Space deſcended in the Time repreſented by AB. And by the ſame Reaſon, the Areas A*bc*, will repreſent the Spaces deſcended in the Times A*b*; ſo then the Spaces deſcended in the Times AB, A*b*, are as the Areas of the Triangle, ABC, A*bc*, which by the 20*th* of the 16*th* of *Euclid* are as the *Squares* of their *Homologous Sides* AB, A*b*, that is to ſay, of the *Times*: Wherefore the *Deſcents* of *Falling Bodies*, are as the *Squares* of the *Times* of their *Fall*. Q. E. D.

Prop. III.] *The Velocity which a falling Body acquires in any ſpace of Time, is double to that, wherewith it would have moved the Space deſcended by an Equable Motion, in the ſame Time.*

Demonſtration.] Draw the Line EC parallel to AB, and AE parallel to BC, and compleat the *Parallelogram* ABCE; it is evident that the *Area* thereof may repreſent the Space, a Body moved *Equably* with the *Velocity* BC, would deſcribe in the Time AB; and the *Triangle* ABC, repreſents the Space deſcribed by the Fall of a Body, in the ſame Time AB, by the *Second Propoſition*. Now the *Triangle* ABC, is Half of the *Parallelogram* ABCE; and conſequently the Space deſcribed by the *Fall*, is Half what would have been deſcribed by an *Equable Motion* with the *Velocity* BC, in the ſame Time: wherefore the *Velocity* BC, at the End of the Fall, is *Double to that Velocity*, which in the Time AB, would have deſcribed the Space fallen, repreſented by the *Triangle* ABC, with an *Equable Motion*. Q. E. D.

Prov. IV.] *All Bodies on or near the Surface of the Earth, in their Fall, deſcend ſo, as at the End of the Firſt Second of Time, they have deſcribed* 16 *Feet, one Inch,* London *Meaſure, and acquired the Velocity of* 32 *Feet, two Inches, in a Second.*

This is made out from the 25*th Prop. Par.* 2. *Porol. Oſcill. Hugen.* wherein he demonſtrates the Time of the leaſt Vibrations of a Pendulum, to be to the Time of the Fall of a Body, from the Height of Half the Length of the Pendulum, as the Circumference of a Circle to its Diameter: whence, as a *Corollary*, it follows, That as the Square of the Diameter to the Square of the Circumference, ſo half the Length of the Pendulum vibrating Seconds, to the Space deſcribed by the Fall of a Body in a ſecond of Time: and the Length of the Pendulum vibrating Seconds, being found 39,125, or ⅛ Inches, the Deſcent in a Second will be found, by the aforeſaid Analogy, 16 Foot and one Inch: and by the *Third Propoſition*, the Velocity will be double thereto; and near to this it hath been found by ſeveral Experiments, which by reaſon of the Swiftneſs of the Fall, cannot ſo exactly determine its Quantity.

From theſe Four Propoſitions, all Queſtions concerning the *Perpendicular Fall* of Bodies are eaſily ſolved, and either *Time*, *Height*, or *Velocity* being aſſigned, one may readily find the other two. From them likewiſe is the *Doctrine of Projects* deducible, aſſuming the two following *Axioms: viz.*

1. *That a Body ſet a moving, will move on continually in a Right Line with an Equable Motion, unleſs ſome other Force or Impediment intervene, whereby it is accelerated, or retarded, or deflected.*

2. *That a Body being agitated by two Motions at a Time, does by their Compounded Forces paſs thro' the ſame Points, as it would do, were the two Motions divided and acted ſucceſſively.* As for inſtance,

Fig. 172. Suppoſe a Body moved in the Line G F, from G to R, and there ſtopping, by another Impulſe ſuppoſe it moved in a Space of Time equal to the former from R towards K to V; I ſay the Body ſhall paſs thro' the point V, tho' theſe two ſeveral Forces acted both in the ſame Time.

Prop. V.] *The Motion of all Projects is in the Curve of a Parabola.*

Demonſtration.] Let the Line G R F be the Line in which the *Project* is directed, and in which by the *firſt Axiom* it would move equal Spaces in equal Times, were it not deflected downwards by the Force of *Gravity*. Let G B be the Horizontal Line, and G C a perpendicular thereto. Then the Line GRF, being divided into equal Parts, anſwering to equal Spaces of Time, let the Deſcents of the Project be laid down in Lines Parallel to G C, proportioned as the *Squares* of the *Lines*, GS, GR, GL, GF, or as the *Squares* of the *Times*, from S to T, from R to V, from L to X, and from F to B; and draw the Lines T H, V D, X Y, B C, Parallel to G F: I ſay, the Points T, V, X, B, are Points in the *Curve* deſcribed by the *Project*, and that that *Curve* is a *Parabola*. By the *ſecond Axiom* they are Points in the *Curve*; and the Parts of the Deſcent G H, G D, G Y, G C, = to S T, R V, L X, F B, being as the Squares of the Times (by the *ſecond Prop.*) that is, as the *Squares* of the *Ordinates* H T, D V, Y X, C B, equal to GS, GR, GL, GF, the Spaces meaſured in thoſe Times; and there being no other *Curve* but the *Parabola*, whoſe Parts of the *Diameter* are as the *Squares* of the *Ordinates*, it follows, that the *Curve* deſcribed by a *Project* can be no other than a *Parabola:* And ſaying, as RV, the Deſcent in Time, to GR, or VD, the direct Motion in the ſame Time; ſo is VD, to a third Proportional; that Third will be the Line called by all Writers of *Conicks*, the *Parameter* of the *Parabola* to the *Diameter* GC; which is always the ſame in *Projects* caſt with the ſame *Velocity:* and the *Velocity* being defined by the number of Feet, moved in a Second of Time, the *Parameter* will be found by dividing the Square of the *Velocity* by 16 Feet 1 Inch, the Fall of a Body in the ſame Time.

Lemma. [*The Sine of the Double of any Arch, is equal to twice the Sine of that Arch into its Co-Sine, divided by Radius; and the verſed Sine of the Double of any Arch, is equal to the Square of the Sine thereof divided by Radius.*

Fig. 173. Let the Arch BC be double the Arch BF, and A the Center; draw the Radii AB, AF, AC, and the Chord BDC, and let Fall BE, perpendicular to AC; and the Angle EBC, will be equal to the Angle BAD, and the Triangle BCE, will be like the Triangle ABD; wherefore it will be, as AB to AD; ſo BC, or twice BD, to BE; that is, as *Radius* to *Co-Sine*, ſo *twice Sine* to *Sine of the double Arch*; and as AB to BD, ſo twice BD or BC, to EC; that is, as *Radius* to *Sine*, ſo *twice that Sine* to the *verſed Sine of the double Arch:* which two *Analogies* reſolved into Equations, are the *Propoſitions* contained in the *Lemma* to be proved.

Prop. VI.] *The Horizontal diſtance of Projections made with the ſame Velocity, at ſeveral Elevations of the Line of Direction, are as the Sines of the doubled Angles of Elevation.*

Fig. 174. Let GB, the horizontal Diſtance be $=z$, the Sine of the Angle of Elevation, FGB, be $=s$, its Co-ſine $=c$, Radius $=r$, and the Parameter $=p$. It will be, as c to s, ſo z to $\frac{sz}{c} =$ FB $=$ GC, and by reaſon of the *Parabola* $\frac{psz}{c}$ $=$ to the Square of CB, or GF. Now as c to r, ſo is z to $\frac{zr}{c} =$ GF, and its Square $\frac{zzrr}{c.c}$ will be therefore $=$ to $\frac{psz}{c}$: which *Equation* reduced, will be $\frac{psc}{rr} = z$. But by the former *Lemma* $\frac{2sc}{r}$ is equal to the Sine of the double Angle, whereof s is the Sine: Wherefore 'twill be, as *Radius* to *Sine* of double the Angle FGB, ſo is half the *Parameter*, to the *horizontal Range* or *Diſtance* ſought: and at the ſeveral *Elevations*, the *Ranges are as the Sines of the double Angles of Elevation.* Q. E. D.

Caroll.] Hence it follows, That *half the Parameter is the greateſt Random, and that that happens at the Elevation of* 45°, *the Sine of whoſe double is Radius.*

Likewiſe, That *the Ranges equally diſtant above and below* 45° *are equal, as are the Sines of all doubled Arches, to the Sines of their doubled Complements.*

Prop. VII.] *The Altitudes of Projections made with the ſame Velocity, at ſeveral Elevations, are as the verſed Sines of the doubled Angles of Elevation.*

As c is to s, ſo is $\frac{pst}{rr}$ = GB, to $\frac{pss}{rr}$ = BF, and VK = RV = $\frac{1}{4}$ BF, the *Altitude* of the *Projection* = $\frac{pss}{4rr}$. Now by the foreging *Lemma* $\frac{2ss}{r}$ = to the *verſed Sine* of the *double Angle*; and therefore it will be, as *Radius* to *verſed Sine* of double the Angle FGB, ſo $\frac{1}{8}$ of *Parameter* to the *height* of the *Projection* VK; and ſo thoſe *Heights* at ſeveral *Elevations* are as the ſaid *verſed Sines.* Q. E. D.

Coroll.] From hence it is plain, That *the greateſt Altitude of the Perpendicular Projection is a* 4*th of the Parameter, or half the greateſt horizontal Range:* The *verſed Sine of* 180 Degrees being = 2 r.

Prop. VIII.] *The Lines* GF, *or Times of the Flight of a Project caſt with the ſame degree of Velocity at different Elevations, are as the Sines of the Elevations.*

As c is to r, ſo is $\frac{psc}{rr}$ = GB (by the 6 *Prop.*) to $\frac{ps}{r}$ = GF, that is, as *Radius* to the *Sine* of *Elevation*, ſo the *Parameter* to the Line GF; ſo the *Lines* GF are as the *Sines* of *Elevation*, and the *Times* are proportional to thoſe *Lines:* wherefore the *Times* are as the *Sines of Elevations*; therefore the *Propoſition* is manifeſt.

Prop. IX. Prob. 1.] *A Projection being made, as you pleaſe, having the Diſtance and Altitude, or Deſcent of an Object, thro' which the Project paſſes, together with the Angle of Elevation of the Line of Direction; to find the Parameter and Velocity*; that is, (having the Angle F G B, G M,) and M X.

Solution.] As Radius to Secant of F G B, ſo G M the diſtance given, to GL; and as Radius to Tangent of FGB, ſo G M to L M. Then L M—M X in *Heights*, or + M X in *Deſcents:* or elſe M X—M L, if the *Direction* be *below* the *Horizontal-Line*, is the *Fall* in the *Time* that the direct Impulſe given in G, would have carried the Project from G to L=L X=G Y; then by reaſon of the *Parabola*; as L X, or G Y, is to G L or Y X; ſo is G L, to the *Parameter* ſought. To find the *Velocity* of the *Impulſe*, by *Prop.* 2 and 4. find the *Time* in Seconds that a Body would *fall* the Space L X, and by that dividing the Line G L, the Quote will be the *Velocity*, or *Space moved* in a Second ſought, which is always a mean *Proportional* between the *Parameter* and 16 Feet, 1 Inch. Fig. 172.

Prop. X. Prob. 2.] *Having the Parameter, Horizontal Diſtance, and Height or Deſcent of an Object; to find the Elevations of the Lines of Direction neceſſary* *to hit the given Object*; that is, having GM, MX, and the greateſt Random equal to half the Parameter; to find the Angles FGB.

Let the Tangent of the Angle ſought be $=t$, the horizontal Diſtance GM $=b$, the Altitude of the Object MX $=h$, the Parameter $=p$, and Radius $=r$; and it will be, as r to t, ſo b to $\frac{tb}{r}=$ ML; and $\frac{tb}{r}\mp h$ $\left\{\begin{matrix}\text{in Aſcents}\\ \text{in Deſcents}\end{matrix}\right\}=$ LX, and $\frac{ptb}{r}\mp ph=$ GL$q.=$ XY$q.$ *ratione Parabolæ*; *but* $bb+\frac{ttbb}{rr}=$ GL$q.$ (47. 1. *Euclid.*) Wherefore $\frac{ptb}{r}\mp ph=bb+\frac{ttbb}{rr}$; which Equation tranſpoſed, is $\frac{ttbb}{rr}=\frac{ptb}{r}\mp ph-bb$; divided by bb, is $\frac{tt}{rr}=\frac{pt}{br}\mp\frac{ph}{bb}-1$. This Equation ſhews the Queſtion to have two Anſwers, and the Roots thereof are $\frac{t}{r}=\frac{p}{2b}\mp\sqrt{\frac{pp\mp 4ph}{4bb}\,1}$; from which I derive the following Rule. Divide half the *Parameter* by the *Horizontal Diſtance*, and keep the Quote, *viz.* $\frac{p}{2b}$; then ſay, as Square of the *Diſtance* given to half the *Parameter*, ſo half Parameter $\left\{\begin{matrix}-\\ +\end{matrix}\right\}$ double $\left\{\begin{matrix}\text{Height}\\ \text{Deſcent}\end{matrix}\right\}$ to the Square of a *Secant* $=\frac{pp+4ph}{4bb}$, the *Tangent* anſwering to that *Secant* will be $\sqrt{\frac{pp\mp 4ph}{4bb}}-1$, or rr: ſo then the Sum and Difference of the afore-found *Quote* and this *Tangent*, will be the Root of the *Equation*, and the *Tangents* of the *Elevations* ſought.

Note here, That in *Deſcents*, if the *Tangent* exceed the *Quote*, as it does when ph is more than bb, the *Direction* of the lower *Elevation* will be below the *Horizon*; and if $ph=bb$, it muſt be *directed Horizontal*, and the *Tangent* of the upper *Elevation* will be $\frac{pr}{b}$: *Note* likewiſe, That if $4bb+4ph$ in *Aſcents*, or $4bb-4ph$ in *Deſcents*, be equal to pp, there is but one *Elevation* that can *hit* the *Object*, and its *Tangent* is $\frac{pr}{2b}$; and if $4bb\mp 4ph$ in *Aſcents*, or $4bb-4ph$ in *Deſcents*, do exceed pp, the *Object* is without the *Reach* of a *Project* caſt with that *Velocity*, and ſo the thing impoſſible.

From this *Equation* $4bb\mp 4ph=pp$, are determined the utmoſt *Limits* of the *Reach* of any *Project*, and the Figure aſſigned, wherein are all the Heights upon each horizontal Diſtance, beyond which it cannot paſs; for by Reduction of that *Equation*, h will be found $=\frac{1}{4}p-\frac{bb}{p}$ in *Heights*, and $\frac{bb}{p}-\frac{1}{4}p$ in *Deſ-*

cents; from whence it follows, that all the Points b are in the *Curve* of the *Parabola*, whoſe *Focus* is the Point from whence the *Project* is caſt, and whoſe *Latus Rectum*, or *Parameter ad Axem* is $= p$. Likewiſe from the ſame *Equation* may the leaſt *Parameter* or *Velocity* be found capable to reach the Object propoſed; for $bb = \frac{1}{4}pp \mp pb$ being reduced, $\frac{1}{2}p$ will be $= \sqrt{bb + bb}$ $\left\{\begin{matrix} +b \text{ in } \textit{Aſcents} \\ -b \text{ in } \textit{Deſcents} \end{matrix}\right\}$ which is the *Horizontal* Range at 45°, that would juſt reach the *Object*, and the *Elevation* requiſite will be eaſily had; for dividing the ſo found *Semi-Parameter* by the *Horizontal Diſtance* given, b, the Quote into *Radius* will be the *Tangent* of the *Elevation* ſought.

But if a *Geometrical Conſtruction* of this *Problem* be required, I think I have one, that is as eaſy as any can be expected, which I deduce from the foregoing *Analytical Solution*, viz. $\frac{t}{r} = \frac{p}{2b} \pm \sqrt{\frac{\frac{1}{4}pp \pm pb - bb}{bb}}$; and it is this: Having made the right Angle L D A, make D A, D F $= p$, or greateſt *Range*, D G $= b$, the *Horizontal Diſtance*, and D B, D C $= b$, the Fig. 174. Perpendicular *Height* of the Object; and draw GB, and make DE $=$ thereto. Then with the Radius A C, and Center E, ſweep an Arch, which if the thing be poſſible, will interſect the Line A D, in H; and the Line D H, being laid both ways from F, will give the Points K, and L, to which draw the Lines G L, G K; I ſay, the Angles L G D, K G D, are the *Elevations* required for *hitting* the *Object*, B. But *Note*, that if B be below the Horizon, its *Deſcent* D C $=$ D B, muſt be laid upon A, ſo as to have A C $=$ A D $+$ D C. *Note* likewiſe, that if in *Deſcents*, D H be greater than F D, and ſo K fall below D, the Angle K G D ſhall be the *Depreſſion* below the Horizon.

n. 216. p. 69. Mar. An. 1695. When I gave the preceding Solution of this Problem, *viz. To hit an Object above or below the Horizontal Line, with the greateſt Certainty and leaſt Force*, I was not aware, that the *Elevation* there ſought did conſtantly *biſect* the Angle between the Perpendicular and the Object, as is demonſtrated from the Difference and Sum of the *Tangent* and *Secant* of any Arch, being always equal to the *Tangent* and *Co-Tangent* of the half Complement thereof to a Quadrant. But having diſcover'd this, I think nothing can be more compendious, or bid fairer to compleat the *Art* of *Gunnery*, it being as eaſy to ſhoot with a *Mortar* at any Object on demand, as if it were on the Level; neither is there need of any Computation, but only ſimply laying the Gun, to paſs in the middle Line between the Zenith and the Object, and giving it its due *Charge*. Nor is there any great need of Inſtruments for this Purpoſe: For, if the *Muzzle* of the *Mortar* be turned truly Square to the Bore of the Piece, as it uſually is, or ought to be, a Piece of Looking-Glaſs Plate applied Parallel to the *Muzzle*, will, by its Reflection, give the true Poſition of the Piece; the *Bombardier* having no more to do, but to look perpendicularly down on the Looking-Glaſs, along a ſmall Thread with a Plummet, and to raiſe or depreſs the *Elevation* of the *Piece*, till the Object appear reflected on the ſame Point of the *Speculum* on which the Plummet falls; for the Angle of *In-*

Incidence and *Reflection* being *Equal*, in this caſe a Line at Right Angles to the *Speculum*, as is the *Axis* of the *Chaſe* of the Piece, will *biſect* the Angle between the Perpendicular and the Object, according as our *Propoſition* requires.

Prop. XI. Prob. 3.] *A Shot being made on an inclined Plane, having the Horizontal Diſtance of the Object it ſtrikes, with the Elevation of the Piece, and the Angle at the Gun between the Object and the Perpendicular; to find the greateſt Horizontal Range of that Piece, laden with the ſame Charge; that is, half the* Latus Rectum *of all the* Parabolæ *made with the ſame* Impetus.

Take half the Diſtance of the Object from the *Nadir*, and take the Difference of the given Elevation from that Half; the *verſed Sine* of twice that Difference ſubtract from the *verſed Sine* of the Diſtance of the Object from the *Zenith*: then ſhall the Difference of thoſe *verſed Sines* be to the Sine of the Diſtance of the Object from the *Zenith*, as the Horizontal Diſtance of the Object ſtruck to the greateſt Horizontal Range at 45°.

Prop. XII. Prob. 4.] *Having the greateſt Horizontal Range of a Gun, the Horizontal Diſtance and Angle of Inclination of an Object to the Perpendicular; to find the two Elevations neceſſary to ſtrike that Object.*

Halve the Diſtance of the Object from the *Nadir*; this Half is always equal to the Half Sum of the two *Elevations* we ſeek. Then ſay, as the greateſt Horizontal Range, is to the Horizontal Diſtance of the Object; ſo is the Sine of the Angle of Inclination, or Diſtance of the Object from the Perpendicular, to a *4th* Proportional; which *4th* being ſubtracted from the *verſed Sine* of the Diſtance of the Object from the *Zenith*, leaves the *verſed Sine* of the Difference of the *Elevations* ſought; which *Elevations* are therefore had, by adding and ſubtracting that half Difference to, and from, the aforeſaid half Sum.

Prop. XIII.] *To determine the Force or Velocity of a Project, in every Point of the Curve it deſcribes.* n. 172. p. 18.

To do this, we need no other *Præcognita*, but only the *Third Propoſition*, viz. *That the Velocity of falling Bodies is double to that, which in the ſame Time would have deſcribed the Space fallen by an equable Motion*: For the *Velocity* of a *Project* is compounded of the conſtant equal *Velocity* of the impreſſed Motion, and the *Velocity* of the Fall, under a given Angle, *viz.* the Complement of the *Elevation*: For inſtance, In the Time wherein a *Project* would Fig. 172. move from G to L, it deſcends from L to X, and by the *third Propoſition* has acquired a *Velocity*, which in that Time would have carried it by an equable Motion from L to Z, or twice the Deſcent L X; and drawing the Line G Z, I ſay the *Velocity* in the Point X, compounded of the *Velocities* G L, and L Z, under the Angle G L Z, is to the *Velocity* impreſſed in the Point G, as G Z is to G L; this follows from our *ſecond Axiom*; and by the *20th* and *21ſt Prop. Lib. 1. Conic. Midorgii*, X O, Parallel and Equal to G Z, ſhall touch the *Parabola* in the Point X. So that the *Velocities* in the ſeveral Points, are as the Lengths of the *Tangents* to the *Parabola* in thoſe

Points, intercepted between any two Diameters: and theſe again are as the *Secants* of the Angles, which thoſe *Tangents* continued make with the Horizontal Line G B. From what is here laid down, may the comparative Force of a Shot in any two Points of the Curve, be either Geometrically or Arithmetically diſcovered.

Coroll.] From hence it follows, that the force of a Shot is always leaſt at V, or the *Vertex* of the *Parabola*; and that at equal Diſtances therefrom, as at T and X, G and B, its force is always equal; and that the leaſt Force in V, is to that in G and B, as *Radius* to the *Secant* of the Angle of *Elevation*, FGB.

The *Tenth Propoſition* contains a *Problem*, untouched by *Torricellius*, which is of the greateſt uſe in Gunnery, and for the ſake of which this Diſcourſe was principally intended. It was firſt ſolved by Mr. *Anderſon*, in his Book of the *Genuine Uſe and Effects of the Gun*, printed in the Year 1674, but his *Solution* required ſo much Calculation, that it put me upon Search, whether it might not be done more eaſily; and thereupon in the Year 1678, I found out the *Rule* I now publiſh, and from it the Geometrical Conſtruction: Since which time, there has a large Treatiſe of this Subject, Intitled, *L' Art de Jetter les Bombes*, been publiſhed in *France* by M. *Blondel*, wherein he gives the *Solutions* of this *Problem*, by Meſſieurs *Bout*, *Romer*, and *de la Heir*: but none of them are the ſame with mine, or in my Opinion more eaſy.

p. 216... 68. It was formerly the Opinion of thoſe concerned in *Artillery*, that there was a certain requiſite of *Powder* for each *Gun*, and that in *Mortars* where their Diſtance was to be varied, it muſt be done by giving a greater or leſſer *Elevation* to the *Piece*. But now our later Experience has taught us, That the ſame thing may be more certainly and readily performed, by *increaſing and diminiſhing the quantity of Powder*, whether regard be had to the Execution to be done, or to the Charge of doing it. For when *Bombs* are diſcharged with great *Elevations* of the *Mortar*, they fall too Perpendicular, and bury themſelves too deep in the ground, to do all that Damage they might, if they came more oblique, and broke upon or near the Surface of the Earth; which is a thing acknowledged by the *beſieged* in all *Towns*, who unpave their Streets to let the *Bombs* bury themſelves, and thereby ſtifle the force of their Splinters. A *Second Convenience* is, that at the Extreme *Elevation*, the *Gunner* is not obliged to be ſo curious in the Direction of his *Piece*, but it will ſuffice to be within a Degree or two of the Truth; whereas in the other method of *Shooting*, he ought to be very curious. But a *Third* and no leſs conſiderable *Advantage* is, in the ſaving of the *King*'s *Powder*, which in ſo great and ſo numerous *Diſcharges*, as we have lately ſeen, muſt needs amount to a conſiderable Value. And for *Sea Mortars* it is ſcarce practicable otherwiſe to uſe them, where the *Agitation* of the Sea continually changes the *Direction* of the *Mortar*, and would render the Shot very uncertain, were it not that they are placed about 45° *Elevation*, where ſeveral degrees above or under makes very little difference in the *Effect*.

Ibid. p. 70. It only remains, by good and valid *Experiments*, to be aſſured of the *Force* of *Gun-Powder*; how to make and conſerve it equal; and to know the Effect ſect thereof in each *Piece*; that is, how far differing *Charges* will caſt the ſame Shot out of it; which may moſt conveniently be Engraven on the outſide thereof, as a ſtanding Direction to all *Gunners*, who ſhall from thence forward have occaſion to uſe that *Piece*: and were this Matter well aſcertained, it might be worth the while to make all *Mortars* of the like *Diameter*, as near as may be alike in *Length* of *Chaſe*, *Weight*, *Chamber*, and all other *Circumſtances*.

Now the foregoing Rules would be rigidly true, were it not for the *Oppoſition* of the *Medium*, whereby not only the direct impreſſed Motion is continually retarded, but likewiſe the Increaſe of the Velocity of the Fall, ſo that the Spaces deſcribed thereby, are not exactly as the *Squares* of the *Times*: But what this *Oppoſition* of the *Air* is, againſt ſeveral *Velocities*, *Bulks*, and *Weights*, is not ſo eaſy to determine. 'Tis certain, That the *Weight* of the *Air* to that of *Water*, is nearly as 1 to 800; whence the *Weight* thereof, to that of any *Project* is given: 'tis very likely, that to the ſame *Velocity* and *Magnitude*, but of different Matter, the *Oppoſition* ſhould be *reciprocally* as the *Weights* of the *Shot*; as likewiſe that to *Shot* of the ſame *Velocity* and *Matter*, but of different *Sizes*, it ſhould be as the *Diameters reciprocally*: Whence generally the *Oppoſition* to *Shot* with the ſame *Velocity*, but of differing Diameters, and Materials, ſhould be as their *Specifick Gravities* into their *Diameters reciprocally*: but whether the Oppoſition to differing *Velocities* of the ſame *Shot*, be as the *Squares* of thoſe *Velocities*, or as the *Velocities* themſelves, or otherwiſe, is yet a harder Queſtion. However it be, 'tis certain, That in large Shot of Metal, whoſe Weight many thouſand times ſurpaſſes that of the *Air*, and whoſe Force is very great in proportion to the Surface wherewith they preſs thereon, this *Oppoſition* is ſcarce *diſcernible*: For by ſeveral *Experiments*, made with all Care and Circumſpection, with a *Mortar-Piece*, extraordinary well fixed to the Earth on purpoſe, which carried a ſolid Braſs Shot of 4½ Inches Diameter, and of about 14 Pound Weight, the Ranges above and below 45°, were found nearly equal; if there were any Difference, the under Ranges went rather the fartheſt, but thoſe Differences were uſually leſs than the Errors committed in ordinary Practice, by the unequal Goodneſs and Dryneſs of the ſame ſort of *Powder*, by the Unfitneſs of the Shot to the *Bore*, and by the Looſeneſs of the *Carriage*. In a ſmaller Braſs Shot of about an Inch and half Diameter, caſt by a *Croſs-Bow*, which ranged it at moſt about 400 Foot, the Force being much more equal than the *Mortar Piece*, this Difference was found more curiouſly, and conſtantly, and moſt evidently, the under *Ranges* out-went the upper. From which Trials I conclude, that altho' in ſmall and light Shot the *Oppoſition* of the *Air*, ought and muſt be accounted for; yet in ſhooting of great and weighty *Bombs*, there need be very little or no Allowance made: and ſo theſe Rules may be put in practice to all Intents and Purpoſes, as if this *Impediment* were abſolutely removed. 179. p. 19.

The Meaſure of the Air's Reſiſtance to Bodies, moved in it; by Dr. Wallis. *n.* 186. *p.* 269. Jan. An. 1687.

IX. 1. In order to compute the *Reſiſtance* of the *Air* to all *Projects*, I firſt premiſe this *Lemma* (as the moſt rational that doth occur, for my firſt footing)

ing) *That* (ſuppoſing other things equal) *The Reſiſtance is a proportional to the Celerity.* For in a double *Celerity*, there is to be removed (in the ſame time) twice as much *Air* (which is a double *Impediment*;) in a treble, thrice as much; and ſo in other Proportions.

2. Suppoſe we then the Force impreſſed (and conſequently the *Celerity*, if there were no *Reſiſtance*) as 1; the *Reſiſtance* as r (which muſt be leſs than the Force, or elſe the Force would not prevail over the Impediment, to create a *Motion*.) And therefore the effective Force at a firſt *Moment*, is to be reputed as $1-r$: That is, ſo much as the Force impreſſed, is more than the *Impediment* or *Reſiſtance*.

3. Be it as $1-r$ to 1, ſo 1 to m (which m is therefore greater than 1.)

4. And therefore the effective Force (and conſequently the *Celerity*) as to a firſt *Moment*, is to be $\frac{1}{m}$ of what it would be, had there been no *Reſiſtance*.

5. This $\frac{1}{m}$ is alſo the remaining Force after ſuch firſt Moment; and this remaining Force is (for the ſame Reaſon) to be proportionably abated as to a ſecond Moment: That is, we are to take $\frac{1}{m}$ thereof, that is, $\frac{1}{mm}$ of the impreſſed Force. And for a third Moment (at equal Diſtance of Time) $\frac{1}{mmm}$; for a fourth $\frac{1}{m^4}$; and ſo onward infinitely.

6. Becauſe the Length diſpatched (in equal Times) is proportional to the *Celerities*; the Lines of Motion (anſwering to thoſe equal Times) are to be as $\frac{1}{m}, \frac{1}{m^2}, \frac{1}{m^3}, \frac{1}{m^4}$, *&c.* of what they would have been in the ſame Times, had there been no *Reſiſtance*.

7. This therefore is a *Geometrical Progreſſion*; and (becauſe of m greater than 1) continually decreaſing.

8. This decreaſing Progreſſion infinitely continued (determining in the ſame Point of Reſt, where the Motion is ſuppoſed to expire) is yet of a finite Magnitude, and equal to $\frac{1}{m-1}$ of what it would have been in ſo much time, if there had been no *Reſiſtance*: As is demonſtrated in my *Algebra*, *Chap.* 95. *Prop.* 8. For (as I have elſewhere demonſtrated) the *Sum* or Aggregate of a *Geometrical Progreſſion* is $\frac{VR-A}{R-1}$, (ſuppoſing V the greateſt Term, A the leaſt, and R the common Multiplier;) That is, $\frac{VR}{R-1}-\frac{A}{R-1}$. Now in the preſent Caſe (ſuppoſing the Progreſſion infinitely continued) the leaſt Term A, becomes infinitely ſmall, or $=0$: And conſequently $\frac{A}{R-1}$ doth alſo vaniſh, and thereby the Aggregate be-

comes $=\frac{VR}{R-1}$. That is (as will appear by Dividing V R by R—1;) $V+\frac{V}{R}+\frac{V}{RR}+\frac{V}{R^3}+$, &c. $=\frac{VR}{R-1}$; (ſuppoſing the Progreſſion to begin at V=1.) That is, (dividing all by R, that ſo the Progreſſion may begin at $\frac{V}{R}=\frac{1}{m}$:) $\frac{V}{R-1}=\frac{V}{R}+\frac{V}{RR}+\frac{V}{R^3}+$ &c. That is, in our preſent Caſe (becauſe of V=1, and R=*m*) $\frac{1}{m}+\frac{1}{mm}+\frac{1}{m^3}$, &c. $=\frac{1}{m-1}$. That is (putting $n=m-1$) $\frac{1}{n}$, of what it would have been, if there had been no *Reſiſtance*.

9. This infinite Progreſſion is fitly expreſſed by an *Ordinate* in the *Exterior Hyperbola*, parallel to one of the *Aſymptotes*; and the ſeveral Members of that, by the ſeveral Members of this, cut in *Continual Proportion*: As is there demonſtrated at *Prop.* 15. For let S H be an *Hyperbola* between the *Aſymptotes*, A B, A F: And let the *Ordinate* D H (in the *Exterior Hyperbola*, parallel to A F) repreſent the impreſſed Force undiminiſhed; or the Line to be deſcribed in ſuch Time, by a *Celerity* anſwerable to ſuch Undiminiſh'd Force. And let B S (a like *Ordinate*) be $\frac{1}{m}$ thereof; which therefore, being leſs than D H (as being equal to a part of it) will be further than it from A F. In A B (which I put =1,) let B*d* be ſuch a part thereof, as is B S of D H. Now becauſe (as is well known) all the inſcribed Parallelograms, in the *Exterior Hyperbola*, A S, A H, &c. are equal; and therefore their Sides reciprocal: Therefore as A*d* $=1-\frac{1}{m}$, (ſuppoſing B*d*, to be taken from B toward A,) to A B=1, (or as $m-1$ to m:) ſo is B S $=\frac{1}{m}$ D H to *d b*, which is therefore equal to $\frac{1}{m-1}$ of D H; that is (as will appear by Dividing 1 by $m-1$,) to $\frac{1}{m}+\frac{1}{mm}+\frac{1}{m^3}$ &c. of D H. *Fig.* 175.

Or if B*d* be taken beyond B: then as A*d* $=1+\frac{1}{m}$, to A B=1, or as $m+1$ to m, ſo is $\frac{1}{m}$ D H, to *d b*, which is therefore equal to $\frac{1}{m+1}$ D H; that is, (as will appear by like dividing of 1 by $m+1$;) $=\frac{1}{m}-\frac{1}{mm}+\frac{1}{m^3}-$, &c. of D H.

10. Let ſuch *Ordinate d b*, or (equal to it in the *Aſymptote*) A F, be ſo divided in L, M, N, &c. (by Perpendiculars cutting the *Hyperbola* in *l*, *m*, *n*, &c.) *Fig.* 176.

n, &c.) as that F L, L M, M N, be as $\frac{1}{m}$, $\frac{1}{mm}$, $\frac{1}{m^3}$, &c. That is, ſo continually decreaſing, as that each Antecedent be to its Conſequent, as 1 to $\frac{1}{m}$, or as m to 1.

11. This is done by taking A F, A L, A N, &c. in ſuch Proportion. For, of continual Proportionals the Differences are alſo continually Proportional, and in the ſame Proportion. For let A, B, C, D, &c. be ſuch Proportionals, and their Differences, a, b, c, &c. That is, A—B=a, B—C=b, C—D=c, &c.

Then becauſe, A, B, C, D, &c. are in continual Proportion;
That is, A : B :: B : C :: C : D ::, &c.
And dividing, A—B : B :: B—C : C :: C—D : D ::, &c.
That is, a : B :: b : C :: d : D ::, &c.
And Alternately; $a.b.c.$ &c. :: B. C. D. &c. :: A. B. C. &c.
That is, In Continual Proportion, as A to B, or as m to 1.

12. This being done; the *Hyperbolick Spaces* Fl, Lm, Mn, &c. are equal, as is demonſtrated by *Gregory San-Vincent*; and as ſuch is commonly admitted.

13. So that Fl, Lm, Mn, &c. may fitly repreſent equal Times in which are diſpatched unequal Lengths, repreſented by F L, L M, M N; &c.

14. And becauſe they are in Number infinite, (tho' equal to a finite Magnitude) the Duration is infinite; and conſequently the impreſſed Force, and Motion thence ariſing, never to be wholly extinguiſhed (without ſome further Impediment) but perpetually approaching to A, in the Nature of *Aſymptotes*.

15. The Spaces Fl, Fm, Fn, &c. are therefore as *Logarithms* (in *Arithmetical Progreſſion* increaſing) anſwering to the Lines, A F, A L, A M, &c. or to F L, L M, M N, &c. in *Geometrical Progreſſion* decreaſing.

16. Becauſe F L, L M, M N, &c. are as $\frac{1}{m}$, $\frac{1}{mm}$, $\frac{1}{m^3}$, &c. infinitely terminated at A; therefore (by *Prop.* 8.) their Aggregate FA, or db, is to DH, (ſo much Length as would have been diſpatched in the ſame Time, by ſuch impreſſed Force undiminiſhed) as 1 to m—1=n.

17. If therefore we take, as 1 to n, ſo A F to D H; this will repreſent the Length to be diſpatched, in the ſame Time, by ſuch undiminiſhed Force.

18. And if ſuch D H be ſuppoſed to be divided into equal Parts, innumerable (and therefore infinitely ſmall;) theſe anſwer to thoſe (as many) Parts unequal in FA, or bd.

19. But, what is the Proportion of r to 1, or (which depends on it) of 1—r to 1, or 1 to m; remains to be enquired by *Experiment*.

20. If the Progreſſion be not infinitely continued; but end (ſuppoſe) at N, and its leaſt Term be A = MN: Then out of $\frac{V}{R-1}+\frac{1}{m}+\frac{1}{mm}=\frac{1}{m^3}$, + *&c.* is to be ſubducted $\frac{A}{R-1}$, (as at *Prop.* 8.) that is, (as by Diviſion will appear) $\frac{A}{R}+\frac{A}{R^2}+\frac{A}{R^3}+$ *&c.* That is, (in our preſent Caſe) $\frac{a}{m}+\frac{a}{mm}+\frac{a}{m^3}+$ *&c.* And ſo the Aggregate will be $\frac{1-a}{m}+\frac{1-a}{mm}+\frac{1-a}{m^3}+$ *&c.* $=\frac{1-a}{n}$.

And thus as to the Line of *Projection*, in which (ſecluding the *Reſiſting*) the Motion is reputed uniform; diſpatching equal Lengths at equal Times. Conſider we next the Line of Deſcent.

21. In the Deſcent of Heavy Bodies, it is ſuppoſed, that to each Moment of Time, there is ſuperadded a new Impulſe of *Gravity* to what was before: And each of theſe, ſecluding the Conſideration of the *Air's Reſiſtance*, to proceed equally (from their ſeveral beginnings) thro' the ſucceeding Moments. As, (in the Erect Lines) IIII *&c.* III *&c.* II *&c.* I. *&c.* and ſo continually, as in the Line of *Projection.*

$$\begin{array}{llll} 1 & & & \\ 1 & 1 & & \\ 1 & 1 & 1 & \\ 1 & 1 & 1 & 1 \\ \&c. & \&c. & \&c. & \&c. \end{array}$$

22. Hence ariſeth (in the Tranſverſe Lines) for the firſt Moment 1, for the ſecond 1 + 1, for the third 1 + 1 + 1, and ſo forth, in *Arithmetical Progreſſion.* As are the *Ordinates* in a Triangle at equal Diſtance.

23. And ſuch are the continual Increments of the Diameter, or of the *Ordinates* in the *Exterior Parabola*, anſwering to the interior *Ordinates*, or Segments of the *Tangent*, equally increaſing; as is known, and commonly admitted.

24. If we take in the Conſideration of the *Air's Reſiſtance*; we are then for each of theſe equal Progreſſions, to ſubſtitute a decreaſing *Progreſſion Geometrical*; in like manner (and for the ſame Reaſons) as in the Line of Projection.

25. Hence ariſeth for the firſt Moment $\frac{1}{m}$; for the ſecond $\frac{1}{m}+\frac{1}{m^2}$; for the third $\frac{1}{m}+\frac{1}{m^2}+\frac{1}{m^3}$; *&c.* And ſuch is therefore the Deſcent of a heavy Body falling by its own Weight. The ſeveral impulſes of Gravity being ſuppoſed equal.

26. That is, as FL, FM, FN, *&c.* in the Line of Deſcent, anſwering to FL, LM, MN, *&c.* in the Line of Projection.

$$\begin{array}{llll} \frac{1}{m} & & & \\ \frac{1}{m^2} & \frac{1}{m} & & \\ \frac{1}{m^3} & \frac{1}{m^2} & \frac{1}{m} & \\ \frac{1}{m^4} & \frac{1}{m^3} & \frac{1}{m^2} & \frac{1}{m} \\ \&c. & \&c. & \&c. & \&c. \end{array}$$

Fig. 176.

27. But tho' the Progreſſions for the Line of Projection, are like to each of thoſe many in the Line of Deſcent: It is not to be thence inferred, that therefore $\frac{1}{m}$ in the one is equal to $\frac{1}{m}$ in the other: But in the Line

of Projection (ſuppoſe) $\frac{1}{m}f$, (ſuch a part of the Force impreſſed, and a Celerity anſwerable:) in the Line of Deſcent $\frac{1}{m}g$, (ſuch a part of the impulſe of Gravity.)

28. Thoſe for the Line of Deſcent (of the ſame Body) are all equal each to other: Becauſe g, (the new Impulſe of Gravity) in each Moment is ſuppoſed to be the ſame.

29. But what is the Proportion of f to g (that is, of the Force impreſſed, to the Impulſe of Gravity, in each Body) remains to be enquired by *Experiment.*

30. This Proportion being found as to one known Force; the ſame is thence known as to any other Force (whoſe Proportion to this is given) in the ſame *uniform Medium.*

31. And this being known as to one *Medium*, the ſame is thence known as to any other *Medium*, the Proportion of whoſe Reſiſtance to that of this is known.

32. If a heavy Body be projected downward in a Perpendicular Line, it deſcends therefore at the rate $\frac{1}{m}$, $\frac{1}{m^2}$, $\frac{1}{m^3}$, &c. of f (the impreſſed Force) increaſe by $\frac{1}{m}$, $\frac{1}{m}+\frac{1}{m^2}$, $\frac{1}{m}+\frac{1}{m^2}+\frac{1}{m^3}$, &c. of g, the Impulſe of Gravity: (by *Prop.* 5. and *P.* 25.) becauſe both Forces are here united.

33. If in a perpendicular Projection upwards; it aſcends in the Rate of the former, abated by that of the latter. Becauſe here the Impulſe of Gravity is contrary to the Force impreſſed.

34. When therefore this latter (continually increaſing) becomes equal to that former (continually decreaſing) it then ceaſeth to aſcend; and doth thenceforth deſcend at the rate wherein the latter continually exceeds the former.

35. In an Horizon or oblique Projection: If to a Tangent, whoſe Increments are as F L, L M, M N, &c. that is, as $\frac{1}{m}f$, &c. be fitted Ordinates (at a given Angle) whoſe Increments are as F L, F M, F N, &c. that is, as $\frac{1}{m}g$, &c. The Curve anſwering to the Compound of theſe Motions, is that wherein the Project is to move.

36. This Curve (being hitherto without a Name) may be called *Linea Projectorum*, the Line of Projects, or things projected; which reſembles a *Parabola deformed.*

37. The *Celerity* and *Tendency*, as to each Point of the Line, is determined by a Tangent at that Point.

38. And that againſt which it makes the greateſt Stroak or Percuſſion, is that which (at that point) is at right Angles to that Tangent.

39. If the Projection (at *pag.* 25.) be not infinitely continued, but ter-

minate (ſuppoſe) at N, ſo that the laſt Term in the firſt Column or Series erect, be a; and conſequently in the ſecond, ma; in the third mma, *&c.* (each Series having one Term fewer than that before it:) then (for the ſame Reaſons as at *P.* 20.) the *Aggregates* of the ſeveral Columns (or erect Series) will be $\frac{1-a}{n}$, $\frac{1-ma}{n}$, $\frac{1-mma}{n}$, and ſo forth, till (the Multiple of a becoming $=1$,) the Progreſſion expire.

40. Now all the Abatements here, a, ma, mma, *&c.* are the ſame with the Terms of the firſt Column taken backward. For a is the laſt, ma the next before it; and ſo of the reſt.

41. And the Aggregate of all the Numerators is ſo many times 1, as is the Number of Terms (ſuppoſe t) wanting the firſt Column; that is, $t-\frac{1-a}{n}$ or $\frac{nt-1+a}{n}$; and this again divided by the common Denominator n, becomes $\frac{nt-1+a}{nn}$. And therefore, $\frac{nt-1+a}{nn}g$, is the Line of Deſcent by its own Gravity.

42. If therefore this be added to a projecting Force downward in a Perpendicular, or ſubducted from ſuch projecting Force upward; that is, to or from $\frac{1-a}{n}f$: The Deſcent in the firſt Caſe, will be $\frac{1-a}{n}f+\frac{nt-1+a}{nn}g$; and the Aſcent in the other Caſe $\frac{1-a}{n}f-\frac{nt-1+a}{nn}g$. And in this latter Caſe, when the Ablative part becomes equal to the Poſitive part, the Aſcent is at the higheſt: And thenceforth (the Ablative part exceeding the Poſitive) it will deſcend.

43. In an Horizontal or Oblique Projection, having taken $\frac{1-a}{n}f$, in the Line of Projection, and thence (at the Angle given) $\frac{nt-1+a}{nn}g$, in the Line of Deſcent; the Point in the Curve anſwering to theſe, is the place of the Project anſwering to that Moment.

44. I am aware of ſome Objections to be made, whether to ſome Points of the Proceſs, or to ſome of the Suppoſitions. But I ſaw not well how to wave it, without making the Computation much more perplexed. And in a manner ſo nice, and which muſt depend upon phyſical Obſervations, 'twill be hard to attain ſuch Accuracy, as not to ſtand in need of ſuch Allowances.

45. Somewhat might have been further added, to direct the *Experiments* ſuggeſted at *P.* 19. and 29. But that may be done at leiſure, after Deliberation had, which way to attempt the *Experiment.*

46 The like is to be ſaid of the different Reſiſtance which different Bodies may meet with in the ſame *Medium*, according to their different Gravities, (extenſively or intenſively conſidered) and their different Figures and

Poſitions in Motion: Whereof hitherto we have taken no account; but ſuppoſed them, as to all theſe, to be alike and equal.

Fig. 176, 177. 47. The Computation (in *P.* 39, 40, 41) may, if that be alſo deſired, be thus repreſented by Lines and Spaces. The Ablatives, *a*, *m a*, *m m a*, *&c.* (being the ſame with the firſt Column taken backward) are fitly repreſented by the Segments of N F, (beginning at N) and therefore by Parallelograms on theſe Baſes, aſſuming the common height of F *b*, or N Q; the Aggregate of which is N *b*, or F Q, and ſo many times 1, by ſo many equal Spaces, on the ſame Baſes, between the ſame Parallels terminated at the *Hyperbola*; the Aggregate of which is *b* F N Q *n*. From whence if we ſubduct the Aggregate of Ablatives F Q; the remaining Trilinear *b* Q *n*, repreſents the Deſcent.

48. If to this of Gravity, be joined a Projecting Force; which is to the impulſe of Gravity, as *b* K to *b* F, (be it greater, leſs or equal) taken in the ſame Line; the ſame Parallels determine proportional Parallelograms, whoſe Aggregate is K Q.

49. And therefore, if this be a Perpendicular Projecting downwards; then *b* K *k m* (the Sum of this with the former) repreſents the Deſcent.

50. If it be a Perpendicular upwards; then the difference of theſe two repreſents the Motion, which, ſo long as K Q is the greater, is Aſcendant; but Deſcendant, when *b* Q *n* becomes greater; and it is then at the higheſt when they be equal.

51. If the Projection be not in the ſame Perpendicular, (but Horizontal or Oblique) then K Q repreſents the Tangent of the Curve; and *b* Q *n*, the Ordinates to that Tangent, at the given Angle.

52. But the Computation before given, I take to be of better uſe than this Repreſentation in Figure. Becauſe in ſuch methametical Enquiries, I chuſe to ſeparate (as much as may be) what purely concerns Proportions; and conſider it abſtractly from Lines or other matter wherewith it is incumbred.

As to the Queſtion propoſed; Whether the reſiſtance of the Medium does not always take off ſuch a proportional part of the Force, moving thro' it, as is the Specifick Gravity of the Medium to that of the Body moved in it (for if ſo, it will ſave us the trouble of Obſervation;) I think this can by no means be admitted; for there be many other things of Conſideration herein, beſide the intenſive Gravity (or, as ſome call it, the Specifick Gravity) of the Medium.

A viſcous *Medium* ſhall more reſiſt than one more fluid, tho' of like intenſive Gravity.

And a ſharp Arrow ſhall bore its way more eaſily thro' the Medium than a blunt-headed Bolt, tho' of equal Weight and like intenſive Gravity.

And the ſame Pyramid with the Point, than with the Baſe forward.

And many other like Varieties intended in my *P.* 46.

And this I think may be admitted, namely, that different Mediums, equally liquid, (and other Circumſtances alike) do in ſuch Proportion reſiſt, as is their intenſive Gravity. Becauſe there is, in ſuch proportion, a heavier Object to be removed, by the ſame Force; which is one of the things to which *P.* 31. refers.

And again: the heavier Project once in motion, (being equally ſwift, and all other Circumſtances alike) moves thro' the ſame Medium in ſuch Proportion more ſtrongly, as is its intenſive Gravity: for now the Force is in ſuch Proportion greater, for the Removal of the ſame Reſiſtance. And this is part of what my *P.* 30 inſinuates.

But where there is a Complication of theſe Conſiderations one with another, and with many other Circumſtances, whereof each is ſeverally to be conſidered; there muſt be reſpect had to all of them.

Experiments, to Determine the Point Blank Diſtance; the Charge of Powder; and beſt Size of Guns. Propoſed; by Sir Rob. Moray. *n.* 26. p. 473. June An. 1667.

X. 1. *To know how far a Gun ſhoots Point Blank, (as they call it) that is, ſo near the Level of the Cylinder of the Piece, that the Difference is either not diſcernible, or non conſiderable.*

On a fit *Platform*, place and point the *Gun* at a Mark as large as the Bullet, ſome 50, 60, or more Yards diſtant, ſo as the underſide of the Mark may be in the ſame Level or Line with the underſide of the *Cylinder* of the *Piece.* Then between the *Gun* and the Mark at convenient Diſtances, place pieces of Canvas, Sheets of Paper paſted together, or the like, upon ſtakes fix'd in the ground; ſo as the underſide, being level with the Horizon, may juſt touch the viſual Line, that paſſeth from the Eye to the upperſide of the Mark, when the Eye is placed in the Line that paſſeth from it to the upperſide of the *Cylinder* of the *Gun*; the Canvas being ſo broad and long, that if the *Bullet* paſs thro' it 2 or 3 Foot higher than the Level of the Mark, or of either hand, the Hole it makes may make it known, how much it flieth higher than the Level of that Place. If the *Bullet* falls lower than the Mark, and touch not the Canvas, the *Gun* may be the next time raiſed a little, and ſo on, till the *Bullet* hit the Mark, or as high as it. If it fall as high as the Mark and cut the Canvas, the Mark and Canvas may be brought nearer the *Gun:* But if it fall as high as the Mark and do not cut the Canvas, the Mark may be removed to greater and greater Diſtances.

If this way of *Experiment* be made for further Diſtances and Raiſings of the *Piece*, as high as conveniently may be above the Level, and the Diſtances meaſured, and then all *Randoms* above theſe likewiſe tried and meaſured; the Diſtance of an Object, to be ſhot at, being known, and other neceſſary Cautions, beneath to be mentioned, carefully obſerved; good *Gunners* may with great Confidence undertake to hit the Mark, be the Diſtance what it will, ſo it exceed not the reach of the *Gun.*

2. *To know what Quantity of Powder is the juſt Charge of any Piece, ſo as it maketh the fartheſt Shot, and Fires totally.*

1. Raiſe the *Gun* to a mean *Random*, as of 20°, or 25°, and ſhoot with the ordinary *Charge* of *Powder*, in ſome convenient Ground where the Fall of the Bullet may be eaſily ſeen, and having made a Shot, meaſure the Diſtances with a Chain between the Hole made by the Bullet and the *Muzzle* of the *Gun.*

2. Then inſtead of a full *Charge* of *Powder* uſed in the firſt Shot, take $\frac{1}{16}$ part leſs, or ſome ſuch Proportion, for the next Trial, doing all things elſe as before.

3. For a third, fourth, or more Trials, diminiſh ſtill the *Quantity* of *Powder*, by $\frac{1}{16}$ at a Time, till the Shot be conſiderably ſhorter than at firſt.

4. Then take $\frac{1}{16}$ more than the firſt *Charge*, and do all things elſe as before, and ſo continue more Trials, increaſing ſtill the *Quantity* of *Powder* in the ſame Proportion every new Trial, till you find the Increaſe of the *Charge* does not make the *Piece* ſhoot further: Only over-charge not ſo far as to endanger the *Gun*.

5. Three or more Shot are to be made with every different *Charge*, and at every ſeveral Trial, that the Certainty may the better appear.

6. The firſt Shot being meaſured and marked, the reſt may all be meaſured from it, or from one another, to ſave Labour.

7. The *Gun* is to be pointed, placed and ordered, every time in one and the ſame Place and Poſition, aiming ſtill at the ſame Mark, or Pointing ſtill at the ſame Line or Azimuth; that ſo all the Shot may fall in the ſame Line as near as is poſſible.

8. The *Powder* (which ought to be all of the ſame *Goodneſs*) muſt be exactly weighed every time the *Piece* is *Charged*, leſt it having been weighed long before, the Weight may be altered; tho' *Experiment* may be made with *Cartridges* and without.

9. The *Powder* and *Bullet* is to be *Rammed* home, equally at every Shot; tho', the looſer the *Powder* lie, it *fire* the better.

10. When the right *Charge* of a *Piece* is found, that makes the *fartheſt Shot* in the ordinary and plain way of *Charging*, M. *de Sons*'s Contrivance of a *Wedge* may be tried, to make it ſhoot farther; which is a piece of Board, ſo long, as being thruſt home to the *Breech* of the *Piece* at one End, the other may reach farther out than the Outſide of the Bullet being *Rammed* up to its place; broad about an Inch, and thin ſo far as the *Wadd* before the *Bullet* reaches on the outſide; there it is to have a Shoulder, from which forward to the end, it is to be cut a-ſlope, like a *Wedge*, being of ſuch thickneſs, as that at the Place, where the Center of the *Bullet* is to be, it may make it ſtick ſo faſt, that the *Powder* finding more Reſiſtance, may at length drive it out with the greater Violence.

11. Another of this Nature is a *Wooden Tampion*, like a piece of a Cylinder, big enough to fill the hollow *Cylinder* of the *Gun*, the length ſomewhat more than the Diameter of it, and hollow'd towards the *Bullet*, ſo as to fit it; and either flat or (which is better) hollow likewiſe towards the *Powder*, and ſerving inſtead of *Wadd*. Theſe, and ſuch others, will probably render the Effect of the *Powder* greater, than otherwiſe it would be: But care muſt be had that they do not endanger the *Piece*.

12. The *Strength* of the *Powder* muſt be examin'd by a *Powder Trier*, that raiſeth a Weight, ſuch an one as hath been contrived by Mr. *Hook*.

13. The ſame *Bullet* is to be made uſe of, if it can be had, till the Figure

of it be marred; otherwiſe another as near of the ſame Size, Shape and Weight, as is poſſible.

14. Obſerve the *Strength* and *Poſition* of the *Wind*, and at what *Azimuth* the Mark ſtands from the *Gun* at every time of Shooting: And take preciſe notice what effect it hath upon the *Bullet* in carrying it further, in hindring, or turning it aſide.

15. Note the Figure, Dimenſions, and Weight of the *Gun*, *Carriage*, and *Wheels*; and record every thing exactly in a Book, as alſo every Accident and Obſervation.

16. After all other *Experiments* are made, every *Piece* may be tried with the right *Charge* of *Powder*, laying every time more and more Weight upon the *Carriage*; and at laſt fixing the *Gun*, ſo as it may not *Recoil* at all, obſerving every Time how far the *Bullet* goes, and how much leſs *Powder* than the full *Charge* will ſerve to ſhoot the *Bullet*, when the *Piece* is Fixt, as far as the whole *Charge* does, when it *Recoils* freely.

17. The *Right Charge* found, the beſt *Random* is to be ſought, by trying all Randoms, by Degrees at a time.

3. *To know what Gun ſhoots fartheſt.*

1. A *Gun* to be prepared of *Culverine-Bore* (as being held the beſt for ſhooting far) but much longer (double the ordinary Length may do well) and without any *Ring* about the *Muzzle*, is to be placed as in the former *Experiments*, and charged with the Ordinary Charge of a *Culverine*, or rather with that Quantity, which by the former *Experiments* ſhall be found the beſt; and being ſhot, the Fall of the *Bullet* is to be mark'd, and Diſtance meaſured.

2. Then try leſs, and more *Powder* in her, as before.

3. Then cut off two Inches of the *Muzzle* with a Saw, and place the Pieces ſo cut off in the *Carriage*, or their *Weight* of Lead in a convenient Figure, that the *Recoil* may ſtill be the ſame: and try as before, doing every thing in the ſame manner: and ſo cut off ſtill for new Trials, till the Shot begin to fall ſhorter than before.

4. The ſame may be done with *Guns* of different *Bores*.

Experiments for Trying the Force of Great Guns; by Mr. Greaves. n. 173. p. 1090.

2. *Mar.* 18. 1651. At 200 Yards diſtance from the Platform for great Ordinance at *Woolwich*, there were raiſed three *Butts*, one behind another: The ſpace between the firſt and ſecond *Butt* was 14 Yards; between the ſecond and the third, eight. The thickneſs of each *Butt* was 19 Inches, whereof 13 was of Beams of Maſſey oak faſtened into the Ground, and ſet ſo cloſe that they touched each other: On each ſide were Planks of Oak, 3 Inches a-piece in Thickneſs, and theſe were joined cloſe, and faſtened on both Sides with Iron bolts, and ſtrong Pins of Wood; and on the Back, at the Ends, and on the Middle there were 3 Braces of Elm, a Foot in Breadth, and 5 Inches in Thickneſs.

The firſt *Experiment* was with an *Iron Demy-Cannon*, having a *Cylinder Bore* of 3500 *lib. Weight*, the *Bullet* 32 *lib.* of Iron, the *Powder* 10 *lib.* which pierced thro' the two firſt *Butts*, and ſtuck in the third, ſo as the Ball was almoſt quite within, but the Timber not ſhivered (ſmall) nor ſcarce ſpilt. The *Butts* being touched by me, felt not warm; the like *Execution* was done when it was charged with 9 *lib.* as alſo when with 8 *lib.* of *Powder*.

The ſecond *Experiment* was with an *Iron Demy-Cannon*, having a *Taper Bore* and being 3600 *lib.* in *Weight*, and 4 Inches longer than the former; the *Iron Bullet* 32 *lib.* and the *Powder* 7. *lib.* which in three Trials ſeemed to have the ſame Force with the firſt. One of the *Shots* piercing thro' the ſecond *Butt*, and lighting near the Edge of the middle * *Butt* of Elm, tore it, but by the yielding of it, the *Bullet* glanced aſide off the third *Butt*, and entered into the Earth.

* *Braſs.*

The third *Experiment* was with a whole *Culverine* in *Braſs*, of 5300 *lib.* in *Weight*, 11 Foot 1 Inch in *Length*, with a *Taper Bore*: the *Iron Bullet* was 18 *lib.* in *Weight*; the *Powder* in the firſt Trial 10 *lib.* in the ſecond 9 *lib.* in the third 8 *lib.* which laſt Proportion did the beſt *Execution*, and paſſed thro' the two *firſt Butts*, entering gently into the Third, which the former two did touch, but not enter.

The fourth *Experiment* was with a whole *Culverine* in *Braſs*, made at *Amſterdam* for the *French*, with this Mark, 3580, being 10 Foot long, and not very thick in the *Breech*; the firſt Shot with 9 *lib.* of *Powder*, 18 *lib.* of *Bullet-Iron*, paſt thro' the three *Butt*, and entered one Foot into the Ground; it paſſed by the Joints of the Timber, two Planks having been beat down before. The ſecond Shot with 8. *lib. Powder*, paſſed thro' two *Butts* and *grazed* between them. The third with 8 *lib.* paſt two *Butts*, and 7 Inches into the third; but the firſt *Butt* was much battered before, where it entered.

The fourth Shot paſſed, with 8 *lib.* of *Powder*, two *Butts*, and in both *Butts* thro' the midſt of a Maſſey ſtrong Beam (below) that had not been battered.

The Fifth *Experiment* was with an *Iron Demy-Culverine*, having 9 *lib. Bullet* in *Iron*, and 4 *lib. Powder*; this paſt one *Butt* (which was torn before) and entered the ſecond.

This $\frac{1}{2}$ *Culverine* was ſhot 8 Times, as faſt as they could *charge* it with *Powder*, and the *Iron-Bullet*, and yet was but ſcarce lukewarm at the *Breech*, a little more in the midſt, moſt at the *Muzzle*, and this laſt ſcarce ſo hot as my Hand; and yet the *Gunners* in charging her, wet not at all the *Scoop*, or *Spunge*.

The Sixth *Experiment* was with a *Braſs Demy-Culverine*, the *Breech* of her was 13 Inches $\frac{5}{8}$, the Mouth $9\frac{5}{8}$. The firſt *Shot*, with 4 *lib.* of *Powder*, 9 *lib. Iron-Bullet*, paſt two *Butts*: The Second *Shot* with 3 *lib.* of *Powder*, paſt almoſt two *Butts*: This proved to be the beſt Shot, becauſe the Timbers were the ſtrongeſt.

Shooting by the Rarefaction of the Air, by Dr. Papin. n. 179. p. 21.

XI. Whereas ordinary *Wind Guns* do their Effect by the *Compreſſion* of the *Air*: *Ottho Ghericke* hath found a New ſort that ſhoots by *Rarefaction*; and he hath publiſhed that Device at large in his Book about *Pneumatick Experiments*. I have contrived another which I take to be better.

A A is a *Pipe*, very equal from one end to the other. Fig. 178. Jan. An. 1686.

B B a ſmall *Pipe* ſolder'd in a Hole near the End of the *Pipe* A A, and apply'd to the *Plate* of the *Pneumatick Engine.*

CCCC ſome kind of *Stool*, to bear up the hinder part of the *Pipe* A A.

D a *Piece* of *Lead* fitted to the Bore of the *Pipe* A A.

The *Pipe* A A is to be ſhut at both Ends by *Valves* outwardly applied, and ſo the ſaid *Pipe* A A, tho' never ſo big, may be *exhauſted* of *Air* by means of the *Pneumatick Engine:* Which done, the *Valve* towards D muſt be ſuddenly opened, ſo that the whole Preſſure of the Atmoſphere acting upon the *Lead* D, may drive it along the *Pipe* A A, with ſuch a Swiftneſs, that it will be able to carry it to a great Diſtance: and becauſe ſuch a *Valve* ſhutting a great Hole, would prove very difficult to be opened, when the *Pipe* A A, is of a great Bore, the Aperture towards D, may be left much ſmaller than the *Pipe*; the Swiftneſs of the Air being ſo great, that even thro' a pretty ſmall Aperture, it preſſed the *Lead* D, as freely almoſt, as if the whole Bore was quite open.

Having prepared a *Barrel* carrying a *Lead* of two Ounces, the *Experiment* was ſhewn before the *Royal Society* and the Effect was found very conſiderable, the Force being little leſs than that of the *Wind-Gun* by *Compreſſion*; the ſame *Experiment* being afterwards repeated with a longer *Barrel*, 'twas found that the length in this way of *Shooting* was very little, if any Advantage.

The Velocity wherewith the Air ruſhes into an exhauſted Receiver; by Dr. Papin. n. 184. p. 193. Oct. An. 1686.

XII. My way of computing the *Velocity* of the *Air* (which I think is better than the Trial made by the *Royal Academy* at *Paris*) is grounded upon this *hydroſtatical Principle*, That *Liquors have a Strength to aſcend as high as their Source is*; and altho' the *Reſiſtance* of the *Medium* does always hinder *Jets d'Eau* in the open Air from reaching quite ſo high, nevertheleſs the *Liquor* at its firſt ſpouting out, hath the neceſſary *ſwiftneſs* to come to that *height.*

Prop. I.] From this Principle may be eaſily deduced this Propoſition, That, *of two different Liquors driven by the ſame Preſſure, that which is in Specie lighter muſt aſcend higher than that which is heavier, and their heights will be reciprocally in the ſame Reaſon as their ſpecifick Gravities are.*

Prop. II.] From the foregoing *Propoſition* another may eaſily be deduced, *viz.* That, *of differing Liquors bearing the ſame Preſſure, thoſe that are lighter in Specie muſt acquire a greater Swiftneſs, and their differing Velocities are to one another* * *as the Roots of the ſpecifick Gravities of the ſaid Liquors.* * *Reciprocally.*

For we have ſeen, *P.* I. That the *Heights* to be attained are * in the ſame Reaſon as the *ſpecifick Gravities:* Now *Gallileus*, *Hugenius*, and others, have demonſtrated, That the *Velocities* of Bodies are to one another, as the *ſquare Roots* of the *Heights* to which they may aſcend: And ſo in this occaſion they are alſo * as the *Roots* of the *ſpecifick Gravities.* * *Reciprocally.* * *Reciprocally.*

If therefore we would know what is the *Velocity* of the *Air* being driven by any degree of *Preſſure* whatſoever, we ought but to find what would be the *Velocity* of *Water* under the ſame *Preſſure:* And then take the *ſquare Roots* of the *ſpecifick Gravities* of theſe two Liquors; becauſe as much as the *ſquare Root* of the *ſpecifick Gravity* of *Water* doth exceed the *ſquare Root* of the *ſpecifick Gravity* of the *Air*; ſo much in proportion will the *Velocity* of *Air* exceed the *Velocity* of *Water.* For Example; when I would compute what ſhould be the *Swiftneſs* of a *Bullet* ſhot by my *Pneumatick Engine,* I ſhould at firſt compute what was the *Velocity* of the Air itſelf that drove the Bullet: I did therefore take notice, That in this Occaſion the *Air* bears a Preſſure much about the ſame as that of *Water* when its *Spring* is 32 Foot high. Now ſuch *Water* would ſpout out with a ſufficient *Velocity* to aſcend 32 Foot Perpendicular, and therefore according to the Rules and Obſervations of *Galliæus, Hugenius,* and others, ſuch Water hath the *Velocity* of 45 Foot in a Second. It remains therefore but to know the Proportion of the *Gravity* of the *Air* to that of *Water:* And we have found it not to be always the ſame; becauſe the Height, the Heat, and the Moiſture, of the *Atmoſphere,* are variable: Nevertheleſs, we may ſay in general, That the Reaſon between the *ſpecifick Gravities* of *Water* and *Air* is much about 840 to 1. Taking then their *Square Roots* as I have ſaid above, which *Roots* are 29 and 1, we may conclude that the *Velocity* of *Air* muſt exceed that of *Water,* by 29 Times: And ſo multiplying 45, the *Velocity* of *Water,* by 29, we ſhall find, that the *Velocity* of *Air* driven by the whole Preſſure of the *Atmoſphere,* is about 1305 Foot in a Second.

Wind produced by the Fall of Water; by Dr. Wal. Pope. n. 2. p. 25. *Fig.* 179. Apr. An. 1665.

XIII. In the *Braſs-Works* at *Tivoli,* the *Waters blow* the *Fire,* not by moving the *Bellows,* but by affording the *Wind.* Thus: A, is the *River.* B, the *Fall* of it. C, the *Tube* into which it falls. L G, a *Pipe.* G, the *Orifice* of the *Pipe,* or *Noſe* of the *Bellows.* G K, the *Hearth.* E, a Hole in the Pipe. F, a Stopper to that Hole. D, a place under ground, by which the Water runs away. Stopping the Hole E, there is a perpetual ſtrong *Wind,* iſſuing forth at G; and G being ſtopt, the *Wind* comes out ſo vehemently at E, that it will, I believe, make a *Ball* play, like that at *Freſcati.*.

The beſt Form of Horizontal Sails for a Mill; by Dr. Rob. Hook, *Phil. Coll:* n. 3. *p.* 61. Dec. Ann. 1681.

XIV. Whatever Men may imagine concerning *Horizontal Sails,* I doubt there never will be found a better, and more advantageous way, for receiving the Strength of the *Wind,* or *Motion* of the *Air,* than *Perpendicular Vanes* made of a true Form, ſo as every part thereof may draw alike. But becauſe I find divers have of late attempted *Horizontal Vanes* for *Mills,* I ſhall explain a way of making *Horizontal Vanes* capable of performing the *moſt* that is *poſſible* with *Vanes* of equal *Extenſion.*

The *Invention* is founded upon the ſame Principle with that of the *Sailing* of *Ships,* and other *Veſſels* upon the Sea; namely, upon diſpoſing and ordering of the Vane or Sail ſo, as *to ſtand in the beſt Poſture 'tis poſſible to move the Arms of the Mill, or the Body of the Ship, in that way it is to be moved, by the Force of the Wind blowing thus or thus againſt them.*

The *Firſt Principle* then common to both, is, that the *Vane* or *Sail* be as near as 'tis poſſible, a perfect *plane* and *ſmooth Superficies,* without any Belly-ing, Bunting, or Curvity in the Superficies thereof, upon which the Motion or Force of the *Wind* is impreſſed.

Secondly, That the *Air* may have as many *Paſſages* between the parts of the *Vane* or *Sail*, as may be, that the moved *Air* may come to it as *freely* as may be, without being intercepted by a *ſtagnant Air* before it, to impede or divert its Force.

Thirdly, That the *Plane* of the *Vane* or *Sail* be put in the *middle Inclination*, between the *way* of the *Wind* and the *way* of the *Arm*, or that of the Body of the Ship.

The *Contrivance* itſelf is *This*.

Fig. 180. Let A B ſignify the *Stream* or *Current* of the *Air* or *Wind*, moving from A to B, and let C repreſent the Center of the *Axis* or *Spindle*, ſtanding perpendicular to the *Horizon*, upon which, at the Top, is fixed at right Angles, the piece D H, making the two *Arms* C D, and C H, upon the Ends of which the *Vanes* M N, are moved on *Spindles*; ſo as that the Plane of the *Vane* doth always paſs thro' the Point D: I ſay, theſe *Vanes* ſo ordered, ſhall be always placed in the moſt *advantageous Poſture* for moving the *Arms* round upon the ſaid *Spindle*, whoſe Center is C, in the Order of DEFGHIKLD.

Firſt, For the *Vanes* placed at D and H, I ſay, They are ſet in the moſt *advantageous Poſture* poſſible, in thoſe two Points: For *Firſt*, the *Vane* M N at D being to move directly againſt the *Wind*, the moſt *advantageous Poſture* is to turn its edge directly againſt the *Wind*, and thereby to give the leaſt *Reſiſtance* poſſible, that being the only Point in which the *Vane*, ſuppoſed only a Superficies, draws not. And *Secondly*, For the *Vane* M N placed at H, it ſtandeth the moſt *advantageouſly*, becauſe its Motion being directly from or before the *Wind*, it ſtandeth full *Croſs*, or oppoſed to the Motion thereof.

Secondly, The *Vanes* at E, F, G, and I, K, L, ſtand the moſt *advantageouſly* becauſe they *divide* the *Angle*, between the *way* of the *Wind*, and that of the *Arms* in thoſe Points into two equal Parts, and conſequently the Wind impreſſeth the greateſt Force in the moſt direct Way: For it is eaſy to be demonſtrated, That the Force impreſſed on the *Vane* by the *Wind*, is perpendicular to the Surface, and conſequently that the Obliquity of the Force to the way of the *Arms*, increaſed by the *Vanes* ſtanding more full againſt the *Wind*, will have a leſs Proportion of Power to promote the Motion thereof, than in the Poſture here ſet. And ſuppoſing the *Vanes* ſet ſharper to the *Wind*, the Diminution of the Force impreſſed by the *Wind* on its Surface, will be greater than the Augmentation of its Power by being moved more directly to the *way* of the *Arms*. This is eaſy enough to be *geometrically demonſtrated*.

The *Vane* may be ſo ordered, as always to ſtand in this *Poſture* by a great many ways: I ſhall only inſtance in one, not the beſt for *Practice*, but the moſt eaſy to be underſtood and *demonſtrated*.

Fig. 181. Let the *Vane* be equally expanded on each ſide of its *Axis*, by which the Preſſure on the Extremes of it are always counterpoiſed; then faſten upon the lower end thereof a Wheel, which may be in a Diameter about ½ of the

length of the *Arms* from Hole to Hole; then fix a Wheel upon the Frame in which the *Spindle* of the *Arms* do move, that ſhall be of half the Diameter with the former, and to contain half the number of Teeth. Then by a third ſmall Wheel, fixed under the *Arms*, of a convenient bigneſs, communicate the Motion of the one to the other; for by this means each *Vane* being ſo provided, they will, being once ſet right, always continue to be moved and diſpoſed in the true Poſture deſired.

This *Contrivance* will not only be uſeful for all manner of common *Wind-Mills*, but alſo for *Water-Mills* in *Rivers*, where there can no *Dam* be made; as may alſo the *perpendicular Vanes* of other *Mills*, neither of which has been ſo much as *hinted*, by any Perſon whatſoever, that I have hitherto heard of.

An Account of Flying; by Dr. Hook. *Phil. Coll.* n. 1. p. 14.

XV. 1. The Art of *Flying* hath been in all Ages *attempted* by many, particularly in the Times of our famous Friar *Roger Bacon*, who lived about 500 years ſince. He was believed a *Magician* or *Conjurer*, and to have performed what was related of him, by the help of *diabolical Magick*; but from the peruſal of ſeveral of his excellent *Works* yet extant, I eſteem him no ſuch Perſon. I rather find him to have been a good *Mathematician*, a knowing *Mechanick*, a rare *Chemiſt*, and a moſt accompliſhed *experimental Philoſopher*, which was a Miracle for that dark Age. This Man affirms the Art of *Flying* poſſible, and that he himſelf knew how to make an *Engine*, in which a Man ſitting, might be able to carry himſelf thro' the *Air* like a *Bird*: And affirms, that there was then another Perſon who had actually tried it with good Succeſs. We have not wanted later Inſtances in *England*, of ſeveral ingenious Men, who have employed their Wits and Time about this *Deſign*. Particularly, I have been credibly informed, that one Mr. *Gaſcoigne* did about 40 Years ſince try it with good Effect; tho' he ſince dying, the Thing alſo died with him. And even now, there are not wanting ſome in *England*, who affirm themſelves able to do it, and that they have proved as much by *Experiment*. We have little or no account of the ways they have taken to effect their Deſigns; but we may conclude them defective in ſomewhat or other, ſince we do not find them brought into *common Uſe*.

The Art of Flying; by S. Beſnier. *Ibid.* p. 15. *Fig.* 182.

2. The Sieur *Beſnier*, a Smith of *Sable* in the County of *Maine*, hath invented an *Engine* for *Flying*. It conſiſts of two *Poles* or *Rods*, which have at each end of them an oblong *Chaſſis* of Taffety; which *Chaſſis* folds from above downwards, as the frame of a folding *Window Chaſſis*. He fits theſe *Poles* upon his Shoulders, ſo that two of the *Chaſſies* may be before him, and the other two behind him. The *order* of *moving* them is thus: When the *right Hand* ſtrikes down the *right Wing* before, A, the *left Leg* by means of the String E, pulls downwards the *left Wing* behind, B; then immediately after, the *left Hand* moves or ſtrikes downwards the *left Wing* before, C; and at the ſame time the *right Foot*, by the String F, moves or pulls down the *right Wing* behind, D; and ſo ſucceſſively, or alternately, the diagonally oppoſite *Wings* always moving downwards, or ſtriking the Air together.

A Flying Chariot; by Fr. Lana. *Ibid.* p. 18.

3. 1. *P. Franciſco Lana* in his *Prodromo*, finding by an *Experiment*, That the *Weight* of the *Air* is $\frac{1}{640}$ part of the *Weight* of a like quantity of *Water*, he concludes certainly, That if we could make a Veſſel of Glaſs or other Matter that might *weigh* leſs than the *Air* that is in it, and ſhould draw out all its *Air*, this Veſſel would be *lighter* in *Specie* than *Air* itſelf, and therefore would ſwim in it and aſcend on high. This he *ſuppoſes* may be done, by making a round Veſſel of thin *Plate Braſs* (*weighing* 3 Ounces in a *ſquare Foot*) of the *Diameter* of 14 Foot. For the *Surface* of the Veſſel will be 616 *ſquare Feet*, and the *Braſs* will *weigh* no more than 1848 Ounces; whereas the *Content* will be 1437$\frac{1}{3}$ *Cub. Feet*, and that Quantity of *Air* will weigh 2155$\frac{2}{3}$ Ounces: So that that Air being evacuated, the Veſſel will be 307$\frac{2}{3}$ Ounces *lighter* than *Air*, and therefore will not only aſcend into the Air, but alſo carry up with it a weight of 307$\frac{2}{3}$ Ounces. And thus by encreaſing the Bulk of the Veſſel, without encreaſing the Thickneſs of the Plates of Braſs, he ſuppoſes a kind of Ship may be made, to ſwim in the Air, and to carry two or three Men in it.

Shewn Impracticable by Dr. Hook. *Ibid.* p. 27.

2. The fallacy of the *Author*'s Reaſoning lies in this; he ſuppoſes *Copper* of 3 Ounces in a Foot Square to be of ſufficient Thickneſs to reſiſt the Preſſure of the *Air* in a *Globe* of 14 Foot *Diameter*, nay of any *Dimenſion*. But in this we can no wiſe aſſent to him: For the Preſſure from without inwards, tho' it be always the ſame upon equal Surfaces, yet upon unequal Surfaces the Caſe is quite otherwiſe, for there the Preſſure will be found not the ſame, but to encreaſe always in the ſame Proportion with the Surface, and thence conſequently the *Thickneſs* of his *Copper*, or any Metal or Material, which he ſhall make uſe of, muſt increaſe in the ſame Proportion, with the *Diameter* of the *Sphere*, and conſequently the *Weight* of his *Copper* muſt always *increaſe* in the ſame Proportion at leaſt to the *Solidity* of his *Sphere*; ſo that by his augmenting the *Quantity* of his *Sphere*, he has no manner of Advantage of making it proportionably *lighter* than the *Air*, and proportionably *ſtrong*, but the contrary: For it is manifeſt, That a bigger *Sphere* ſo made of any Matter we yet know, has leſs Power of reſiſting the ſame *Preſſure* of the *Air* than a leſs, becauſe of finite Reſiſtance of Matter to Preſſure, there being ſome degree of *Preſſure* that will *cruſh* every Body.

An Engine to make Linnen Cloth; by M. de Gennes. n. 140. p. 1007. *Fig.* 183. July, An. 1678.

XVI. This *Engine* is compoſed of four principal *Parts*; the *Serpent* A A two Foot-Steps or *Treddles* B B, one *Clapper* C, and two *Arms* D D, D D. The *Serpent* or Iron Bar A A, has two Elbows, E E, whereto the Ends of the Ropes are fix'd that raiſe and put down the *Foot-ſteps* B B; F F are two fourths of a Circle, that ſucceſſively reſt upon two Arches or Bows of Iron, G G, which are above the *Clappler* C, to raiſe it. H H are two Teeth of Iron, added to the *Serpent*, making an Angle of 25 *Deg.* with F F, and K K; which ſerve to put down a *Baſcule*, or *Sweep*, which is in the Arm that carries the *Shuttle*. The Foot-ſteps or *Treddles* differ in nothing from thoſe that are uſually made uſe of, only the Cords that hold them pendent from the Ground are fix'd in the Elbows of the *Serpent*, which in turning raiſes and puts them down by the help of two little Pullies, upon which the Ropes turn.

The *Clapper* is ſupported between two Pillars, with a Rope double twiſted, which occaſions it to make a kind of Spring, and cauſes it naturally to give forwards to beat the Cloth.

L M, is one of the *Arms* which paſſes freely into the Canal or Pipe N N, ſupported by four Pillars of Wood O O O O. The Motion of it proceeds from the following Parts. P Q, is a *Baſcule*, which tho' unequally divided by its Supporter R, is yet *in Æquilibrio*, the end P R being made to weigh exactly as much as R Q.

At the Extremity of this *Baſcule* is ty'd a Cord which paſſes thro' the Pulley S, and terminates at the Extremity of the *Arm*, where it is faſtened, to a little Bow M. At the other Extremity of the ſame *Arm*, that is to ſay, towards L, is alſo faſtened underneath, a Cord, which paſſes thro' the Pulley T, and which carries the Weight V.

At the ſame end of the *Arm* is added a little *Niche* Z, about the bigneſs of half the *Shuttle:* Then over a little Bar X Y, which paſſes a-thwart the *Arm*, there are two other little pieces of Wood, having at the end of them two Teeth which enter into the *Niche* Z, thro' two Holes which are there, of the one ſide and t'other.

To the Ends of theſe little pieces of Wood, there is a little *Bow* of Whalebone or Steel, which keeps the two Ends aſunder, and forces the Teeth, which are at the other end, to enter into the *Niche* before the ſaid pieces can themſelves. At the Points 1 1, are two Ropes that paſs thro' the Pullies 2 2, faſtened to the Pillars, *o* 3 *o* 4, and have each of them a little weight at the end big enough to keep it from paſſing thro' a little Bowl which is under each Pulley.

This *Arm* thus diſpoſed, goes and comes in the Hole N N, in the following manner. One Tooth of the *Serpent*, already deſcribed, ſtrikes upon the Extremity of the *Baſcule* P Q, and ſo cauſes the End Q to riſe up; which drawing the Cord faſtened to the Point Q, makes the *Arm* L M, to advance forward. But when afterwards the Tooth of the *Serpent* is come forth again, then the Weight V, tied to the other End of the ſame *Arm*, by a Cord that paſſes thro' the Pulley T, forces the ſaid *Arm* by its own Weight to return again.

When the *Arm* L M is in its ordinary place, the two little pieces of Wood into which enters the Bar XY, encloſe the *Shuttle* by means of the Whalebone-*Spring*. But when the ſaid *Arm* approaches the other oppoſite Arm, then the Cords tied to the Points 1 1, being a little too ſhort, and the Weight which is at the end of them not being able to paſs thro', the *Spring* gives way a little, and ſo the *Shuttle* is no longer encloſed by the *Arm* which carries it, but is wholly received and graſp'd by the other; which likewiſe in its turn, delivers it back again in the ſame manner.

The Motion of the whole *Machine* is made at the rate as you move the Handle of the *Serpent*, for then the *Arms* cauſe the Threads to open, and immediately one of the *Arms* begins to ſlide in towards the oppoſite Arm, to which it carries the *Shuttle*, and retires immediately: At the ſame time, one of the Quarters of a Circle, which held the *Clapper* elevated, forſakes it, and leaves it for to flap, and then the oppoſite Quarter of a Circle elevating itſelf, the other Elbow changes the Threads, and the other *Arm* retires; and ſo ſucceſſively.

The *Advantages* of this *Engine* are theſe. 1. One *Mill* will ſet 10 or 12 of theſe *Looms* at Work. 2. You may make the *Cloth* of what *Breadth* you pleaſe. 3. There will be fewer Knots in the *Cloth*, ſince the Threads will not break ſo faſt as in other *Looms*, becauſe the *Shuttle* that breaks the greateſt Part, can never touch them. In ſhort, The *Work* will be carried on *Quicker*, and at leſs *Charge*, in regard that inſtead of ſeveral Work-Folks which are required in making very large *Cloths*, One Boy will ſerve to tie the Threads at the ſeveral *Looms* as faſt as they break, and to order the *Quills* about the *Shuttle*.

XVII. I ordered a *Model* of a part of a *Waggon* to be made conſiſting of four *Wheels*, two *Axes*, and a *Board* nailed upon the *Axes*. The *Leſſer Wheels* were $4\frac{1}{3}$ Inches high, and the *Bigger-Wheels* $5\frac{2}{3}$ Inches high, *viz*, $\frac{1}{12}$ of the ordinary Height of the *Wheels* of a *Waggon*: The Weight of the *Model* was almoſt $1\frac{1}{2}$ *lib.* I had alſo two other *Wheels* made $5\frac{2}{3}$ Inches high to be put on inſtead of the Leſſer. The Middle of the two *Axes* were $6\frac{1}{4}$ Inches aſunder. All the *Wheels* turned very eaſily upon the *Axes*.

***Advantages of High Wheels experimented; by a Member of the Oxford Society.* n. 167. p. 856. Jan. An. 1685.**

A piece of Lead $50\frac{1}{4}$ *lib. Averdupoiſe*, was laid upon the *Model*, ſo forward, that the *Leſſer Wheels* ſeemed to bear above $\frac{2}{5}$ parts of the *Weight*. Then the *Model* was drawn with a String laid over a Pulley, the Top whereof was $\frac{1}{4}$ of an Inch higher than the Top of the *Hinder Axis*, and the Middle of this Pully was $7\frac{1}{2}$ Inches from the Middle of the *Fore Axis*.

The *Leſſer Wheels* being put on, and the String being tied to the Top of their *Axis*,

1. *Three* Pound drew the *Model* on the ſmooth Level Table.
2. *Twenty* Pound drew the *Leſſer Wheels* over a Squared Rod $\frac{1}{4}$ of an Inch thick.
3. *Thirty* Pound drew *them* over a round Rod a little more than $\frac{1}{2}$ an Inch thick.
4. *Thirty One* Pound drew *them* over a Square Rod half an Inch thick.
5. *Twelve* Pound drew the *Hinder Wheels* over the bigger Square Rod.

The String being laid under the *Axis*, *viz.* $\frac{5}{8}$ of an Inch lower than before,

6. *Twenty nine* Pound drew the *Leſſer Wheels* over the Bigger Square Rod.

Then the two *Bigger Wheels* being put on inſtead of the *Leſſer*, and the String lying over the *Axis*,

7. *Three* Pound drew the *Model* on the Table.
8. *Twenty five* Pound drew the *Fore Wheels* over the Round Rod.
9. *Twenty five* Pound drew them over the Bigger Square Rod.
10. The String lying under the *Axis*, 16 Pound drew them over the leaſt Rod.
11. *Twenty three* Pound drew them over the round Rod.

12. *Twenty three* Pound drew them over the Bigger Square Rod.

13. *Thirteen* Pound drew the *Hinder Wheels* over the Bigger Square Rod.

In all theſe *Experiments*, the *Lead* was laid exactly upon the ſame part of the *Board*, but yet when the *Leſſer Wheels* were taken off, the *Lead* did not lean ſo much forward, ſo that the *Hinder Wheels* were ſomewhat more preſſed than they were before.

By comparing the ſecond, third, and fourth *Experiments*, with the tenth, eleventh and twelfth, it appears how much more eaſily a *Waggon*, *&c.* might be drawn in rough Ways, if the *Fore Wheels* were as high as the *Hinder Wheels*, and if the *Thills* were fixed under the *Axis*. Such a *Waggon* as this, would likewiſe be drawn more eaſily, where the *Wheels* cut in *Clay* or *Sand*, or any Soft Ground. And moreover, *High Wheels* would not cut ſo deep as *Low Wheels*.

Low Wheels indeed are better for Turning in a narrow Compaſs than high Ones: But it ſeems probable that *Waggons* with four *High-Wheels*, might be ſo contrived, that there ſhould be no great Inconvenience in that reſpect; at leaſt, ſuch *Waggons* as ſeldom have occaſion to turn ſhort, as *Carriers Waggons*, and ſuch like.

The Difference which you may obſerve in the eighth and eleventh *Experiments*, is agreeable to what is ſaid by *S. Stevinus*, and *Dr. Wallis*, *viz.* That if a *Coach*, *&c.* muſt be drawn over rough, uneven Places, it is beſt to fix the *Traces* to the Coach lower than the Height of the Horſes Shoulders.

14. A Table $2\frac{1}{2}$ Foot long, was ſet with one End $8\frac{1}{2}$ Inches higher than the other End, and the *Model* being loaded as before, leſs Weight by 6 Ounces drew it up the Table, when the four *Bigger Wheels* were on, than when two *Bigger* and two *Leſs* were on. Becauſe, in the firſt Caſe there was almoſt the ſame *Direction* of the Motion of the *Model* and of the String that drew it; but not in the ſecond Caſe, when the *Fore Axis* was ſo much lower than the Top of the Pulley.

A new ſort of Caleſh deſcribed by Sir R. B. n. 172. *p.* 1028. June. An. 1685.

XVIII. This *Caleſh* goes on two Wheels; carries one Perſon, is light enough; tho' it hangs not on *Braces*, yet it is eaſier than the Common Coach: A common Coach will overturn, if one Wheel go on a Superficies a Foot and a half higher than that of the other, but this will admit of the Difference of $3\frac{1}{2}$ Foot in Height of the Superficies, without danger of Over-turning: We choſe all the irregular Banks, and ſides of Ditches to run over; and I have this Day ſeen it at five ſeveral Times turn over and over, and the Horſe not at all diſordered. If the Horſe ſhould be in the leaſt unruly, with the help of one Pin, you diſingage him from the *Caleſh* without any Inconvenience. I myſelf have been once overturned, and knew it not till I looked up, and ſaw the Wheel flat over my Head; and if a Man went with his Eyes ſhut, he ſhould imagine himſelf in the moſt ſmooth way. tho' at the ſame time there be three Foot Difference in the Height of the Ground of each Wheel.

The Contrivance of a perpetual Motion; by M. .... *explained; by Dr.* Papin. n. 177. *p.* 1240. *Fig.* 184. Dec. An. 1685.

XIX. Let DEF, be a pair of *Bellows* 40 Inches long, that may be opened by removing the Part F, from E: Let them be exactly ſhut every where, but at the Aperture E; and Let a Pipe E G, 20 or 22 Inches long, be ſodered to the ſaid Aperture E, having its other end in a Veſſel G, full of *Mercury*, and placed near the middle of the *Bellows*.

A, is an *Axis* for the *Bellows* to turn upon.

B, a Counterpoiſe faſtened to the lower end of the *Bellows*.

C, a Weight with a Claſp to keep the *Bellows* upright.

Fig. 185.

Now if we ſuppoſe the *Bellows* opened only to $\frac{1}{3}$, or $\frac{1}{4}$, ſtanding upright, and full of *Mercury*; it is plain that the ſaid *Mercury* being 40 Inches high, muſt fall, as in the *Torricellian Experiment*, to the Height of about 27 Inches, and conſequently the *Bellows* muſt open before F, and leave a *Vacuity* there. This *Vacuity* muſt be filled with *Mercury* aſcending from G thro' the Pipe GE, the ſaid Pipe being but 22 Inches long: By this means the *Bellows* muſt be opened more and more till the *Mercury* continuing to aſcend, makes the upper part of the *Bellows* ſo heavy, that the lower part muſt get looſe from the Claſp C, and the *Bellows* ſhould turn quite upſide down; but the Veſſel G, being ſet in a convenient place, keeps them Horizontal, and the part F, engageth there in another Claſp C; then the *Mercury* by its Weight runs out from the *Bellows* into the Veſſel C, thro' the Pipe E G, and the *Bellows* muſt ſhut cloſer and cloſer until the part E F comes to be ſo light, that the Counterpoiſe B is able to make the part F, get looſe from the Claſp C; then the *Bellows* comes to be upright again as before; the *Mercury* left in them falls again to the height of 27 Inches, and conſequently all the other Effects will follow, as we have already ſeen, and the *Motion* will *continue for ever*.

And ſhewn inſufficient by him. *N. p.* 241. *n.* 182. p. 178.

Upon this, it is to be obſerv'd, That the *Bellows* can never be opened by the internal Preſſure, unleſs the ſaid Preſſure be ſtronger than the external. Now in the Caſe before us, it is plain, That altho' the lowermoſt part of the *Bellows* be preſſed outward by 40 Inches of *Mercury*, yet the upper part having no *Mercury* above it, bears none at all; the parts that lie in the middle near the Axis of the *Bellows* bear but 20 Inches, and ſo all the reſt muſt bear more or leſs, according as they lie higher or lower: It is evident therefore, That there are as many parts that bear leſs than 20 Inches, as there are that bear more, and the Increaſe of Preſſure following an *Arithmetical Progreſſion*, it is undeniable, that all theſe Preſſures added together, will do more than one uniform Preſſure, that would be equal to 20 Inches every where. It is alſo plain, that the Weight of the *Atmoſphere*, cannot come at the inward part of the *Bellows*, but thro' the Pipe G E, which containing 22 Perpendicular Inches of *Mercury*, doth counterpoiſe ſo much of the Weight of the *Atmoſphere*; ſo that this being ſuppoſed to be 27 Inches of *Mercury*, it cannot preſs the inward part of the *Bellows*, but with a weight equivalent to 5 Perpendicular Inches of *Mercury*. So that we find, the Inward Preſſure both of the *Mercury* and the *Atmoſphere*, is equivalent but to 25 Inches of *Mercury* in all: whereas the Preſſure of the *Atmoſphere* upon the Outſide is every where equal to 27 Inches; from whence it appears, That the Preſſure without is ſtronger than the Preſſure within. From this we may conclude, that the *Bellows* ſtanding upright will rather ſhut than open.

n. 186. *p.* 267. I ſhall ſay nothing to the Alterations this *Author* may make in his *Engine*, reſolving to leave it to others to ſhew him, that upon that Principle all he can do ſignifies nothing. And I doubt not, but if he pleaſes to conſult *M. Perault*, *De la Hire*, or any other at *Paris*, he will find them of the ſame Opinion with Mr. *Boyle*, and Mr. *Hook*, and others *here*.

The Speaking Trumpet improved; by Mr. J. Conyers. *n.* 141. *p.* 1027. Sept. An. 1678. *Fig.* 186.

XX. This *reflecting Trumpet* conſiſts of two Parts. The utmoſt *B b*, is a large Concave Pyramid, about a Yard long, (or may be of any manageable Length) open at the Baſe B, and cloſed not with a Flat, but a Concave Head, at the Cone B. Within this is faſtened a bended Tube A *a*. This *Trumpet* did at a Meeting of the *Royal Society* at *Arundel Houſe*, diſtinctly deliver ſome Words, croſs the Garden and the River *Thames*, and that againſt the Wind which was then ſtrong; and the words were written down by one, that was ſent over for that purpoſe: Whereby it appeared, That a *reflecting Trumpet*, after this, or ſome other like manner of Wood, Tin, Pewter, Stone, or Earth, or which may be beſt, of Bell-Metal, will carry the Voice as far, if not farther, than the long one invented by Sir *Samuel Moreland*. *Fig.* 187. Beſides that, it ſeems to take off from the aſtoniſhing Noiſe near at hand, which happens in uſe of the ſaid long *Trumpet*: *Fig.* 188. By Sir *Sam. Moreland*'s *Trumpet* angularly arched in the middle, the delivery of Sound to any diſtant Place was much ſhortened; and by another with three large angular Arches, reaching almoſt from one end to the other, the Sound was almoſt wholly obſtructed.

The Swiftneſs of Sounds and their Reflections or Eccboes; by Mr. Walker. n. 247. *p.* 433. Dec. An. 1698.

XXI. I provided a *Pendulum*, of ſmall virginal Wire, with a Piſtol Bullet at the end of it, which had two Vibrations in one Second of Time. I took this *Pendulum*, and ſtanding over-againſt a high Wall, I clapt two pieces of ſmall Boards together, and obſerved how long it was ere the *Eccho* returned; and I removed my Station till I found the place whither the *Eccho* returned in about half a Second. But that I might diſtinguiſh the Time more nicely, I clapt every Second of Time, 10 or 15 times together; ſo that by this means, I could the better diſcover whether the diſtances betwixt the Claps and the *Eccboes*, and the following Claps, were equal. And tho' it be very difficult to be exact, yet I could come within ſome few Yards of the place I ſought for, thus I obſerved the two Places, where I could but juſt diſcover that I was too near, and where I was too far off; and from the midway betwixt them I meaſured to the Wall, which Meaſure doubled, was the Space that the *Sound moved* in half a Second.

Here follow the Numbers of *Engliſh* Feet which a *Sound moved* in one Second of Time at ſeveral Trials.

Trials

Trials.	*Feet.*	*Trials.*	*Feet.*	*Trials.*	*Feet.*
1	1256	5	1292	9	1278
2	1507	6	1378	10	1290
3	1526	7	1292	11	1200
4	1150	8	1185		

Merſennus mentions an *Experiment* wherein he found the *Motion* of the *Sound* to be 1474 *Feet* in a *Second.* The *Academy del Cimento* cauſed 6 Harquebuſſes, and 6 Chambers to be fired one after another at the Diſtance of 5739 *Engliſh Feet*, and from the Flaſh to the Arrival of the Report each was 5″: And repeating the *Experiment* at the Midway, the *Motion* was exactly in half the time; and Mr. *Boyle* obſerved, that the *Motion* of *Sound* paſſes above 400 *Yards* in a *Second.*

When the *Firſt Trial* was made, there was ſome Wind ſtirring, tho' not much; the 2d, 3d and 6th were made in a Calm Morning. In the 8th, the *Eccho* was returned from a Wall at 395 *Yards* Diſtance in two *Seconds*; and in the 9th and 10th, at 213 and 215 *Yards* Diſtance, in one *Second.* The 4th was made at one end of St. *John's Cloiſter*, in *Oxford*, which is 104 *Feet* 7 *Inches* long, where the *Sound* was *Reflected* 11 *times* in two *Seconds:* And the 5th, on the North ſide of *New College Cloiſter* (which is 160 *Feet* 8 *Inches* long) where there are about 7¼ *Ecchoes* in two *Seconds.*

By ſome of thoſe *Experiments* that I tried, I am inclined to think, That the *Sound* moved quicker when it was Calm, than in a Wind, even when the *Sound* moved half way with the Wind; and that it moves ſwifter at firſt, than afterwards.

There is ſeldom any *Eccho*, where there is not ſome Wall, Wood, Bank, or ſuch like, directly oppoſite, that may Reflect the *Sound* to the Perſon that makes it; but in St. *John's Grove*, if you ſtand near the Gate leading from the *College* to the *Grove*, and Clap, the *Eccho* will return to you from the *Ball Court*, tho' a Line drawn from you to the *Ball Court* be not perpendicular to the Wall there, but as much oblique as the Line A B is to the Line B C; where A repreſents the Gate, B C the *Ball Court* Wall, and B D another Wall. *Fig.* 180. Or, if you ſtand at E, the Corner of the *Grove* next to *Trinity*, and clap, the *Eccho* will return to you from the *Ball Court.*

In the ſame *Grove* I ſtood about 20 Yards from the ſame Gate, and the Gate being ſhut, Clapt, and at other times Stamped, and the *Eccho* returned from the Gate as loud, if not louder than the Clap or Stamp.

An *Eccho* reflected from a Gate or Door, has uſually a baſer and duller Sound than that which is returned from a Wall, this being much briſker.

As I have been walking towards a Wall, I have clapped my Hands together ſeveral times, and I could diſtinguiſh the *Eccho* from the Clap, till I came within 7 or 8 Yards of the Wall.

In the *Cloiſters*, where, as we ſaid before, the *Eccho* was repeated ſeveral times, the firſt Repetition ſeemed to be ſlower than the ſecond or third;

but of all the Repetitions, beſides the firſt, the ſubſequent ſeemed ſlower than the precedent.

I have obſerved the Toſſing of a *Sound* forward and back again, in very many Places where there are Parallel Walls; and where the Diſtance of the Walls is leſs, there the *Ecchoes* follow one another quicker.

Whereſoever a *Sound* was thus toſſed betwixt two Walls, if I ſtood about the Middle, I could hear the *Sound* twice as quick, that is twice as often repeated in one Second, as if I ſtood near one Wall: The *Sound* being reflected to me from both ends, when I ſtood in the middle.

Fig. 190. In *Trinity Ball Court*, when I ſtood and clapt at B, three or four Yards from the End of the Wall C, or at A, which is oppoſite to B, the *Sound* was toſſed betwixt the oppoſite Walls, but not half ſo long as when I ſtood betwixt the Walls. In Places where there are Parallel Walls, not above ſix or eight Yards aſunder, as in *Trinity Ball Court*, and at the Entrance into St. *John*'s *Grove*, *&c.* I have heard the Ecchoes of a Clap following one another diſtinctly enough: But there the *Ecchoes* of a *muſical Note*, which was longer than a Clap, were ſo confuſed, that they ſeemed one continued *long Sound:* which makes me think, that the *Eccho* in ſome Vaults, is nothing elſe but the *Sound* toſſed betwixt the ſide Walls, and betwixt the top and bottom. This alſo makes me conjecture, That the Reaſon why *ſtringed Muſical Inſtruments* give a greater and longer *Sound* to the *Strings* than if the *Strings* were fixt to a ſingle Board, may be this; becauſe the *Sound* is toſſed from ſide to ſide in the Belly of the *Inſtrument*.

The Doctrine of Sounds; by Narciſſus, *Biſhop of* Ferns *and* Leighlin. n. 156. *p.* 472. *Nov.* An. 1683.

XXII. I cannot better explain the *Uſefulneſs* of this *Theory* of *Sounds*, than by making a Compariſon 'twixt the Faculty of *Seeing* and *Hearing*, as to their Improvements. In order to which, I obſerve, That *Viſion* is threefold, *direct*, *refracted*, *and reflex'd*; anſwerable whereunto we have *Opticks*, *Dioptricks*, and *Catoptricks*:

In like manner *Hearing* may be divided into *direct*, *refracted*, and *reflex'd*; whereto anſwer three Parts of our Doctrine of *Acouſticks*, which are yet nameleſs, unleſs we call them *Acouſticks*, *Diacouſticks*, and *Catacouſticks* (or in another Senſe, but to as good Purpoſe) *Phonicks*, *Diaphonicks*, and *Cataphonicks*.

Direct Viſion has been improved two ways.

1. *Ex parte Objecti*, by the Arts of *producing*, *conſerving*, and *imitating*, and duly *applying*, Light and Colours.

2. *Ex parte Organi vel Medii*, by making uſe of *Tubes without Glaſſes*, or, a Man's cloſed Hand to look thro'. So likewiſe *direct Hearing*, partly has, and partly may further receive great and notable Improvements, both *ex parte Objecti*, and *ex parte Organi vel Medii*.

1. As to the Object of *Hearing*, which is *Sound*, Improvement has been and may be made, both as to the *begetting*, and as to the *conveying* and *propagating* (which is a kind of *Conſerving*) of *Sounds*.

1. As to the *begetting* of *Sounds*. The Art of imitating any *Sound* whether by ſpeaking (that is pronouncing) any kind of Language (which really

is an *Art*; and the *Art* of Speaking perhaps one of the greateſt) or by Whiſtling, or by Singing (which are allowed *Arts*) or by Hollowing, or Luring, (which the *Huntſman* and *Falconer* would have to be an *Art* alſo) or by imitating with the Mouth (or otherwiſe) the Voice of any Animal; as of Quails, Cats, and the like; or by repreſenting any Sound begotten by the Colliſion of ſolid Bodies, or after any other manner; theſe are all *Improvements* of *direct Hearing*, and may be *improved*.

Moreover, the Skill to make all ſorts of *muſical Inſtruments*, both antient and modern, whether *Wind Inſtruments* or *ſtring'd*, or of any other ſort, whereof there are very many (as *Drums*, *Bells*, the *Syſtrum* of the *Egyptians*, or the like) that *Beget* (and not only *Propagate*) *Sounds*: The Skill of making theſe, I ſay, is an *Art*, that has much improv'd *direct Hearing*; and an *harmonious Sound* exceeds a ſingle and rude one, that is an *immuſical Tune*: which *Art* is yet capable of farther Improvement And I hope, That by the Rules which may happily be laid down, concerning the *Nature*, *Propagation*, and *Proportion* or *adapting* of *Sounds*, a way may be found out, both to improve *Muſical Inſtruments* already in uſe, and to invent new ones, that ſhall be more ſweet and luſcious than any yet known. Beſides, that by the ſame means *Inſtruments* may be made, that ſhall imitate any *Sound* in Nature, that is not *Articulate*; be it of Bird, Beaſt, or what thing elſe ſoever.

2. The *Conveying* and *Propagating* (which is a kind of *Conſerving*) of *Sounds*, is much helped by duly *Placing* the *ſonorous Body*, and alſo by the *Medium*.

For if the *Medium* be *Thin* and *Quieſcent*, and the *ſounding Body placed* conveniently, the *Sound* will be eaſily and regularly *propagated* and mightily *conſerved*.

1. The *Medium* muſt be *Thin* and *Quieſcent*: Hence in a ſtill Evening, or the Dead of the Night (when the Wind ceaſes) a *Sound* is better ſent out, and to a greater Diſtance, than otherwiſe.

2. The *ſonorous Body* muſt be *placed* conveniently, *viz.* Near a *ſmooth Wall*, either *Plain* or *Arched* (*Cycloidically* or *Elliptically*, rather than otherwiſe; tho' a *Circular* or any *Arch* will do; but not ſo well.) Hence in a *Church*, the nearer the *Preacher* ſtands to the *Wall* (and certainly it's much the beſt way to place *Pulpits* near the *Wall*) the better is he heard, eſpecially by thoſe who ſtand near the *Wall*; alſo, tho' at a greater Diſtance from the *Pulpit*, thoſe at the remoteſt End of the Church, by laying their Ears ſomewhat cloſe to the Wall, may hear him eaſier than thoſe in the middle.

Hence alſo do ariſe *Whiſpering Places*. For the Voice being applied to one end of an *Arch*, eaſily rowls to the other. And indeed were the Motion and *Propagation* of *Sounds* but rightly underſtood, 'twould be no hard matter to contrive *Whiſpering Places* of infinite variety and uſe. And perhaps there could be no better or more pleaſant hearing a *Concert* of *Muſic*, than at ſuch a Place as this; where the *Sounds* rowling long together before they come to the Ear, muſt needs conſolidate and imbody in one; which becomes a true *Compoſition* of *Sounds*, and is the very Life and Soul of *Concert*.

2. If the *ſonorous Body* be placed near *Water*, the *Sound* will eaſily be *convey'd*, yet mollify'd; as Experience teacheth us from a *Ring* of *Bells* near

a River, and a great Gun ſhot off at Sea; which differ much in the ſtrength, and yet Softneſs and Continuance, or *Propagation* of their *Sounds*, from the ſame at Land; where the *Sound* is more harſh and more periſhing, or much ſooner decays.

3. In a *Plain* a Voice may be heard at a far greater Diſtance, than in uneven Ground. The Reaſon of all which laſt nam'd *Phænomena* is the ſame; becauſe the *ſonorous Air* meeting with little or no Reſiſtance upon a *Plane* (much leſs upon an *Arch'd*) ſmooth *Superficies*, eaſily rowls along it, without being let or hindred in its Motion, and conſequently without having its Parts disfigured, and put into another kind of Revolution, than what they had at the firſt *begetting* of the *Sound*, which is the true Cauſe of its *Preſervation* or *Progreſſion*; and fails much when the *Air* paſſes over an uneven Surface, according to the degrees of its Inequality; and ſomewhat alſo, when it paſſes over the *Plain Superficies* of a Body that is hard and reſiſting.

Wherefore the *ſmooth* Top of the *Water* (by reaſon of its yielding to the *Arched Air*, and gently riſing again with a kind of *Reſurge*, like to *Elaſticity* tho' it be not ſo; by which *Reſurge* it quickens and haſtens the *Motion* of the Air rowling over it, and by its yielding preſerves it in its *Arched Cycloidical* or *Elliptical* Figure) the *ſmooth* Top of the *Water*, I ſay, for theſe Reaſons, and by theſe Means, *conveys* a *Sound* more entire, and to a greater Diſtance, than the *Plane Surface* of a piece of Ground, a Wall, or any other ſolid Body whatever, can do.

2. The *Organ*, which is the *Ear*, is helpt much by *placing* it near a Wall, (eſpecially at one end of an *Arch*, the *Sound* being *begotten* at the other) or near the Surface of Water, or of the Earth; along which the *Sounds* are moſt eaſily and naturally conveyed; as was before declared. And 'tis Incredible, how far a *Sound* made upon Earth (by the Trampling of a Troop of Horſes, for Example) may be heard in a ſtill Night, if a Man lays his Ear cloſe to the Ground in a large *Plain*.

Otacouſticks here come in for helping the *Ear*; which may be ſo contrived (by a right underſtanding the *Progreſſion* of *Sounds*, which is the principal Thing to be known for the due regulating all ſuch kinds of Inſtruments) as that the *Sound* might enter the *Ear* without any *Refraction*.

2. *Refracted Viſion* (which is always made *ex parte Medii*) ariſes from the different *Denſity*, *Figure*, and *Magnitude* of the *Medium*; which is ſomewhat altered by the divers *Incidence* of the *Viſive Rays*, and ſo it is in *Refracted Hearing*, all theſe Cauſes concur to its *Production*; and ſome others to be hereafter conſidered.

Now as any Object (a Man for Example) ſeen thro' a Thickened Air, by *Refraction* appears greater than really he is: So likewiſe a *Sound* heard thro' the ſame Thickened part of the *Atmoſphere*, will be conſiderably vary'd from what it would ſeem to be, if heard thro' a Thinner *Medium*. And this I call a *Refracted Sound*.

Improvements of *Refracted Viſion* have been made, by Grinding or Blowing Glaſſes into a certain Figure, and placing them at due Diſtances; whereby the Object may be (as 'twas) enabled to ſend forth its Rays more Vigorouſly, and the *Viſive Faculty* impowered the better to receive them. Thus,

1. A fine Glaſs Bubble, filled with clear Water, and placed before a burning Candle or Lamp, does help it to dart forth its Rays to a prodigious Length and Brightneſs.

2. The *Viſive Faculty* is much helped,

1. By *Spectacles* and other Glaſſes, which are made to help the *Purblind* and *Weak Eyes*, to ſee at any competent Diſtance.

2. By *Perſpective Glaſſes* and *Teleſcopes*, which help the *Eye* to *ſee* Objects at a very great *Diſtance*, which otherwiſe would not be diſcernible.

3. By *Microſcopes* or *Magnifying Glaſſes*, which help the *Eye* to ſee *near Objects*, that by reaſon of their Smallneſs were *Inviſible* before.

4. By *Polyſcopes* or *Multiplying Glaſſes*, whereby one thing is repreſented to the *Eye* as many, whether in the ſame or different Shapes.

After the ſame manner, Inſtruments may be contrived for aſſiſting both the *Sonorous Body* to ſend forth its *Sound* more ſtrongly, and the *Acouſtick Faculty* to receive and diſcern it more eaſily and diſtinctly. And thus;

1. An Inſtrument may be invented, that applied to the *Mouth*, (or any *Sonorous Body*) ſhall ſend forth the *Voice* diſtinctly as to a prodigious *Diſtance* and *Loudneſs*. For if the *Stentoro-phonicon* (which is but a Rude and Inartificial Inſtrument) does ſuch great Feats; what might be done with One compoſed according to the Rules of Art, whoſe *Make* ſhould comply with the Laws of *Sonorous Motion*, which that does not?

2. There are ſome *Inſtruments*, and more ſuch may be Invented to help the *Ear*: As,

1. *Otacouſticks* (and better may be made) to help *Weak Ears* to hear at a reaſonable Diſtance alſo. Which would be as great a help to the *Infirmity* of *Old Age*, as the other Invention of *Spectacles* is, and perhaps greater; foraſmuch as the *Hearing* what's ſpoken is of more daily Uſe and Concern to ſuch Men, than to be able to *read* Books, or to view Pictures.

2. A ſort of *Otacouſticks* may be ſo contrived, as that they ſhall receive in *Sounds* made at a very great *Diſtance*, which otherwiſe would have been *Inaudible*: and theſe *Otacouſticks*, in ſome Reſpects, would be of greater uſe than *Perſpectives*.

1. In Time of *War*, for diſcovering the *Enemy* at a good *Diſtance*, when he marches or lies incamp'd behind a Mountain or Wood, or any ſuch Place of Shelter, which hinder the Sight from reaching very far.

2. At *Sea*, when in dark *Hazy Weather* the Air is too thick, or in *Stormy* Tempeſtuous *Weather*, the Waves riſe too high, for the *Perſpective* to be made uſe of.

3. In Dark Nights, when *Perſpectives* become almoſt inſignificant, and yet at ſuch times, generally, Soldiers take their March, when they would ſurprize their Enemies.

4. *Microphones*, or *Micracouſticks*, that is, *Magnifying Ear Inſtruments*, which may be contrived after that manner, that they ſhall render the moſt *Minute Sound* in Nature diſtinctly Audible by *Magnifying* it to an uncon-

ceiveable Loudneſs: By the help whereof we may hear the different Cries and Tones of the ſmalleſt *Animals*.

5. A *Polyphone*, or *Polycouſtick*, ſo ordered that One *Sound* may be heard, either of the Same, or a different *Note*: Inſomuch that who uſes this *Inſtrument*, he ſhall at the Sound of a *Single Viol* ſeem to hear a whole *Concert*, and all True *Harmony*. By which means this *Inſtrument* has much the Advantage of the *Polyſcope*.

I have called it *Refracted Hearing*, becauſe made thro' a *Medium*, *viz.* Thick Air, or an Inſtrument, thro' which the *Sound* paſſing is broken or *Refracted*,

3. *Reflected Viſion* (which is always made *ex parte Objecti*) hath been improv'd by the Invention of *Looking Glaſſes* and *Poliſh'd Metals*, whether *Plane*, *Concave* or *Convex*, of ſeveral Figures, and placed at Determinate Diſtances.

In like manner *reflex'd Audition* (which is only made *ex parte Corporis Oppoſiti*) may be improved by contriving ſeveral ſorts of *Artificial Ecchoes*. For (ſpeaking in general) any *Sound* falling *directly* or *obliquely* upon any denſe Body of a ſmooth (whether Plane or Arch'd) Superficies, is beat back again and reflected, or does *Eccho* more or leſs.

I ſay, (1.) *Falling directly* or *obliquely*; becauſe, if the *Sound* be ſent out and Propagated Parallel to the Surface of the denſe Body, there will be no *Reflection* of *Sound*, no *Eccho*.

I ſay, (2.) Upon a Body of a *ſmooth Superficies*; becauſe if the Surface of the *Corpus Obſtans* be uneven, the Air by *Reverberation* will be put out of its regular Motion, and the *Sound* thereby broken and extinguiſh'd: So that, tho' in this caſe alſo the Air be beaten back again; yet *Sound* is not reflected, nor is there any *Eccho*.

I ſay, (3.) It does *Eccho more or leſs*, to ſhew, that when all things are, as is before deſcrib'd, there is ſtill an *ecchoing*, tho' it be not always heard, either becauſe the *direct Sound* is too weak to be beaten quite back again to him that made it; or that it does return home to him, but ſo weak, that without the help of a good *Otacouſtick* it cannot be diſcerned; or that he ſtands in a wrong Place to receive the *reflected Sound*, which paſſes over his Head, under his Feet, or to one ſide of him; which therefore may be heard by a Man ſtanding in that place, where the *reflected Sound* will come, provided no interpos'd Body does intercept it; but not by him that firſt made it.

Theſe *Ecchoes* (like *Reflected Viſion*) may be ſeveral ways *produced*, as;

1. A *Plane Corpus Obſtans reflects the Sound* back in its due *Tone* and *Loudneſs*; if allowance be made for the porportionable Decreaſe of the *Sound* according to its Diſtance.

2. A, *Convex Corpus Obſtans repels* the *Sound* (inſenſibly) ſmaller: but ſomewhat quicker (tho' weaker) than otherwiſe it would be.

3. A *Concave Corpus Obſtans ecccboes* back the *Sound* (inſenſibly) bigger, ſlower, (tho' ſtronger) and alſo *inverted*; but never according to the order of Words. Nor do I think it poſſible for the Art of Man to contrive

a ſingle *Eccho*, that ſhall invert the *Sound* and repeat backwards; becauſe then the Words laſt ſpoken, that is, which do laſt occur to the *Corpus Obſtans*, muſt firſt be *repell'd*; which cannot be. For where in the mean time ſhould the firſt Words hang and be conceal'd or lie dormant? Or how, after ſuch a Pauſe be reviv'd and animated again into Motion? Yet in complicated or compound *Ecchoes*, where many receive from one another, I know not whether ſomething that way may not be done.

From this determinate *Concavity* or *Archedneſs* of theſe *reflecting Bodies*, it comes to paſs, that ſome of them from a certain Diſtance or Poſiture, will *eccho* back but one determinate *Note*, and from no other Place will they *reverberate* any; becauſe of the undue Poſition of the *ſounding Body*. Such an one (as I remember) is the *Vault* in *Merton College* in *Oxford*.

4. *The Echoing Body*, being removed farther off, *reflects* more of the *Sound*, than when nearer. And this is the Reaſon, why ſome *Ecchoes* repeat but one Syllable, ſome one Word, and ſome many.

5. *Echoing Bodies* may be ſo contriv'd and plac'd, as that *reflecting* the *Sound* from one to the other, either *directly* and *mutually*, or *obliquely* and by Succeſſion, out of one *Sound* ſhall many *Echoes* be begotten; which in the firſt Caſe will be all together and ſomewhat involv'd or ſwallow'd up of each other; and thereby confuſed (as a Face in Looking-Glaſſes obverted;) in the other they will be diſtinct, ſeparate and ſucceeding one another, as moſt *multiple Echoes* do.

Moreover, a *multiple Echo* may be made, by ſo placing the *Echoing Bodies*, at unequal Diſtances, that they *reflect* all one way, and not one on the other; by which means a *manifold ſucceſſive Sound* will be heard (not without Aſtoniſhment; one Clap of the Hands like many; one *Ha* like a Laughter; one ſingle Word like many of the ſame *Tone* and *Accent*; and ſo one *Viol* like many of the ſame kind, imitating each other.

Furthermore, *Echoing Bodies* may be ſo ordered, that from any one *Sound* given, they ſhall produce many *Echoes* different both as to their *Tone* and *Intention*. By this means a *muſical Room* may be ſo contrived, that not only one *Inſtrument*, played on in it, ſhall ſeem many of the ſame Sort and Size; but even a *Concert* of (ſomewhat) different ones; only by placing certain *Echoing Bodies* ſo, as that any *Note* (played) ſhall be returned by them in *3ds*, *5ths*, and *8ths*, which is not poſſible to be done otherwiſe than was mentioned before in *refracted Audition*.

I have been thus large, that I might give you a little Proſpect into the Excellency and Uſefulneſs of *Acouſticks*, and that thereby I might excite others to bend their Thoughts, towards the making of *Experiments* for the compleating this (yet very Imperfect tho' Noble) *Science*; a *Specimen* whereof I will give in theſe three *Problems*.

Prob. I.] *To make the leaſt Sound (by the help of Inſtruments) as loud as the greateſt; a Whiſper to become as loud as the Shot of a Cannon.*

Prob.

By the help of this *Problem* the moſt minute Sounds in Nature may be clearly and diſtinctly heard.

Prob. II. *To propagate any* (*the leaſt*) *Sound to the greateſt Diſtance.*

By the help thereof any *Sound* may be conveyed to any and therefore heard at any *Diſtance*, (I muſt add, within a certain, tho' very large Sphere.)

Moreover by this means a *Weather-Cock* may be ſo contrived, as that with an ordinary blaſt of Wind it ſhall cry (or whiſtle) loud enough to be heard many Leagues. Which haply may be found of ſome Uſe, not only for Pilots in mighty tempeſtuous Weather, when *Light Houſes* are rendered almoſt uſeleſs: But alſo for the meaſuring the Strength of Winds, if allowance be made for their different Moiſture. For I conceive, That the more dry any Wind is, the louder it will whiſtle, *cæteris paribus*: I ſay, *cæteris paribus*, becauſe, beſides the ſtrength and dryneſs of *Winds* or Breath, there are a great many other things (hereafter to be conſider'd) that concur to the increaſe of *magnifying of Sounds*, begotten by them in an Inſtrument expoſed to their Violence, or blown into.

Prob. III.] *That a Sound may be convey'd from one extreme to the other* (*or from one diſtant Place to another*) *ſo as not to be heard in the middle.*

By the Help of this *Problem* a Man may talk to his Friend at a very conſiderable Diſtance, ſo that thoſe in the middle Space ſhall hear nothing of what paſſed betwixt them.

Fig. 197. I ſhall here add a *Semiplane* of an *Acouſtick* or *Phonical Sphere*, as an Attempt to explicate the great Principle in this *Science*, which is the *Progreſſion* of *Sounds*.

You are to conceive this (rude) *Semiplane* as *Parallel* to the *Horizon*; for, if it be *Perpendicular* thereunto, I ſuppoſe the upper *Extremity* will be no longer Circular, but *Hyperbolical*, and the lower part of it ſuited to a greater Circle of the Earth. So that the whole *Phonical Sphere* (if I may ſo call it) will be a ſolid *Hyperbola*, ſtanding upon a *Concave Spherical Baſe*. I ſpeak this concerning *Sounds* made (as uſually they are) nigh the *Earth*, and whoſe *ſonorous Medium* has a free Paſſage every way. For if they are generated high in the Air, or directed one way, the Caſe will be different; which is partly deſigned in the Inequality of the Draught.

XXIII A Paper, *of leſs General Uſe*, omitted, *viz.*

Carriages. n. 161. p. 666. E*Xperiments to be made, relating to* Carriages; *propoſed by Sir* William Petty.

XXIV. Ac-

XXIV. Accounts *of Books* and Additions, omitted.

1. D*E vi Percuſſionis*, Joh. Alphonſ. Borelli. Bononiæ, 1667. *in 4to.* n. 32. p. 626.

2. *De Motionibus à* Gravitate *dependentibus Liber*, Jo. Alphonſi Borelli, *in Academia* Piſana *Mathefeos* Profeſſoris, Regio Julio 1670. *in 4to.* n. 73. p. 2210.

3. *Dialogi Phyſici, quorum Primus de Lumine*; *Secundus & Tertius de* Vi Percuſſionis & Motu; *Quartus de Humoris Elevatione per Canaliculum*; *Quintus & Sextus de Variis Selectis. Auth.* Honor. Fabry, *S. Jeſu.* Lugduni Galliarum. 1669. *in 8vo.* n. 67. p. 2057.

4. Mechanica, *ſive de* Motu, *Tractatus Geometricus*: *Auth.* Joa. Wallis, *S. S. Th. D.* Londini, 1670. 1671. *in 4to. The* Author *here makes ſome* Additions *to* Prop. I. Chap. XV. p. 753. *concerning the* Center of *Gravity of the* Hyperbola. n. 54. p. 1086. n. 61. p. 2005. n. 76. p. 2286. n. 87. p. 5074.

5. *Exercitationes Mechanicæ*, Alexandri Marchetti. Piſis, 1669. *in 4to.* n. 61. p. 2008.

6. *De Reſiſtentia Solidorum*, Alexandri Marchetti *in* Piſana *Academiâ Phil. Prof.* Florentiæ. 1665. *in 4to.* n. 82. p. 4050.

7. *Hypotheſis, Phyſica nova, ſive Theoria* Motus Concreti, *unà cum Theoria* Motus Abſtracti. *Auth.* Gothfredo Gulielmo Lebnitio. *J. V. D.* Lond. 1621 *in* 12°. n. 73. p. 2213. *Of this Book Dr.* Wallis *here gives his Opinion.* n. 74. p. 2227.

8. *La Statique ou la Science de* Forces Mouvantes, *par le P.* Ignace Gaſton Pardies, *S. J. à* Paris 1673. *in* 12°. *The firſt Part being of* Local Motion. *Printed at* Paris, 1670. *was* Engliſhed *and printed at* London, *the ſame Year, in* 12°. n. 94. p. 6042. n. 65. p. 2010.

9. Chriſtiani Hugenii Zulichemii *Horologium Oſcillatorium.* Pariſiis, 1673. *in* Fol. n. 95. p. 6068.

10. *A Diſcourſe made before the* Royal Society *concerning the Uſe of* Duplicate Proportion *in ſundry important Particulars*; *together with a* New Hypotheſis *of* Elaſtique *or* ſpringy Bodies: *By Sir* William Petty. n. 109. p. 2096.

11. *Traité de la* Percuſſion *ou* Choq *des* Corps, *&c. par M.* Marriotte, *de l'* Academie Royal *des* Sciences. *A* Paris, 1673. *in* 12°. n. 134. p. 859.

12. *Philoſophiæ Naturalis Principia Mathematica. Authore* If. Newton *Lond. in 4to.* n. 186. p. 291. n. 226. p. 445.

13. *Traité de* Mouvement *des* Eaux *& des autres* Corps Fluids. *Par feu M.* Mariotte, *à* Paris, 1686. *in 8vo.* n. 181. p. 191.

14. *Mechanick Exerciſes*; *or*, The Doctrine of *Handy Works. By Mr.* Joſ. Moxon. Lond. 1677. *in 4to.* n. 138. p. 967. n. 139. p. 967.

15. *The* Speaking-Trumpet, *as it hath been contrived, and Publiſhed, by Sir* Samuel Moreland; *together with its* Uſes *both at* Sea *and* Land. Lond. 1671. n. 79. p. 3056.

CHAP. VI.

Hydroſtaticks. Hydraulicks.

To weigh Water, or other Fluids; by . . . . A new Areometer; by M. Homberg. n. 262. p. 530. *Fig.* 192.

I. 1. TAKE a *Viol* with a very *narrow Body*, and, when it is almoſt full, the Water is to be dropt into it, drop by drop, till it can hold no more. Then *weigh* it exactly, and deduct the *weight* of the *empty Viol.*

2. A is a Glaſs Bottle like a little *Matracium*, of which the *Neck* BC is ſo ſmall that a drop of Water therein takes up the Space of 5 or 6 Lines; near that *Neck* is a little Capillar Tube D, about 6 Lines long, and Parallel to the *Neck* B C; the opening B is a little dilated, in the Faſhion of a Tunnel, for pouring more eaſily the Liquors into the Bottle, and the little Tuble D is for giving a way to the Air contained in that Veſſel to go out, when the Liquor is poured in at B; the Point C is a little Mark at the ſame height as the end of the little Tube D.

When we fill the Veſſel, we pour the Liquors into it, by the opening B, until it goes out by the little Tube D, and, if the Height of the Liquor is even to the Mark C, 'tis well; if it is lower, we muſt fill more to that Point; if it is higher, we muſt ſtrike ſoftly upon the opening B, till the Overplus of the Liquor be even to the Point C in the *Neck* of the Bottle. By that means we have always exactly the ſame *Volume* of *Liquor*, and we can know how the ſame *Volume* of the ſeveral Liquors weighs more one than another preciſely. But we muſt conſider the variation of the Weather, when we compare the weight of a Liquor which we weigh in Summer time, with the weight of another which we have weighed in the Winter; for the ſame Liquor, being more rarified in the hot time, and condenſed in the cold, the ſame Volume of it will be more weighty in cold Weather than in warm.

A New Eſſay Inſtrument; by M. Boyle. n. 24. p. 447. n. 115. p. 329. *Fig.* 193.

II. 1. Many Years ago I made uſe of a little *Glaſs Inſtrument*, conſiſting of a *Bubble*, and furniſhed with a *long* and *ſlender Stem* to compare the *ſpecifick Gravities* of *different Liquors* by its more or leſs *ſinking* in *them*: And I have ſince employed it to diſcover the *ſpecifick Gravities* of *ſeveral Solids, appended*, by its being more or leſs depreſſed by them in the *ſame Liquor*. For 'tis clearly deducible from the Grounds of *Hydroſtaticks*, that any ſolid Body heavier than Water, loſes in the Water as much of the weight it had in Air, as Water of equal Bulk to the immerſed Solid would weigh in the Air; and conſequently, ſince Gold is by far the moſt ponderous of Metals, a piece of Gold, and one of equal Weight of Copper, Braſs, or any other Metal, being propoſed, the Gold muſt be leſs in bulk, than the Copper or Braſs. And by this means, if both of them be weighed in the Water, the Gold muſt loſe in that Liquor leſs of its former Weight than the Braſs or Copper; becauſe the baſer Metal, as well as the Gold, grows lighter by the weight of a bulk of Water equal to it: and the baſer Metal being the more voluminous, the correſpondent Water muſt weigh more than that which is correſpondent to the Gold. Whence I concluded, that the *floating Inſtrument* abovementioned would be made to ſink deeper by an Ounce, for Inſtance of Gold, hanging at it under Water, than by an Ounce of Braſs, or any other Metal, which, by reaſon of its greater Bulk than Gold, loſing more of its weight by the Immerſion, muſt needs retain leſs, and ſo have leſs power to depreſs the Inſtrument 'twas faſtened to. Which Concluſion will alſo hold (tho' the Diſparity be not ſo great and conſpicuous) in reference to other Metals, as Lead and Tin, that differ in *ſpecifick Gravity*.

This *Inſtrument* may be of *Glaſs*, *Copper*, *Silver*, or almoſt any other ſolid *Body*, that is, or may be made, fit to *float* on the Water, with a *Guinea*, *&c. hanging* at it, and of a Texture cloſe enough to keep out the Water. It conſiſts of three Parts: The *Ball* or Globulous part; the Stem or Pipe; and that which *holds* the *Coin*.

The *Ball* or Round part of BCDE (if of *Metal*) conſiſts of two thin concave Plates, exactly ſoldered together in the middle; and at the diſtanteſt parts from the Commiſſure, there ought to be left two oppoſite *Holes*, one in each Plate, for the two other parts of the Inſtrument. This middle part, tho' for Brevity's ſake we name it the *Ball*, ſhould not be exactly round; but of any Shape that ſhall be found fit to make the *Inſtrument* keep to its erect poſture ſteadily in the Water. It muſt contain as much Air as may ſerve to keep the whole *Inſtrument*, when loaded, from ſinking beneath the top of the *Stem*. Fig. 194.

The *Stem* AB is to be ſoldered on to the *Ball* at the uppermoſt of the two mention'd *Holes*. It may be either hollow or ſolid: But it ought to be made very ſlender, that the different Depreſſions of the Inſtrument in the Water may be the more notable. And, for the ſame reaſon, it ought not to be too ſhort, eſpecially if it be to be applied to other Uſes than the Examining of *Guineas*.

At the undermoſt of the two *Holes* in the *Ball*, is inſerted and ſolder'd the undermoſt part of the *Inſtrument*, which I call the *Screws*, or the *Stirrup*. The *Screw* F is a very ſhort Piece of Braſs with a broad Slit in it, capable of receiving the Edge of the *Guinea*, which with one turn or two of a ſmall and ſlight lateral Screw may be kept faſt in it, and readily, the Operation being ended, taken out again. The *Stirrup* G is made of a piece of Wire, that, a little beneath the bottom of the *Ball*, is bent round, ſo as to ſtand Horizontally, that the *Guinea* may be laid on it. Fig. 195.

It would be convenient, that the *undermoſt Stem* and the *Screw* be made by itſelf, that it may be at pleaſure thruſt upon the Stem, and taken off again. For, by this means, if the *Ball* of the *Inſtrument* be made large enough, you may have room to put on for Ballaſt, as occaſion ſhall require, one, two, or three flat and round pieces of Copper, Lead, *&c.* with each of them a Hole in Fig. 196.

in the middle fitted to the Size of the Stem, ſo that they may be put on as near the lower part of the *Ball* as you think fit, and then the *Screw* may be thruſt on after them, not only to take Hold of the *Coin* or metalline Mixture to be *examin'd*, but to ſupport the thin Plates.

To adjuſt this *Inſtrument* for the uſe of examining *Guineas*, which are by far the moſt uſual *Gold Coins* that paſs in *England*, you muſt by the help of the *Stirrup* or *Screw*, hang, at the bottom of it, a piece of that *Coin* which you know to be genuine, and having carefully ſtopt the Orifice of the Stem (if it be a Pipe) that no Water may get in at it, immerſe the *Inſtrument* leiſurely and perpendicularly into a Veſſel full of clean Water, 'till it be depreſt almoſt to the top of the Stem, and then letting it alone; if, being ſettled, it continue in the ſame Station and Poſture, your work is done, if it emerge, you muſt add a little weight to it, either by putting into the *Stem*, if it be Hollow, ſome Duſt Shot, Filings of Lead, or ſome other minute and heavy Body, or elſe by putting on the ſhort Stem abovementioned, that comes out beneath the *Ball*, a flat, round and perforated piece of Lead, of Weight ſufficient to enable the *Guinea* to depreſs the Weight as low as 'tis deſired: But, if it ſink quite under Water, you muſt lighten it either with a File, or by ſcraping or grating off a little of the Ballaſt Plate abovementioned; or, if you have put any Weight into the Cavity to poize it, by taking out ſome of that, till you have made it light enough: This being done, a Mark, H, is to be made juſt at the place where the Surface of the Water touches the *Fig.* 193. *Stem*, and then taking out your *Inſtruments*, ſubſtitute, in the place of your *Fig.* 194. *Guinea*, a little round Plate of Braſs, of the ſame Weight, or a Grain or two heavier, in the Air; and, putting the *Inſtrument* into the Water, as before, ſuffer it to Settle, and make another Mark I, at the Interſection of the *Stem* and the Horizontal Surface of the Water.

There may (tho' 'tis like there very ſeldom will) happen a Caſe wherein, tho' the Principle our *Inſtrument* is framed on, will hold good, yet the Practical Application may be Unſecure. For, if a *Falſifier* of *Money* have the Skill, by *Waſhing* or otherwiſe, to take off much of the Quantity or Subſtance of the *Guinea*, without altering or impairing either the *Figure* or *Stamp*, the piece of Coin will not be able to depreſs our Inſtrument to the uſual Mark, and may thereby make it to be judged *Counterfeit*, when 'tis indeed but too *Light*. But it preſently ſhews, that the propoſed *Guineas*, if it be not *Counterfeit*, is otherwiſe *Abuſed*; and, tho' it does not clearly determine, whether that likewiſe proceed from the want of *Specifick Gravity* in the Metal, or from the Coins having been *waſhed*, or otherwiſe fraudulently *leſſened*; yet it probably reſolves the doubt, becauſe, if the want of Weight appear by the *Inſtrument* to be very great, as it uſually does, where the piece has been robbed of ſome of its Subſtance, 'tis a ſtrong Preſumption, that 'tis rather *Waſhed*, &c. than *Counterfeited*. However, it will be ſure to prompt him that uſes it, to employ the Balance, which will preſently aſſiſt him to reſolve his Doubt. For if the Suſpected Coin have in the Air its due Weight, 'twill argue that the great Lightneſs of it in the Water proceeds from its not being of the requiſite Fineneſs; and, if it want much of its due Weight in the Air, 'tis very probable, that 'tis *Waſhed*, *&c.* rather of another Metal than *Gold*.

Any other kind of *Gold Coin*, that is near about the Weight of a *Guinea*, may be examined by our *Inſtrument* after the Manner above deliver'd. If the *Coin* be heavier than a *Guinea*, as is a twenty Shilling piece of *Broad Gold*, the *Ballaſt*, whether internal or external, of the *Inſtrument*, muſt be taken off, that ſo heavy a *Coin* may not quite ſink it. But, if it be lighter than a *Guinea*, one may add as much *Gold* (of the ſame *Alloy*) beaten into thin Plates, as with the Coin propoſed, will make up in the Air the Weight of a *Guinea*. For then this Aggregate, being examined, as if it were a *Guinea*, will diſcover in the Water, whether the *Coin* be *Right* or *Counterfeit*.

This *Inſtrument* may be alſo made to ſerve to examine ſome ſorts of *white Money*, leſs heavy than Half-Crowns. And, becauſe it may be uſeful to know in General, what Coins may, and what may not, be examined by this or that particular *Inſtrument* propoſed, I ſhall here add a general way that is not difficult for finding this out; namely, firſt by Weighing the piece of *Gold* or *Silver* in the Air, and afterwards in the Water, and Subtracting the latter from the former, to obtain the Difference of the two Weights: And next by Weighing alſo in the Air and in the Water a piece of Copper or Braſs, if this be the likelieſt to be employed in *Counterfeiting* the *Coin*, and obſerving likewiſe the Difference between thoſe Weights. For, if the leſſer of theſe Differences being ſubtracted from the greater, the Remains will ſhew, how much the true piece of Coin will out-weigh the other in the Water, and conſequently, if ſo many Grains, as this reſidue amounts to, being added to the Weight of the lighter Metal, do make a ſufficiently manifeſt Depreſſion of it below the Mark it would ſtay at without that Addition, one may probably conclude, that the Difference between a True and Counterfeit piece of Coin propoſed, will be diſcoverable by the *Inſtrument*. But it may be Expedient, for thoſe that have frequent Occaſions to examine various ſorts of Coin, to have a ſeveral *Inſtrument* adjuſted for each of them, to ſave themſelves ſome Pains and Trouble.

With this *Inſtrument*, Pure *Tin* may be certainly diſtinguiſh'd from ſuch as is adulterated. For, as *Gold*, being the Heavieſt of Metals, cannot be allay'd by any other that will not depreſs our *Inſtrument* leſs than *Gold* can do; ſo *Tin*, being the lighteſt of Metals, cannot be mixed with any other that will not ſink it lower than unmix'd *Tin*, (ſtill ſuppoſing the Weight to be the ſame in the Air.)

After the ſame manner may *Pewter* be compared and examin'd. For, having once obſerv'd how much the *Inſtrument* is depreſt by a piece of two, three, or four Drams, or even an Ounce Weight of *Pewter*, which is known to be good, and to contain ſuch a proportion of Lead in reference to the Tin, if you load the *Inſtrument* with an equally Heavy piece of any other Maſs of *Pewter* propounded, if the *Inſtrument* ſink deeper, 'twill be a ſign that the former Proportion of Lead may be very probably argued to exceed in the mixture; I ſay probably, becauſe perhaps 'tis poſſible to embaſe *Pewter* by mixing not only Lead, but other Mineral Subſtances, whoſe *Specifick Gravity* is not well known: But yet I ſay very probably, becauſe the Addition of too much Lead is the moſt Gainful way of Adulterating *Pewter*.

This *Inſtrument* may alſo aſſiſt us, to make ſuch an Eſtimate as will not much deceive us of the *Fineneſs* of *Gold* and its differing *Allays* with Silver, or ſome other determinate Metal.

In order to this, the *Inſtrument* may be fitted to ſink to the tip of the Pipe with ſome determinate Weight of the *Fineſt Gold*, as of 24 *Carats*, as they call that which is moſt *Pure* and *Fine*. But 'twill be convenient, that this Metal, in the Air, be juſt an Ounce, or half an Ounce, or ſome ſuch Determinate Weight, that is commodiouſly Diviſible into many aliquot Parts. Then you may make a Mixture, that contains a known Proportion of the Metal wherewith you *allay* the *Gold*; as if it hold 19 or 15 parts of *Gold*, and one of *Silver*; and, letting the *Inſtrument* ſettle in the Water, mark the place where the Surface of the Water cuts the *Stem* or *Pipe*. And then putting in another Mixture, wherein the Silver has a new and greater Proportion to the *Gold*; as if the former be an 18*th* or 14*th* part of the latter, you may obſerve, how much leſs than before this depreſſes the *Inſtrument*, and ſo you may proceed with as many Mixtures or Degrees of *Allays* as you think fit, or can be diſtinguiſh'd conveniently on the *Stem*; being always careful, that, whatever be the Proportion of the two Ingredients, the Weight of the Maſs in the Air be juſt the ſame with that of the *Pure Gold*, which we may have lately ſuppoſed to be an Ounce, or half an Ounce.

By the ſame Method may be examined the differing *Allays* of *Pure Silver*, upon the Admixture of ſuch and ſuch determinate Proportions of Copper, or any other Metal lighter in Specie than *Silver*; and by the ſame way, with a ſlight Variation, 'twill not be difficult to eſtimate, how much divers *Coins*, whether of *Silver* or *Gold*, are more or leſs embas'd by the known ignobler Metal that is mix'd in the piece propoſed. Theſe Eſtimates (which may be made without much Trouble) will come nearer the Truth, not only than the Eſtimates wont to be made by the *Touch-Stone*, but perhaps too, than ſome of thoſe that divers make with Trouble, Inconvenience, and Charge.

It may be alſo employ'd to examine other *Mixtures* beſides *Allay'd Coins*, and that if the *Inſtrument* be adjuſted to an Ounce, for inſtance, of Pur Copper, it may help Men to make an Eſtimate of the *Allay* of *Tin*, or th Quantity of it that is oftentimes added to *Copper*, to make different Sort of *Bell-Metal*, and of thoſe *Metalline Specula*, whether Plane or Concave that are call'd Steel Glaſſes, as alſo of Solders conſiſting of certain Proportions of Silver aud Braſs, or Copper; in all which, and divers others, th Diſcovery of the Proportion of the Ingredients may, on ſome Occaſions, b uſeful to Tradeſmen, as well as deſirable to *Virtuoſi*. And tho' I have obſerved, that by Mixture, Tin and Copper acquire a *Specific Gravity* ſom what differing from what their Ingredients promiſe; yet, ſince the *Inſtr* *ment* is to be fitted for ſuch Eſtimates, not by Calculation, but by Trials, the Eſtimates may be made near enough to the Truth.

Further conſidered; by . . . .
n. 116. *p.* 553.

2. Long ſince I took Notice, how light and Silver-like the Pewter was, which deſcended to us; but, as ſoon as, to follow the Faſhion, we changed it, the Weight and the very Colour was altered; and is in every Change more and more embaſed. And, if our *Silver-Smiths* hold on their degrading Mixtures, I ſhall queſtion, whether our Silver-Plate may not ſhortly come down to approach our Fore-Fathers Pewter: I mean, in the Country, where 'tis never or ſeldom tried.

The Weight of Water in Water; by Mr. Boyle.
n. 50. *p.* 6001.

III. A Glaſs Bubble, of about the Bigneſs of a Pullet's Egg, was purpoſely blown at the Flame of a Lamp, with a ſomewhat long Stem turn'd up at the End, that it might the more conveniently be broken off. This Bubble being very well heated to rarify the Air, and thereby drive out a good part of it, nimbly ſealed at the End, and, by the help of the Figure of the Stem, was by a convenient Weight of Lead depreſſed under Water, the Lead and Glaſs being tied by a String to one Scale of a good Balance, in whoſe other there was put ſo much Weight, as ſufficed to Counterpoiſe the Bubble, as it hung freely in the midſt of the Water. Then with a long Iron Forceps I carefully broke off the Seal'd End of the Bubble under Water, ſo as no Bubble of Air appear'd to emerge or eſcape thro' the Water, but the Liquor by the Weight of the Atmoſphere ſprung into the unrepleniſh'd part of the Glaſs-Bubble, and filled the whole Cavity about half full; and preſently, as I foretold, the Bubble ſubſided, and made the Scale it was faſtened to, preponderate ſo much, that there needed 4 Drachms and 38 Grains to reduce the Balance to an *Æquilibrium.* Then, taking out the Bubble with the Water in't, we did, by the help of the Flame of a Candle, warily applied, drive out the Water (which otherwiſe is not eaſily excluded at a very narrow Stem) into a Glaſs counterpoiſed before; and we found it, as we expected, to weigh about 4 Drachms and 30 Grains, beſides ſome little that remained in the Egg, and ſome ſmall matter that may have been rarify'd into Vapours, which added to the Piece of Glaſs that was broken off under Water and loſt there, might very well amount to 7 or 8 Grains. By which it appears not only, that Water hath ſome Weight in Water, but then it weighs very near or altogether as much in Water, as the ſelf ſame Portion of a Liquor would weigh in the Air. We repeated the Experiment with another ſeal'd Bubble as big as a great Hen-Egg, with like Succeſs.

The Preſſure of Water in great Depths; by a Perſon *of* Honour.
n. 193. p. 504.

IV. *Apr.* 7. 1680. Being off of *Pantalara* near *Sicily* in a Calm, I let down a *Bottle* 70 *Fathom*, ſtop'd with an excellent good tender Cork, well fitted, and the Cork came up in the Bottle $\frac{3}{4}$ full of Salt Water. The Bottle was again fitted with an excellent good Cork, but of a Woodineſs or Hardneſs as ſome Corks are; with the which, being let down in like manner, the Cork continued in its Place; but as it were bruiſed, and the Bottle, as before, about $\frac{3}{4}$ full of Salt Water: Whereupon I took a good Ox Bladder, and bound it four fold over the Mouth of the Bottle without any Cork at all, only I put a piece of Leather to keep the Glaſs from cutting the Bladder; and ſo ordered, it was let down as before, but taken up without any Water, or the leaſt Moiſture in it.

May 18. 1680. Being in a ſtark Calm ſome Leagues diſtant from the Coaſt of *South Spain*, off the great Hills of *Granada*, we took a Bottle and clapt a Leather on the mouth of it, tying over that a ſingle part of the Bladder, the which we let down 75 *Fathom*, but it came up again entire: We then made a Hole in the Leather, about the bigneſs of a large Pea, and let the ſame down again 75 *Fathom*, but it came up perforated in the vacant place where the Leather had the Hole in it, and almoſt full of Water: we then bound over another part of the Bladder ſingle, and let it down but 30 *Fathom*, but it came up whole and entire; whereupon immediately we let it down 50 *Fathom*, but it came up broke and full of Water. Then, we again fitted the Bottle with the ſaid perforated piece of Leather and a double Bladder, and let it down 50 *Fathom*; but it again came up entire: So again, immediately we let it down 75 *Fathom*, but then it came up broken and full of Water.

June 24. 1680. Being in 39¼ Degrees of *Latitude*, and by the Ship's Account 150 Leagues Weſtward of *Portugal*, I cauſed a *Florence Flaſk* to be well ſtopped with a Bladder over the Mouth of it, and lower'd it down 30 *Fathom*, but it was taken up broken. Whereupon, imagining that the roughneſs of the Lead's halling ſo tender a Body ſo violently thro' the Water might be the breaking thereof, I cauſed another *Flaſk* in like manner to be fitted, and cloſe by it I tied likewiſe another *Flaſk*, ſo as to be borne with the Mouth downwards, as were the other, but which was not ſtopp'd; and theſe I cauſed to be taken up when they had been but 10 *Fathom* under Water, and found them both entire; but the open *Flaſk* almoſt full of Water; the which being emptied, were both let down again and taken up at 20 *Fathom*, when the open *Flask* was entire, tho' full of Water, but the other broken to pieces.

By Dr. Oliver. n. 204. p. 908. *Jun.* 8. 1693. In the *Bay* of *Biſcay*, when we had 100 *Fathom* of Water, we took a Quart-Glaſs-Bottle ſtopt with a large Cork: And, faſtening it to our Plumbing-Rope with the Lead at the end, we ſunk it to the *Bottom* of the Sea, which as ſoon as we perceived, we drew it up again, and found the Cork quite preſſed thro' the Neck of the Bottle into its Cavity, and the Bottle full of Salt Sea-Water. We repeated our *Experiment* with another Bottle and Cork in the ſame manner as before; but, the Cork being not ſound, the Sea-Water ſoaked thro' it, and the Bottle was half full of Water; ſo the Cork remained in the Mouth of the Bottle, not preſſed down at all. We repeated our *Experiment* a third time in 90 *Fathom* of Water, with a very ſound Cork, and much larger than the Mouth of the Bottle. We beat it in with a Hammer as far as it would go, leaving about an Inch of the Cork above the Mouth of the Bottle. The Cork at this *Trial* was preſſed down only into the Neck, and became level with the Mouth of the Bottle: But I really believe, had we had 10 or 20 *Fathoms* of Water more, it would have ſucceeded as at our firſt *Trial.*

The Weight of divers Bodies try'd by the Direction of the Phil. Society *at* Oxford. n. 169. p. 926. Mar. An. 1685.

V. 1. The following Bodies were poured gently into a Veſſel of well ſeaſoned Oak, whoſe Concave was an exact *Cubick Foot.* Thoſe in the Twelve firſt Experiments were weighed in *Scales* turning with two Ounces, but the laſt Seven were weighed in *Scales* turning with one Ounce. The *Pounds* and *Ounces* here mentioned are *Averdupois.*

		℔	℥
1	A Foot of *Wheat* (worth 6*s.* a Buſhel.)	47	8
2	*Wheat* of the beſt ſort (worth 6*s.* 4*d.* a Buſhel.)	48	4
3	The ſame ſort of *Wheat* meaſured the ſecond time.	48	2
	Both ſorts were *Red Lammas Wheat* of the laſt Year.		
4	*White Oats* of the laſt Year.	29	8
	The beſt ſort of *Oats* were 2*d.* in a Buſhel better than theſe.		
5	*Blue Peaſe* (of the laſt Year,) and much Worm-eaten.	49	12
6	*White Peaſe* of the laſt Year but one.	50	8
7	*Barley* of the laſt Year: (the beſt ſort ſells for 1*s.* 6*d.* in a Quarter more than this.)	41	2
8	*Malt* of the laſt Year's *Barley,* made two Months before.	30	4
9	Field *Beans* of the laſt Year but one.	50	8
10	*Wheaten Meal* (unſifted.)	31	0
11	*Rye Meal* unſifted.	28	4
12	*Pump Water.*	62	8
13	*Bay Salt.*	54	1
14	White *Sea Salt.*	43	12
15	*Sand.*	85	4
16	*Newcaſtle Coal.*	67	12
17	*Pit Coal* from *Wedneſbury* 63, but this is very uncertain in the filling the Interſtices between the greater pieces	63	0
18	*Gravel.*	109	5
19	Wood-*Aſhes.*	58	5

2. *The Specifick Gravities of ſeveral Bodies, by the Direction of the Phil. Society at Oxford.* *Ib. p.* 927.

Pump-Water.	1000
Fir Dry.	546
Elm Dry.	600
Cedar Dry.	613
Walnut-Tree Dry.	631
Crab-Tree meanly Dry.	765
Aſh meanly Dry, and of the Out-ſide lax part of the Tree.	734
Aſh more Dry, about the Heart.	845
Maple Dry.	755
Yew of a Knot or Root 16 Years Old.	760
Beech meanly Dry.	854
Oak very Dry almoſt Worm-eaten.	753
Oak of the Out-ſide ſappy Part, Fell'd a Year ſince.	870
Oak Dry; but of a very ſound cloſe Texture.	929
The *ſame* tried another Time.	932
Logwood.	913
Claret.	993
Moil Cyder, not Clear.	1017
Sea-Water ſettled Clear.	1028
College Plain *Ale* the ſame.	1028
Urine.	1030
Milk.	1031
Box the ſame.	1031
Redwood the ſame.	1031
Sack.	1033
Beer Vinegar.	1034
Pitch.	1150
Pit-Coal of Staffordſhire.	1240
Speckled Wood of *Virginia.*	1313
Lignum Vitæ	1327
Stone Bottle.	1777
Ivory.	1826
Alabaſter.	1872
Brick.	1979
Heddington Stone, the Soft Lax kind.	2029
Burford Stone, an old Dry Piece.	2049
Paving Stone, a hard Sort from about *Blaidon.*	2460
Flint.	2542
Glaſs of a Quart Bottle.	2666
Black Italian Marble.	27
White Italian Marble tried twice.	270
White Italian Marble of another ſort, of a viſibly Cloſer Texture.	27
Block *Tin.*	7
Copper.	8
Lead.	113
Quick Silver.	1401
Quick Silver.	1359

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The laſt *Experiment* was tried with another quantity of *Quick-Silver*, which had been uſed in Water in the preceding *Experiment:* However, I rather truſt the laſt, for that I found a ſmall miſtake (tho' here in the Calculation allow'd for) in the Weight of the *Glaſs* containing the *Quick-Silver*, in the Trial before.

The *Solids* here mentioned, were examined *Hydroſtatically* by weighing them in Air and Water; but the *Fluids*, by weighing an equal Portion of each in a Glaſs holding about a Quart. The Numbers ſhew the Proportion of *Gravity* of equal Portions of theſe Bodies: But if of theſe Bodies we take Portions equally heavy, their *Magnitudes* will be *reciprocally* proportional to their correſpondent Numbers: *e. g.* a Cubic Foot of *Water* is to a Cubic Foot of *Alabaſter* in *Gravity* as 1000 to 1872; but a Pound Weight of *Water*, is to a pound Weight of *Alabaſter* in *Magnitude*, as, 1872 to 1000. So that, knowing by the former Table, the Weight of a Cubic Foot of *Water*, and by this Proportion in *Gravity* betwixt *Water* and *Alabaſter*, we may by the Rule of Three find the *Weight* of a Cubic Foot of *Alabaſter*, and ſo of any other of theſe Bodies; or we may know their *Magnitude* by knowing their *Gravity*. So that, an irregular piece or quantity of theſe Bodies being offered, 'tis but *weighing* them, and we may know their juſt *Magnitude* without farther trouble.

Pump-Water.	1000
Cork.	237
Saſſafras Wood.	482
Juniper Wood (Dry.)	556
Plum-Tree (Dry.)	663
Maſtic.	849
Santalum Citrinum.	809
Santalum Album.	1041
Santalum Rubrum.	1128
Ebony.	1177
Lignum Rhodium.	1125
Lignum Aſphaltum.	1179
Aloes.	1177
Succinum Pellucidum.	1065
Succinum Pingue.	1087
Jet.	1238
The Top part of a *Rhinoceros Horn.*	1242
The Top part of an *Ox Horn.*	1840
The (Blade) *Bone* of an *Ox.*	1656
An *Human Calculus.*	1240
Another *Calculus Humanus.*	1433
Another *Calculus.*	1664
Brimſtone, ſuch as commonly ſold.	1811
Borax.	1720

3.
By Mr. J. C.
n. 199. p. 904.
Apr. An. 1693.

A Spotted *Factitious Marble.*	1822
A *Galley-Pot.*	1928
Oyſter Shell.	2092
Murex Shell.	2590
Lapis Manati.	2270
Selenitis.	2322
Wood *Petrified* in *Lough Neagh.*	2341
Onyx Stone.	2510
Turcois-Stone.	2508
Engliſh Agat.	2512
Grammatias Lapis.	2515
A *Cornelian.*	2568
Corallachates.	2605
Talo.	2657
Coral.	2689
Hyacinth (Spurious.)	2631
Jaſper (Spurious.)	2666
A Pellucid *Pebble.*	2641
Rock Cryſtal.	2659
Cryſtallum Diſdiaclaſticum.	2704
A Red *Paſte.*	2842
Lapis Nephriticus.	2894
Lapis Amiantus from *Wales.*	2913
Lapis Lazuli.	3054
An *Hone.*	3288
Sardachates.	3598
A *Granat.*	3978
A Golden *Marcaſite.*	4589
A Blue *Slate* with ſhining Particles.	3500
A *Mineral Stone*, yielding 1 part in 160 *Metal.*	2650
The *Metal* thence extracted.	8500
The (reputed) *Silver Ore* of *Wales.*	7464
The *Metal* thence extracted.	11087
Biſmuth.	9859
Spelter.	7065
Spelter Solder.	8362
Iron of a Key.	7643
Steel.	7852
Caſt *Braſs.*	8100
Wrought *Braſs.*	82
Hammer'd *Braſs.*	834
A Falſe *Guinea.*	90
A True *Guinea.*	188
Sterling Silver.	105
A *Braſs* Half Crown.	94

Electr

Electrum a Britiſh Coin.	12071
A *Gold Coin* of *Barbary.*	17548
A *Gold Medal* from *Morocco.*	18420
A *Mentz Gold Ducat.*	18261
A *Gold Coin* of *Alexander*'s.	18893
A *Gold Medal* of Queen *Mary*'s.	19100
A *Gold Medal* of Queen *Elizabeth*'s.	19125
A *Medal* eſteem'd to be near *Fine Gold.*	19636

VI. M. *Homberg* has given us the following Table of the *Various Weights* of ſome more uſual Liquors in the *Coldeſt* Time, and in the *Hotteſt*.

The Different Weight of ſeveral Liquors in Winter *and* Summer; *by M.* Homberg, n. 262. p. 530. *Vid. Sup.* §. I. 2. Mar. An. 1700.

	In Summer.			In Winter:		
	℥	ʒ	gr.	℥	ʒ	gr.
The *Aerometer* full of *Mercury.*	11	0	6	11	0	32
Oil of *Tartar.*	1	3	8	1	3	31
Spirit of *Urine.*	1	0	32	1	0	43
Oil of *Vitriol.*	1	3	58	1	4	4
Spirit of *Nitre.*	1	1	40	1	1	70
Spirit of *Salt.*	1	0	39	1	0	47
Aqua Fortis.	1	1	38	1	1	55
Vinegar.	0	7	55	0	7	60
Spirit of Wine.	0	6	47	0	6	61
River Water.	0	7	53	0	7	57
Diſtilled Water.	0	7	50	0	7	54
This Empty *Aerometer* Weighs				0	1	28

Experiments about the Superficial Figures of Fluids, *eſpecially Liquors Contiguous to other Liquors, and their Reflective Powers*; *by Mr.* Boyle. n. 131. p. 775. Jan. An. 1676.

VII. 1. Having poured a ſtrongly *Alcalizat Menſtruum* (I uſed that made of *Fix'd Nitre*, diſſolved by the moiſture of a Cellar) into a Pipe of Glaſs, ſealed at one end, and not full a quarter of an Inch in Bore; that the Cavity, which in a greater depth would ſeem leſs deep, might be the more conſpicuous: We gently poured on it ſome highly Dephlegm'd *Spirit* of *Wine*, which we knew would not mix with it, but ſwim above it, and preſently, as we had gueſs'd, we found the *Figure* of the *Surface* of the lower Liquor changed, and the Cavity quite deſtroyed; the *Surface* that ſeemed, as it were, common to the two *Contiguous Liquors*, appearing flat or Horizontal. And ſuch a level Superficies we had, by putting theſe two *Liquors* together in a much wider Glaſs.

2. We found alſo, that by Employing *Oyl* of *Turpentine*, inſtead of the *Spirit* of *Wine*, the Liquor did almoſt totally loſe its Cavity.

3. But, if, inſtead of *deliquated Tartar*, we put common Water into the Pipe, we found this Liquor to retain its Concave Surface, tho' we put to it ſome *Oil* of *Turpentine*, and left it to reſt upon the Water a good while.

4. Having provided ſome pure *Oil* of the *Gum* of *Guaiacum*, and poured a little of it into a ſlender Pipe, we found the upper Superficies of it to be Concave almoſt, if not altogether, like that which Water would have had in the ſame Pipe. But when I put a little Water upon this *Oil*, it preſently changed the Figure of its Surface, which became viſibly, tho' not very much, Protuberant or Convex.

5. Having put ſome *Oil* of *Tartar* into the ſlender Pipe, and put ſome Drops of the *Oil* of *Guaiacum* to it, we found, that this Liquor did not manifeſtly alter the Concave Figure of the Surface of the Liquor *Alcali*, as the *Oil* of *Turpentine* had done: And having for Curioſity's ſake warily poured a little Water upon the *Oil* of *Guaiacum*, I found, as I had reaſon to ſuſpect, that the upper Superficies of it changed preſently from a Concave Figure to a Convex; ſo that this *Oil* in the midſt of the other two Liquors appeared like a little red Cylinder; which, inſtead of having Circular Baſes, was protuberant at both ends; but more at that which touched the *Oil* of *Tartar*.

6. I put ſome *Eſſential Oil*, (as *Chymiſts* call it) of *Cloves* into a new ſlender Pipe, and, having obſerved it to be ſomewhat Concave at the Top, where it was Contiguous to the Air, we cauſed a little common *Water* (perhaps a quarter of a Spoonful or leſs) to be put to it, and found, as we expected, the Surface of this *Oil* alſo to be tumid. And in regard this Liquor, as well as the forementioned *Oil* of *Guaiacum*, tho' it were ſo heavy as to ſink into Water, would not do ſo in *Deliquated Salt* of *Tartar*, we did, into another ſlender Pipe, put firſt ſome of this laſt named Liquor, then ſome of the *Aromatic Oil*, and laſtly, a little common Water; by which means we found, that the little *Cylinder* of *Oil*, did, like that of the *Oil* of *Guaiacum*, appear Convex at both ends, but was unlike it in one Circumſtance, that the *Oil* of *Cloves* appear'd more Convex at the upper end where 'twas Contiguous to the *Water*, than at the lower, that leaned upon the Surface of the *Oil* of *Tartar*.

7. Having taken a little ſlender Glaſs, that was much longer, but of the like Bore with the former, we put into it a ſmall quantity of *Quick-Silver*; and having taken notice how the upper Superficies ſwelled in the middle above the Level of the Parts where it touched the Glaſs, we poured ſome Water upon it, and found a manifeſt and conſiderable Depreſſion of the Surface, tho' the Protuberance were not quite ſuppreſſed.

8. This *Phænomenon*, having been for greater ſecurity ſeveral times rep ed, ſometimes it ſeem'd, that when the *Aqueous Cylinder* was much long , the Depreſſion of the *Mercurial* Surface was ſomewhat greater. But this d not ſo conſtantly happen; but we often obſerved, that tho' a very li e Water ſufficed by its Contact to make, in the Judgment of the Eye, a ma i-feſt Abatement of the Protuberance of the *Quik-Silver*; yet it had not t e ſame effect on that ponderous Fluid, that it had, when being increaſed alm ſt as high as the length of the Pipe would permit, a greater Weight of it w s

incumbent on the *Mercury*, for then I manifeſtly perceived, and ſhewed to others, that the Surface of the *Quick-Silver* being depreſſed almoſt to a Level in thoſe Parts of it that were near the inſide of the Glaſs, there was about the Middle of the Surface an Elevation of *Mercurial Matter*, that appeared to be rather more than a half Globe, and was, to the Height of its full Semidiameter, raiſed above the reſt of the *Mercurial* Surface, and in that State it continued, as long as I thought fit to let it do ſo. And, leſt this Trial ſhould impoſe upon me, I cauſed it to be more than once repeated; and, the better to confirm it, I afterwards cauſed the incumbent *Water* to be little by little ſuck'd up, and found as I expected, that, when the incumbent *Water* began to be too much ſhorten'd, the little Teat or Segment of a Sphere, lately mentioned, began to be ſomewhat flatned, and ſubſided more and more as the *Water* was further taken off.

9. Having conveyed into one of our *Pneumatical Receivers*, a Couple of ſuch ſlender Pipes as have been already deſcribed, one of them furniſh'd with *Common Water*, and the other with *Quick-Silver*, we cauſed the Common Air to be diligently pump'd out, without obſerving any ſenſible Change in the Concave Figure of the *Water*: but as for the *Quick-Silver*, I knew not what to conclude about it. For, having repeated the Trial twice or thrice, the *Mercury* ſometimes ſeem'd manifeſtly to ſwell, to be more protuberant upon the Exhauſtion of the *Receiver*, than when it was put in, eſpecially when its Figure was attentively viewed, and the external Air, that was pump'd out but ſlowly, was ſuffer'd to re-enter with all convenient Celerity. But that which yet kept me doubtful was, that I obſerved, That, upon the diligent withdrawing of the Air's Preſſure upon the *Quick-Silver*, there diſcloſed themſelves ſome little Bubbles, which, I fear'd, we had not been able to free it altogether from, and which might be ſuſpected to have ſome Intereſt in the *Phænomenon*. We alſo conveyed into our *Receiver*, a clear *Chymical Oil* that was heavier than *Water*, and, whilſt it was contiguous to it, had not a Concave but a Convex Surface, and, having placed the Pipe furniſh'd with both Liquors in the *Pneumatical Receiver*, we pump'd out the Air, without finding that the *Oil* ſenſibly altered its protuberant Surface, as neither did the Water loſe the concave Figure of its Surface.

10. I took *Fix'd Nitre*, (or which is Analogous to it, *Salt* of *Tartar*) reſolved *per Deliquium* into a Tranſparent Liquor, and having filled a clear Viol half full with this, I poured on it a convenient Quantity of *Vinous Spirit* exactly *Rectified*, that there might be no Phlegm to occaſion an Union betwixt the two Liquors, which ought, as ours did, to retain Diſtinct Surfaces, and ſpeedily regain them, though the Glaſs were well ſhaken. Then, having found by a Trial formerly mentioned, that Common *Oil* of *Turpentine*, if employ'd in a competent Quantity, will not totally (and much leſs will readily) diſſolve in *Spirit* of *Wine*, and alſo having obſerved (what may ſeem ſomewhat ſtrange) that if this *Spirit* of *Wine* be exquiſitely dephlegm'd, the *Oil*, tho' a *Chymical* One, will not ſwim on it, but ſink in it; I warily let fall ſome drops of *Oil* into the *Spirit*, and had the pleaſure to ſee, as I expected, that they fell towards the bottom of the Glaſs, till their Deſcent was ſtop'd by

the Horizontal (for it was not Concave) Surface of the *Alcalizat Liquor* of *fix'd Nitre*. And, becauſe my deſign was chiefly to obſerve the ſuperficial Figure of a Fluid encompaſſed by other Fluids without touching any ſolid Body, I ſhall here take notice of the chief *Phænomena* that were produced of that kind, without ſtaying to enquire into the Cauſes or the Conſequences of them.

1. If the *Oily Drops* were but ſmall, they ſeem'd to the Eye exactly enough *Spherical*. For, the *Oil* differing but very little in the *ſpecifick Gravity* from the *Spirit* of *Wine*, the Drops did but juſt touch the Surface of the ſubjacent *Alcali*; and, the ſame Drops being but ſmall, their own Weight was not great enough viſibly to depreſs them, and hinder that Roundneſs which the Preſſure of the Ambient *Spirit*, or their own Viſcoſity, endeavoured to give them.

2. If an Aggregate of Drops were conſiderably bigger than thoſe newly mentioned, as if it had about a third part of an Inch in Diameter, it would then manifeſtly lean upon the *Alcalizat Liquor* as upon a Floor, and appear ſomewhat *Elliptical*, (for ſome little part of the bottom was a Plane;) the Weight of the upper parts depreſſing the Drops, and making the horizontal Diameter ſomewhat longer than the Tranſverſe.

3. If a yet greater Portion of *Oil* were let fall upon the heavy Liquor, it would for a pretty while appear in the Form of a ſomewhat imperfect *Hemiſphere*, or ſome other large Section of a *Sphere*, the lower part being cut off; (as if a Globe were divided by a Plane) by the horizontal Surface of the *deliquated Salt*.

4. But if the Quantity of *Oil* were not too great, 'twas pretty to obſerve, that, tho' at firſt putting in, it did perhaps ſpread itſelf over the ſubjacent Liquor, and lie as it were flat upon it; yet, by little and little, (for 'twas but ſlowly) it would by the Action of the ambient, concurring with it's own Tenacity, be raiſed above the Surface of the Fluid *Nitre*, and be reduced to the Figure, either of half a Globe, or of a greater Segment of a Globe, or even of an Imperfect Ellipſis, according to the Bulk or Weight of the *Oil*.

5. Tho' theſe Globules, or Portions of *Oil*, did oftentimes readily mingle, when they touched one another, yet divers times alſo we obſerved, that, having warily approach'd them, we were able to make them touch without mingling, inſomuch that we have with pleaſure made them ſo far bear againſt one another's Surfaces, as manifeſtly to preſs them inwards, tho' being parted they would preſently reſume their former Figure. But, in caſe any of theſe *Oily Portions* came by a more preſſing Contact to be united, they would then alter the Figures they had whilſt ſeparate, and take another ſuitable to the Bulk of the Aggregate.

9. When a large Portion of *Oil* reſted upon the *Saline Liquors*, if then the ambient *Spirit* was moderately and warily agitated, 'twas not unpleaſant to obſerve the various Figurations, which the Convex and Protuberant part of the mutilated Globe would be put into by the Shakes, without any viſible Solution of Continuity, or conſiderable Motion of the whole Body, which would very quickly recover its former Figure. Tho', if the Agitation were too

ſtrong, ſome Portions would be quite broken off, and preſently turned into little Globes.

11. I tried to produce another *Phænomenon*, that would not have been unpleaſant, by putting together in a ſomewhat large Veſſel, with other Liquors, two *Oils*, (whereof one, if I miſtake not, was from *Turpentine*) which firſt by reaſon of the Oleaginous Nature wherein they agreed, might exactly mingle and make a compounded Liquor ; and then, by reaſon of their being one heavier, and the other lighter in *Specie* than Water, might by this Liquor be again ſeparated, and include betwixt them the Liquor that had divided them. But I found that the *Oils* being once united would not be eaſily parted, but according to the Prevalency of the lighter or heavier Ingredient, in the mixture, the compounded *Oil*, would almoſt totally either emerge to the top of the Water, or lie beneath at the bottom of it ; I ſay, almoſt totally, becauſe ſome Part of the *Oil*, which was not perhaps all uniformly mix'd, did not keep in a Body with the reſt ; but either was ſeparated from the Maſs in the form of Globules, or elſe ſticking to the Side of the Glaſs, had the other Part of its Superficies, which was contiguous to the Water, very *Variouſly Figured*, according as the bulk and degree of Gravity of the adhering *Oil*, and other Circumſtances happen'd to determine.

Theſe are ſome of the *Phænomena* I obſerved in *Oil* of *Turpentine*, when 'twas invironed only with Fluids ; but, if it were permitted to be contiguous to the Inſide of the Glaſs, and ſo to faſten part of its Surface to a Solid, the greater part of the Surface, which remained expoſed to one or both of the contiguous Liquors, would partly by their Action, and partly by the Gravity of the *Oil* itſelf, be put into *Figures* ſo *Various* and ſometimes ſo *Extravagant*, that 'twas much more pleaſant to behold them, than it would be eaſy to deſcribe them.

12. *Confining Fluids* may have diſtinct Surfaces, without having, at leaſt in many Poſitions, *Refractions* differing enough, or *Reflections* ſtrong enough, to make the Plane that diſterminates them, obvious to the Eye. Thus, when the *Oil* of *Tartar*, or *Nitrous Alcali*, that I employed, happen'd to be very clear and colourleſs, I have more than once made highly *Rectified Spirit* of *Wine* float upon it ſo, that in moſt Poſitions the Vial ſeem'd to have in it but one uniform Liquor ; the Plane that divided the two Fluids being unapt to be diſcerned, but in a Poſition, wherein the Rays of Light paſſing thence to the Eye, fell very obliquely on it ; and indeed, when there was no little Duſt or other Feculency, ſwimming upon the Surface of the *Oil* of *Tartar*, I had ſometimes much ado to convince ordinary Spectators, that the Vial, in two diſtinct Regions of it, contained two *Unſociable Liquors*.

13. We took a *Deliquated Alcali*, made of *Nitre* and *Tartar*, and deeply tinged with *Cochineal* ; and, that the Liquors might not only be heterogeneous, but as differing in *Gravity* and *Denſity*, as we could make them, we poured on it a peculiar kind of *Oil* lighter than *Spirit* of *Wine*, and holding the Plane where the two Liquors were contiguous in a convenient Poſition, in reſpect of the Light and the Eye, I obſerved it to make a ſtrangely vivid

Reflection of the incident Beams of Light: So that this Phyſical Surface which was flat, look'd almoſt, for 'twas not ſo *Specular*, like that of *Quickſilver*) and, when I kept it till Night, and conſidered it by the Light of a Candle, the bright Figure of the Flame was ſtrongly reflected almoſt as from a cloſe ſpecular Body; which tempted me to ſuſpect, that there might be ſomething elſe than the bare Smoothneſs of the Surface of the *Alcalizat Liquor* to produce ſo briſk a *Reflection*; and the rather, becauſe I did not obſerve, that the Remains of the ſame Tinged *Alcali*, which I kept in another Glaſs, nor a Portion of the ſame *Oil*, which I had alſo by me in a ſeparate Vial, did either of them afford ſo vivid a *Reflection* from its Surface; tho' I did the leſs wonder at this, becauſe of the great Diſpoſition to *reflect Light*, which I had formerly the Curioſity to obſerve in the foremention'd *Oil*, when I joined it with other Liquors. I ſhall add, that looking on this Liquor, as a Body which tho' it have all the neceſſary Qualities of an *Oil*, does in regard of its Origin, and ſome Properties I have found in it, differ from common *Chymical Oils*; I was invited the more to obſerve its *Phænomena* in reference to *Reflection*; and I found among other Things (not pertinent to this Place,) *Firſt* that the confining Plane often mentioned between the tinged *Alcali*, and this *Liquor*, did not appear Red itſelf, nor communicate that Colour to the Image of the Flame of a Candle reflected from it. *Secondly*, That when I warily ſhook the Vial, which contained the two Liquors, the uppermoſt would be reduced into a ſeeming Froth, conſiſting of a great number of imperfectly Globular Bodies, which after a while would make a kind of a rude Phyſical Plane; which, tho' neither very Horizontal, nor ſenſibly Smooth, would at its upper Superficies, ſend back the *incident Light* with more Briſkneſs than one would expect; and, when the ſeeming Froth conſiſted of ſmaller Particles, theſe, when they were of a certain Size, and conveniently placed, in reference to the Flame of a Candle and the Eye, would (as more than one Trial informed me) reflect the *incident Light* ſo many ways, and ſo viſibly, that they ſeemed, for Multitude and Splendor, like little ſparkling Corpuſcles of poliſhed *Silver*; or almoſt like thoſe Gliſtering ones, that appear when a clean Plate of *Copper* is firſt immerſed into a much allayed Solution of good *Silver*, made in *Aqua Fortis*. *Thirdly*, That tho' pure *Spirit* of *Wine* be ſo thin a Liquor, and our *Oil* is nevertheleſs ſo light as to ſwim upon it, yet I found the confining Surface very ſtrongly *Reflexive*.

I have alſo found, that ſome other *Eſſential Oils* (as *Chymiſts* call thoſe that are diſtilled with Water in *Limbecks*) and particularly an Unſophiſticated *Oil* of *Lemons*, did with our *tinged Alcali* afford moſt of the ſame Phænomena; but not ſo briſk a *Reflection*: I ſay moſt, chiefly becauſe with *Spirit* of *Wine*, their *Subtile Oils*, as I formerly noted, will readily be confounded, tho' our Anomalous *Oil* be unſociable with it.

N. 132. p. 799. 14. In Cold Weather we took *Eſſential Oil* of *Anniſeeds*, whoſe Property it is to Coagulate in ſuch Weather, and, having in a gentle Warmth brought it to be fluid, we poured it into a ſlender Vial more than half filled with *Com-*

mon Water; that had been alſo a little warmed, that the *Oil* might not be too haſtily reduced to its former State. This Oil being lighter than ſo much *Water*, and being poured on in a convenient Quantity, had its upper Surface ſomewhat Concave, as that of the Water was; but the lower Surface, ſurrounded by the Water was very Convex, appearing almoſt (for it was not perfectly) of the Figure of a great Portion of a *Sphere*. This being done, the Vial was ſtopt, and ſuffered to reſt for ſome time in a cold Place, by which means the *Water* continuing Fluid as before, the *Oil* of *Anniſeeds* was, as I expected, found *coagulated*, in a Form approaching to that it had whilſt in a Fluid State; I ſay approaching, becauſe it was not eaſy to diſcern the exact Figure in the Vial I was fain to make uſe of: And I ſuſpected that the *Oil* grown conſiſtent was become leſs Convex than before. But 'twas worth obſerving, how great a Difference there was between the dull *Reflection* it made when it was *coagulated*, and the fine *Reflection* it had made whilſt 'twas a Liquor. The latter of which *Reflections* brought into my mind, how vivid the *reflective Power* of ſome *Fluids* is in compariſon of that of the Generality of *Solid Bodies*.

15. Having obſerved, That *Quick-Silver*, and rectified *Oleum Petræ*, are, the Former of them the Heavieſt, and the Latter the Lighteſt, of all the Viſible Fluids that are yet known to me; I put ſome (Diſtill'd) *Quick-Silver* into a ſmall Vial, and held it in ſuch a poſture, that the *incident Light* was ſtrongly remitted to my Eye: I then ſlowly put to it ſome *Petroleum*, that being well rectified was very clear, and obſerved, that, as this Liquor covered the *Quick-Silver*, there was at the imaginary Plane, where they both confined, a briſker *Reflection* than the *Quick-Silver* alone had given before. On this occaſion it will not be amiſs to take notice, that either the Surface of the Air itſelf, as thin and yielding a Fluid as it is, or the Surface of a Solid contiguous to included Air, or ſome interpoſed *Subtile Matter*, may *reflect the incident* Beams of *Light* more ſtrongly than moſt Men would expect. To this purpoſe, I remember, that a curious Perſon having one Day brought me a couple of Rarities, which he told me were two pieces of a Solid, but tranſparent Body, that he had caſually found; in one of which there was a *Pearl*, Large, Round, and Orient, and in the other a leſs perfect One: One of them was opened, and that which had appeared a *Pearl* was found to be but a Cavity, that contained no groſſer Subſtance than Air. And I have by me a well ſhap'd piece of Glaſs of a good Thickneſs, with an Aereal Bubble in the middle, which by ſome Qualities, particularly its Pearl-like Shape and vivid *Reflection*, does not ill reſemble a fair, tho' not *orient Pearl*. But in ſuchlike Obſervations, the Poſition of the Eye, and that wherein the Body receives the Beams of Light, may be very conſiderable. For I have by me a Small Stone (with which I have puzzled a ſkillful Jeweller to determine what kind of *Gem* it was) that being laid flat upon ones Hand, or a piece of Paper, and look'd on directly downwards, looks almoſt like a piece of common Glaſs, and is tranſparent: But, if the Eye be ſo placed, that the *incident* Beams of *Light*, by whoſe *Reflection* it's ſeen, fall with a conveni-

ent Degree of Obliquity upon the Stone, it makes an exceeding pretty ſhew, ſometimes appearing like a fine *Opal*, and ſometimes not very unlike an *orient Pearl*.

16. We made a competent Quantity of a *Reſinous* or *Gummous Subſtance*, that looked like high-coloured *Amber*, but was eaſy to melt. This we put into a deep round Glaſs with a wide Mouth, and held it by the Fire-ſide in a moderate Warmth, till it was brought into a fluid State; then we tranſferr'd it into one of our *Pneumatical Receivers*, where we preſumed, that this temporary Liquor, would, as well as Liquors that are conſtantly ſuch, diſcloſe *Aereal Bubbles*, when the Preſſure of the Air was withdrawn from it; and accordingly having cauſed the Air to be pumped out by degrees, we found, that ſtore of Bubbles appeared at the Top of the Liquor, and made there a copious Froth, many of them being by reaſon of the Viſcoſity of the Fluid, very large, and divers of them, becauſe of the Nature and Texture of it, and the thinneſs of the Films, being adorned with the Colours of the Rainbow, whoſe Vividneſs made them pleaſant to behold, and ſuggeſted to us ſome *optical Conſiderations*. But, notwithſtanding this Froth, I cauſed the Pumping to be continued, that thoſe Bubbles that had moſt of common Air in them, and which therefore are wont to riſe firſt, might get to the Top, and the ſubſequent Bubbles might meet with more Reſiſtance from the Liquor ſtill tending to grow cold, and ſo might be the more expanded, and yet kept from emerging by the Concretion of the reſinous Subſtance; and anſwerable to this we found, That, when this Subſtance had reſumed its conſiſtent Form, there were intercepted, between the upper and the lower Surfaces of it, ſome Bubbles that were not ſmall, which yet had a conſiderable *Reflection*, notwithſtanding the ſmall Quantity of the groſſer Particles of the Air, that may be ſuppoſed to be contained in Bubbles ſo very much expanded.

17. 'Tis taken for granted, That the *falling Drops* of *Rain* are *Spherical*, yet their Deſcent is ſo ſwift, that I fear 'tis rather *ſuppoſed* than *obſerved* that their Figure is *Spherical*; which will be the more queſtionable, if it be true, which is vulgarly thought, That *Hail* is but *Rain Frozen* in its Paſſage thro' the *Air*. For 'tis evident, That the Grains of *Hail* are frequently of other Figures than truly Orbicular. But the *Surface* of *Water* may have differing Figures, according as 'tis totally encompaſs'd with heterogeneous Fluids, or as 'tis only in ſome places contiguous to one or more of them. In the former caſe we found it not ſo eaſy to make an Obſervation, becauſe, we know not of any two Liquors (ſetting *Mercury* aſide) that will not mingle either with one another, or with Water. We therefore cautiouſly convey'd into ſome *chymical Oil* of *Cloves* ſome Portions of *common Water* of differing Bigneſſes, taking care, as far as we could, that they might not touch one another; by which means the *Oil* being tranſparent, and yet ſomewhat coloured, 'twas eaſy to obſerve, that the ſmaller Portions of *Water* were ſo near totally invironed with the *Oil*, that they were reduced into almoſt perfect *Globes*; Thoſe Portions, that were ſomewhat bigger (as about twice the Bigneſs of a

Pea,) would be of a Figure ſomewhat approaching to that of an *Ellipſis* (for 'twas not the ſame) and thoſe Portions that were yet ſomewhat larger, tho' they ſeemed to be ſunk almoſt totally beneath the *Oil*; yet, they held to it by a ſmall Portion of themſelves, whoſe Surface was eaſily enough diſtinguiſhable from that of the *Oil*. Theſe larger Portions of *immerſed Water* being almoſt wholly inviron'd with the other Liquor, were by it reduced into a round Figure, which was ordinarily ſomewhat *Elliptical*, but more depreſſed in the Middle than that Figure requires.

18. Having into a ſlender Pipe, of that ſort that has been deſcribed before, put a little *Oil* of *Cloves*, and upon this ſome *Oil* of *Turpentine*, that ſo the *Water* might both above and beneath be touched by Heterogeneous Liquors, I obſerved not the *Oil* of *Cloves* to be very manifeſtly Tumid at the top, nor the lower Surface of the *Oil* of *Turpentine* (for the upper was Concave) to be very Convex: For ſomewhat Convex it was downwards. And, from this 'twill be eaſy to conclude the Figure of the Cylindrical Portion of *Water* intercepted between theſe two Oils.

19. I took *Oil* of *Anniſeeds*, thaw'd by a gentle Warmth, and *Common Water*, and, having put them together in a convenient ſhap'd Glaſs, they were ſuffered to ſtand in a Cold Place, till the *Oil* was coagulated; which done, it was parted from the *Water*, and by the Roughneſs of its Superficies manifeſted, as I expected, that, when its Parts were no longer agitated and kept eaſily Diſplaceable, by the ſubtile permeating Matter, or whatever other Agent or cauſe it were to which it owed its *Fluidity*, when the contiguous *Water* grew unable to inflect, or otherwiſe place them after the manner requiſite to conſtitute a *ſmooth Surface*. And, what happen'd to that part of the *Oil*'s Surface that was touched by the *Water*, happen'd alſo to that which was contiguous to the *Air*, ſave that the *Aſperity* of the laſt named Surface was differing from the other, which, whether it were an accidental or conſtant *Phænomenon*, farther Trial muſt determine. But I have often obſerv'd, that the upper Surface of *Oil* of *Anniſeeds*, when this Liquor comes to be Coagulated by the cold Air, was far enough from being *Smooth*, being variouſly aſperated by many flaky Particles, ſome of which lay with their broad, and others with their edged Parts upwards.

20. An Inequality and Ruggedneſs of Superficies I have alſo obſerved in *Water*, when, having covered it with *chymical Oil* of *Juniper*, and expoſed it in very cold Weather, tho' the *Oil* continued *Fluid*, yet the *Water* being Frozen had no longer a *ſmooth Superficies*, as whilſt in its Liquid State 'twas contiguous to the *Oil*. And the like Inequality, and rather a greater, we obſerved in the Surface of *Water* Frozen, which had *chymical Oil* of *Turpentine* ſwimming over it, yet a no leſs, if not a much greater, Roughneſs may be oftentimes obſerved in the Surfaces of divers Liquors that abound with *Water*, when, thoſe Liquors being Frozen, their Surfaces have an immediate Contact with the Air. I ſhall here add, that, having purpoſely cauſed a Strong and Blood red Decoction of the Soot of Wood to be expoſed in a large Glaſs in a very cold Night, I was more pleaſed than ſurprized, to find

in the Morning a Cake of Ice, that was curiouſly Figured, being full of large flakes ſhap'd almoſt like the broad Blades of Daggers, but neatly fringed at the Edges. But that which I chiefly mention theſe Figures for, is, that they ſeem to be as it were Imboſt, being both to the Eye and the Touch raiſed above the horizontal Plane or Level of the other Ice.

21. I have ſometimes obſerved the like *Phænomenon* in one and the ſame Liquor, and particularly not long ſince in froſty Weather, on a Vial where I had long kept *Oil* of *Vitriol*, I perceived that the Cold had reduced far the greateſt part of the *Menſtruum* into a conſiſtent Maſs, whoſe upper Surface was very rugged and oddly figur'd, tho' it lay covered all over with a pretty deal of high-coloured Liquor, that was not frozen or coagulated, nor ſeem'd to be diſpoſed to be ſo, at leaſt, in that degree of Cold.

22. This may be alſo obſerv'd in the beſt ſort of what the *Chymiſts* call *Regulus Martis Stellatus*, where the Figure of a Star, or a Figure ſomewhat like that of the Decoction of the Soot lately mentioned, will frequently appear imboſt upon the upper Superficies of the *Regulus*; and ſuch a raiſed Figure I have ſeen on a Maſs of *Regulus* made of *Antimony* without *Mars*. But if to thoſe two Bodies *Copper* be alſo ſkilfully added, the Superficies will be oftentimes adorned with new Figures according to the Circumſtances; tho' the moſt uſual I took notice of was that of a Net, that ſeemed to cover the Surface of the compounded *Regulus*. But this is not ſo conſtant, but that I have by me a Maſs of the Conical Figure conſiſting of two very contiguous, but eaſily ſeparable Parts, whereof the lowermoſt, which abounds more in *Metal*, hath its upper Surface covered with round Protuberances, in ſhape and bigneſs not unlike to ſmall Peaſe cut in two: and theſe are ſo really imboſt and elevated above the reſt of the Superficies, that the other part of the Cone which is of a more ſcorious Nature, as in its lower Surface, which exactly fits the upper of the *Regulus*, Cavities, for number, Shape and Bigneſs, anſwering to the Protuberances lately mentioned; which argues that the *Regulus* cooled firſt with that Inequality of Surface we have deſcribed, and that the Lighter and more Recrementitious Subſtance, continuing longer Fluid, had thereby opportunity to accommodate itſelf to the Superficial Figure of the *Regulus*, on which it firſt leaned, and afterwards coagulated.

Why Bodies Diſſolved Swim in Menſtrua Specifically Lighter than themſelves; by Mr. Will. Molyneux. n. 181. p. 88. *May* An. 1686.

VIII. 1. My Brother, Mr. *Tho. Molyneux* (in the *Nouvelles de la Republique des Lettres*) has given this Reaſon for the *Phænomenon*, *viz.* That the *Internal Motion* of the Parts of the Liquor does keep up the Particles of the *diſſolved Solid*, for they, being ſo very minute, are moveable by the leaſt Force imaginable, and the Action of the *Particles* of the *Menſtruum* is ſufficient to drive the Atomes of the *Diſſolved Solid Body* from place to place; and conſequently, notwithſtanding their *Gravity*, they do not *Sink in the Liquor Lighter than themſelves*.

But I conceive another Account may be given of this *Appearance*, and that the Prime Law of *Hydroſtaticks* is a little deficient. 'Tis true indeed, if we conſider only the *Specifick Gravity* of a Liquor, and the *Specifick Gravity* of a

Solid Particle Floating therein, the forementioned Rule is exact; but in *Sinking* there is requiſite a *Separation* of the Parts of the Liquor by the *Sinking* Body; and there being a Natural Inclination in the Parts of all Liquors to *Union*, ariſing from an Agreement or *Congruity* of their Parts, there is a Reſiſtance therein to any thing that ſeparates this Conjunction: Now, unleſs a Body have Weight enough to overcome this *Congruity* or Union of Parts, ſuch a Body will float in a Liquor *Specifically Lighter* than itſelf. But that a Heavy Body, as *Mercury* or *Iron*, may have its Parts reduced to that Minutneſs that their *Gravity* or Tendency downwards, is not ſtrong enough to Separate the *Coheſion* or Union of Parts of a Liquor, will be manifeſt, if we conſider, that the Reſiſtance made by the Medium to a *Falling* Body, is according to the Superficies of the Body: But, as the Body decreaſes in Bulk, its Superficies does not proportionably Decreaſe: thus a Sphere of an Inch Diameter has not *eight* times leſs Superficies than a Sphere of two Inches Diameter, tho' it have *eight* times leſs bulk, and conſequently paſſing thro' a Medium, as ſuppoſe Air or Water, the Sphere of an Inch Diameter, is proportionably to its Bulk, more *Reſiſted*, than a Sphere of two Inches Diameter in proportion to its Bulk; and hence it will come to paſs, that at laſt a Body may be reduced to that Minuteneſs, that its Gravity *Preſſing* downwards (which is according to its Bulk) may be leſs than the *Reſiſtance* of the Medium, which operates on the Surface of the Body; ſeeing, as I ſaid before, the Surfaces of Bodies do not decreaſe ſo faſt as their Bulks, theſe decreaſing in a Triplicate, but thoſe in a Duplicate *Ratio* of the Bodies Diameters.

But, becauſe I have ſaid that the forementioned Law of *Hydroſtaticks* is a little defective, I deſire to explain myſelf a little further in that point. In Weights *Falling* thro' the Air, were Gravity only conſidered, the Proportions of their Deſcents would be exactly as *Galileo* has demonſtrated; but it is allowed by all, that the *Reſiſtance* of the Air, not being conſidered in thoſe Demonſtrations, they are not *Mathematically* True in Practice, but that really there is ſomething of that Proportion hindred by the Air's *Reſiſtance*. Now, what is this leſs than to ſay, that the *Reſiſtance* of the Air takes off ſome of the Operation of Gravity, or is able to withſtand or oppoſe part of its Action? And if ſo, what ſhall we ſay, were an *Iron* Sphere let thro' a Medium of Water? Surely the Proportions of its Deſcents, would be much more diſturbed herein, as Water is much more ſolid and difficult to be ſeparated or paſſed thro' than Air, and conſequently we muſt needs grant, that more of the Operation of Gravity is taken off or reſiſted by this Oppoſition of the Water, than that of the Air. And if ſo, ſurely there may be a certain Degree of Gravity, that may be quite taken off by the *Reſiſtance* of the Water. Were a Piſtol Bullet let fall thro' the Air, it would deſcend imperceptibly nigh the Proportions that *Galileo* has aſſigned; but were a ſingle Grain of Sand ſo let fall, it would be much hindered in its Courſe, and half of this Grain would be more obſtructed; what ſhall we then ſay of the Ten thouſandth part, or of a part of the Ten thouſand Millionth of this, and again of the infinite Subdiviſions of that, till at laſt we come to a Part that would be wholly *reſiſted*; or kept up; ſuch I conceive the minute Particles of a Body *diſſolved* in a *Menſtruum*.

On this Account, 'tis, I ſay, that the forementioned Principle of *Hydroſtaticks* is a little defective; for it conſiders not the natural *Congruity* of the Parts of a Liquor, whereby they deſire as 'twere to unite and keep together, juſt as we ſee two drops of Water on a dry Board being brought together, do jump and coaleſce, and therefore Liquors have an innate Power of *reſiſting* a certain degree of Force that would *ſeparate* them; ſuch as I ſuppoſe the degree of Gravity, in the moſt minute Particles of a Body *diſſolved* in a *Menſtruum*.

The forementioned Rule holds true to the moſt nice Senſe in great Bodies; but in thoſe that are by many Millions of Diviſions ſmaller, it ſeems to fail.

I would not however be thought wholly to reject my *Brother's Solution* of this *Problem*: for certainly that *Motion* (whatſoever it is) in a *Menſtruum*, which is able to diſſolve ſuch a ſolid Body as Iron, that is, which is able to diſturb the cloſe and ſtrong Coheſion of the Parts of Iron, may very well be ſuppoſed ſufficient to diſturb or keep up theſe Parts from reſting in the Bottom of the Veſſel, wherein the *Solution* was made; and certainly no better Account can poſſibly be given of ſuch *Solutions*, than by ſuppoſing ſuch an *internal Motion* in the parts of the *Menſtruum* inſinuating themſelves into the ſolid Body, and looſening its parts. But I leave to others to conſider what kind of *Motion* and peculiar Conformation of Parts is requiſite both in the *Menſtruum* and in the diſſolved Body, that a Solution may reſult from their Commixture.

Conſidered; by
Mr. Tho. Molyneux.
Ibid. p. 93.

2. Tho' Liquors conſiſt of Parts united, and tho' this Union be eaſily deſtroyed, yet of neceſſity it requires ſome degree of Force for effecting it; yet this *Property* ought not to be rely'd on as the ſole Cauſe of this *Appearance*: For, in this *Solution* of the *Problem*, We firſt ſuppoſe the minute Particles of a heavy Body rais'd, and then give the Reaſon of their not ſinking; whereas 'tis not to be queſtioned, but that that Force which raiſed them, is the ſame that keeps them from falling to the bottom.

An Undertaking
for Raiſing of
Water; by Sir
Sam. Moreland.
n. 102. p. 25.
Apr. An. 1674.

IX. Sir *Sam. Moreland* undertakes to demonſtrate, (contrary to the common and received Opinion thro' *England* and all *Europe*)

1. That he will force Water 60 Foot high with treble the Weight that ſhall raiſe it 20 Foot; and ſo proportionably, *in infinitum*.
2. That by how much wider the Barrel is, in which the Forcer works, than the Pipe thro' which the Water is forced up, by ſo much is the *Engine* preſſed with unneceſſary Weight.

A Siphon *per-*
forming the ſame
things with the
Sipho Wurtem-
burgicus; *by Mr.*
J. Davis.
n. 167. p. 846.
Nov. An. 1684.

X. 1. The laſt Summer, *Ann.* 1684. a Treatiſe fell into my Hands, called *Sipho Wurtemburgicus*, or an inverted Siphon with Legs of the ſame Height, running and running back again, ſuch as had never been heard of before. The Author ſpeaks wonderful Things of this Machine, but begs the Reader's Pardon for his moſt Serene Patron; who had a mind to preſerve the Structure of it for himſelf. When I read this I conſider'd how a Siphon might be conſtructed, which ſhould perform the ſame Things as are ſaid of that of *Wurtemburg*. Having therefore in my Hands a certain Siphon of Glaſs, I erected it as nearly perpendicular as I could over two Veſſels; and when it was fixt in this Situation, I poured Water into one of the Veſſels, till the Orifice of the Siphon was a little immerſed in it, and preſently as I expected the Water run out into the other Veſſel. Then that Veſſel being emptied into which I had firſt poured Water, I poured it into the other, and immediately that Water run back into the firſt Veſſel. Tho' I would not venture to compare the Artifice of this Siphon of mine with that of *Wurtemburg*, yet as to its uſe I doubt not but that it will not much yield to that, eſpecially if a certain Inſtrument be added to it, which I have contrived in a particular manner.

2. In the Treatiſe concerning the *Wurtemburg* Siphon, which was lately printed at *Stutgard* by Dr. *Salomon Reiſelius*, certain wonderful Things, and before unheard of, are told of that new Siphon, of which the Characteriſtic Properties are declared in theſe Words. *By Dr. Papin. Ibid. p. 847.*

1. *The Orifices of the two Legs of his* Siphon *being placed Horizontally, are dipped into the Brims of the Veſſels; whereas in thoſe of the antient Invention the longer Leg always deſcends below the Brim or the Equilibrium.*

2. *The Orifices being either partly or half fill'd with Water, yet the Water flows out when drawn over a Mountain. Whereas in other* Siphons *the whole Orifice muſt be fill'd with Water, or immerged in the Water.*

3. *Tho' the Machine be at reſt and in continual Dryneſs, yet it will produce its Effect, Water being apply'd again.*

4. *Either of the Eyes or Orifices being open, and the other after ſome Hours or a Day being ſtop'd by a Cone or Stopper, yet the Water will run out; whereas in others both the Eyes muſt be open at the ſame time.*

5. *The Orifices being placed in an Horizontal Line, and the Legs being equal in height, yet the Liquor will run out; but in the Machines of* Porta *and others, the Legs muſt be unequal, and the Perpendicular greater.*

6. *The Water being poured into either Veſſel will aſcend and run into either; whereas in thoſe of the Antients it runs out at one Leg only, and that the longer, but never runs back.*

Theſe are the Words of the Author; but by what Method, or what Artifice theſe notable Effects may be produced, he adds not a Syllable. Therefore the Royal Society order'd me to prepare a Machine, which might exhibit the ſame Phenomena as theſe deſcribed in the Book. I have performed the thing after three different manners; but to avoid being tedious, I judge the following, as the eaſieſt and ſimpleſt, will be ſufficient.

Fig. 197. A A are two Veſſels of Metal, in which are inſerted the two Extremities of the *Siphon*. BCDEDCB is a *Siphon*, the Eyes of which BB are to be diſpoſed in the ſame Horizontal Line. F is a little Tube ſoder'd into a

Hole in the upper part of the *Siphon*, and carefully cloſed, after the *Siphon* is totally repleniſhed with Water. Now it is plain, that the Water contain'd in the Parts C D will hinder the Influx of the external Air, ſo that it cannot penetrate to the upper Part of the *Siphon* E. So that the *Siphon* being always full of Water, (provided it does not exceed a due height,) will moſt ſurely produce its Effect, as ſoon as the Water in the Veſſels A A ſhall fill any part of either of the Orifices B. And when both the Orifices being partly fill'd with Water, in either Veſſel A the Surface of the Water ſhall come to the ſame Horizontal Line, if you pour never ſo little Water to either Veſſel, part of it will preſently be carry'd through the *Siphon* into the other Veſſel; and in the ſame Method the other *Phenomena* may be exhibited as are deſcribed in the *Book*.

By D. Salomon Reiſelius *himſelf*. n. 178. *p.* 1272. Dec. An. 1685.

3. That I may not any longer be wanting in ſatisfying the Deſire of the moſt ſerene Society, I confeſs that the *Siphon* of the moſt excellent Dr. *Papin* to be the very *Wurtemburgic Siphon*, alſo made with a Recurvation of the Feet. Nor is there any other Myſtery at the Top, as the Inventer has deſcribed, than that it muſt be fill'd by a Funnel, without which Impletion it cannot run away. This ſhall ſoon be confirm'd farther by the Preſs, becauſe it would be long and tedious to ſet down every thing here.

Now that I may ſhew at preſent that I have already perform'd ſomething about the Effects of the *Siphon*, here is ſomething that has occur'd to me in making my Experiments. For I have learn'd how the Water may run out either at the top, or at the ſides, which many have promis'd to do, but hardly any one has perform'd.

A New way of Raiſing Water Enigmatically propoſed; by Dr. Papin. n. 173. *p.* 1093. July An. 1685. *Fig.* 198.

XI. 1. A A is a great Glaſs made like a Tumbler, but much bigger, and laid upon the Chimney Board, B B.

C C is the *Engine* like a ſmall *Rock*, that doth conſtantly *ſpout* out Water by the two Holes D D: This *Rock* is kept at a diſtance from the bottom of the *Glaſs* A A; ſo that it may plainly be ſeen that it cannot receive any Water by ſubterraneal Tubes.

E E is a factitious *Coral*, reaching from the Center of the *Rock* C C, to the Center of the *Crown* F F.

F F is a *Crown* bearing upon the Aperture of the *Glaſs* A A, and holding the *Rock* C C, ſuſpended at a conſiderable diſtance from the Bottom.

G G a Glaſs open at both ends, apply'd to the *Rock* C C, to keep the Water upon it from falling down.

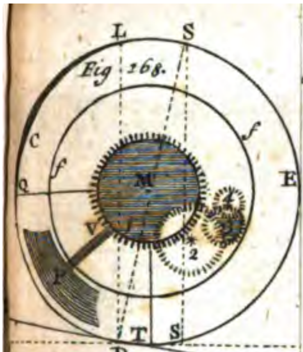
The Water in this Engine runs conſtantly. H H, Two *Shells* to receive the Water from the *Jetto's*.

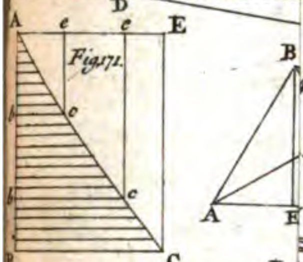
Solv'd; by Dr. Nath. Vincent. n. 177. *p.* 1238. Dec. An. 1686.

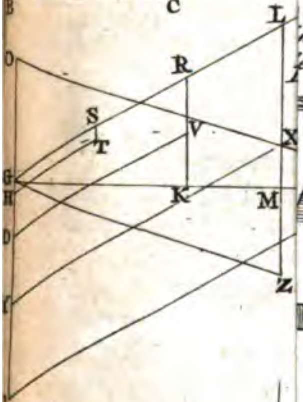
2. Within the *Rock* C C, there may be a Veſſel placed, which ſhall be made like the Body of a Pair of *Bellows*, or thoſe *Puffs* heretofore uſed by *Barbers*, which being filled with Water, a Piece of *Clockwork* put under it may produce the *Jetto's*; the Water being received into the *Shell* H H, and running thence into the hollow of the *Coral* E E, may be thereby conveyed into the follicular Cavity in the ſame quantity it is ejected from the

two

Plate V. Vol.1. pag.538.







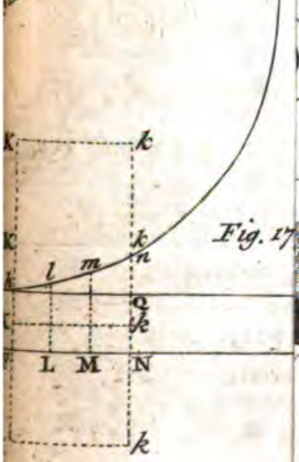


Fig. 17

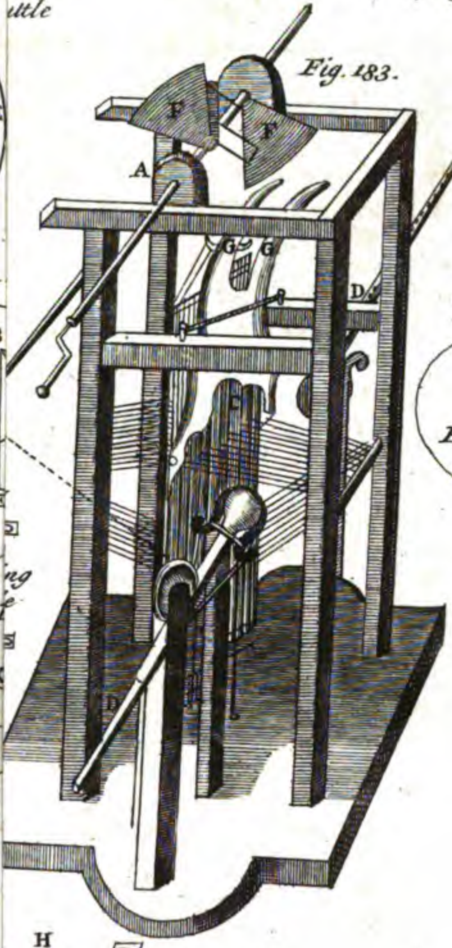


Fig. 183.

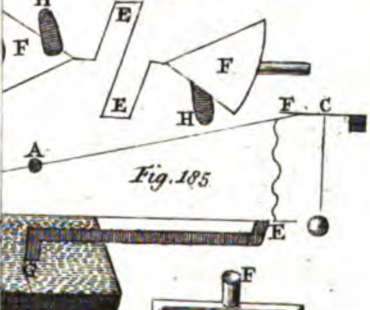


Fig. 185

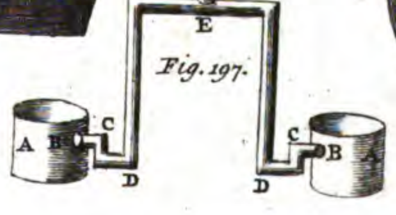


Fig. 197.

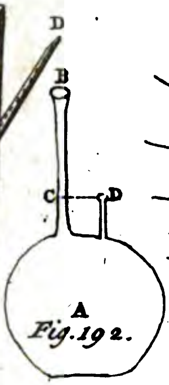


Fig. 192.

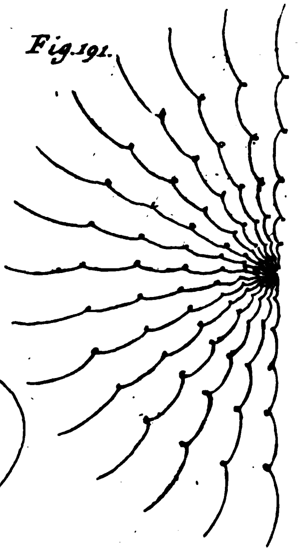


Fig. 191.

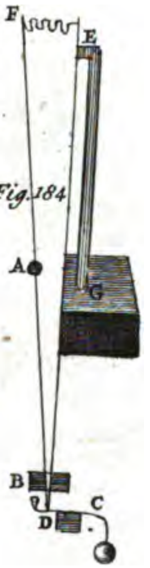


Fig. 184

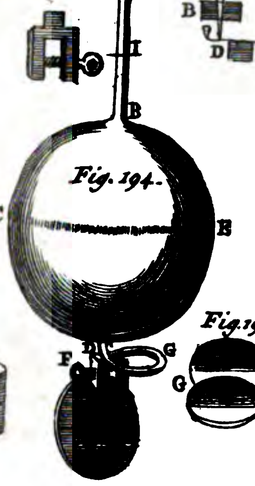


Fig. 194.

Fig. 196.

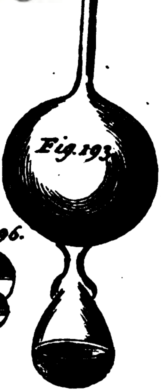


Fig. 193.

two emerging Tubes; and it will circulate according to the going of the *Clock-work*.

3. I conceive that the Air is forced into the *outer Glaſs* at the bottom thereof. *By Mr.* R. A. *ib. p.* 1259.

That it then paſſes up between the two *Glaſſes*.

That the *outer Glaſs* or Caſe being cloſe luted at the Head or *Crown* to which the *inner Glaſs* is hung by the *Coral*, the Air is forced into the Mouth of the *inner Glaſs*.

That the Air ſo forced, preſſing on the Surface of the Water that covers the *Rock*, forces the Water to riſe thro' thoſe two extream Parts that are not at all clogg'd, or covered with Water.

4. ABDE, ſignifies a Cylindrical Veſſel, cloſed on every ſide, and divided into two Rooms by the Floor EF. *In a Letter Subſcribed*, W. Tenon. n. 178. *p.* 1254. *Fig.* 199. Dec. An. 1685.

GLMH, is another Cylindrical Veſſel within that upper Room, cemented with its Mouth downwards to the Floor, and full of Water up to the Surface IK; the upper Part thereof GIKH, being full of Air.

QO, RP, Two Pipes, open above and below, and let thro' the Upper Room into this Veſſel, and reaching almoſt down to the Floor EF.

VW, a Pipe open above and below, and let into the Upper Room. Theſe Pipes muſt be cloſe joined round about them to the Floors CD and GH.

XY, Two little Hemiſpherical Bladders prepared with Oil or ſome Oily Subſtance (as Butter and Turpentine) againſt Water, and Cemented with their Mouths upward to the Floor EF, underneath.

a, β Two Valves Opening out of the Upper Room in the Bladders.

γ, δ Two other Valves Opening out of the Bladders into the inner Veſſel above.

NZ, a *Pendulum* playing upon the Centre N, and having two Battledoor Arms *a*, *b*, to ſqueeze alternately the Bladders which reſt upon them.

Let the Upper Room be filled with Water at the Pipe VW, and if the *Pendulum* be made to play by *Clock-work*, the Bladders will perpetually pump it thence into the Inner veſſel, and the compreſt Air GIKH in the Upper part of that Veſſel preſſing upon the Surface of the Water IK, will force it thence into the Pipes OQ, PR, out of which ſpouting with a perpetual even Stream into the Spoons ST, it will run down by the Pipe WV, into the Upper Room again: the *Pendulum* will play moſt eaſily when the upper Room is filled to the Top of the Pipe WV. Inſtead of the Bladders may be other Contrivances, as of Suckers or little Organ Bellows, playing alternately with two Leaves about an Axis in the middle.

5. AA, is the *Great Tumbler*, that muſt have ſome little Hole in the bottom, as I. *By Dr.* Papin. *Ibid. p.* 1274. *Fig.* 200.

ILL, a ſlender Pipe hidden by the Chimney Board BB, whereby the *Tumbler* AA, hath Communication with the *Pump* or *Bellows* MM.

MM, ſome kind of *Pump* or *Bellows* well ſhut, and having no other Aperture, but thro' the Pipe I L L. Theſe are put in ſome ſecret Place where a Body may play the ſame and not be ſeen.

N N, a ſlender Pipe, that makes a Communication between the *Glaſs* AA, and the *Crown* F F; this Pipe reacheth near to the cover of the *Crown*, that the Water contained in it may not run down by that Aperture.

E E, the *Factitious Coral*, hollow within, ſhut at the Bottom and open at the Top.

D D, D D, Two crooked Pipes ſoldered to the Sides of the *Coral* E E, ſo that the Water running down the *Coral* may ſpout out at the Holes D D.

O O, a Pipe hidden in the *Coral* E E, paſſing thro' the bottom of the ſame, where it muſt be well ſolder'd, and reaching near to the Bottom of the *Rock* C C.

P P, a Pipe to convey the Water from the *Glaſs* G G, into the *Rock* CC; this Pipe is well ſolder'd to the *Cover* of the ſaid *Rock*.

Q, a *Valve* working by a Spring at the Bottom of the Pipe P P, to keep the Water, that gets in that Way, from returning back.

R, another *Valve* at the top of the Pipe O O, that the Water getting up that way, may not fall thro' the ſame.

Now it is plain, that the *Rock* C C, being filled partly with Water, partly with Air; if we open the *Bellows* M M, the Air from the *Crown* F F, muſt run thro' the Pipe N N, into the *Tumbler* A A; and thence thro' the Pipe I L L, into M M, to fill the Vacuity made therein: The Air in the *Crown* F F, being thus rarified, gives liberty to the Air in the *Rock* C C, to rarify too, by driving the Water thro' the Pipe O O. The Water being got up into the *Crown* F F, runs down the *Coral* E E, and thro' the crooked Pipes D D, D D, ſpouts out at their upper Apertures, and from the *Shells* H H falls upon the *Rock* C C: If we come afterwards to ſhut the *Bellows* M M, the Air got into their Vacuity muſt run back into the *Tumbler* A A, and preſs upon the Water at the Top of the *Rock* C C: But the Air in the ſaid *Rock* having been rarified, its Spring is not ſufficient to reſiſt this Preſſure, and ſo the Water is forced into the ſaid *Rock* thro' the Pipe P P: And thus opening and ſhutting the *Bellows* M M, the Water muſt conſtantly circulate by the Ways aforeſaid.

The Uſe of this Contrivance.

As for the Uſes this way for *raiſing Water* may be applied to, this I do conceive: The Glaſſes, being merely to conceal the Secret, muſt be left out; and their may be made ſeveral Receptacles above one another to receive the raiſed Water, ſo as doth the *Crown* F F: And there ſhould be as many *Bellows* to communicate every one with one *Receptacle*: Theſe *Bellows* ſhould be moved by an Axis, ſo that, when the *Firſt* is open, the *Second* ſhould be ſhut; the *Third* open, the *Fourth* ſhut; and ſo forth, alternatively; which may be eaſily done: By this Means, the firſt or loweſt *Receptacle* would give the neceſſary Supply of Water to the *Second*, the *Second* to the *Third*, and the *Third* to the *Fourth*, &c. till the Water would be *raiſed* to the intended Height. Such Receptacles might eaſily be ſet at 12 or 15 Foot above one another, and ſo but few of them might *raiſe Water* to a conſiderable Height, as well as ordinary *Pumps* do: But this new Way would have this advantage, that in the ordinary Pumps the Strength to be applied lieth near the Water to be raiſed, but by this Contrivance the Stream of a River may be applied to draw Water out of a Mine far diſtant from it. By the ſame way the Stream of the *Thames* might keep conſtant *Water-Works* in *Windſor-Caſtle*, as eaſy almoſt as in the loweſt Fields: The River *Seine* might do the ſame at St. *Germain*, and perhaps at *Verſailles* too, notwithſtanding the great Diſtance. For it is to be obſerved, That the Pipes of Communication between the *Bellows* and the *Engine*, being merely for the conveying of the Air, which moves very ſwiftly, they may be ſlender enough, and ſo contain but a ſmall Quantity of Air to be rarified; and beſides, they will not be ſubject to burſt or leak, ſince the Preſſure they bear, being all external to the Pipe, will rather ſtrengthen than break the ſame. From whence it follows, That the ſaid Pipes need not be ſtrong, but may be made at very ſmall Charges. It is alſo to be obſerved, That thoſe *Bellows* which are open, have the Air in them very much rarified, ſo that the outward Air lieth heavy upon (to ſhut) them; by which means the Motion of the *Engine* muſt be help'd in Lifting up the oppoſite *Bellows*, that are to be opened: And this Obſervation may anſwer the greateſt difficulty that might be objected againſt this Contrivance. So that I don't queſtion but this way for *raiſing Water*, may on ſeveral Occaſions be of great Advantage.

Further Explained. *Fig.* 201.

A B, A B, Are ſeveral *Receptacles* ſet above one another, which muſt be well ſhut and ſoldered every where.

CDD, CDD, Are two ſlender Pipes, whereby the Firſt and Third *Receptacles* have a Communication with the *Pump* H H.

EFF, EFF, Two other ſlender Pipes, whereby the Second and Fourth *Receptacles* have a Communication with the *Pump* I I.

HH, I I, Two *Pumps* whoſe Plugs are ſo moved by the Axis L L, that when one goeth down the other goeth up.

MM, a Wheel faſtened to the Axis L L, that it may be moved by the Stream of a River.

NO, PQ, NO, PQ, Are big Pipes for the Water to go up, from a lower into a higher *Receptacle*.

O, Q, O, Q, Are *Valves* fitted to the Top of the aforeſaid Pipes, that the Water may not go down thro' the ſame.

Now it is plain, that, when the *Plug* in the *Pump* HH, is going up, the Air comes in thro' the Pipes C D D, and ſo it is rarified in the firſt and third *Receptacles* marked A, A: And by that Means the Water may be driven up into the ſaid *Receptacles* thro' the Pipes N O, becauſe at the ſame time the *Plug* in the *Pump* II, going down, cauſeth the Air to return to its ordinary Preſſure in the ſecond and fourth *Receptacles*, that it may be able to drive up the Water thro' the ſaid Pipes N O, and the loweſt Pipe draws the Water that lies open to the Air. By the ſame reaſon when the *Plug* in the *Pump* II, goeth up, the Air muſt come in thro' the Pipes

EFF: And ſo it is rarified in the ſecond and fourth *Receptacles* marked B, B, and by that means the Water may be driven up into the ſaid *Receptacles* thro' the Pipes P Q, P Q, becauſe at the ſame time the *Plug* in the *Pump* AH, going down, cauſes the Air to return to its ordinary Preſſure in the firſt and third *Receptacles*, ſo that it is able to drive up the Water thro' the ſaid Pipes P Q.

Several Objections made by M. Nuis, Anſwered; by Dr. Pavin. n. 186. *p.* 263. Jan. An. 1687.

6. 1. To keep the *Receptacles* from being fill'd too much, the Water may be let out by inſerting into each a crooked Pipe, reaching a pretty way downwards, and having its lower Aperture ſhut up with a Valve, whereby the Water may run out when the *Receptacle* ſhall be fill'd to a certain Height: And I may add, to prevent new difficulties, that, leſt the *Pumps* ſhould be fill'd too much, a Valve may be made that ſhall open as ſoon as the Air in the *Pump* ſhould be more compreſt than the outward Air: So the Air getting in thro' any Pores would be conſtantly let out.

2. I have not poſitively promiſed a good Succeſs, but for *Windſor* and St. *Germain*; but when I ſpoke of *Verſailles* I uſed the Word *perhaps*, thereby ſhewing that, before any one ſhould go about ſuch a great Undertaking, he ſhould reflect upon it more than I would then do, not having occaſion for ſuch Work. But I now make the following computation.

Let the Diſtance of *Verſailles*, as *M. Nuis* ſuppoſeth, be 12000 Foot, and the Capacity of each *Receptacle* be about one half of a cubic Foot: I might make the Wheel with the Axis to make their *Revolution* in one Minute of Time, and ſo order all things that the Air under the aſcending *Plugs* might come to be rarified to ſuch a Degree, that by its Elaſticity it might not counterpoiſe more than 7 Foot of Water; but at the ſame time the Air in the *Receptacles* A,A,B,B, would, even in its greateſt Dilatation, be able to counterpoiſe 17 Foot; So it is plain, that the Air will be driven from the *Receptacles* into the *Pumps* by a Strength equivalent to 10 Foot of Water: Now if we compute the * *Velocity* of Air driven by ſuch a Preſſure we ſhall find that the ſaid *Velocity* will be about 740 Foot in a Second: So that in half a Minute, during which the Plug goeth up, this Air might paſs above 22000 Foot, altho' it were not rarified at all; but being rarified, as we do ſuppoſe it to be, it might go a great deal further.

* *Vid. Sup. Cap.* v. §. XII.

I muſt now take notice, that according to the Honourable Mr. *Boyle*'s Experiments, the *Rarefaction* of the *Air* is much leſſer than *M. Nuis* takes it to be: for the Water contained in the Pipe N O, is ſo far from cauſing the *Air* to fill up a Space four times bigger, that it will not extend itſelf to a Space once bigger than before: Conſidering therefore the *Velocity* of the Air, and the ſmall Dilatation it doth ſuffer, if any one will take the Trouble to compute, he will find, that if the *Pumps* have in Diameter the Diagonal of a ſquare Foot, and the ſame Height; and if the ſmall Tubes of Communication be made of $\frac{1}{9}$ part of an Inch in Diameter; ſo that being 12000 Foot long, they may contain about one cubic Foot of Air; that would be more than ſufficient to make the neceſſary *Rarefaction* in the *Receptacles*.

But for the good Succeſs of the *Engine* it is not enough to make the Air paſs from the *Receptacles* into the *Pumps*, it muſt alſo return from the *Pumps* into the *Receptacles:* Now for this Intent it would be neceſſary to ſet the *Receptacles* but five Foot above one another; ſo to drive the Water up the Pipe N O, it would be enough that the Air in the *Receptacle* B, ſhould preſs with a Strength equivalent to 23 Foot of Water: for it is plain, that five Foot in the Pipe N O, together with a Preſſure equivalent to 17 Foot, which I have ſuppoſed to be in the upper *Receptacle* A, will make but 22 Foot in all; and therefore 23 Foot depreſſing in the *Receptacle* B, muſt prevail, and cauſe the Water to aſcend: Now the Preſſure in the *Receptacle* being but 23 Foot, and the Air in the *Pump* returning to its ordinary Preſſure, which is about 33 Foot, it is plain that the Air going back to the *Receptacle*, will be driven by a Strength equivalent to 10 Foot, as well as it had been in coming from the *Receptacle* towards the *Pump:* and ſo the Bigneſs aſſigned for the *Communication Pipes* will alſo prove more than ſufficient to this Effect.

From what I have been ſaying, it is plain, That in great diſtances there ſhould be made as many *Pumps* as *Receptacles*, as hath been already propounded: And for to raiſe Water but 16 Foot High, there ſhould be required 13 or 14 *Receptacles*, and as many *Pumps* of the Bigneſs aforeſaid. Some People may take this for a great Difficulty: But I anſwer, That in this *Engine*, this is not ſo much as it ſeems at firſt; becauſe, the Preſſure being all from without, there is no need of any great Strength to reſiſt it; and ſo the Metal for the *Pump* will coſt but little: There may alſo be found Occaſions where to make ſo good uſe of them, that ſuch an *Engine* as I have deſcribed would in a Year's time ſave Labour enough to pay for many *Pumps*, ſince it might every hour raiſe about 1800 Pounds of Water to the Height of 60 Foot. Mean while I don't pretend to have given here the beſt Proportion for the Bigneſs of every part of the *Engine*; but it may be, by altering the Capacity of the *Pumps*, of the *Pipes*, or of the *Receptacles*, a much more conſiderable Effect might be produced.

3. The Water doth not at any time aſcend higher than from a lower *Receptacle* into the next upper *Receptacle*; which Height is but 12 Foot: So that it is plain enough, that the Preſſure of the Air may be ſufficient to drive it up. It is indifferent, whether it be by Rarefaction or otherwiſe that the Water comes into the *Receptacle* A; it is enough that the Water is there, and that the Air preſſes upon it with ſuch a Strength as will prevail againſt all that oppoſeth it.

4. Tho' the Uſe of the Pipes be merely for conveying of Air, they may nevertheleſs be eaſily fill'd with Water, when need requires; and ſo the Defects in them may as well be found out as in Pipes that are uſed for the conveying of Water.

An Engine for Raiſing Water, by the Help of Fire; by Mr. Tho. Savery. n. 253. p. 228. Fig. 202. June An. 1699.

XII. A, The *Furnace*.
B, The *Boyler*.
CC, Two *Cocks*, which convey the *Steam* by turns to the *Veſſels* D D.

DD, The *Veſſels* which receive the Water from the Bottom, in order to Diſcharge it again at the Top.

EEEE, *Valves.*

FF, *Cocks* which keep up the Water, while the *Valves* on Occaſion are cleans'd.

G, The *Force* Pipe.

H, The *Sucking* Pipe.

I, The *Water.*

An Hydraulique Engine; by . . . . n. 128. *p.* 679. *Fig.* 203. An 1675.

XIII. This *Engine* is a *Cheſt* of Copper A, pierced with many Holes above BB, and holds within it the Body of a *Pump* EFM, whoſe *Sucker* DE, is raiſed and abaſed by two *Levers* C, O; theſe *Levers* having each of them two Arms, and each Arm being fitted to be laid hold on by both Hands of a Man. Each *Lever* is pierced in the middle by a Mortiſe *a a*, in which an Iron Nail, which paſſes thro' the Handle of the *Sucker*, turns when the *Sucker* is raiſed or lower'd. Near the Body of the *Pump* there is a *Copper Pot* IHL, joined to it by the Tube G, and having another Tube KNL, which in N may be turned every way.

To make this *Engine* play, Water is pour'd upon the *Cheſt* to enter in at the Holes that are in the Cover thereof. This Water is drawn into the Body of the *Pump*, at the Hole F, at the time when the *Sucker* is raiſed: and when the ſame is let down, the *Valve* of the ſame Hole F ſhuts, and forces the Water to paſs thro' the Hole M into the Tube G, of which the *Valve* H being lifted up, the Water enters into the *Pot*, and filling the bottom, it enters thro' the Hole K, into the Tube KNL, in ſuch a manner, that when the Water is higher than the Tube KNL, and the Hole of the Tube G is ſhut by the *Valve* H, the Air encloſed in the *Pot* hath no Iſſue; and it comes to paſs, that, when you continue to make the Water enter into the *Pot* by the Tube G, which is much thicker than the Aperture of the end L, at which it muſt iſſue, it muſt needs be, that the ſurplus of the Water that enters into the *Pot*, and exceeds that which at the ſame time iſſues thro' the ſmall end of the *Jet*, compreſſes the Air to find Place in the *Pot*: which makes that, whilſt the *Sucker* is raiſed again, to make new Water to enter into the Body of the *Pump*, the Air which has been *Compreſs'd* in the *Pot* drives the ſurplus of the Water by the Force of its *Spring*, mean time that a new *Compreſſion* of the *Sucker* makes new Water to enter, and cauſes alſo a new *Compreſſion* of Air. And thus the Courſe of the Water, which iſſues by the *Jet*, is always entertained in the ſame State.

A cheap Pump: by Mr. Conyers. n. 136. *p.* 888. June An. 1677.

XIV. AA, the Body of a ſquare Taper *Pump*, made of Oak, Elm, or Deal Planks; with a *Valve* at bottom, *a a*.

BB, the *Bucket*, in the midſt of which there is a *Valve* *b*, not viſible in the Figure, being concealed by the Sides of the Leather, *b b*.

CCC, the Iron to raiſe the *Bucket*.

D D, the Wood at the bottom of the *Bucket* containing the *Valve.*

E E, the *Handle* for raiſing the *Bucket*, to be managed by fewer Hands than ordinary *Pumps* are; which may be altered ſo as to employ a Horſe, or Mill, or other ſuch-like way, more advantageous than that of this Handle managed by the ſtrength of Men. Fig. 204.

F F, a Square *Taper-Box* with Holes in the Sides, and open at the Bottom; into the narrower part of which is encloſed the narrower End of the Body of the *Pump.*

G G, an additional *Bucket* of a larger Dimenſion, to be placed on the Iron Work of the *Pump* about H, when it ſhall be needful to lengthen the Taper of your *Pump*, and thereby to raiſe the Water more forcibly to a greater height.

I I, the *Spout* of the *Pump*, to caſt out the Water, of the ſame Breadth with the ſide of the *Pump.*

K K, the Iron or Wooden Work ſet off, or bent back (if need be) and placed at the back of this *Pump*, for the eaſier and more capacious Motion of the *Pump-Handle*, in which it moves.

This *Pump* was by me contrived in 1673. when the *New Canal* of *Fleet-River* in *London* was enlarged: It was found to raiſe at leaſt twice as much Water proportionably as thoſe of the ſame, or rather bigger Bore, that were firſt made uſe of and caſt by. It was 8½ Foot long, and 1 Foot 8 Inches broad at the top, and about 8 Inches broad at the bottom, where it is inſerted in the *Box*; and did caſt out 8 Gallons at a *Stroke*, and 21 *Strokes* being made in one *Minute*, there were delivered about 169 Gallons in a *Minute's* time; whence it is eaſy to compute, what Quantity is thrown out in an *Hour*. This kind of *Pump* may by the ſame Contrivance be made of a Tree bored thro' with a Taper Bore: and a Baſket may be uſed at the bottom of the *Pump* inſtead of the *Box-Colender.*

XV. Papers *of leſs General Uſe*, (*Extracted from a Book of* Jo. Alph. Borellius *de Motu Animalium*) omitted.

1. A *Way how a Man may* Swim *under* Water, *and* breathe *by the Help of a Bag about his Head.* Phil. Coll. n. 2. p. 35.

2. *Another way of* Breathing *under* Water *by the help of a Leathern Pipe kept open by wreathed Wires, and extended by the* Swimmer's *Head to the top of the Water.*

3. *A way to make a* Submarine Veſſel *accommodated with ways to* Row *it, and to make it* Riſe *and* Sink *in the Water.*

XVI. Accounts *of Books* omitted.

n. 8. p. 145. n. 10. p. 173. 1. HYdroſtatical Paradoxes, *made out by New* Experiments, (*for the moſt part Phyſical and Eaſy) by the Honourable* R. Boyle, *Eſq;*

n. 226. p. 481. 2. Recueil de diverſes Pieces touchant quelques *Nouvelles Machines*, &c. Par *D. Papin, M. D. A Caſſel.* 1695. *in 8vo.*

CHAP. VII.

Geography. Navigation.

A new place for the firſt Meridian propos'd; by a Profeſſor *of* Math. *at* Sevill. n. 118. p. 425. Oct. An. 1675.

I. THE *Longitude* of a Place upon the *Earth* is an Arch of the Equinoctial intercepted between two Meridians; or the Space of Time which is number'd by the Equinoctial between two Places. Wherefore it would be proper to fix the beginning of *Longitude* in the Equinoctial it ſelf. Moreover, ſince the Equinoctial Circle divides the *Globe* into Northern and Southern Hemiſpheres, if the fixing of this *Primary Meridian* is appointed in it, there will be an Equality and Conformity between the Northern and Southern Parts. Beſides it is neceſſary for exactneſs, that the Place of this firſt *Meridian* ſhould be ſmall, that the Reckoning of the *Longitude* may be expreſs'd the more exactly; not as ſome have done, who have aſſumed all the *Fortunate Iſlands* for their Beginning of Longitude, and did not regard the Diſtance of two Degrees which they have from one another; which ſurely is very abſurd. Again care muſt be taken that the *Primary Meridian* may not be confounded with the Land and Draughts of Places which are deſcribed on the *Globes* or *Maps*, which muſt be if it paſſes thro' the midſt of Countries. And if it divides the principal Parts of the Earth, as *America*, *Africa*, *Europe*, paſſing over the Seas, it will be ſo much the fitter and more convenient in the Repreſentation of the *Terraqueous Globe*. Now conſidering all theſe Reaſons here mentioned, I have found, that as Nature, which provides nothing in vain, has placed a certain Iſland under the very *Equinoctial Circle*, near *Braſile*, formerly called *Abroxos*, which Iſland is diſtant from the *Pike* of *Teneriff* 9 Degrees to the Weſt, and from *Uraniburg* 42 Degrees to the Weſt, in which all the Circumſtances are found proper to conſtitute the firſt *Meridian*, as by me have been enumerated.

II. 1. Thoſe

M. Huygen's *Inſtructions for finding the Longitude with Pendulum Watches; Enlarged by* . . . . n. 47. p. 937. May An. 1669.

II. 1. Thoſe that intend to make uſe of *Pendulum Watches* at *Sea*, muſt have two of them at leaſt; that, if one of them ſhould by miſhap or neglect come to ſtop, or (being by length of time become foul) need to be made clean, there may likely always remain one in Motion.

2. The *Watches* on *Shipboard* are to be hung in a Cloſe Place, where they may be freeſt from Moiſture or Duſt, and out of Danger of being diſordered by knocking or touching.

3. Before the *Watches* be brought on *Shipboard*, 'tis convenient they be adjuſted to a *Middle* or *Mean Day*; the Uſe of them being then moſt eaſy.

To adjuſt the Watches.

4. Here take notice; That the Sun paſſeth the 12 Signs, or makes one entire Revolution in the *Ecliptick* in 365 Days, 5^h^ 48′. or thereabout; and that thoſe Days, reckoned from Noon to Noon, are of different Lengths; as is known to all that are verſed in *Aſtronomy*. Now between the Longeſt and the Shorteſt of thoſe Days, a Day may be taken of ſuch a Length, as 365 ſuch Days, 5^h^ 49′. *&c.* make up, or are equal to that Revolution; and this is called the *Equal* or *Mean Day*, according to which the *Watches* are to be Set; and therefore the Hour or Minute ſhewed by the *Watches*, tho' they be perfectly juſt and equal, muſt needs differ almoſt continually from thoſe that are ſhew'd by the *Sun*, or are reckoned according to its Motion. But this Difference is Regular, and is otherwiſe called the *Æquation*; which is accounted from the firſt of *February* in the following *Table*.

	Januar.		*Februar*		*March.*		*April.*		*May.*		*June.*	
	′	″	′	″	′	″	′	″	′	″	′	″
1	6	10	0	0	4	46	14	23	19	25	16	24
2	5	47	0	2	5	03	14	39	19	28	16	13
3	5	24	0	4	5	21	14	55	19	29	16	01
4	5	02	0	8	5	39	15	10	19	29	15	49
5	4	41	0	12	5	57	15	25	19	29	15	37
6	4	21	0	16	6	15	15	39	19	28	15	24
7	4	02	0	21	6	33	15	53	19	26	15	11
8	3	44	0	26	6	51	16	07	19	24	14	58
9	2	27	0	32	7	09	16	21	19	21	14	45
10	3	11	0	40	7	27	16	34	19	18	14	32
11	2	55	0	48	7	45	16	47	19	15	14	19
12	2	39	0	57	8	03	16	59	19	11	14	06
13	2	23	1	06	8	22	17	11	19	07	13	53
14	2	07	1	16	8	41	17	22	19	02	13	40
15	1	52	1	26	9	01	17	33	18	57	13	27
16	1	38	1	37	9	21	17	43	18	51	13	15
17	1	25	1	49	9	41	17	53	18	45	13	03
18	1	13	2	02	10	01	18	03	18	39	12	52
19	1	02	2	15	10	21	18	13	18	33	12	41
20	0	51	2	28	10	40	18	23	18	26	12	30
21	0	41	2	42	10	59	18	32	18	18	12	19
22	0	32	2	56	11	18	18	39	18	10	12	08
23	0	24	3	11	11	37	18	46	18	01	11	58
24	0	18	3	26	11	56	18	53	17	51	11	48
25	0	13	3	41	12	15	18	59	17	41	11	38
26	0	9	3	56	12	34	19	04	17	30	11	28
27	0	6	4	12	12	53	19	09	17	19	11	18
28	0	3	4	29	13	12	19	14	17	08	11	09
29	0	1			13	31	19	18	16	57	11	00
30	0	0			13	49	19	22	16	46	10	52
31	0	0			14	06			16	35		

July

	July.		*Auguſt.*		*Septem.*		*October.*		*Novem.*		*Decemb.*	
	′	″	′	″	′	″	′	″	′	″	′	″
1	10	45	11	07	19	41	29	16	31	13	21	14
2	10	38	11	16	20	01	29	30	31	03	20	44
3	10	31	11	25	20	22	29	43	30	53	20	14
4	10	25	11	36	20	43	29	56	30	43	19	44
5	10	19	11	48	21	04	30	09	30	32	19	14
6	10	13	12	01	21	25	30	22	30	20	18	44
7	10	07	12	14	21	47	30	34	30	08	18	14
8	10	02	12	28	22	09	30	45	29	55	17	44
9	9	58	12	42	22	31	30	55	29	40	17	14
10	9	54	12	57	23	52	31	04	29	23	16	44
11	9	51	13	12	23	13	31	12	29	06	16	14
12	9	49	13	27	23	33	31	19	28	48	15	44
13	9	47	13	43	23	53	31	26	28	30	15	14
14	9	46	13	59	24	13	31	32	28	11	14	43
15	9	46	14	16	24	33	31	38	27	51	14	12
16	9	46	14	33	24	53	31	43	27	30	13	41
17	9	47	14	50	25	13	31	47	27	08	13	10
18	9	49	15	08	25	33	31	50	26	45	12	40
19	9	52	15	26	25	52	31	53	26	22	12	10
20	9	56	15	45	26	11	31	55	25	58	11	40
21	10	00	16	04	26	30	31	55	25	34	11	10
22	10	04	16	23	26	49	31	55	25	10	10	40
23	10	08	16	42	27	08	31	55	24	45	10	10
24	10	13	17	01	27	26	31	54	24	20	9	41
25	10	18	17	21	27	43	31	52	23	55	9	13
26	10	23	17	41	28	00	31	50	23	30	8	45
27	10	28	18	01	28	16	31	47	23	04	8	17
28	10	34	18	21	28	32	31	43	22	38	7	50
29	10	41	18	41	28	47	31	37	22	11	7	23
30	10	49	19	01	29	02	31	30	21	43	6	58
31	10	58	19	21			31	22			6	34

By the help of the foregoing *Table* you will always know what a Clock it is by the *Sun* preciſely, and conſequently, whether the *Watches* have been ſet to the right Meaſure of the *Mean Day*, or no; uſing the *Table* as follows.

When you firſt Set your *Watch* by the *Sun*, you are to ſubduct from the Time obſerved by the *Sun*, the *Æquation* adjoined to that Day of the Month in the *Table*, and to Set the *Watches* to the remaining Hours, Minutes and Seconds; that is, the *Watches* are to be Set ſo much Slower than the Time of the *Sun*, as (in the *Table*) is the *Æquation* of that Day; ſo that the *Æquation* of the Day added to the Time of the Clock, is the true Time by the *Sun*. And when after ſome Days, you deſire to know by the *Watch* the Time by the *Sun*, you are to add to the Time ſhewed by the *Watch*, the *Æquation* of that Day; and the Aggregate ſhall be the Time by the *Sun*, if the *Watch* hath been perfectly well adjuſted after the Meaſure of the *Mean Days*; for the Doing of which, this will be a convenient Way.

Draw a *Meridian Line* upon a Floor, and then hang two Plummets, each by a ſmall Thread or Wire, directly over the ſaid *Meridian*, at the diſtance of ſome two Foot or more one from the other, as the Smallneſs of the Thread will admit. When the Middle of the *Sun* (the Eye being placed ſo, as to bring both the Threads into one Line) appears to be in the ſame Line exactly (for the better and more ſecure diſcerning whereof, you muſt be furniſh'd with a Glaſs of a dark Colour, or ſomewhat Black'd with the Smoak of a Candle) you are then immediately to Set the *Watch*, not preciſely to the Hour of 12, but by ſo much leſs as is the *Æquation* of that Day; *e. g.* If it were the 12*th* of *March*, the *Æquation* of that Day being by the Table, 8′ 3″ theſe are to be ſubducted from 12 Hours, and the Remainder will be 11ʰ, 51′ 57″, to which Hours, Minutes, and Seconds, you are to Set the *Index* of the *Watch* reſpectively: Then after ſome Days you are to obſerve again in the ſame manner, and likewiſe to note the Hour, Minute, and Second of the *Watch*; to which you are to add the *Æquation* of theſe Days, taken out of the Table; and if the Aggregate do juſt make 12 Hours, the *Watch* is adjuſted to the Right Meaſure; but if it differ, you are to divide the Minutes and Seconds of that Difference by the Number of the Days between both the Obſervations, to get the daily Difference.

Let us ſuppoſe this ſecond Obſervation to have been made the 20ᵗʰ of *March*, *viz.* Eight Days after the firſt, and finding that the Middle of the *Sun*, being ſeen in the Meridian in the ſame Line with the two Threads, as before,

The *Watch* points, — ——— ——— ——— ———	11	51′	07″.
The *Æquation* of the 20*th* of *March*, by the *Table*, is —	00	10	40
Which being added to the Time ſhow'd by the *Watch*, gives	12	01	47

If this had been juſt 12 Hours, the *Watch* would have been well adjuſted, but being 1′ 47″, more than 12, it hath gone ſo much too faſt in eight Days. And theſe 1′ 47″, that is 107″, being divided by 8, there comes $13\frac{3}{8}$ Seconds for the Difference of every 24 Hours; which Difference being known, if you want time, or have no mind to take the Pains to adjuſt the *Watch* to its right meaſure, (this being not neceſſary, ſince you may bring it thus on *Ship-Board*) note only the daily Difference, and regulate yourſelf accordingly. But if you will adjuſt it better, you muſt remove the leſs Weight of the *Pendulum* a little downwards, which will make it go ſlower, and then you muſt begin a-new to obſerve by the *Sun*, as before. If it had gone too ſlow, you muſt have removed the mentioned Weight ſomewhat upwards. And this is of that Importance in the finding out of *Longitudes*, that if it be not obſerved, you may ſometimes in the ſpace of Three Months miſreckon 7 Degrees, and more yet (without any fault in the *Watches*;) which under the *Tropicks* will amount to above 400 *Engliſh Miles*.

The *Watch* may alſo be adjuſted *on Board*, when a Ship Rides at Anchor, thus: In the Morning, when the *Sun* is juſt half above the *Horizon*, Note what Hour, Minute and Second, the *Watch* points at, if it be going; if not, ſet it a going, and put the *Indexes* at what Hour, Minute, and Second you pleaſe. Let them go till *Sun-Set*, and when the Body of the *Sun* is juſt half under the *Horizon*, ſee what Hour, Minute, and Second the *Indexes* of the *Watch* point at, and note them too; and reckon, how many Hours, &c. are paſſed by the *Watch* between the one and the other. Then take the half of that Number, and add it to the Hours, &c. of the Morning Obſervation, and you ſhall have the Hours, &c. which the *Watch* did ſhow, when the *Sun* was in the *South*; whereunto add the *Æquation* in the Table belonging to that Day, and note the Sum. Then, ſome Days being paſs'd, (the more the better) you are to do juſt the ſame: And if the Hour of this laſt Day be the ſame that was noted before, your *Watch* is well adjuſted; but if it be more or leſs, the Difference divided by the Number elapſed between the two Obſervations, will give the daily Difference. And if you will, you may let it reſt there, or otherwiſe, removing the leſſer Weight of the *Pendulum*, you may adjuſt it better. You may alſo, inſtead of the *Sun's Riſing* and *Setting*, take two equal Altitudes of the *Sun*, before, and after Noon, and having noted the time given by the *Watches* at the time of both the Obſervations, proceed with it in the ſame Manner, as was juſt now directed for obſerving the *Sun* in the *Horizon*. In either of which ways there may be ſome Error, cauſed by the *Sun's Refraction*; which is inconſiderable, and therefore needs not to be taken notice of.

To find by them the Longitude at Sea.

5. Give to each of the *Watches* a Name, or a Mark, as A, B, C; and before you ſet *Sail*, ſet them to the Time obſerved by the *Sun* in the Place where you are, and whence you are departing, allowing for the *Æquation* of the Day whereon you make your Obſervation; which Day you are to note, if the *Watches* be not well adjuſted; otherwiſe it is not neceſſary.

Then afterwards being at *Sea*, and deſiring to know the *Longitude* of the Place where you are; that is, How many degrees the Meridian of that Place is more Eaſterly or Weſterly, than the Meridian of that Place where you did ſet the *Watches*; you muſt obſerve by the *Sun* or *Stars*, what Time of the Day it is, as preciſely as is poſſible, and Note at the ſame time, to what Hour, Minutes and Seconds the *Watches* do point (which Time, if the *Watches* be not ſet to the Right Meaſure, is by the known Daily Difference to be adjuſted,) adding thereunto the *Æquation* of the preſent Day, which gives you the time of the Day, ſhewed by the Sun, at the Place where the *Watches* were ſet: And, if this time of the Day be the ſame with that obſerved where you are, then you are under the ſame Meridian with the Place where the *Watches* were Set by the *Sun*; but, if the time of the Day, obſerved where you are, be greater than that ſhewed by the *Watches*, you may be aſſured, that you are come under a more Eaſterly Meridian; and if leſs you are come under a more Weſterly. And counting for every hour of difference of Time, 15 Degrees of *Longitude*, and for every Minute, 15 Minutes, or ¼ of a Degree, you ſhall then know, how many Degrees, Minutes, &c. the ſaid *Miridians* do differ from one another. *E. g.* Suppoſe the *Watches* A, B, C, were Set at the Place, whence you parted, on the 20*th* of *February*, to the time of Day obſerved by the *Sun*, abating the *Æquation* of the 20*th* of *February*, (*viz.* 2′ 28″.) and ſuppoſe that the *Watch* A, be ſet to its Right Meaſure, but that B goes every Day 7″ too Slow, and C every Day 12″ too faſt: Some Days after, ſuppoſe the 5*th* of *May*, deſiring to know the *Longitude* of the Place where you are at *Sea*.

You obſerve the Time of the Day there to be	05^{h}	18′	10″
And you find the *Watch* A to point at	02	06	00
But the *Watch* B to point at	1	57	22
Going too ſlow by 7″ every Day, which makes in 74 Days (*viz.* From the 20*th* of *Feb.* to the 5*th* of *May*,)	00	08	38
Which being added to its own Time, gives the ſame with that of the *Watch* A, *viz.*	02	06	00
You find alſo the *Watch* C to point at	02	20	48
Going 12″ too faſt every Day, which makes in 74 Days	00	14	48
Which being Subducted from its own Time, gives again	02	06	00
The Time of the Day therefore by the *Watches* being	02	06	00
Add thereunto the *Æquation* of the 5*th* of *May*	00	19	29
And ſo you have for the Time of Day at the Place where the *Watches* were ſet	02	25	29
But the Time obſerved being	05	18	10
Exceeds this by	02	52	41
Wherefore the *Meridian* of the Place, where you are *May* 5. is more Eaſterly, than the place where the *Watches* were ſet by	02	52	40
Which being reduced to Degrees, reckoning 15 Degrees for an Hour, comes to	43°	10′	15″

'Tis true, that from the ſame Reckoning it may be concluded, that you are 180 Degrees more Eaſterly; which happens, becauſe the *Hour Index* goes round in the ſpace of 12 Hours in the *Watches*; but the Difference is ſo great, that one cannot be deceived in it; elſe the *Watch* might be ſo made, that the *Index* ſhall go round about once in 24 Hours.

To find the Time of the Day at Sea.

6. Since that for finding the *Longitude*, the Time of the Day, at the Place where you are, muſt be known, (as hath been ſaid above you muſt have a

care to obſerve that Time as preciſely as poſſible. For every Minute of Time, that you miſreckon, makes ¼ Degree in *Longitude*, which amounts, near the *Æquator*, to above 15 *Engliſh Miles*, but leſs elſewhere. Wherefore to find the Time of the Day with Certainty, the beſt way is to obſerve the *Sun's Altitude* when it is in the Eaſt or Weſt, (the nearer the better :) for, being there, its *Altitude* changes in a ſhort time more ſenſibly than before or after; and thus from the Height of the Pole and the Declination of the Sun, the Hour may be calculated.

7. At the *Riſing* and Setting of the *Sun*, when it is half above the *Horizon*, *An Eaſier Way.* mark the time of the Day, which the *Watches* then ſhew; and tho' ye have in the mean time ſailed on, it is not conſiderable. Then reckon by the *Watches* what Time is elapſed between them, and add the half thereof to the Time of the *Riſing*, and you ſhall have the Time by the *Watches*, when the *Sun* was at *South*, to which is to be added the *Æquation* of the preſent Day by the Table. And, if this together makes 12 Hours, then was the Ship at Noon under the ſame *Meridian* where the *Watches* were ſet with the *Sun*. But, if the Sum be more than 12, then was ſhe at Noon under a more *Weſterly Meridian*; and, if leſs, then under a more *Eaſterly*; and that by as many Times 15 *Deg.* as that Sum exceeds or comes ſhort Hours of 12; as the Calculation thereof hath been already deliver'd.

Suppoſe, *e.g.* that the *Watches* A and B, as before, were ſet with the *Sun* at the Place whence you parted, the 20 of *February*; and the *Indexes* ſet to the Hour, Min. and Sec. ſhewed by the *Sun*, abating the *Æquation* of that Day, *viz.* 2′ 20″; the *Watch* A being reduced to the right Meaſure, and B going too ſlow by 7″ a day. Afterwards on the 22*d* of *May* deſiring to know the *Longitude* of the Place to which you are come, you obſerve in the

Morning the *Sun* half above the Horizon, when the *Watch* points at	2h	30′	10″
And in the *Evening*, the *Sun* being half under the *Horizon*, when the ſame *Watch* points at	3	8	40
To find the Time elapſed between them, ſubducting the Time of the *Riſing*	2	30	10
From	12	0	0
There Remains	9	29	50
Adding thereunto the Time of the *Setting*	3	8	40
You have for the Time elapſed between the Obſervations	12	38	30
Whereof the Half	6	19	15
Being added to the Time of *Riſing*	2	30	10
You have the Time by the *Watch* A, when ☉ was in the *South*	8	49	25
And after the ſame manner you are to ſeek the Time by the *Watch* B, when the *Sun* was in the *South*; which let be	8	38	48
But this *Watch* going 7″ a day too ſlow, it is retarded in 19 days (from the 20*th* of *Febr.* to the 22*d* of *May*)	0	10	37
Which therefore added to the ſaid Time gives	8	49	25
That is the ſame Time given by the *Watch* A. Now adding to this Time of the *Watches*, the *Æquation* of the 22*d* of *May*	0	18	10
You have	9	7	35

Which is the ſame time of the Day with that of the Place, where the *Watches* were ſet when the *Sun* was in the ſame *Meridian* with the Ship, or where the Ship was at Noon. h ′ ″

The Difference is —— —— —— —— —— 2 52 25

Wherefore this laſt *Meridian* is by ſo much more *Eaſterly* than the firſt, which being reduced to Degrees, as (as hath been formerly directed) make —— —— —— —— —— ° ′ ″ 43 6 15

'Tis manifeſt, that by this way you find preciſely enough the *Longitude* of the Place, where you were at Noon, or the Time of the Sun's being in the South: Which altho' it differs from the *Longitude* of the Place, where you are when you obſerve the *Setting* of the *Sun*; yet you may eſtimate near enough, how much you have advanc'd or chang'd the *Longitude* in theſe few Hours, by the *Log-Line*, or other ordinary Practices of *Reckoning* the Ship's Way; or (which is the ſurer Way) by the Degrees paſſed in 24 Hours by a former day's Obſervation.

You may alſo, inſtead of obſerving the *Sun's Riſing* and *Setting*, obſerve the *Setting* firſt, and then the next Morning the *Riſing*; marking at both Times the Time ſhew'd by the *Watches*; and find thence, after the ſame manner as before, the *Longitude* of the Place where the Ship was at Midnight.

Finally, You may alſo, inſtead of the *Riſing* and *Setting* of the *Sun*, obſerve before and after Noon two *Equal Altitudes* of the *Sun*, noting the Time ſhown by the *Watches*, and reckoning in the ſame manner, as hath been ſaid of the *Riſing* and *Setting*: Yet it is to be conſider'd, that the *Altitudes* of the *Sun* are beſt taken, when it is about Eaſt and Weſt, as hath been already intimated. But note, that in Sailing North and South you make not the Obſervations at the Sun's *Riſing* and *Setting*, but at its being due *Eaſt* and *Weſt*.

8. But you may put the Rule here preſcribed in Practice, by taking two *Equal Altitudes* of ſome known *Star*, that riſeth high above the Horizon. For you ſhall thence, according to the mentioned Rule, know at what time by the *Watches* the Star hath been in the South; and ſo the *Right Aſcenſion* of that Star being known, as alſo the *Right Aſcenſion* of the *Sun*, you may thence eaſily calculate, what Time it then was: Which, being compared with the Time of the *Watches*, as before, ſhall give the *Longitude* of the Place where you were, when you had the Star in the Meridian.

9. If the *Watches* that have gone exactly for a while, ſhould come to differ from one another (as in length of time it may well happen, that the one or other fail a Minute, more or leſs;) in that caſe it will be beſt to reckon by that which goes faſteſt; unleſs you perceive an apparent Cauſe, why it goes too faſt (as it may happen when the *Cheeks* retain not their proper Figures) ſeeing it is not ſo eaſy for theſe *Pendulum-Watches* to move faſter than at firſt, as it is to go ſlower. For the *Wire*, on which the *Pendulum* hangs, may perhaps by the violent Agitation of the Ship, come to ſtretch a little, but it cannot grow ſhorter; and the little weight of the *Pendulum* may perhaps ſlip downwards, but cannot get up higher.

n. 48. p. 976.

If it ſhould be ſaid, that upon any Foulneſs the *Watch* will go faſter by reaſon of the ſhorter Vibrations of the *Pendulum*, it is to be conſidered, that this is only True when the *Watches* have no *Cheeks*, but when they have them 'tis not ſo.

n. 47. p. 952.

10. When you get Sight of any known Country, Iſland or Coaſt, be ſure to note the *Longitude* thereof as exactly as you can by the help of the Rules here preſcribed. Firſt, thereby to correct the *Sea-Maps*, after the *Longitude* of a Place ſhall have been found at divers times to be the ſame, ſo that you doubt no more of it. For all *Maps* are very defective as to the Situation of Places in reſpect of Eaſt and Weſt, chiefly where Seas are interpoſed. Secondly, to be able always to know in the Proſecution of your Journey, how far you have ſailed from any Place to the Eaſt or Weſt. And, if by any notable Miſchance or Careleſſneſs all the *Watches* ſhould come to ſtand ſtill, yet you may at any Place, whereof the *Longitude* is certainly known, ſet them a going again, and adjuſt them there by the *Sun*, and ſo reckon the *Longitudes* from that ſame Meridian. For you are to know, that you are not at all obliged to put one certain Meridian of any known Place as a Beginning of the *Longitude Reckoning*; this happening only in *Maps*, or Tables of *Longitude:* As when you take for that purpoſe the *Meridian* of the *Pico* in *Teneriffe*, or that of the Iſlands of *Corvo* and *Flores* (the moſt Weſterly of the *Azores*) or any others. Yet it were very fit, that all *Geographers* agreed and pitched upon one and the ſame *Firſt Meridian*, that ſo all Places might be known by the ſame Degrees as well of *Longitude* as *Latitude*; tho' in Voyaging, it is ſufficient to obſerve only the Difference of *Longitudes*, beginning to reckon from the *Meridian* of any Place, you pleaſe, as if it were the *Firſt*.

11. If it happen that being at *Sea* all the *Watches* ſtop, you muſt as ſpeedily as is poſſible, ſet them a moving again, that you may know how much you advance from that Place towards the *Eaſt* or *Weſt*.

The Journal for the Watches.

12. The *Watches* being diſtinguiſhed by Marks, as A, B, or the like, every Day about Noon, or when moſt conveniently you can, obſerve the Time of the Day by the *Sun*, or by the *Stars* at Night, and ſubduct thence the Minutes and Seconds, that are adjoined to that Day in the Table, and write the Remainder down in the Paper, wherein 9 Columns or more are marked, placing them in the ſecond Column, having placed the Day of the Month in the Firſt; and at the ſame time write down the Hours, Minutes, and Seconds, of each *Watch* in a diſtinct Column, all oppoſite one to another. Then in another Column write down the Difference between the Time taken by Obſervation, and that given by the *Watches*, or one of them. Then one Column for the *Latitude:* One for the *Longitude* by the Ordinary Way of *Reckoning:* Another for the *Longitude* taken from the Difference between the Time found by Obſervation, and that given by the *Watches:* And at laſt a large Column to note the Accidents, that befal the *Watches*, &c.

The Succeſs of Pendulum Watches; by Maj. Holmes. n. 1. *p.* 13.

2. Major *Holmes* having left the Coaſt of *Guinea*, and being come to the Iſle of St. *Thomas* under the *Line*, he adjuſted his *Watches* there, and put to Sea, and Sailed Weſtward, 7 or 800 Leagues, without Changing his Courſe;

after which finding the Wind favourable, he ſteered towards the Coaſt of *Africk* N. N. E. But, having ſailed upon that Line, a matter of two or three hundred Leagues, the Maſters of the other Ships under his Conduct, apprehending that they ſhould want Water, before they could reach that Coaſt, did propoſe to him to ſteer their Courſe to *Barbadoes*, to ſupply themſelves with Water there. Whereupon the ſaid *Major*, having called the Maſters and Pilots together, and caus'd them to produce their Journals and Calculations, it was found that thoſe Pilots did differ in their Reckoning from that of the *Major*, one of them about 80 Leagues, another about a hundred, and the third more; but the *Major*, judging by his *Pendulum Watches* that they were only ſome thirty Leagues diſtant from the Iſle of *Fuego*, which is one of the Iſles of *Cape Verde*, and that they might reach it next day, and having a great Confidence in the ſaid *Watches*, reſolved to ſteer their Courſe thither; and having given Order ſo to do, they got the very next Day about Noon a Sight of the ſaid Iſle of *Fuego*, finding themſelves to ſail directly upon it, and ſo arrived that Afternoon as he had ſaid.

M. Huygens being informed of this Succeſs, wrote to *Paris* to this effect; Ib. p. 14. I did not magine that the *Watches* of this Firſt Structure would ſucceed ſo well, and I had reſerved my main Hopes for the New ones. But, ſeeing that thoſe have already ſerved ſo ſucceſsfully, and that the other are yet more juſt and exact, I have the more Reaſon to believe, that the Invention of *Longitudes* will come to its Perfection. In the mean time I ſhall tell you, that the States did receive my Propoſition, when I deſired of them a Patent for theſe New *Watches*, and the Recompence ſet apart for the Invention, in Caſe of Succeſs; and that without any difficulty they have granted my Requeſt, commanding me to bring one of theſe *Watches* into their Aſſembly, to explicate unto them the Invention, and the Application thereof to the *Longitudes*; which I have done to their Contentment.

Longitudes from the Moon's Places; by Math. Profeſſor *at* Seville. n. 118. p. 427.

III. 1. I have at laſt found a Method of knowing the *Moon's Place*, by the Aſſiſtance only of a ſmall Inſtrument, within a Scruple or two; and which is wonderful, neither the *Refractions* nor the *Parallaxes* are any hindrance to my Obſervations, becauſe the Contrivance of the Method delivers me from theſe Niceties. I can uſe this Method either at Sea or Land; therefore the *Tables* of the *Moon* being corrected, a Method will no longer be wanting of taking the ſo much deſired *Longitudes* of Places, wherever they are, either at Sea or Land,

Conſidered by Mr. Flamſteed. Ib. p. 431.

2. What the Profeſſor of *Seville* writes about his little Inſtrument, with his leave I muſt own exceeds my belief. In our *Horizon* the *Moon* cannot be clear'd either of *Refraction* or *Parallax*, unleſs ſhe could aſcend as high as the *Zenith*. For the *Refractions* are extended ſo far, and make her Place troubleſome to find. Once only in a Day the *Parallax* of *Longitude* ceaſes in the Nonageſim Degree of the *Ecliptic*, but not ſo that of *Latitude*. Nor can I comprehend how an Inſtrument ſhall be conſtructed, which together with the *Parallax* takes notice of the *Refraction*, the Increaſe of which is owing to very different Principles.

IV. The Obſervation of *Lunar Occultations* is of Singular Uſe to determine the *Longitude* of Places, eſpecially thoſe that are far remote.

Longitudes *from Lunar Occultations; by* M. Halley. n. 181. *p.* 87.

V. The *Revolution* of *Jupiter* upon his *Axis* being the ſwifteſt, and the moſt Regular Motion that is hitherto known in the Heavens, a Traveller alone, even without having any Correſpondence with other Obſervers, may make uſe of it to find the *Longitudes* of the moſt remote Places of the Earth.

Longitudes *by the Revolution of* Jupiter *upon his Axis; by M.* Caſſini. n. 82. *p.* 4042.

VI. 1. I am ſomething in doubt whether the *Eclipſes* of *Jupiter*'s Satellites are equally convenient for inveſtigating the Difference of *Meridians*, as the Occultations of the fixt Stars by the *Moon*. Particularly becauſe of the too ſlow Motion of the *Jovials*, tho' the Obſervations were perform'd with a more accurate Tube.

Longitudes *by the Satellite Eclipſes; by M.* Hevelius. n. 78. *p.* 3030.

2. The *Eclipſes* of the *Satellites* of *Jupiter*, which happen almoſt every Day, afford a fair Way for eſtabliſhing the *Longitudes* over all the Earth. For, beſides that theſe *Eclipſes* are very frequent, the *Emerſion* and *Immerſion* of theſe *Satellites*, eſpecially in the Shadow of *Jupiter*, is ſo Momentary and ſo Senſible, that they may be obſerved with the greateſt Exactneſs, being altogether exempt from thoſe Eſſential Inconveniences that accompany the *Eclipſes* of the *Sun* and *Moon*, which alſo are rare, and whoſe *Beginning* and *End* are always doubtful by reaſon of a certain *Ambiguous Light*.

By M. Borelli. n. 128 *p.* 691.

The *Longitude* of Places at *Sea*, *Capes*, *Promontories*, and divers *Iſlands*, being once exactly known by this Means, would doubtleſs be of Great Help, and conſiderable Uſe to *Navigation*.

3. The *Eclipſes* of *Jupiter*'s *Satellites* have been eſteemed, and certainly are a much better Expedient for the Diſcovery of the *Longitude* than any yet known, by reaſon that they happen frequently, and are eaſily obſervable with a Teleſcope of 12 Foot, or for need with one of eight.

By Mr. Flamſteed *n.* 151. *p.* 322.

The *Longitude* might be alſo attained by Obſervations of the *Moon*, if we had Tables that would anſwer her Motions exactly; but after 2000 Years Experience (for we have ſome Obſervations of *Eclipſes* much Ancienter) we find the beſt Tables extant erring ſometimes 12 Minutes or more in her apparent Place, which would cauſe a Fault of half an Hour, or 7½ *deg.* in the *Longitude* deduced, by comparing her *Place* in the Heavens with that given by the *Tables*. I undervalue not this Method, for I have made it my Buſineſs, and have ſucceeded in it, to get a large Stock of good *Lunar Obſervations* in order to the Correction of Her *Theory*, and as a Ground-Work for better *Tables*; but, if we ſhould happily attain what we ſeek, yet the Calculation will be ſo perplexed and tedious, that it will be found much more inconvenient and difficult than that I propoſe by obſerving the *Eclipſes* of *Jupiter's Satellites*, which however at preſent I muſt prefer. For I am perſuaded, that the *Eclipſes* of the *Firſt* will ſcarcely be found above 4 Minutes of Time different from my Calculations, and I hope it will ſcarce ever be found to err ſo much. But, if the ſame *Eclipſe* may be obſerved in two diſtant

n. 154. *p.* 404. n. 165. *p.* 760.

diſtant Places at the ſame time, or compared with an Obſervation of the ſame *Satellite*, made within a Week elſewhere, the *Difference* of *Meridians* will be had ſomething better, than by comparing two Obſervations of the ſame *Phaſis* of a *Lunar Eclipſe*, made in diſtant Places. For, whereas it is ſomewhat difficult by reaſon of the *Penumbra* to determine the True Time of the Application of either of the *Moon*'s Limbs to the Shadow, the *Satellite Eclipſes*, eſpecially thoſe of the *Firſt*, are almoſt Momentary.

And, whereas there can rarely happen 4 *Eclipſes* of the *Moon* Viſible, the ſame Year, thoſe of the *Satellites* happen ſo frequently that there are more of them Viſible in one Year than we count Days in it, tho' the Planet *Jupiter* lie hid under the *Sun*'s Rays every Year a whole Month together.

I know our *Navigators* will object againſt this Method, that it is difficult to practiſe at *Sea*, becauſe long Teleſcopes are required, which the Motion of the *Ship* will not permit them to manage aboard. But, if it be not practicable at *Sea*, they cannot deny but that it is at *Land*; and that the *True Longitude* of remote Coaſts from us is the firſt thing deſired for the Correction of their Charts; (Let them attempt theſe firſt, and I doubt not but the Succeſs will encourage them ſo much, that they will readily find means to put it in Practice at *Sea*;) That the *French* have uſed this Method ſucceſsfully both in *Denmark* and in their *Own Country*; That a Teleſcope of 14 Foot Long at moſt, or for need one of 8 Foot, with broad *Eye Glaſſes*, will be ſufficient for this purpoſe; That the Difficulty cannot be known till it be tried, and that Uſe renders may things eaſy, which our firſt Thoughts conceived impracticable.

If it be required to know whether any of thoſe *Eclipſes* which are inviſible with us, be viſible in any other given Place, convert the *Difference* of *Meridians* betwixt it and *London* into Time; and, if the Place lie to the *Eaſt* of *London*, add it to, if to the *Weſt*, ſubtract it from, the Time of the Appearance at *London*; the Sum or Difference accordingly ſhall be the true Time of the *Eclipſe* under that *Meridian*, at which, if *Jupiter* be above the Horizon, and the *Sun* beneath it, the *Eclipſe* is there viſible, otherwiſe not.

Or, By the help of the *Ephemerides* of the *Planet*'s Places, and a *Terreſtrial Globe*, the ſpace on it, in which any of theſe *Eclipſes* will be viſible, may be found thus:

Firſt ſeek the true Places of the *Sun* and *Jupiter* with his *Latitude* in the *Ephemerides*, whereby you may find their Declinations and Right Aſcenſions, either by the Vulgar Tables, or the Globe itſelf, exactly enough for this Method.

Bring *London* on the *Globe* to the *Meridian*, and, detaining it there, note what *Deg*. of the *Æquator* is cut by it. From this ſubtract the Time of the *Eclipſe* after Noon converted into *deg*. and *min*. the Remainder ſhews you the *Longitude* of that *Meridian* on the Earth, where it is then Noon when the *Satellite* is *Eclipſed*, which I therefore call the *Meridional Longitude* of the *Eclipſe*. Bring this *Meridional Longitude* under the *Meridian*, and elevate the nearer Pole to the *Sun* as much as is his *Declination*: keep the

Globe in this Poſition, and, if *Jupiter* be in *Conſequence* of the *Sun*, draw a Line of the *Globe* along the *Eaſtern Horizon*, it paſſes over all thoſe Places where the *Sun* is Setting at that Time; but, if *Jupiter* be in *Antecedence* of the *Sun*, draw the ſaid Line on the *Globe* by the *Weſtern* Edge of the *Horizon*, it paſſes over all thoſe Places where the *Sun* is then Riſing. *Jupiter* being in *Conſequence* of the *Sun*, add the Difference of *His* and the *Sun*'s Right Aſcenſions to the *Meridional Longitude* afore-mentioned, bring the *Deg.* of the *Æquator* anſwering their Sum under the Meridian, raiſe the *Pole* next *Jupiter* equal to his Declination, and, detaining the *Globe* in this Poſition, draw a Line again to the *Eaſtern Horizon*; the Space intercepted betwixt this and the Line of the *Sun*'s Setting, before deſcribed on the *Globe*, comprehends all thoſe Places on the Earth where this *Eclipſe* is ſeen from *Sun* Setting till *Jupiter* is Set. But, if *Jupiter* were in *Antecedence* of the *Sun*, Subtract the difference of his and the *Sun*'s Right Aſcenſions from the *Meridional Longitude*, ſet the *Degree* of the Æquator anſwering the Remainder under the *Meridian*, and elevate the *Pole* next *Jupiter* equal to his Declination: Keeping the *Globe* in this Poſition, draw a Line by the *Weſtern* Edge of the *Horizon*, the Space included betwixt this and the Line of the *Sun*'s Riſing contains all thoſe places on the Earth, where this *Eclipſe* is viſible betwixt *Jupiter*'s *Riſing* and *Sun-riſe*.

When any *Eclipſe* of theſe is obſerved, the Difference betwixt the noted Time and that given by the *Tables* ſhall be the *Difference* of *Meridians* betwixt the Place of the Obſervation and *London*.

As the *Sun* removes from the Conjunction of *Jupiter*, the *Ingreſſes* of the *Satellites* into his Shadow become obſervable. When he is about 30° from it, the *Emerſions* of the *Fourth*, and at 60°, of the *Third*, began to be ſeen betwixt the Shadow and Body, continuing ſo till the *Sun* be arrived within 60° of the Oppoſition of *Jupiter*, when the *Emerſions* of the *Third* fall behind his Body, but the *Emerſions* of the *Fourth* continue viſible till he be leſs than 30° diſtant from ☍: at which Time they alſo are hid behind him, all the Appearances being made really to the Right Hand, or in Antecedence of *Jupiter*, tho' with inverting Teleſcopes, they appear on the contrary, to the Left.

After the Oppoſition of the *Sun* and *Jupiter* we begin to ſee the *Immerſions* of the *Satellites* from the Shadow now on the Left Hand, or in Conſequence of *Jupiter*, but thro' inverting Glaſſes on the Right; when the *Sun* is near 30 *deg.* from the Oppoſition, the *Ingreſſes* of the *Fourth*, when 60 *deg.* from it, of the *Third*, begin to be obſervable betwixt the Body and Shadow, continuing ſo till the *Sun* arrive at the ſame or rather within ſomething a wider Diſtance for the Conjunction of *Jupiter*.

4. By this very eaſy Method, to be perform'd with a very little Apparatus of Inſtruments, the Foundations for reſtoring *Geography* may be laid. *By Mr.* Halley, n. 121. p. 435. The Precepts leading to this cannot be unknown to *Aſtronomers*. It may be proper to take notice of one thing, that with a Tube of 7 or 8 feet, which is eaſily portable, the Moments of theſe *Eclipſes* may be obſerved diſtinctly enough, particularly in the exterior *Satellites*; eſpecially if the Aperture of the Object-glaſs be 2½ or 3 Inches. For thus the greateſt Quantity of

the refracted Rays will come to the Eye, whence theſe very little Stars near *Jupiter* may be ſeen, which otherwiſe would be extinguiſh'd by his greater Light. And tho' they may be tinged with Colours, and the Limb of *Jupiter* may ſeem not very bright, yet ſince we are only concern'd about the Moment of loſing or of recovering the Light, it is ſufficient that they ſtrike our Eyes ſurely, increaſed in their Light as much as may be.

n. 214. *p.* 237.

The *Eclipſes* of the *Firſt Satellite* of *Jupiter* are found by the *Royal Academy* at *Paris*, in Aſcertaining the Geographical Site of the Principal Ports of *France*, almoſt *Inſtantaneous*, and with good Teleſcopes diſcernible almoſt to the very Oppoſition of *Jupiter* to the *Sun*. So that, could the *Satellites* be obſerved with Teleſcopes manageable on *Shipboard*, a *Ship* at *Sea* might be enabled to find the Meridian ſhe was in to a very great exactneſs, beyond what we can yet hope to do by the *Moon*, tho' they ſeem to afford us the only Means practicable for the *Seamen*. However, before *Sailors* can make uſe of the Art of *Finding* the *Longitude*, it will be requiſite that the *Coaſt of the whole Ocean* be firſt laid down truly; for which Work this Method by the *Satellites* is moſt appoſite.

Long. and Lat. of Derby; *by Mr.* Flamſteed. n. 55. *p.* 1103, 1106. n. 111. *p.* 237.

VII. The *Longitude* of *Derby* from *London* W. is 5 or 6 *min.* the *Latitude* 52° 57′ or 59′.

Lat. of Ecton. n. 76. *p.* 2272.

VIII. The *Latitude* of *Ecton* in the County of *Northampton* is 52° 15′.

Long. and Lat. of Townley. n. 127. *p.* 664.

The obſerved *Latitude* of *Townley* in the County of *Lancaſter*, (as Mr. *Townley* writes,) is 53° 44′. and its *Longitude* from the Meridian of *London*, is about 9 Minutes to the Weſt.

Lat. of Tredagh. n. 164. *p.* 749.

X. The *Latitude* of *Tredagh* in *Ireland* is 53° 40′.

Long. of Oxford *and* Dantzick; *by Mr.* Haley. n. 129. *p.* 724. *vid ſup. Cap.* IV. § LXII.

XI. Having carefully conſidered the *Parallaxes* of the *Moon* in the Obſervations of the Occultations of *Mars*, *Aug.* 21. 1676 at *Dantzick* and *Greenwich*, I find from the *Immerſion* the *difference* of *Meridians* between *Greenwich* and *Oxford* $4'\ 57''$, between *Greenwich* and *Dantzick* $1^{h}\ 14'\ 50''$: By the *Emerſion* the *firſt* of theſe *Differences* is found $4'\ 59''$, the latter $1^{h}\ 14'\ 41''$; which near Agreement ſhews the exactneſs of the Obſervations.

Long. of Paris; *by M.* Caſſini. n. 117. *p* 390. *vid. ſup. Cap.* V. XLV.

XII. 1. I took much Pleaſure in comparing the Obſervations of Mr. *Flamſteed*, about the Eclipſe of the *Moon*, *July* 7. New Style 1675. with ours made in the *Royal Obſervatory*. For from them I ſee confirm'd within a few Seconds, by ten new Compariſons, the *Difference* of *Meridians* which formerly I had defined to be 11 Minutes, by a Compariſon of our Obſervations.

Mr. *Flamſteed*'s Obſervations.				Our Obſervations.				Diff. of Meridians.		
	h.	′	″		h.	′	″		′	″
Pentadactylus cover'd	1	55	15	The ſame, or *Seleucus*	2	06	15		11	00
Porphyrites cover'd	2	02	20	The ſame, or *Ariſtarchus*	2	12	40		10	20
The firſt Limb of *Sinai*	2	05	30	Of the ſame, or of *Tycho*	2	16	30		11	00
				or *		16	25		10	55
The firſt Limb of *Ætna*	2	06	00	Of the ſame, or of *Copernicus*	2	16	30		10	30
				or *		16	40		10	40
The firſt Limb of *Beſbicus*	2	23	05	Of the ſame, or of *Manilius*	2	34	15	Leſs than	11	05
Horminius cover'd	2	26	03	To the ſame, or *Dionyſius*	2	36	15	Greater than	10	12
It touch'd the firſt Limb of *Corocondometus*	2	39	30	Of the ſame, or *Palus Somni*	2	50	20		10	50
It touch'd *Palus Mæotis*	2	45	00	The ſame, or *Mare Caſpium*	2	55	20		10	20
				or *		55	40		10	40
Mæotis wholly cover'd	2	50	40	The ſame	3	01	10		10	30
Immerſion	2	56	55	*Immerſion*	3	07	45		10	50
				or *	3	07	40		10	45

This Mark * denotes the particular Determination of Mr. *Caſſini*; in the reſt he agreed with Mr. *Pichard* and Mr. *Romer*.

2. The Middle of the Lunar *Eclipſe*, *Jan.* 1. New Style, 1675. h. ′ ″ n. 123. p. 562.
is derived from a Compariſon of the Beginning and End 3 20 00
Of two equal *Phaſes* ——— ——— ——— ——— 3 20 15 vid. ſup. Cap. IV. §. XLVI.

From the Obſervations of Mr. *Flamſteed* the Middle of the *Eclipſe* may be derived in a like manner. For he at $2^h\ 29'\ 30''$, obſerved the Diſtance of the Cuſpids to be $17'\ 16''$. and at $3^h\ 52'\ 45''$, the *Eclipſe* decreaſing he obſerved the ſame Diſtance $18'\ 57''$, that is, $1'\ 41''$. greater. Therefore the Middle of the *Eclipſe* is nearer the latter Obſervation than the former. The middle Time between both the Obſervations was $3^h\ 11''\ 7''$. Therefore the Middle of the *Eclipſe* is derived from hence ſomething later. Whence the Difference of *Meridians* would come out leſs than $9'$; which but little agrees with more certain Obſervations of the preceding Summer *Eclipſe*, from whence I determin'd it to be $10\frac{1}{4}$ Minutes. Our firſt Obſervation compared with the former of Mr. *Flamſteed*, which was a little later, exhibits the Difference of *Meridians* greater than $8'\ 35''$. Our latter, being later than Mr. *Flamſteed*'s latter Obſervation, would exhibit the Difference of *Meridians* leſs than $9'\ 40''$.

	h.	′	″
The End as eſtimated by Mr. *Flamſteed* ——— ——— —	4	07	15
And by us ——— ——— ——— ——— ——— ——— —	4	15	25
Would infer the Difference of *Meridians* to be ——— ———	0	08	10
The Beginning obſerved by Dr. *Halley* at *London* ——— ———	2	16	00
With that obſerved by us ——— ——— ——— ——— —	2	24	35
Would make the Difference of *Meridians* to be ——— ———	0	08	35

Therefore from this *Eclipſe* the Difference of *Meridians* would come out leſs by about two Minutes than by the *Eclipſe* of the foregoing Summer, which yet I prefer far before this. Not only in reſpect of the greater Facility of determining the Times of the Appulſes and Emerſions in that total *Eclipſe* than in this partial one; but alſo becauſe of the clearneſs of the Air, which we enjoy'd alike in that *Eclipſe*, whereas in this the Air was very ſerene at *Paris*, but at *London* it was cloudy. Therefore I judge we ought to ſtand by the former, till we can determine the Matter with greater exactneſs, by Obſervations of the *Immerſions* and *Emerſions* of the *Satellites* of *Jupiter*, which I think better fitted for this purpoſe. *Ib. p.* 564.

By Mr. Flamſteed. *Ib. p.* 565.

3. I can hardly truſt to the Difference of *Meridians*, derived from the *Eclipſe* of the *Moon* of *June* 27. 1675. obſerved at *London* and at *Paris*; for tho' I believe the Times of the *Phaſes* obſerved by you to be very accurately determin'd; yet as I was obliged to make uſe of a Quadrant of only 20 Inches, becauſe a larger was not at hand, which had only plain Sights, in in order to correct my Clock; therefore I can hardly be ſure of the Moment of any Phaſis, nearer than to a Minute. I was better furniſhed when I obſerved the laſt *Eclipſe* of *Dec*. 22. 1675. but as the Air was ſomething cloudy, and becauſe of the oblique Incidence of the *Moon* into the *Earth*'s Shadow, its Appulſe to the Spots was very ſlow, and thence this *Eclipſe* was not ſo proper for this purpoſe. Therefore the Difference of our *Meridians* ſtill remains uncertain within two Minutes, which I do not queſtion but we ſhall ſometime determine to our Wiſhes.

Long. of Straſburg *and* Paris; *by* M. Bullialdus. n. 125. p. 610. *vid. ſup. Cap.* IV. § XLVI.

XIII. The Meridian of *Paris* is diſtant from that of *Straſburg* 22′ 48″. from the End of the *Lunar Eclipſe* of *Jan.* 1. New Style 1676. And by this *Eclipſe* the *Meridian* of *Paris* is diſtant from that of *London* 6′ 38″ Eaſterly. From the Obſervation of the *Eclipſe* of *July* 7. 1675. this appear'd to be 10′, as alſo from the *Eclipſe* of *Jan.* 11. of the ſame Year.

Long. of Avignon; *by Mr.* Halley. *Ph. Col.* n. 5. p. 126.

XIV. *Avignon* is 19′ 40″, or 4° 50′ to the Eaſtward of *London*.

Long. & Lat. of ſeveral Places in France n. 163. p. 718, 719, 720. *vid. ſup. Cap.* IV. § XXXIV.

XV. *M. Caſſini* having compared together the Obſervations of the *Solar Eclipſes* of *July* 12. *ſt. n.* 1684. and made ſuch Reductions as the Parallax requires, lays down the *Longitudes* from *Paris* to

Aix in *Provence* 14′ E. The *Lat.* by M. *Gautier* is 43° 30′.
Avignon 8½′ E.
Lyons 8′, or 13′ E.
Roſes 4′ E. The *Lat.* by M. *Chaſſeles* 42° 10′.
Honfleur 7′ W.
Pau 11′. W. The *Lat.* by *P. Richaud* 43° 30′.

Long. of Lisbon n. 146. p. 151. *vid. ſup. Cap.* IV. §. XLIX.

XVI. Mr. *Jacobs* an *Engliſh* Merchant reſiding at *Lisbon*, informed Mr. *Flamſteed*, that he obſerved the *Beginning* of the *Lunar Eclipſe*, Feb. 11. $168\frac{2}{3}$ there at 8h 31′, *p. m.* which gives the *Difference* of the *Meridians* betwixt the *Obſervatory* at *Greenwich* and *Lisbon*, 41½ *Minutes* of Time, or 10° 2 conſiderably different from our *Maps* and *Sea-Charts*.

Lat. of Madrid; *by* E. *of* Sandwich. n. 22. p. 390.

XVII. The *Earl* of *Sandwich* eſteem'd, by the *Sun*'s Altitude in the *ſtice*, and by other Meridian Altitudes, the *Latitude* of *Madrid* to be 40° 1 which differs conſiderably from that aſſigned by others, the General Cha of *Europe* giving to it 41° 30′; the General Map of *Spain* 40° 27′, and large Provincial Map of *Caſtile* 40° 38′.

XVI

XVIII. 1. By many concurring Obſervations of the *Moon* and other Planets, I have found that the Diſtance of the City of *Seville* from *Uraniburg*, as to *Longitude*, is 90′ or $1\frac{1}{2}$ Hour, the Diſagreement being within 2′.

2. The Profeſſor ſhould conſider again, how he can make the Diſtance of the *Meridians* of *Seville* and *Uraniburg* to be 90′. For the Obſervations of the Eclipſe of the *Moon*, *Jan.* $\frac{1}{11}$, 1675. make the Middle at *London* $7^h\ 11'\frac{1}{2}$ after Noon; to which the Obſervations at *Paris* agree. The Obſervations of the ſaid *Profeſſor* fix the Middle at *Seville* at $6^h\ 47'$. Therefore the Difference of our *Meridians* is $24'\frac{1}{2}$. But between us and *Uraniburg* there are only 52′. Therefore the Difference of *Meridians* between *Seville* and *Uraniburg* can be only $1^h\ 16'\frac{1}{2}$. I am afraid theſe Obſervations of the *Profeſſor* are made only with his naked Eye; for the Times of *Incidence* and *Emerſion* make $1^h\ 5'$. whereas our Obſervations, thoſe of *Paris*, and of *Hevelius*, do not make thoſe Times greater than $1^h\ 1'\frac{1}{2}$. or perhaps ſomething leſs.

Long of Seville *and* Uraniburg; *by* . . . . Math. Profeſſor *at* Seville. *n.* 118. p. 427. *by Mr.* Flamſteed. *ib.* p. 431. *vid ſup. Cap.* IV. §. XLIV.

XIX. The *Difference* of *Meridians* of *Copenhagen* and *Paris* by the Obſervations of *Jupiter*'s *Satellites*, is found by Mr. *Picard* to be $0^h\ 41'\ 40''$.

Long. of Copenhagen; *by M.* Picard. n. 146. p. 145.

XX. *An.* 1680. *Oct.* 23. *St. v. S. Joſ. Ponthia*, and *Marco Antonio Cellio*, with a Teleſcope of 25 Palms, obſerved the Total *Immerſion* of the *firſt Satellite* in *Jupiter*'s Shadow at *Rome*, at $10^h\ 7'\ 53''$; *p. m.* which in our *Obſervatory* here I noted at $9^h\ 15'\ 41''$, whoſe Difference is the *Difference* of our *Meridians* = 52′ 12″, or 13° 03′. Again, *Jan.* 28. 1685. S. *Francis Blanchini* obſerved the Total *Immerſion* of the *Firſt* at *Rome*, at $11^h\ 19'\frac{3}{4}$ which I ſaw not here, but my Numbers give at $10^h\ 27'\frac{1}{4}$: Therefore the *Difference* of *Meridians* is $52'\frac{1}{2}$, and *Rome* lies ſo much more *Eaſterly* than the *Obſervatory* at *Greenwich*; agreeing with the former Obſervation.

The Noble *Tycho* judged therefore not much amiſs, when he placed *Uraniburg* and *Rome* under the ſame *Meridian*; for by ſeveral Obſervations of *Satellite Eclipſes* it is evident, that the *Difference* of *Meridians* betwixt *Uraniburg* and our *Obſervatory* is 51′ 10″ of Time; ſo that *Rome* lies only one *Minute* of Time, or ¼ of a *Deg.* to the *Eaſt* of *Uraniburg*.

Long. of Rome *and* Uraniburg; *by Mr.* Flamſteed. n. 177. p. 1215.

XXI. 1. *Dantzick* is by many and undoubted Obſervations proved to be $1^h\ 15'\ 30''$, more *Eaſterly* than *London*.

2. *An.* 1683. On the very Day of the *Summer Solſtice*, *Jun.* 21. New Style, at *Dantzick*, the *Sun*'s Altitude at Noon was 59° 7′. by a certain ſmall Braſs Quadrant, but very exact. But on the Day of the *Autumnal Equinox* the *Sun*'s Altitude at Noon was found 35° 27′.

Long. of Dantzick; *by Mr.* Halley. *Ph. Col.* n. 5. p. 124.
Lat. of Dantzick; *by M.* Hevelius. n. 151. p. 330. n. 154. p. 424.

XXII. The *Longitude* of *Nuremburg* has been formerly ſtated 11° from *London*, and ſince found to be ſo by Obſervations of the *Eclipſe* of the *Sun* *July* 2. 1684. which made it $44'\frac{1}{2}$ of Time.

Long. of Nuremburg; *by* . . . . . n. 182. p. 147.

Long. of Moſco, Leipſick *and* Aleppo; *by* . . . . . n. 192. *p.* 453. *vid. ſup. Cap.* V. §. LIII.

XXIII. The *Duration* of the *Lunar Eclipſe*, *Apr.* 5. 1688. is made by M. *Timmerman* from 7^{h} 38′ about 10^{h} 45′, which agrees within 8 or 10 *Minutes* with our *Tables*, that never err ſenſibly in the Continuance of *Eclipſes*; and ſo much ought to be allowed to an Obſerver not ſufficiently inſtructed to diſtinguiſh the *Penumbra* from the true Shadow, tho' a ſmall *Teleſcope* were uſed in this Obſervation. Let us conclude then, that the *End* was at 10^{h} 40′ at *Moſco*. We do not find that this *Eclipſe* was obſerved at *London:* However this defect is in good part ſupplied by an Obſervation thereof made at *Leipſick*, by M. *Gotfrid Kirck*, and publiſhed in his *Ephemerides* for the Year 1689; where the *End* is determined at 8^{h} 54′ *p. m.* Hence *Moſco* will be 1^{h} 46′ to the *Eaſtward* of *Leipſick*; and the *Difference* of *Meridians* between *London* and *Leipſick* being already determin'd 49′, it will follow that *Moſco* is 2^{h} 35′ to the *Eaſt* of *London*, or 38° 45′ of Longitude, which from other Accounts we find to be very near that of the City of *Aleppo* in *Syria*.

n. 181. *p.* 86. *it is* 52 *Min.*

Lat. of ſeveral Places in Ruſſia. *ib. p.* 454.

By the ſame hand we have procured the Latitudes of the following Places, obſerved, as 'tis ſaid, with a large Quadrant.

Moſco	55°	34′
Tereſlaw	57	44
Wologda	59	19
Woſtak	61	15
Arch-Angel	64	30

Latitudes of ſome remarkable Places; *by Mr.* Francis Vernon. n. 134. *p.* 582. Jan. An. 1675.

XXIV. I have been as curious as I could in taking the Latitudes of ſome remarkable Places: As I find them, I ſhall give them you.

Athens	38°	05′	*Patras*	38°	40′
Corinth	38	14	*Delphos*	38	50
Sparta	37	10	*Thebes*	38	22
Corone	37	02	*Negropont* or *Chalcis*	38	31

Latitudes of Conſtantinople *and* Rhodes; *directed to* A. B. Uſher; *by Mr.* Greaves. n. 178. *p.* 1295. Dec. An. 1685.

XXV. Upon Intimation of your *Grace*'s Deſires, and upon Importunity of ſome Learned Men, having finiſhed a Table, as a Key to your *Grace*'s exquiſite Diſquiſition, touching *Aſia properly ſo called*; I thought myſelf obliged to give both you and them a Reaſon, why, in the ſituation of *Byzantium* and the Iſland *Rhodes*, (which two eminent Places I have made the παραπέγματα and Bounds of the *Chart*) I diſſent from the Traditions of the *Antients*, and from the Tables of our *late* and beſt *Geographers*; and conſequently, Diſſenting in theſe, have been neceſſitated to alter the Latitudes
not Longitudes) of moſt of the remarkable Cities of this diſcourſe. A
firſt for *Byzantium*, the received Latitude of it by *Appianus*, *Mercator*, C
telius, *Maginus*, and ſome others, is 43° 5′. And this alſo we find in t
Baſil Edition of *Ptolemy*'s *Geography*, procured by *Eraſmus* out of a Greek M
of *Pettiobius*. The ſame likewiſe is confirmed by another choice MS. Greek, of the moſt Learned and Judicious Mr. *Selden*, to whom for this Favo

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and ſeveral others I ſtand obliged. And as much is expreſſed in the late *Edition* of *Ptolemy* by *Bertius*, compared and corrected by *Sylburgius*, with a Manuſcript out of the *Palatine Library*. Wherefore it cannot be doubted, having ſuch a croud of Witneſſes, but that *Ptolemy* aſſigned to *Byzantium*, as our beſt *Modern Geographers* have done, the Latitude of 43° 5′. And this will farther appear, not only out of his *Geography*, where it is often expreſſed, but alſo out of his Μεγάλη Σύταξις, or *Almageſt* as the *Arabians* term it, where deſcribing the Parallel paſſing διὰ Βυζαντίου, he aſſigns to it 43° 5′. What was the Opinion, concerning *Byzantium*, of *Strabo* preceding *Ptolemy*, or of *Hipparchus* preceding *Strabo*, or of *Eratoſthenes* antienter, and it may be accurater than all of them, (for *Strabo Lib.* 2. calls him τελευταῖον πραγματευσάμενον περὶ τῆς γεωγραφίας) tho' *Tully* (*Lib. Ep. ad Att.*) makes *Hipparchus* often reprehend *Eratoſthenes*, as *Ptolemy* after him doth *Marinus*, their Writings not being now extant, (unleſs thoſe of *Strabo*) cannot be determined by us. But as for *Strabo*, in our Inquiry we can expect little Satisfaction; for his Deſcription of Places having more of the *Hiſtorian* and *Philoſopher*, (both which he hath performed with ſingular Gravity and Judgment) than the Exactneſs of a *Mathematician*, who ſtrictly reſpects the Poſition of Places, without Inquiſition after their Nature, Qualities and Inhabitants, (tho' the beſt *Geography* would be a Mixture of them all, as *Abulfeda*, an *Arabian Prince* in his *Rectification of Countries* above 300 Years ſince hath done;) I ſay, for theſe Reaſons we can expect little Satisfaction from *Strabo*, and leſs may we hope for from *Dionyſius Afer*, *Arrianus*, *Stephanus Byzantius*, and others. Wherefore, next having recourſe to the *Arabians*, who in *Geography* deſerved the ſecond Place after the *Græcians*, I find in *Naſſir Eddin* the Latitude of *Byzantium*, which he terms *Buzantiya*, *Conſtantiniya*, to be 45°, and in *Uleg. Beg*'s *Aſtronomical Tables* the ſame to be expreſſed. *Abulfeda* chiefly follows four Principal Authors as his Guides, in the compiling of his *Geographical Tables*; thoſe are, *Alfaras*, *Albiruny*, *Hon Saiid Almagraby*, laſtly *Ptolemy*, whoſe *Geography* he terms a *Deſcription of the Quadrant*, (or the fourth Part of the Earth) inhabited; and all theſe, according to his Aſſertion, place *Byzantium* in 45° of Latitude. And here it may juſtly be wondred, how this Difference ſhould ariſe between the *Greek Copies* of *Ptolemy*, and thoſe tranſlated into *Arabick* by the Command of *Almamon*, the Learned *Caliph* of *Babylon*; for *Abulfeda* expreſly relates, that *Ptolemy* was firſt interpreted in his Time, that is, in the Computation of *Almecinus* in *Erpenius*'s *Edition*, and of *Emir Cond* a *Perſian Hiſtoriographer*, more than 800 Years ſince: Concerning which *Abulfeda* writes thus, *This Book* (diſcourſing of *Ptolemy*'s *Geography*) *was tranſlated out of the Grecian Language into the Arabick for Almamon:* And in this I find (by three fair MSS of *Abulfeda*) *Byzantium* to be conſtantly placed in 45°. and as conſtantly in the *Greek Copies* in 43° 5′. But in the πρόχειροι Κάνονες of *Chryſococca*, out of the *Perſian Tables*, made about the Year 1346. in *Scaliger*'s Calculation, it is placed in 45°. To reconcile the Difference between the *Greeks* and *Arabians* may ſeem impoſſible, for the common Refuge of flying to the Corruption of Numbers by Tranſcribers, and laying the Fault on them which ſometimes is the Author's, will not help us in this particular; ſeeing the *Greek Copies* agree among themſelves, and the *Arabick Copies* amongſt themſelves. The beſt way to end the Diſpute will be, to give Credit concerning the *Latitude* of *Byzantium*, neither to the *Greeks* nor *Arabians*. And that I have reaſon for this Aſſertion, appears by ſeveral Obſervations of mine at *Conſtantinople*, with a Braſs Sextant of above 4 Foot *Radius*. Where taking, in the *Summer Solſtice*, the Meridian Altitude of the *Sun*, without uſing any προσαφάιρεσις for the Parallax and Refraction, (which at that time was not neceſſary,) I found the *Latitude* to be 41° 6′. And in this *Latitude* in the *Chart* I have placed *Byzantium*, and not in that either of the *Greeks* or *Arabians*. From which Obſervation, being of ſingular Uſe in the *Rectification* of *Geography*, it will follow by way of Corollary, that all *Maps* for the *North Eaſt* of *Europe*, and of *Aſia*, adjoining upon the *Boſphorus Thracius*, the *Pontus Euxinus*, and much farther, are to be corrected; and conſequently the Situation of moſt Cities in *Aſia*, *properly ſo called*, are to be brought more Southerly than thoſe of *Ptolemy* by almoſt two entire Degrees, and than thoſe of the *Arabians* by almoſt four.

Concerning *Rhodes*, it may be preſumed, that, having been the Mother and Nurſe of ſo many eminent Mathematicians, and having long flouriſhed in Navigation, by the Direction of theſe, and by the Vicinity of the *Phœnicians*, they could not be ignorant of the preciſe *Latitude* of their Country, and that from them *Ptolemy* might receive a true Information. Tho' it cannot be denied, but that *Ptolemy* in Places remoter from *Alexandria* hath much erred. I ſhall only inſtance in our own Country, where he ſituates Λονδίνιον, that is *London*, in 54° of *Latitude*; and the τὸ μέσον or the middle of the *Iſle* of *Wight*, (which in the printed Copies is falſely termed 'Οὐίκλεσις, but in the MSS. rightly 'Οὐῆκλις,) in 52° and 20′ of *Latitude*. Whereas *London* is certainly known to have for the Altitude of the *Pole*, or *Latitude* of the Place, only 51° and 32′, and the Middle of the *Iſle* of *Wight* not to exceed 50° and ſome Minutes.

But in my Judgment *Ptolemy* is very excuſable in theſe and the like Errors, of ſeveral other Places far diſtant from *Alexandria*; ſeeing he muſt for their Poſition neceſſarily have depended either upon Relations of *Travellers*, or Obſervations of *Mariners*, or upon the *Longitude* of the *Day* meaſured in thoſe times by *Clepſydræ*: All which how uncertain they are, and ſubject unto Error, if ſome Celeſtial Obſervations be not joined with them, and thoſe exactly taken with large Inſtruments, (in which kind the *Antients* had not many, and *Our Times* (excepting *Tycho Brahe*, and ſome of the *Arabians*) but a few,) I ſay no Man, that hath converſed with *Modern Travellers* and *Navigators*, can be ignorant. Wherefore to excuſe thoſe Errors of
(or rather of others fathered by him) with a greater Abſurdity, by
ſerting the *Poles* of the *World* ſince his time to have Changed their
and conſequently all Countries their *Latitudes*, as *Mariana* the Maſte
Copernicus, and others after him have imagined: Or elſe to charge *Ptole*
being ſo excellent an Artiſt, with Ignorance, and that even of his own Cou
try, as *Cluverius* hath done, (from which my Obſervations at *Alexandria*, a

2 *M*

Memphis, may vindicate him,) the former were too great a Stupidity, and the latter too great a Preſumption. But to return to *Rhodes*, an Iſland (in *Euſtathius*'s *Comment* upon *Dionyſius*'s Περιήγησις) of 920 Furlongs Circuit, where according to *Ptolemy* the Parallel paſſing δία Ῥόδȣ hath 36° of *Latitude*, and ſo hath *Lindos*, and Ἰηλυσσός, the Chief Cities of the Iſland; the ſame is confirmed by the MS. but where the printed Copy and *Euſtathius* read Ἰηλυσσός, which *Mercator* renders *Talyſſus*, the MS. renders Ἰλισός. *Abulfeda* in ſome Copies ſituates the Iſland *Rhodes*, (for he mentions no Cities there) in the *Latitude* of 37 *Deg.* and 40 *Min.* and the *Geography* of *Said Ibn Aly Algiorgany*, commended by *Gilbertus Gaulmyn*, in 37°, if it be not by a Tranſpoſition in the MS. of the Numerical Letters in *Arabick*, 37 for 36. which by reaſon of their Similitude are often confounded in *Arabick* MSS. By my Obſervations under the Walls of the City *Rhodes*, with a fair Braſs *Aſtrolable* of *Gemma Friſius*, containing 14 Inches in the Diameter, I found the *Latitude* to be 37° and 50′. A larger Inſtrument I durſt not adventure to carry on Shore in a Place of ſo much Jealouſy. And this *Latitude* in the Chart I have aſſigned to the City *Rhodes*, (from the Iſland ſo denominated, upon which on the North Eaſt ſide it ſtands ſituated) better agreeing with the *Arabians* than with *Ptolemy*, whom I know not how to excuſe.

Long. and Lat. Cape of good Hope. n. 185. *p.* 253.

XXVI. In the Second Book of the *Voyage de Siam des Peres Jeſuites*, are related two Obſervations of the *Satellites* of *Jupiter*, capable, if well made, to aſcertain the *Longitude of the Cape of good Hope*. The firſt was there made *June* 2d. *ſt. n.* 1685, when at 11h 29′ 20″, the *Firſt* or innermoſt *Satellite* touched the *Weſtern Edge* of *Jupiter*, and at 11h 30′ 50″ it appeared no more: This Obſervation is ſaid to be made with an excellent Teleſcope of 12 Foot. The other was on *June* the 4*th* following *ſt. n.* when the *Emerſion* of the ſame *Satellite* was obſerved at 9h 37′ 40″, from which latter is concluded, that the *Longitude* of the *Cape* is 18° to the *Eaſt* of *Paris*; for that the ſaid *Emerſion*, according to the *Calculus* of *Caſſini*, in the *Meridian* of *Paris*, ought to have happen'd at 8h 26′. This ſame *Emerſion* is computed by Mr. *Flamſteed*, at 8h 19′, at *London*, that is 3 *min.* later than by *S. Caſſini*; and conſidering that neither is verified by Obſervation in *Europe*, the *Longitude* hence deduced is doubtful at leaſt 3 *min.* if this had been the only Obſervation. But the former being conſidered will yet ſhew that there is a much greater Doubt ſtill remaining: For from certain Aſtronomical Principles the *Parallax* of the *Orb*, or Difference between the Place of *Jupiter* ſeen from the *Sun* and *Earth* was, at the Time of the firſt Obſervation, 9° 9′; which Arch that *Satellite* moves in 1^{h} 6′. and the utmoſt Duration of an *Eclipſe* hereof in this Poſition of *Jupiter* being ſcarce 2^{h} 20′ (as appears by the accurate Obſervations of M. *Caſſini* and M. *Flamſteed*) it will follow, that from the *Immerſion* behind *Jupiter*'s Weſtern Edge to the *Emerſion* out of the Shadow, there could not be full 3^{h} 26′. Wherefore the *Emerſion* out of the Shadow, on *June* 2*d*, ought according to the time of *Immerſion*, to be at 14^{h} 56′, at the lateſt at the *Cape*; which by

Mr. *Flamſteed's Calculus* was at *London* 13^{h} 51′. or according to *S. Caſſini* at 13^{h} 58′ at *Paris*. Hence the Longitude of the Cape will be found but 14 *deg.* and a half at moſt to the *Eaſt* of *Paris*; ſo that theſe 2 Obſervations will differ in the Reſult about a quarter of an Hour, which is a little too much. However there are ſome Reaſons that ſeem to argue for this latter Longitude rather than the former; for it is much eaſier to obſerve what becomes of a luminous Object that appears, than to wait upon the firſt Appearance of a Star eclipſed: And it is probable that the *Satellite* might in the latter Time be ſeveral Minutes *emerged* out of the Shadow, when they might firſt perceive it; but they could not but ſee the Application to the Body of *Jupiter* in the Former, if we may ſuppoſe their Teleſcopes ſo good as they are ſaid to be. And that the *Cape of good Hope* is not more than an Hour to the *Eaſt* of *Paris*, is proved by the conſtant Conſent of our *Navigators*, who find by their Reckonings that the Iſland of St. *Helena* is about 22 or 23 *deg.* of *Longitude* to the *Weſtward* of the *Cape*: (and that Sailing both backwards and forwards, 'tis the ſame, which takes away the Objection of Currents) Now by accurate Obſervations made at St. *Helena*, and compared with others made in *Europe* at the ſame time, the *Longitude* of that Iſle is certainly about 8 ½ *deg.* to the *Weſt* of *Paris*; it follows therefore that the *Cape* cannot be much more than 14 or 15 *deg.* to the *Eaſt* of *Paris*; and undoubtedly it muſt be leſs than 18°; for 3 *deg.* is much too great an Error to be committed in ſo ſhort a Diſtance of Sailing.

Long of St. Helena. *B.*

The Long. of Madagaſcar; *by* Mr. Flamſteed. n. 143. *p.* 15. *vid ſup. Cap.* IV. § XLVIII.

XXVII. Mr. *Thomas Heathcot* was Chirurgeon to a Ship, which, *Aug.* 19. 1681. lay at the bottom of a deep Bay on the Weſtern Shore of *Madagaſcar*, and that part which the *Portugueſe* and our *Maps* call the *Terra del Gada*; He had then with him on Shore, a Quadrant of two Foot Radius, and a Teleſcope of 9 Foot, but no Clock; to ſupply which Defect, he made a *Pendulum* of a String and a Bullet 39 Inches long, that each ſingle Vibration might anſwer a Second of Time. Waiting the *Beginning* of the *Eclipſe* with his Glaſs, as ſoon as he ſaw the true Shadow enter on the *Moon's* Limb, he cauſed his Friends who aſſiſted him, to make the Pendulum vibrate, and count its Vibrations; of which they had numbred 140 = 2′ 20″ of Time, when he took the height of *Procyon* (then *Eaſt* of the Meridian) 25° 39′. The next day he obſerved the *Sun's* Meridional Height with the ſame Quadrant, whence he found the Latitude of the Place 19° 29′ *South*: hence the time when he took the height of *Procyon* is found 4^{h} 51′ *mane*, and ſubtracting the 2′ 20″, paſt ſince the obſerved *Beginning* of the *Eclipſe*, its
true *Beginning* was at ———————— 4^{h} 48′ 40″
Which at the *Obſervatory* here I noted at ———————— 1 50
Therefore this Part of *Madagaſcar* is more *Eaſterly* ———— 2 58
or 44° 30′, which our *Maps* make 52°; that is 7 ½ *deg.* more remote fr
it than it really is.

The Long. and Lat. of Balla.

XXVIII. Taking the Obſervations of the *Occultation* of the *Bull's* E
Oct. 28. 1680. under the Examination of a *Calculus*, I find that at 8

or the *Immerſion* at *London*, the true Place of the *Moon* correct by *Parallex* was ♊ 4° 32′ 24″; but at 16^{h} 00′, at *Ballaſore Road* (in the *Lat.* of 21° 23′, N. and about 20 Miles E. S. E. from the Town) the true place of the *Moon* was ♊ 5° 54′, that is 1° 21′ 36″, more than at 8^{h} 6′, at *London*: Now according to the Moon's Velocity at that Time, ſhe paſſed an Arch of 1° 21′ 36″, in 2^{h} 8′ 40″, of time; ſo then at 10^{h} 14′ 40″, at *London*, the *Moon* was in the ſame Place as at 16^{h} 00′, at *Ballaſore Road*; whence the Difference of Longitude will be 5^{h} 45′ 20″, or 86° 20′. *Ballaſore* being ſo much to the *Eaſtward* of *London*.

ſore in India; *by Mr.* Edm. Halley, *Ph. Col.* n. 5. p. 124. *vid. ſup. Cap.* IV. § LXVI. Feb. An. 1682.

2. By the Calculation of the *Immerſion* of the *Bull's Eye*, *Dec.* 1680. I find that at the 14^{h} 49′, at *Ballaſore* the *Moon*'s true Place was ♊ 6° 30′ 30″, and at 7^{h} 46′ 12″, the correct Time of the *Immerſion* at *Dantzick*, the true Place was ♊ 4° 55′ 11″; that is 1° 35′ 20″, ſhort of the Place deduced from the Obſervation at *Ballaſore Road*; which make in Time 2^{h} 32″ 40″. Whence it follows, that 10^{h} 18′ 52″, at *Dantzick* make 14^{h} 49′, at *Ballaſore Road*, and the Difference of Longitude 4^{h} 30′ 8″; and *Dantzick* being 1^{h} 15′ 30″, more *Eaſterly* than *London*, *Ballaſore Road* will be from *London* 5^{h} 45′ 38″, or 86° 24′. and the ſame Difference of *Meridians* will be found 86° 14′, if you make uſe of the *Immerſion* at *Dantzick*.

Vid. ſup. Cap. IV. § LXVII.

3. For farther Confirmation hereof, Mr. *Benj. Harry*, being aſhore at *Ballaſore* Town, obſerved with very great Care and Exactneſs, *Nov.* 18, 1680. that at 9^{h} 13′, the Star, which *Tycbo* calls, *in Cotyla dextra* Aquarii *duarum præcedens*, (and which was then in *Aquarius* 28° 52′, and *Lat.* 2° 46′, *N.*) was in a right Line with the *Cuſps* of the *Moon*, then near the firſt Quarter. The Star's place is confirmed by the Agreement of *Hevelius*'s Obſervations with thoſe of *Tycbo*; and the *Theory* of the *Moon* cannot be conſiderably faulty in that Part of the *Orb*, it falling preciſely on her greateſt *Equation*: wherefore by the Theory and Numbers of *Horrox*, the true Place of the *Moon* at 2^{h} 53′ at *London*, is found ♒ 29° 22′ 10″. but at 9^{h} 13′, at *Ballaſore*, her Place was in ♒ 29° 41′ 17″; that is, 19′ 7″ more than at *London*, which in Time gives 36′; ſo that 3^{h} 29′ at *London*, was 9^{h} 13′, at *Ballaſore*, and the Difference of Long. 5^{h} 44′ or 86° 00′ preciſely; which the *Dutch Maps* make full out 99°: And the *French Maps* of *Sanſon*, pretending to correct them, have made them 5° worſe, and the Error 18° completely. What then is to be thought of the Deſcriptions of thoſe Places which have been but ſeldom viſited?

XXIX. I have deduced 7^{h} 23′. for the Difference of Longitudes between *Canton* and *Paris*, from the Exit of *Mercury* out of the *Sun*'s Diſk obſerved at *Canton* and *Nurimburg*; and from the *Eclipſes* of the *Moon* obſerved at *Nurimburg* and *Paris*.

Long. of Canton; *by M.* Caſſini. n. 5. *p.* 371. *Vid. ſup. Cap.* IV. § XCVIII.

XXX. From the greateſt Meridian Altitude of the Polar Star, obſerved by the Fathers of the Society of *Jeſus*, *Dec.* 31. 1694. with a correct Inſtrument, which was 42° 16′ 50″. the Refraction being ſuppoſed to be 1′ 17″, and the Diſtance of the Pole *Star* from the *Pole* at that time was 2° 19′ 57″, whence the *Altitude* of the *Pole* is 39° 54′ 56″.

Lat. and Long. of Pekin; *by M.* Ja. Caſſini. 2 37. *p.* 53.

From the leaſt Meridian Altitude of the ſaid *Polar Star*, obſerved on the 7, 8, and 13 Days of *May*, 1695. with a correct Inſtrument to be 37° 36′ 40″. ſuppoſing the Refraction to be 1′ 28″. and the Diſtance of the *Polar Star* from the *Pole* to be 2°. 19″ 50″. The *Altitude* of the *Pole* is found 39° 55′ 2″.

Neglecting the Refraction, the greateſt Altitude of the *Pole-Star* derived from the Obſervation of *Dec.* 31. aforegoing, was

	°	′	″
at the Beginning of *May*	42	16	43
And the leaſt Altitude of the ſame was then	37	36	40
Therefore the Difference of Altitudes	04	40	03
And the Diſtance of the *Polar Star* from the *Pole*	02	20	01½
And the apparent Altitude of the *Pole*	39	56	41½
To this apparent Altitude the Refraction by my Table is	00	01	10
Wherefore the Altitude of the *Pole* at *Peking* is	39	55	31½

For the Longitude of the City of *Peking*, the Immerſion of the firſt of *Jupiter*'s *Satellites* into the Shadow of *Jupiter* was obſerved *Jan.* 18. 1695. at 12^{h} 51′ 14″.

Our Tables on that Day repreſent this *Immerſion* to have been at 5^{h} 18′ 49″. Now the Obſervations made in the ſame Month in the *Royal Obſervatory* at *Paris* ſhew that the Tables were retarded at that time 2′. 30″.

Therefore that *Immerſion* was at *Paris* at 5^{h} 16′ 19″. Therefore the Difference of Meridians between the Cities of *Paris* and *Peking* will be 7^{h} 34′ 55″.

But as from other Obſervations the ſame Difference of Meridians has been formerly found to be 7^{h} 36′. we may aſſume it 7^{h} 35′½.

Lat. of St. Salwadore. n. 105. *p.* 91.

Lat. of Bridge-Town. n. 189. *p.* 370.

A Deſcription of Nova Zembla; *by* M. Nich. Witſen. n. 101. *p.* 3. *Fig.* 205. Mar. An. 1674.

XXXI. St. *Salvadore* in *Braſil* is in the *Southern Latitude* 12° 47′.

XXXII. *Bridge-Town* in *Barbadoes* is in the *Northern Latitude* of 12° 58′.

XXXIII. 1. I herewith ſend you what I have received out of *Muſcovy*, which is a *New Map* of *Nova Zembla* and *Weigats*, as it hath been diſcover'd by the expreſs Order of the *Czar*; and drawn by a Painter, call'd *Panelapoetſki*, who ſent it me from *Moſco* for a Preſent; by which it appears, that *Nova Zembla* is not an *Iſland*, as hitherto it hath been believed to be; and that the *Mare Glaciale* is not a Sea, but a *Sinus*, or Bay, the Waters whereof are freſh: Which is the ſame with what the *Tartars* do alſo aſſure us, who have taſted theſe very Waters in the midſt of the *Sinus*. The *Samojeds*, as well as the *Tartars*, do unanimouſly affirm, that paſſing on the back of *Nova Zembla*, at a conſiderable Diſtance from the Shore, *Navigators* may well paſs as far as *Japan*. And 'tis a great Fault in the *Engliſh* and *Dutch*, that, ſeeking to get to *Japan* on the South-ſide of *Nova Zembla*, they have almoſt always paſs'd the *Weigats*.

The Letter O in the great River *Oby* marks the Place of a Cataract or Fall of Waters. The Letter K denotes the Conjunction of *Zembla* with the *Continent*. The River marked L runs from *China*, called *Kitaie*; which is not every where Navigable, by reaſon of the *Rocks*, and other Inconveniences that obſtruct the paſſing of Veſſels. *Weigats* itſelf is very difficult to paſs, becauſe of the great Quantity of *Ice* continually falling into it out of the River *Oby*, whereby that ſtreight Paſſage is ſtopped up. The *Sa-*

mojeds go every year a Fiſhing upon the ſaid freſh Sea, and that on *Nova Zembla*'s ſide.

2. I formerly thought *Nova Zembla* had been a *Continent:* But I have ſince been better informed, and retracted that Error. And, whereas the late M. *Voſſius* would needs perſuade himſelf, as well as he did others, to their Ruin, that there was a Paſſage to *Japan* by the *North*, and that the *Tartarian* Countries behind *Nova Zembla* did decline immediately towards the South; I did always oppoſe it, and think I can even demonſtrate the Impoſſibility thereof. So that what he wrote, to encourage Mariners to that Attempt, was even directing them to the point of Death; as it afterwards enſued. 193. *p.* 494.

XXXIV. What is noted with a ſingle Line, is exactly copied from the *Map*, which *M. Sanſon*, one of the moſt illuſtrious *Geographers* of this Age preſented to the *Dauphin*, *An.* 1679. The Names of Cities, whoſe Situation is alſo taken from this *Map*, are written in *Italian* Characters; the Correction of the Poſition of Coaſts (which is deduced from the Obſervations which were made to that End) is marked with a Stroke a little ſhadow'd towards the *Sea*, as is commonly done; and the Names of Cities, whoſe Situation is corrected, are ſet down in *Roman* Characters. *A Map of* France; *by M.* Picard; *and M.* De La Hire. n. 226. *p.* 443. *Fig.* 205. Mar. An. 1697.

The Degrees of *Latitude* are marked on both ſides of the Border, and the Degrees of *Longitude* in the ſame Border, above and below; but the Diviſion of them begins at the *Meridian* that paſſes thro' the *Obſervatory* at *Paris*, by going to *Eaſt* and *Weſt*, and not at the *Meridian* of the Iſle of *Fer*, as hath been eſtabliſhed, becauſe we do not exactly know the Situation of this Iſland in reſpect of the *Obſervatory*.

XXXV. 1. What *Arithmetick*, in whole Numbers and Fractions, as alſo in Decimals and Logarithms, is neceſſary for the ſame: And what Books are beſt for Teaching ſo much thereof. 2. What Vulgar, Practical, Mechanical *Geometry*, performable by the Scale and Compaſs, is ſufficient. 3. What *Trigonometry*, right Lined and Spherical, will ſuffice. 4. How many *Stars* are to be known. 5 What *Inſtruments* are beſt for Uſe at *Sea*, with the Conſtruction of them, and the Manner of uſing them. 6. The whole Skill of the *Magnet*, as to the directive Virtues thereof, and all the Accidents that may befall it. 7. The *Hydrography* of the *Globe* of the Earth, the *Perſpective* of the Coaſts, and the Deſcription of the under-water-bottom of the *Sea*. 8. The Knowledge of Winds and *Meteors*, ſo far as the ſame is attainable. 9. The Hiſtory and Skill of all Sorts of *Fiſhings*. 10. The Art of *Medicine* and *Chirurgery*, peculiar for the *Sea*. 11. The *Common Laws* of the *Admiralty*, and *Juriſdiction* of the *Sea*. 12. The ſeveral *Victuallings* and *Cloathings* fit for Seamen. 13. The whole Science of *Ebbing* and *Flowing*, as alſo of Currents and Eddies at Sea. 14. *Dromometry*, and the Meaſures of a Ship's Motions at Sea. 15. *The Building of Ships* of all Sorts, with the ſeveral Riggings and Sails for each Species, and the Uſe of all the Parts and Motions of a Ship. 16. *Naval Oeconomy*, according to the ſeveral *What a Compleat Treatiſe of* Navigation *ſhould contain; by* Sir W. Petty. n. 198, *p.* 657. Mar. An. 1693.

Voyages and Countries. 17. The Art of *Conning*, *Rowing* and *Sailing*, of all the ſeveral ſorts of Veſſels. 18. The *Gunnery*, Fireworks and other Armatures peculiar to Sea and to Sea-Fights. 19. The Art of *Loading* and *Unloading* the Chief Commodities to the beſt Advantage. 20. The Art of *weighing ſunk Ships* and Goods, as alſo of *Diving* for ſunk Goods in deep Water. 21. The General *Philoſophy* of the *Motion* and Figures of the *Air*, the *Sea*, and of Seaſons; of *Timber*, *Iron*, *Hemp*, *Brimſtone*, *Tallow*, &c. And of their ſeveral Uſes in *Naval* Affairs. 22. An Account of 5 or 6 of the beſt *Navies* of *Europe*, with that of the *Arſenals*, *Magazines*, *Yards*, *Docks*, &c. 23. An account of all the *Shipping* able to croſs the *Seas* belonging to each Kingdom and State of *Europe*. 24. An account of all the chief *Commercial* Parts of the World; with mention of what *Commodities* are originally carried from, and ultimately to, any of them. 25. An account of the chief *Sea-Fights*, and all other *Naval Expeditions* and Exploits, relating to *War*, *Trade*, or *Diſcovery*, which have happened in this laſt Century. 26. Of the moſt advantageous Uſe of *Teleſcopes* for ſeveral Purpoſes at *Sea*. 27. Of the ſeveral *Depths* of the *Sea*, and *Heights* of the *Atmoſphere*. 28. The Art of making *Sea-water freſh* and *potable*, and fit for all Uſes in Food and Phyſick at Sea.

The Collection of Secants, and the true Diviſion of the Meridian *in the* Sea-Chart; *by Dr.* Wallis. n. 176. p. 1193. *Fig.* 206. Nov. An. 1685.

XXXVI. 1. Tho' it be well known, that, in the *Terreſtrial Globe*, all the *Meridians* meet at the *Pole*, (as E P, E P,) whereby the Parallels to the Equator, as they be near to the Pole, do continually decreaſe.

2. And hereby a degree of *Longitude* in ſuch Parallels is leſs than a degree of *Longitude* in the Equator, or a degree of *Latitude*:

3. And that in ſuch Proportion, as is the Co-Sine of *Latitude* (which is the Semidiameter of ſuch Parallel) to the Radius of the *Globe*, or of the Equator.

4. Yet hath it been thought fit (for ſome Reaſons) to repreſent theſe *Meridians*, in the *Sea-Chart*, by parallel ſtraight Lines; as E p, E p.

5. Whereby, each Parallel to the Equator (as L A) was repreſented in the *Sea-Chart*, (as l a) as equal to the Equator E E; and a Deg. of *Longitude* therein, as large as in the Equator.

6. By this means, each Degree of *Longitude* in ſuch *Parallels* was increaſed, beyond its juſt proportion, at ſuch rate as the Equator (or its Radius is greater than ſuch *Parallel*, or the Radius thereof.)

7. But, in the Old *Sea-Charts*, the degrees of *Latitude* were yet repreſented (as they are in themſelves) equal to each other, and to thoſe of the Equator.

8. Hereby, amongſt many other inconveniencies, (as Mr. *Ed. Wright* obſerves, in his *Correction of Errors* of *Navigation*, firſt publiſhed in the Year 1599) the Repreſentation of the Places remote from the Equator was ſo diſtorted in thoſe Charts, as that (for Inſtance) an *Iſland* in the *Latitude* of 60 degrees, (where the Radius of the *Parallel* is but half ſo great as that of the Equator) would have its Length (from *Eaſt* to *Weſt*) in Compariſon of Breadth (from *North* to *South*) repreſented in a double Proportion of what indeed it is.

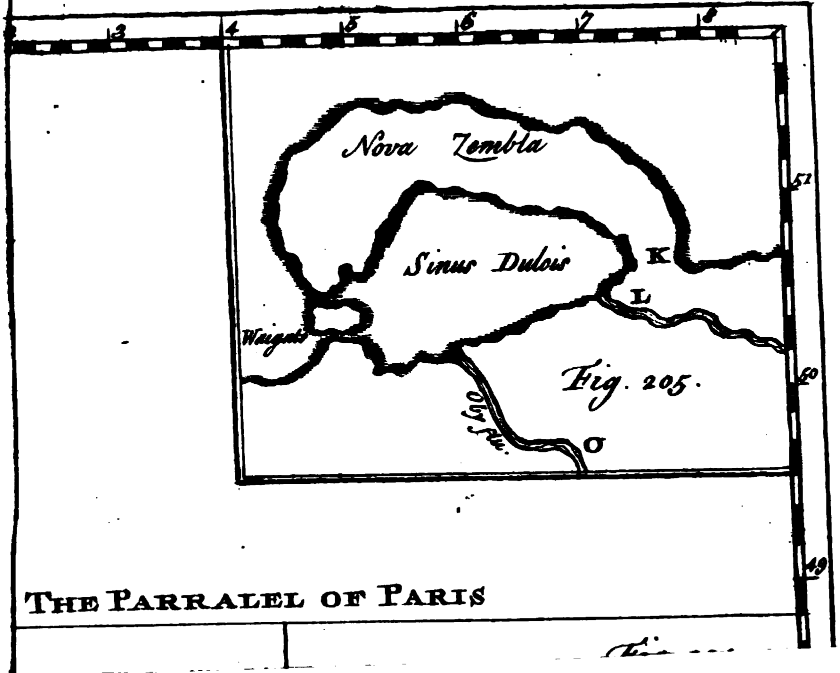


Fig. 205.

ı, except the leaſt, (anſwering to the
s that he follows:) And it would be
nitting all theſe) we take the Inter-
:. or (the double of theſe) *Min.* 1, 3,
Convex-ſide of the Curve) would be

19. But

1599) the Repreſentation of the
diſtorted in thoſe Charts, as tha
60 degrees, (where the Radius
of the Equator) would have its
of Breadth (from *North* to *Sou*
what indeed it is.

9. For rectifying this in ſome Meaſure, (and of ſome other Inconveniencies) Mr. *Wright* adviſeth, that (the *Meridians* remaining Parallel, as before) the degrees of the *Latitude* remote from the Equator, ſhould at each Parallel be protracted in like Proportion with thoſe of *Longitude*.

10. That is; as the Co-Sine of *Latitude*, (which is the Semidiameter of the Parallel) to the Radius of the *Globe*, (which is that of the Equator:) So ſhould be a degree of *Latitude* (which is every where equal to a degree of *Longitude* in the Equator,) to ſuch degree of *Latitude* ſo protracted (at ſuch Diſtance from the Equator;) and ſo to be repreſented in the Chart.

11. That is, every where, in ſuch Proportion, as is the reſpective Secant (for ſuch *Latitude*) to the Radius. For, as the Co-Sine, to the Radius; ſo is the Radius, to the Secant of the ſame Arch or Angle; or Σ : R :: R : ſ. *Fig.* 207.

12. So that (by this means) the Poſition of each Parallel in the *Chart* ſhould be at ſuch Diſtance from the Equator, compared with ſo many Equinoctial Degrees or Minutes, (as are thoſe of *Latitude*) as are all the Secants (taken at equal Diſtances in the Arch) to ſo many times the Radius.

13. Which is equivalent (as Mr. *Wright* there notes) to a Projection of the Spherical Surface (ſuppoſing the Eye at the Centre) on the Concave Surface of a *Cylinder* erected at right Angles to the Plane of the Equator.

14. And the Diviſion of *Meridians*, repreſented by the Surface of a *Cylinder* erected (on the Arch of *Latitude*) at Right Angles to the Plane of the *Meridian* (or a Portion thereof:) The Altitude of ſuch Projection (or Portion of ſuch Cylindrick Surface) being (at each Point of ſuch Circular Baſe) equal to the Secant (of *Latitude*) anſwering to ſuch Point. *Fig.* 208.

15. This Projection (or Portion of the Cylindrick Surface) if expanded into a Plane, will be the ſame with a plain Figure, whoſe Baſe is equal to a Quadrantal Arch extended (or a Portion thereof) on which (as Ordinates) are erected Perpendiculars equal to the Secants, anſwering to the reſpective Points of the Arch ſo extended: The leaſt of which (anſwering to the Equinoctial) is equal to the Radius; and the reſt continually increaſing, till (at the *Pole*) it be infinite. *Fig.* 209.

16. So that, as E R S L, (a Figure of Secants erected at right Angles on E L, the Arch of *Latitude* extended) to E R R L, (a Rectangle on the ſame Baſe, whoſe Altitude E R is equal to the Radius;) ſo is E L (an Arch of the Equator equal to that of *Latitude*,) to the Diſtance of ſuch Parallel, (in the *Chart*) from the Equator.

17. For finding this Diſtance, anſwering to each Degree and *Minute* of *Latitude*, Mr. *Wright* (as the moſt obvious way) adds all the Secants (as they are found calculated in the *Trigonometrical Canon*) from the Beginning to the Deg. or Min. of Latitude propoſed.

18. The Sum of all which, except the greateſt, (anſwering to the Figure inſcribed) is too little: The Sum of all, except the leaſt, (anſwering to the circumſcribed) is too great; (which is that he follows:) And it would be nearer to the Truth than either, if (omitting all theſe) we take the Intermediates; for *Min.* $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, *&c.* or (the double of theſe) *Min.* 1, 3, 5, 7, *&c.* Which yet (becauſe on the Convex-ſide of the Curve) would be ſomewhat too little.

19. But any of theſe ways are exact enough for the Uſe intended, as creating no ſenſible Difference in the *Chart*.

20. If we would be more exact; Mr. *Oughtred* directs, (and ſo had Mr. *Wright* done before him) to divide the Arch into Parts yet ſmaller than Minutes, and calculate Secants ſuiting thereunto.

21. Since the *Arithmetick* of *Infinites* introduced, and (in purſuance thereof) the Doctrine of *Infinite Series*, (for ſuch Caſes as would not, without them, come to a determinate Proportion;) Methods have been found for ſquaring ſome ſuch Figures.

Fig. 207. 22. In order to a *Quadrature* for this Figure of Secants (by an infinite Series fitted thereunto) put we, for the *Radius* of a Circle, R; the Right Sine of an Arch or Angle, S; the verſed Sine, V; the Co-Sine, (or Sine of the Complement) $\Sigma = R - V = \sqrt{Rq - Sq}$; the Secant, ſ; the Tangent, T.

23. Then is $\Sigma : R :: R : ſ$. That is, $\Sigma)\, R^2\, (ſ = \frac{R^2}{\Sigma}$; the Secant.

24. And $\Sigma : S :: R : T$. That is, $\Sigma)\, SR\, (T = \frac{SR}{\Sigma}$; the Tangent.

Fig. 210. 25. Now, if we ſuppoſe the Radius CP, divided into equal Parts, and each of them $= \frac{1}{\infty} R$;) and on theſe, to be erected the Co-Sines of Latitude L A.

26. Then are the Sines of Latitude in Arithmetical Progreſſion.

27. And the Secants anſwering thereunto, $Lſ = \frac{R^2}{\Sigma}$.

28. But theſe Secants, (anſwering to the Right Sines in Arthemetical Progreſſion,) are not thoſe that ſtand at equal Diſtances on the Quadrantal Arch extended. *Fig.* 209.

29. But, ſtanding at unequal Diſtances (on the ſame extended Arch;) Namely, on thoſe Points thereof, whoſe right Sines (whilſt it was a Curve) are in Arithmetical Progreſſion. As *Fig.* 211.

Fig. 211. 30. To find therefore the Magnitude of R E L ſ, *Fig.* 209. which is the ſame with that of *Fig.* 211, (ſuppoſing E L of the ſame Length in both; however the Number of Secants therein may be unequal) we are to conſider the Secants, tho' at unequal Diſtances, *Fig* 211, to be the ſame with thoſe of equal Diſtances, in *Fig.* 210. anſwering to Sines in Arithmetical Progreſſion.

31. Now, theſe Intervals, (or Portions of the Baſe) in *Fig.* 211. are the ſame with the intercepted Arches, (or Portions of the Arch) in *Fig.* 210. For this Baſe is but that Arch extended.

32. And theſe Arches, (in Parts Infinitely ſmall) are to be reputed equivalent to the Portions of their reſpective Tangents intercepted between the ſame Ordinates. As in *Fig.* 210, 212.

Fig. 212. 33. That is equivalent to the Portions of the Tangents of Latitude.

34. And theſe Portions of Tangents are to the equal Intervals in the Baſe, as the Tangent (of Latitude) to its Sine.

35. To find therefore the true Magnitude of the Parallelograms, or Segments of the Figure; we muſt either protract the Equal Segments of the Baſe, *Fig.* 210. (in ſuch Proportion as are the reſpective Tangents to the Sine) to make them Equal to thoſe of *Fig.* 211.

36. Or elſe, (which is equivalent) retaining the equal Intervals of *Fig.* 210, protract the Secants in the ſame Proportion. (For either way the Intercepted Rectangle, or Parallelograms will be equally increaſed) as L M. *Fig.* 212.

37. Namely, as the Sine (of *Latitude*) to its Tangents; ſo is the Secant, to a Fourth; which is to ſtand (on the Radius equally divided) inſtead of that Secant. *Fig.* 212.

$$S : \frac{SR}{\Sigma} (: : \Sigma : R.) : : \frac{R^2 \; R^3}{\Sigma : \Sigma^2 = R^2 - S^2} = LM.$$

38. Which therefore are as the Ordinates in (what I call *Arith. Infin. Prop.* 104.) *Reciproca Secundanorum:* Suppoſing Σ^2 to be Squares in the Order of Secundans.

39. This (becauſe of $\Sigma^2 = R^2 - S^2$; and the Sines S, in Arithmetical Progreſſion) is Reduced (by Diviſion) into this *Infinite Series:*

$$R : + \frac{S^2}{R} + \frac{S^4}{R^3} + \frac{S^6}{R^5} \; \&c.$$

40. That is, (putting $R = 1$,) $1 + S^2 + S^4 + S^6$, &c.

41. Then, according to the *Arithmetick* of *Infinites,* we are to interpret S, ſucceſſively by 1 S, 2 S, 3 S, &c. till we come to S, the greateſt. Which therefore repreſents the Number of all.

And, becauſe the firſt Member doth repreſent a Series of Equals, the ſecond of Secundans; the third of Quartans, &c. Therefore the firſt Member is to be multiplied by S; the ſecond by $\frac{1}{3}$ S; the third by $\frac{1}{5}$ S; the fourth by $\frac{1}{7}$ S; &c.

43. Which makes the Aggregate, $S + \frac{1}{3} S^3 + \frac{1}{5} S^5 + \frac{1}{7} S^7 + \frac{1}{9} S^9$, &c. $= ECLM$.

44. This, (becauſe S is always leſs than $R = 1$) may be ſo far continued, till ſome *Power* of S become ſo ſmall, as that it (and all which follow it) may be ſafely neglected.

45. Now, (to fit this to the *Sea-Chart*, according to Mr. *Wright*'s Deſign) having the propoſed Parallel (of *Latitude*) given; we are to find (by the *Trigonometrical Canon*) the Sine of ſuch *Latitude*; and take Equal to it, $CL = S$. And by this find the Magnitude of E C L M, *Fig.* 212. that is, of REL *ſ*, *Fig.* 211, that is, of REL *ſ*, *Fig.* 209. And then, as RRLE (or ſo many times the Radius) to REL *ſ*, (the Aggregate of all the Secants;) ſo muſt be a like Arch of the Equator (equal to the Latitude propoſed,) to the Diſtance of ſuch Parallel, (repreſenting the Latitude in the *Chart*) from the Equator: Which is the Thing required.

46. The ſame may be obtained in like manner, by taking the verſed Sines in Arithmetical Progreſſion. For, if the right Sines (as here) beginning at the Equator, be in Arithmetical Progreſſion, as, 1, 2, 3, &c. Then will

the verſed Sines, beginning at the *Pole*, (as being their Complements to the Radius) be ſo alſo.

47. The ſame may be applied in like Manner, (tho' that be not the preſent Buſineſs) to the Aggregate of *Tangents*, anſwering to the Arch divided into equal Parts.

48. For thoſe anſwering to the Radius ſo divided are $\frac{SR}{\Sigma}$; (taking S in Arithmetical Progreſſion.)

49. And then inlarging the Baſe (as in *Fig.* 211.) or the Tangent (as in *Fig.* 212.) in Proportion of the Tangent to the Sine;

$$S : \frac{SR}{\Sigma} (: : \Sigma : R) : : \frac{SR}{\Sigma} : \frac{SR^2}{\Sigma^2} = \frac{SR^2}{R^2 - S^2}.$$

50. We have by Diviſion this Series, $S + \frac{S^3}{R^2} + \frac{S^5}{R^4} + \frac{S^7}{R^6} + \frac{S^9}{R^8}$, *&c.*

51. That is, (putting $R = 1$) $S + S^3 + S^5 + S^7 + S^9$, *&c.*

52. Which (multiplying the reſpective Members by $\frac{1}{2}S$, $\frac{1}{4}S$, $\frac{1}{6}$, $\frac{1}{8}S$, $\frac{1}{10}S$, *&c.*) becomes $\frac{1}{2}S^2 + \frac{1}{4}S^4 + \frac{1}{6}S^6 + \frac{1}{8}S^8 + \frac{1}{10}S^{10}$, *&c.*

Which is the Aggregate of Tangents to the Arch whoſe right Sine is S.

53. And this Method may be a Pattern for the like *Proceſs* in other Caſes of like Nature.

Two Problems in Navigation, *propos'd; by Mr.* Nich. Mercator. n. 13. *p.* 215. Jun. An. 1666.

XXXVII. The Line of *Artificial Tangents*, or the *Logarithmical Tangent Line*, beginning at 45°, and taking every half Deg. for a whole one, is found to agree pretty near with the *Meridian-Line* of the *Sea-Chart*, they both growing, as it were, after the ſame proportion. But the Table of *Meridional* Degrees being calculated only to every ſexageſimal Minute of a Degree, ſhews ſome ſmall Difference from the *ſaid Logarithmical Tangent-line.* Hence it may be doubted, whether the Difference do not ariſe from that little Error which is committed by calculating the Table of *Meridional Degrees* only to every Minute.

But, if a certain Rule could be produced, by which the Agreement or Diſagreement of the ſaid two Lines might be ſhewn, the *Helix* or *Spiral* Line of the *Ship's Courſe* would be reduced to a more preciſe Exactneſs than ever was pretended by any.

The ſame Rule would alſo diſcover a far eaſier way of making *Logarithms*, than ever was practiſed or known; and therefore might ſerve, whenever there ſhould be Occaſion, to extend the *Logarithms* beyond the number of Places that are yet extant.

Moreover, ſuch a Rule would enable Men to draw the *Meridian Line* Geometrically, that is, without Tables or Scales; which indeed might alſo be done by ſetting of the *Secants* of every whole or half Degree, if there were not this Inconveniency in it, which is not in my Rule: That a Line compoſed of ſo many ſmall Parts, would be ſubject to many Errors, eſpecially in a ſmall Compaſs.

The ſame Rule will alſo ſerve to find the Courſe and Diſtance between two Places aſſigned, as far as Practice ſhall require it; and that without any Table of *Meridional Parts*, and yet with as much Eaſe and Exactneſs.

And, ſeeing all theſe things do depend on the Solution of this Queſtion, *Whether the Artificial Tangent Line be the true Meridian Line*; it is therefore that I undertake, by God's Aſſiſtance, to reſolve the ſaid Queſtion. And, to let the World know the Readineſs and Confidence I have to make good this Undertaking, I am willing to lay a Wager againſt any one or more Perſons that have a Mind to engage, for ſo much as another Invention of mine (which is of leſs Subtilty, but of a far greater Benefit to the Publick) may be worth to the *Inventor*.

As for the great Advantage, that all *Merchants*, *Mariners*, and conſequently the *Common-Wealth*, may receive from this other *Invention*, it is, in my Judgment, highly valuable, ſeeing it will oftentimes make a *Ship* ſail, tho', according to the common way of *Sailing*, the Wind be quite contrary, and yet as near to the Place intended, as if the Wind had been favourable: Or, if you will, it will enable one to gain ſomething in the intended Way, whether the Wind be good or no, (except only when you go directly *South* or *North*) but the Advantage will be moſt where there is moſt need of it, that is, when the Wind is contrary: ſo that one may very often gain a fifth, fourth, third Part, or more of the intended Voyage, according as it is longer or ſhorter, *viz.* always more in a longer Voyage, where the Gain is more conſiderable, and more welcome, not only by ſaving Time, but alſo Victuals, Water, Fuel, Mens Health, and ſo much Room in the *Ship*.

The Analogy of Logarithmick Tangents to the Meridian Line *Demonſtrated; by Mr.* Edm. Halley. n. 219. p. 202. Feb. An. 1696.

XXXVIII. It was firſt diſcovered by chance, and, as far as I can learn, firſt publiſhed by Mr. *Henry Bond* as an Addition to *Norwood*'s *Epitome of Navigation*, about 50 Years ſince, that the *Meridian Line was analogous to a Scale of Logarithmick Tangents of half the Complements of the Latitudes.*

For the Demonſtration of that *Propoſition*, it is requiſite to premiſe theſe four *Lemmata*.

Lemma 1. *In the Stereographick Projection of the Sphere upon the Plane of the Æquinoctial, the Diſtances from the Centre*, which in this Caſe is the Pole, *are laid down by the Tangents of half thoſe Diſtances*; that is, of half the Complements of the Latitudes. This is evident from *Eucl.* 3. 20.

Lemma 2. *In the Stereographick Projection, the Angles, under which the Circles interſect each other, are in all Caſes equal to the Spherical Angles they repreſent*; which is a very valuable *Property* of this Projection.

Fig. 215.

Demonſt. Let EPBL be any great Circle of the *Sphere*, E the Eye placed in its Circumference; C its Center, P any Point thereof; and let FCO be ſuppoſed a Plane erected at Right Angles to the Circle EPBL, on which FCO we deſign the Sphere to be projected. Draw EP croſſing the Plane FCO in *p*, and *p* ſhall be the Point P projected. To the Point P draw the Tangent APG, and on any Point thereof, as A, erect a Perpendicular AD, at right Angles to the Plane EPBL, and draw the Lines PD, AC, DC; and the Angle APD ſhall be equal to the Spherical Angle contained between the

Planes APC, DPC. Draw alſo AE, DE, interſecting the Plane FCO in the Points *a* and *d*; and join *a d*, *p d*: I ſay, the Triangle *a d p*, is ſimilar to the Triangle ADP; and the Angle *a p d* equal to the Angle APD. Draw PL, AK, Parallel to FO; and, by reaſon of the Parallels, *a p* will be to *a d*, as AK, to AD: But, (by *Eucl.* 3. 32.) in the Triangle AKP, the Angle AKP=LPE, is alſo equal to APK=EPG: wherefore the Sides AK, AP, are equal; and 'twill be, as *a p* to *a d*, ſo AP to AD. Whence the Angles DAP, *d a p*, being right, the Angle APD will be equal to the Angle *a p d*; that is, the Spherical Angle is equal to that on the *Projection*, and that in all Caſes. Q. E. D.

This *Lemma* I lately received from Mr. *Ab. de Moivre*, tho' I ſince underſtand from Dr. *Hook*, that he long ago produced the ſame thing before the *Society*. However, the *Demonſtration*, and the reſt of the Diſcourſe is my own.

Lemma. 3. *On the Globe, the Rhumb Lines make equal Angles with every Meridian*, and, by the foregoing Lemma, they muſt likewiſe make *equal Angles with the Meridians in the Stereographick Projection on the Plane of the Equator:* They are therefore in that *Projection*, *Proportional Spirals* about the Pole Point.

Fig. 214. Lemma 4. *In the Porportional Spiral it is a known Property, that the Angles* BPC, *or the Arches* BD, *are Exponents of the Rationes of* BP *to* PC: For, if the Arch BD be divided into innumerable equal parts, Right Lines drawn from them to the Center P, ſhall divide the Curve B *c c* C into an Infinity of *Proportionals* between PD and PC, whoſe number is equal to all the Points *d*, *d*, in the Arch BD: Whence, and by what I have delivered concerning the *Conſtruction* of *Logarithms*, it follows, that, as BD to B*d*, or, as the Angle BPC, to the Angle BP *c*, ſo is the *Logarithm* of the *Ratio* of PB to PC, to the *Logarithm* of the *Ratio* of PB to P *c*. vid. ſup. Cap. 1. § XXVIII.

From theſe *Lemmata* our Propoſition is very clearly *Demonſtrated:* For, by the *Firſt*, PB, P*c*, PC, are the Tangents of half the Complements of the Latitudes in the *Stereographick Projection:* And, by the *Laſt* of them, the Differences of Longitude, or Angles at the Pole between them, are *Logarithms* of the *Rationes* of thoſe Tangents one to the other. But the *Nautical Meridian Line* is no more than a Table of the *Longitudes*, anſwering to each Minute of *Latitude* on the *Rhumb Line*, making an Angle of 45 Degrees with the *Meridian*. Wherefore, the *Meridian Line* is no other than a *Scale of Logarithmick Tangents of the Half Complements of the Latitudes.* Q. E. D.

Corol. 1. Becauſe that in every Point of any *Rhumb Line*, the Difference of *Latitudes* is to the *Departure*, as the Radius to the Tangent of the Angle that *Rhumb* makes with the Meridian; and thoſe equal *Departures* are every where to the *Differences* of *Longitude*, as the Radius to the Secant of the *Latitude*; it follows, that the *Differences* of *Longitude* are, on any *Rhumb*, *Logarithms* of the ſame Tangents, but of differing Species; being proportioned to one another, as are the Tangents of the Angles made with the *Meridian*.

Corol. 2. Hence any Scale of Logarithm Tangents (as thoſe of the vulgar Tables made after *Briggs*'s Form; or thoſe made to *Napier*'s, or any other Form whatſoever) is a Table of the Differences of Longitude, to the ſeveral Latitudes, upon ſome determinate *Rhumb* or other: And therefore, As the Tangent of the Angle of ſuch Rhumb, to the Tangent of any other Rhumb; ſo the difference of the *Logarithms* of any two Tangents, to the difference of Longitude on the propoſed Rhumb, intercepted between the two Latitudes, of whoſe half Complements you took the Logarithm Tangents.

Now the *momentary Augment*, or *Fluxion* of the *Tangent-Line* at 45 Degrees, is exactly double to the *Fluxion* of the Arch of the Circle, (as may eaſily be proved) and the Tangent of 45° being equal to the Radius, the *Fluxion* alſo of the *Logarithm Tangent* will be double to that of the Arch, if the *Logarithm* be of *Napier*'s Form: But, for *Briggs*'s Form, it will be as the ſame doubled Arch multiplied into 0,43429, *&c.* or divided by 2,30258, *&c.* yet this muſt be underſtood only of the Addition of an indiviſible Arch, for it ceaſes to be true, if the Arch have any determinate Magnitude.

Hence it appears, that, if one Minute be ſuppoſed Unity, the Length of the Arch of one Minute being 0,000290888208665721596154, *&c.* in parts of the Radius, the Proportion will be as Unity to 2,908882, *&c.* So Radius to the Tangent of 71° 1′ 42″, whoſe *Logarithm* is 10,46372611720718325204, *&c.* and under that Angle is the *Meridian interſected* by that *Rhumb-line*, on which the Differences of *Napier*'s Logarithm Tangents of the Half Complements of the Latitudes are the true Differences of Longitude, eſtimated in Minutes and Parts, taking the firſt 4 Figures for Integers. But for *Vlacq*'s Table, we muſt ſay,

As 2302585, *&c.* to 2908882, *&c.* ſo Radius to 1,2633114387244569212, *&c.* which is the Tangent of 51° 38″ 9″, and its Logarithm 10,10151042850772094 1162, *&c.* Wherefore in the *Rhumb-line*, which makes an Angle of 51° 38′ 9″ with the Meridian, *Vlacq*'s *Logarithm Tangents* are the true Differences of Longitude. And this, compared with our *ſecond Corollary*, may ſuffice for the Uſe of the Tables already computed.

But, if a Table of *Logarithm Tangents* be made by Extraction of the Root of the *infiniteth Power*, whoſe Index is the Length of the *Arch* you put for Unity, (as for the Minutes the 0,0002908882*th*, *&c. Power*) which we will call *a*; ſuch a *Scale* of Tangents ſhall be the true *Meridian Line*, or Sum of all the Secants taken infinitely many. Here the Reader is deſired to have Recourſe to my little Treatiſe of *Logarithms*, that I may not need to repeat it. By what is there delivered it will follow, that putting *t* for the Exceſs or Defect of any Tangent above or under the Radius or Tangent of 45°: the *Logarithm* of the *Ratio* of Radius to ſuch Tangent will be,

vid. ſup. Cap. I. § XXVIII.

$$\frac{1}{m} \text{ into } t - \tfrac{1}{2}t^2 + \tfrac{1}{3}t^3 - \tfrac{1}{4}t^4 + \tfrac{1}{5}t^5, \text{ \&c.}$$

when the *Arch* is greater than 45°, or

$$\frac{1}{m} \text{ into } t + \tfrac{1}{2}t^2 + \tfrac{1}{3}t^3 + \tfrac{1}{4}t^4 + \tfrac{1}{5}\,{}^5, \text{ \&c.}$$

when it is leſs than 45°. And by the ſame Doctrine putting T for the Tangent of any Arch, and t for the Difference thereof from the Tangent of another Arch, the Logarithm of their *Ratio* will be $\frac{1}{m}$ into $\frac{t}{T}+\frac{tt}{2TT}+\frac{t^3}{3T^3}+\frac{t^4}{4T^4}+\frac{t^5}{5T^5}$, &c. when T is the greater Term; Or, $\frac{1}{m}$ into $\frac{t}{T}-\frac{t^2}{2T^2}+\frac{t^3}{3T^3}-\frac{t^4}{4T^4}+\frac{t^5}{5T^5}$ &c. when T is the leſſer Term.

And, if m be ſuppoſed, 0,0002908882, &c. $=a$, its reciprocal $\frac{r}{a}$ will be 3437,74677078493925 26, &c. which multiplied into the aforeſaid *Series* ſhall give preciſely the difference of the Meridional Parts between the two Latitudes, to whoſe half Complements the aſſumed Tangents belong.

Nor is it material from whether Pole you eſtimate the Complements, whether the elevated or depreſſed; the Tangents being to one another in the ſame *Ratio*, as their Complements, but inverted.

In the ſame Diſcourſe I alſo ſhewed, that the Series may be made to converge twice as ſwift, all the even Powers being omitted; and that putting τ for the Sum of the two Tangents, the ſame *Logarithm* would be $\frac{2}{m}$ or $\frac{2r}{a}$ into $\frac{t}{\tau}+\frac{t^3}{3\tau^3}+\frac{t^5}{5\tau^5}+\frac{t^7}{7\tau^7}+\frac{t^9}{9\tau^9}$, &c. but the Ratio of τ to t, or of the Sum of two Tangents to their Difference, is the ſame as that of the Sine of the Sum of the Arches, to the Sine of their Difference. Wherefore, if S be put for the *Sine Complement* of the *middle Latitude*, and s for the Sine of half the Difference of *Latitudes*, the ſame *Series* will be $\frac{2r}{a}$ into $\frac{s}{S}+\frac{s^2}{3S^3}+\frac{s^5}{5S^5}+\frac{s^7}{7S^7}+\frac{s^9}{9S^9}$, &c. wherein, as the Differences of *Latitude* are ſmaller, fewer Steps will ſuffice. And, if the *Equator* be put for the middle Latitude, and conſequently S=R, and s the *Sine* of the *Latitude*, the *Meridional Parts* reckoned from the *Equator* will be $\frac{s}{a}+$
Vid. § XXXVI. $\frac{s^3}{3rra}+\frac{s^5}{5r^4a}+\frac{s^7}{7r^6a}$, &c. which is co-incident with Dr. *Wallis*'s *Solution*. And this ſame *Series*, being half the *Logarithm* of the *Ratio* of R+s to R—s, that is of the *verſed Sines* of the Diſtances from both Poles, does agree with what Dr. *Barrow* had ſhewn in his XI. *Lecture*.

The ſame *Ratio* of r to t may be expreſſed alſo by that of the Sum of the Co-ſines of the two *Latitudes*, to the *Sine* of their Difference: As likewiſe by that of the *Sine* of the Sum of the two *Latitudes*, to the Difference of their Co-ſines: Or by that of the *verſed Sine* of the Sum of the *Co-Latitudes*, to the Difference of the *Sines* of the *Latitudes*: Or as the ſame Difference of the *Sines* of the *Latitudes*, to the *verſed Sine* of the Difference of the *Latitudes*; all which are in the ſame *Ratio* of the Co-ſine of the *middle Latitude*, to the *Sine* of half the Difference of the *Latitudes*. As it were eaſy to demonſtrate, if the Reader were not ſuppoſed capable to do it himſelf, upon the bare Inſpection of a Scheme duly repreſenting theſe Lines.

This variety of *Expreſſion* of the ſame *Ratio* I thought not fit to be omitted, becauſe by help of the *Rationality* of the *Sines* of 30°, in all Caſes where the Sum or Difference of the *Latitudes* is 30°, 60°, 90°, 120°, or 150°; ſome one of them will exhibit a ſimple *Series*, wherein great Part of the Labour will be ſaved. But the former ſeems for all Uſes the moſt convenient, whether we deſign to make the whole *Meridian-Lines*, or any Part thereof, *viz.* $\frac{2r}{a}$ into $\frac{s}{S}+\frac{s^3}{3S^3}+\frac{s^5}{5S^5}+\frac{s^7}{7S^7}+\frac{s^9}{9S^9}$, *&c.* Wherein a is the Length of any Arch, which you deſign ſhall be the Integer or *Unity* in your *Meridional Parts* (whether it be a Minute, League or a Degree, or any other) S the Co-ſine of the *Middle Latitude*, and s the *Sine* of half the Difference of *Latitudes*; but, the *Secants* being the Reciprocals of the Co-ſine $\frac{s}{S}$ will be equal to $\frac{ſs}{rr}$ putting $ſ$ for the *Secant* of the *Middle Latitude*; and $\frac{2r}{a}$ into $\frac{s}{S}$ will be $=\frac{2ſs}{ra}$. This multiplied by $\frac{ss}{3SS}$ that is by $\frac{ſſss}{3rrrr}$, will give the ſecond Step; and that again by $\frac{3ſſss}{5rrrr}$, the third Step; and ſo forward till you have compleated as many Places as you deſire. But, the *Squares* of the *Sines* being in the ſame *Ratio* with the *verſed Sines* of the double Arches, we may inſtead of $\frac{ss}{3SS}$ aſſume for our Multiplicator $\frac{v}{3V}$, or the verſed Sine of the Difference of the *Latitudes* divided by thrice the verſed Sine of the Sum of the *Co-Latitudes*, &c. which is the utmoſt *Compendium* I can think of for this Purpoſe; and the ſame *Series* will become $\frac{2sr}{aS}$ into $1+\frac{v}{3V}+\frac{v^2}{5V^2}+\frac{v^3}{7V^3}+\frac{v^4}{9V^4}$. Hereby we are enabled to eſtimate the Default of the Method of making the *Meridional Line*, by the continual Addition of the Secants of equi-different Arches, which, as the Differences of thoſe Arches are ſmaller, does ſtill nearer and nearer approach the Truth. If we aſſume, as Mr. *Wright* did, the Arch of one Minute to be Unity, and one Minute to be the common Difference of a Rank of *Arches*; it will be in all Caſes, as the Arch of one Minute to its Chord, ſo the Secant of the middle Latitude to the firſt Step of our Series. This by Reaſon of the near Equality between a and $2s$, which are to one another in the *Ratio* of Unity to 1—0,00000000352566457713, *&c.* will not differ from the Secant ſ, but in the 9*th* Figure; being leſs than it in that Proportion. The next Step being $+\frac{2ſ^3 s^3}{3arr}$ will be equal to the Cube of the Secant of the *middle Latitude* multiplied into $\frac{2sss}{3arr}$=0,0000-0000705132908715; which therefore, unleſs the Secant exceed ten Times Radius, can never amount to 1 in the fifth Place. Theſe two Steps ſuffice to make the *Meridian Line*, or *Logarithm Tangent*, to far more Places than any of natural Secants yet extant, are computed to; but, if the third Step be required, it will be found to be +ſs into $\frac{2s^5}{5ar^4}$ =0,00000-0000000000089498; by all which it appears that Mr. *Wright*'s Table does no where exceed the true *Meridian Parts* by fully half a Minute; which ſmall Difference ariſes by his having added continually the Secants 1′, 2′, 3′, *&c.* inſtead of $0\frac{1}{2}'$, $1\frac{1}{2}'$, $2\frac{1}{2}'$, $3\frac{1}{2}'$, *&c.* But, as it is, it is abundantly ſufficient for *Nautical* Uſes. That in Sir *Jonas Moor*'s *New Syſtem of the Mathematicks* is much nearer the Truth; but the Difference from *Wright* is ſcarce ſenſible, till you exceed thoſe Latitudes where *Navigation* ceaſes to be practicable; the one exceeding the Truth about half a Minute, the other being a very ſmall Matter deficient therefrom.

For an Example eaſy to be imitated by whoſo pleaſes, I have added the true *Meridional Parts* to the firſt and laſt Minutes of the *Quadrant*.

The *Firſt Minute*, 1,00000001410265862178.

The *Second*,——2,00000005641063806707.

The *Laſt*, or 89° 59′. 30374,96343114142228643, and not 32348, 5279, as Mr. *Wright* has it, by the Addition of the Secants of every whole Minute: Nor 30249,8, as Mr. *Oughtred*'s Rule makes it, by adding the Secants of every half Minute. Nor 30364,3, as Sir *Jonas Moor* had concluded it by I know not what Method, tho' in the reſt of his Table he follows *Oughtred*.

The ſame may be deduced independently from the *Arch* itſelf. For if the Latitude from the Equator be eſtimated by the Length of its *Arch* A, Radius being Unity, and the *Arch* put for an Integer be a, as before; the *Meridional Parts* anſwering to that *Latitude* will be $\frac{1}{a}$ into A, $+\frac{1}{6}A^3+\frac{1}{24}A^5+$

$\frac{\frac{1}{84}}{60}A^7$ or $\frac{61}{5040}A^7 + \frac{\frac{11}{2880}}{1386}A^9$ or $\frac{1385}{362880}A^9$, *&c.* which converges much ſwifter than any of the former *Series*, and beſides has the Advantage of A increaſing in Arithmetical Progreſſion; which would be of great Eaſe if any ſhould undertake *de novo* to make the Logarithm Tangents, or the *Meridian-line* to any more Places than now we have them; the Logarithm Tangent to the Arch of 45° $+ \frac{1}{2}A$ being no other than the aforeſaid Series $A + \frac{1}{6}A^3 + \frac{1}{24}A^5$, *&c.* in *Napier*'s Form, or the ſame multiplied into 0,43429, *&c.* for *Briggs*'s.

But, becauſe all theſe *Series* towards the latter End of the Quadrant do *converge* exceeding ſlowly, ſo as to render this Method almoſt uſeleſs, or at leaſt very tedious: It will be convenient to apply ſome other Arts, by aſſuming the Secants of ſome intermediate Latitudes; and you may for s, or the *Sine* of n, the Arch of half the Difference of Latitudes, ſubſtitute $a - \frac{1}{6}a^3 + \frac{1}{120}a^5 - \frac{1}{5040}a^7 + \frac{1}{362880}a^9$, *&c.* according to Mr. *Newton*'s Rule for giving the Sine from the *Arch*; and, if a be no more than a Degree, a very few Steps will ſuffice for all the Accuracy that can be deſired.

And, if a be commenſurable to a, that is, if it be a certain Number of thoſe Arches which you make your Integer, then will $\frac{a}{a}$ be that Number, which if we call n, the Parts of the Meridional Line will be found to be

$$\frac{sn}{r} \text{ into } \left\{ \begin{array}{l} 1 + \frac{ssaa}{3r^4} + \frac{s^4a^4}{5r^8} + \frac{s^6a^6}{7r^{12}}, \text{ \&c.} \\ -\frac{aa}{6rr} - \frac{ssa^4}{6r^6} - \frac{s^4a^6}{6r^{10}}, \text{ \&c.} \\ +\frac{1a^4}{120r^4} + \frac{13s^2a^6}{360r^8}, \text{ \&c.} \\ -\frac{1a^6}{5040r^6}, \text{ \&c.} \end{array} \right.$$

In this the firſt two Steps are generally ſufficient for nautical Uſes, eſpecially when neither of the Latitudes exceed 60 Degrees, and the Differences of Latitudes do not paſs 30 Degrees.

To conclude, I ſhall only add, that Unity being *Radius*, the Co-ſine of the Arch A, according to the ſame Rules of Mr. *Newton*, will be $1 - \frac{1}{2}A^2 + \frac{1}{24}A^4 - \frac{1}{720}A^6 + \frac{1}{40320}A^8 - \frac{1}{3628800}A^{10}$, *&c.* from which and the

I former

former *Series* exhibiting the *Sine* by the Arch, by Diviſion it is eaſy to conclude, that the *natural Tangent* to the Arch A is $A + \frac{1}{3}A^3 + \frac{2}{15}A^5 + \frac{17}{315}A^7 + \frac{62}{2835}A^9$, *&c.* and the *natural Secant* to the ſame Arch $1 + \frac{1}{2}A^2 + \frac{5}{24}A^4 + \frac{61}{720}A^6 + \frac{277}{8064}A^8$, *&c.* And from the *Arithmetick of Infinites*, the Number of theſe *Secants* being the Arch A, it follows, that the Sum total of all the *Infinite Secants* on that Arch is $A + \frac{1}{6}A^3 + \frac{1}{24}A^5 + \frac{61}{5040}A^7 + \frac{277}{72576}A^9$, *&c.* the which, by what foregoes, is the Logarithm Tangent of *Napier*'s Form, for the Arch of $45^\circ + \frac{1}{2}A$, before.

And collecting the infinite Sum of all the *natural Tangents* on the ſaid Arch A, there will ariſe $\frac{1}{2}AA + \frac{1}{12}A^4 + \frac{1}{45}A^6 + \frac{17}{2520}A^8 + \frac{31}{14175}A^{10}$, *&c.* which will be found to be the Logarithm of the Secant of the ſame Arch A.

To find the Variation *of the* Compaſs *at Sea; by* .... n. 24. *p.* 435. Apr. An. 1667.

XXXIX. The Height of the Pole, and the *Sun*'s Declination being known, a large Ring-Dial, truly wrought, having a Box with a Compaſs or Needle fixt to its Meridian below, may go as near as any other Inſtrument, to ſhew the Variation of the *Needle* at *Sea*. For, when it is ſet to the juſt Hour and Minute of the Day, the Meridian of it ſtands juſt in its due Place; and ſo ſhews how far the *Needle* varies from it, as exactly as the Largeneſs of the Card will permit.

But, becauſe theſe Dials are ſo rarely juſt, *&c.* tho' they may be uſed and taken notice of, yet they are not to be relied on. The Thing therefore is to be performed, as followeth:

Find out the *Sun's Azimuthal* diſtance from the Meridian ſome Hours before or after Noon, and then its *magnetical Azimuth*, or diſtance from the Meridian pointed at by the *Needle*; and the difference of thoſe two diſtances is the variation of the *Needle*.

To find the *Sun*'s true *Azimuth*, or by how many Degrees, *&c* of the Horizon, it is diſtant from the Meridian: Its Declination, its Altitude, and the Elevation of the Pole, muſt all three be known; and thence the true *Azimuth* may be eaſily calculated. The true *Azimuth* of the *Sun* being thus found, and the *magnetical Azimuth* of it, according to your *Needle*, being obſerved, ſubtract the leſſer number from the greater, and the Remander the Variation of the *Needle*. If the *Magnetical Azimuth* be leſs than the ther, then the Variation is towards the ſame ſide of the Meridian, where t *Sun* is; if greater, on the other.

To obſerve the *Sun*'s *Azimuth* by the *Needle*, and the *Needle*'s Variation, to Degrees, any *Needle* long enough to afford upon a Card under it a Circle divided into Degrees, put into a ſquare Box, after the ordinary manner of *Clinatories*, will ſerve turn; by placing the Box ſo, as the Sun may ſhine upon any two oppoſite ſides of it, at the ſame time when the Sun's Height, &c. are taken. For then the *Needle*'s Diſtance from the Diameter of the Circle on the Card, that is parallel to thoſe Sides, is the *Magnetick Azimuth* required.

The ſame may be done with an ordinary *Sea-Compaſs*, ſo it have a Circle towards the Limb of the Card, divided into degrees, by faſtning a ſmall Thread, Lute-ſtring or Wire (not of Iron) ſo upon it, as to paſs juſt over the Centre of that Circle; and placing a ſtrait Piece of Wood or Braſs-wire perpendicular on the Edge of the Box at the end of the Thread, and turning it to the Sun till the Shadow of it fall juſt upon the Thread: Then obſerve what Degrees of the Circle on the Card the Thread cuts, by looking plum upon it; and that is the Sun's *Magnetical Azimuth*.

But to have the *Variation* to *Degrees* and *Minutes*, (which is moſt deſireable) then the Obſervation laſt mentioned muſt be made with a *Quadrant*, *Sextant*, or ſome ſuch other Inſtrument, ſo large as to admit of the Diviſion of a *Degree* into *Minutes*: which will require the Radius to be about 3 Foot; the larger the better. If a *Quadrant*; then, it being laid flat, and the ſquare Box with the *Needle* placed upon it, move the *Quadrant* to and again, till that ſide of it, on which the Box is placed, lie parallel to the *Needle* when at quiet: Then the Sight of the *Quadrant* being ſlid along the Limb of it, till the Sun ſhine on both its ſides at the ſame time, the Mid-line, that divides equally the Sight, when the Sun ſhines upon it through the Slit, will mark the *Degree* and *Minute* of the *Sun*'s *Magnetical Azimuth*. All which is eaſy to be put in Practice.

To find this Variation by the Stars, is ſo eaſy, that every Maſter can do it.

A Caution for Obſerving the Variation at Sea; *by Mr.* Edm. Halley, n. 195. p. 571. Oct. An. 1692.

XL. It is a received Error, in the Practice of obſerving the Variation at *Sea*, to take it by the Amplitude of the *Riſing* and *Setting Sun*, when his Centre appears in the Viſible Horizon, whereas he ought to be obſerved when his Under-Limb is ſtill above the Horizon about $\frac{2}{3}$ of his Diameter, or 20 Minutes, upon the Score of the *Refraction*, and the Height of the Eye of the *Obſerver* above the Surface of the *Sea*: Or elſe they are to work the *Amplitude* as they do the *Azimuth*, reckoning the *Sun*'s Diſt. from the *Zenith* 90° 36′.

This, though it be of little Conſequence near the Equinoctial, will make a great Error in *High Latitudes*, where the *Sun* riſes and ſets obliquely.

A Caution to Seamen *bound up the* Engliſh Channel; *by Mr.* Halley, n. 267. p. 725. Nov. Ann. 1700.

XLI. The *Latitudes* of the *Lizard* and *Scilly*, are laid down too far Northerly by near 5 Leagues: For, from undoubted Obſervation, the *Lizard* lies in 49° 55′, the middle of *Scilly* due *Weſt* therefrom, and the South Part thereof neareſt 49° 50′. Whereas in moſt Charts and Books of *Na-*

Navigation they are laid down to the Northward of 50°, and in ſome full 50° 10′. Nor was this without a good Effect, as long as the *Variation* continued *Eaſterly*, as it was when the *Charts* were made. But ſince it is become conſiderably *Weſterly*, (as it has been ever ſince the Year 1657.) and is at preſent about 7½ *Deg.* all Ships ſtanding in, out of the Ocean, *Eaſt* by the *Compaſs*, go two thirds of a Point to the *Northward* of their true Courſe, and in every 80 Miles they ſail, alter their *Latitude* about 10′. So that if they miſs an Obſervation for two or three Days, and do not allow for this *Variation*, they fail not to fall to the *Northward* of their Expectation, eſpecially if they reckon *Scilly* in above 50°, and to run up the *Briſtol Channel*, not without great Danger of all, and the Loſs of many of them. This has been by ſome attributed to the *Indraught* of St. *George*'s *Channel*: But, the *Variation* being allowed, it hath been found, that the ſaid *Indraught* is not ſenſible. It is therefore recommended to all Maſters of Ships, that they ſteer two Watches E. by S. for one E. which will exactly keep their Parallel; as alſo, that they come in, out of the Sea, on a Parallel not more *Northerly*, than 49° 40′, which will bring them fair by the *Lizard*.

XLII. Papers of Leſs General Uſe, omitted.

Pendulum Watches. *n.* 118. *p.* 440. *vid. ſup. Cap.* V. *Sect.* V.

Mr. *Oldenburg* having publiſhed from *Journal de Scavans*, an Account of M. *Huygen*'s Portable *Watches*, Dr. *Hook*, in the Poſtſcript to his *Deſcription of Helioſcopes*, complains of it, for not having taken Notice, *That this Invention was firſt found out by an* Engliſhman, *and long ſince publiſhed to the World.* To this Mr. *Oldenburg* anſwers, by relating the *Plain Truth of the Matter*: Whereupon Dr. *Hook*, in a Poſtſcript to his *Lampus*, further complains, and reflects on Mr. *Oldenburg*'s Integrity and Faithfulneſs in his *Management of the Intelligence of the Royal Society.* This gave Occaſion to the *Council* of that Society to declare, *That Mr.* Oldenburg *had carried himſelf Faithfully and Honeſtly, and given no juſt Cauſe of ſuch Reflections*: To which Mr. *Oldenburg* adds Part of a Letter from Mr. *Huygens* to him, *Offering* (*if Mr.* Oldenburg *believes a Patent in* England *might be worth ſomething*) *all he might there pretend to.* So that if Mr. *Oldenburgh* had a Deſire to take out a Patent, it was for no other Contrivance than Mr. *Huygen*'s.

n. 128. p. 710.

n. 129. p. 749.

XLIII. Account of Books and Emendations omitted.

n. 231. p. 670. 1. Volumen Primum *Geographorum Gr.* Minorum. *Oxon* in 8*vo.*

n. 231. p. 671. 2. *Dionyſii* Periegeſes, Græce & Latine, cum Scholiis Gr. tam Editis quam Ineditis. Cura *Edw. Thwaits*, M. A. *Oxon.* in 8*vo.*

n. 91. p. 5172. 3. *Bernhardi Vareni*, M. D. *Geographia* Generalis; aucta & illuſtrata ab *Iſaaco Newtono*, R. S. S. *Cantab.* 1672. in 8*vo.*

n. 231. p. 666. 4. *Philippi Cluverii* Introductio in Univerſam *Geographiam*, tam Veterem quam Novam: Tabulis *Geographicis* 46. ac Notis olim ornata, à *Joanne Bunone*; jam vero locupletata Additamentis & Annotationibus *Jo. Frid. Hekelii* & *Jo. Reiſkii. Amſt.* 1697. in 4*to.*

5. *Geography Anatomiz'd*; or a Compleat *Geographical Grammar*. Being a ſhort and exact *Analyſis* of the whole Body of *Modern Geography*, after a New and Curious Method; by *Pat. Gordon*, M. A. F. R. S. The *Second Edition*. n. 256. p. 335.

6. An Account of the Meaſure of a *Degree* of a *Great Circle* of the *Earth*; by M. *Picart. Paris*. 1671. *Fol.* Tranſlated into *Engliſh*; by Mr. *Waller, R. S. Sec. Lond.* 1687. *This Book is here abridg'd, and the Sum of the whole amounts in ſhort to this,* M. Picart *meaſured on a Plain and ſtraight Ground a ſpace of* 5663 Toiſes, *to ſerve for the firſt* Baſis *to divers Triangles; by which he hath concluded the* Length *of a* Meridian Line *equivalent to a* Degree *of* Latitude, *to be* 57060 Toiſes *or* Fathoms, *that is,* 28½ Leagues *and* 60 Toiſes. n. 112. p. 261. n. 124. p. 596. n. 189. p. 376. n. 126. p. 636.

7. The Seaman's Practice; by Mr. *Richard Norwood, Lond.* 1636. in 4to. *The Meaſure of a* Degree *is here extracted from that Book. Mr.* Norwood, *An.* 1635. *having actually meaſured, for the moſt Part, the Way from* York *to* London, *and having obſerved the Meridian Altitudes of the* Sun *in both Places, he found the* Difference *of* Latitude *to be* 20° 28', *and the* Diſtance *of their* Parallels 905751 Engliſh *Feet; and therefore one* Degree *of a* Great Circle *is* 367196 *Feet, or* Numero Rotundo 367200 *Feet, which is equal to* 69½ Engliſh Miles, and 14 Poles; *Whereas the* French *make it no more than* 365000 *ſuch Feet.*

8. Longitude found; by *Hen. Bond*, Sen. *Lond.* 1676. *in 4to. A Miſtake in that Book is here corrected.* n. 95. p. 6065. n. 130. p. 774.

9. A Book publiſhed by Mr. *Jo. Moxon*, deſcribing a new ſort of Terreſtrial Globes, invented by the E. of *Caſtlemain*. n. 139. p. 988. *Ph. Col.* n. 1. p. 43.

10. The *Engliſh Atlas. Oxford*, for *Moſes Pitt*. 1680. *Fol.* *Ph. Col.* n. 28. p. 39.

11. A New Map of *England*, full 6 Foot Square, wherein *Computed* and *Meaſured* Miles are entered in Figures; by Mr. *Jo. Adams*. n. 155. p. 586.

12. A large and curious Map of *Great Tartary*; by M. *Nich. Witſon*. n. 193. p. 492.

CHAP. VIII.

Architecture. Ship-Building.

Stones fit for Building; by . . . . n. 93. p. 6010. Apr. An. 1673.

I. THERE is a ſort of grey *Freeſtone* at *Paris*, every where on the South ſide of the River *Sein*, which is of a reaſonable coarſe Greet, and ſo ſoft when firſt taken out of the *Quarry*, that 'tis dreſt and hewn with broad ſharp Axes, almoſt as eaſily as dried Clay, but grows harder and harder in the Air; 'tis very durable, and exceeding fit for *Building*. The *Portland* Stone is of a fine Chalky Greet, fit for all curious hewn and carved Work, tho' not fit for Water or Fire. On the contrary, the *Freeſtone* in *Kent*, of a whitiſh grey Colour, laſts well in Air and Water; the Greet thereof leſs fine and chalky than that of *Portland*. The *Derbyſhire Freeſtone*, tho' it endure the fierceſt Fire, is yet brittle, and ſo unfit for fine and curious Workmanſhip.

The Choice and Characters of Slate for covering Houſes; by Mr. Sam. Colepreſs. n. 50. p. 1009. Aug. An. 1669.

II. 1. Take the thin cleft Stone, Slat or Shindle, and ſo knock it againſt any hard Matter, as to make it yield a Sound; if the Sound be good and clear, that ſort of Stone is not crazy, but firm and good. Or,

2. If in hewing it does not break before the Edge of the *Setts*, (the Hewing Inſtrument of the *Slatters*) you may not much doubt of the Firmneſs of the Slate. But,

3. If after it hath been exactly weighed (and the Accompt thereof laid by) it be put, and for 2, 4, or 8 Hours left to remain all under Water in a Veſſel; and afterward taken up and wiped very clean with Cloths, if then it weigh more than before, 'tis of that kind which imbibes Water, and therefore not ſo fit to endure any conſiderable time without rotting the Lathes and Timber.

4. Theſe Stones may be pretty well gueſſed at, whether they be of a cloſe or looſe Texture, by their Colour: For the over blackiſh Blue is apteſt to take in Water; but the lighter Blue is always the firmeſt and cloſeſt. To which may be added the Touch; for a good Stone feels ſomewhat hard and rough; whereas an open Stone feels very ſmooth, and as it were Oily.

5. Place your Stone long-ways perpendicular in the midſt of a Veſſel of Water, (no matter how ſhallow the Water be, ſo it exceed half a Foot depth;) and be ſure, the upper un-immerſed Part of the Stone be not accidentally wetted by the Hand, or otherwiſe; and ſo let it remain a Day, or half a Day, or leſs. If it be a good firm Stone, it will not draw (as they ſpeak)

ſpeak) Water more than half an Inch above the Level of the Water, and that perhaps but at the Edges only, the Parts of which might be ſomewhat looſened in the Hewing. But a bad Stone will draw Water up to the very top, be the Stone as long as it will, all over.

As for the Charge of covering Houſes with Slate, they may be thus computed.

	s.	d.
1000 of *Efford Small Blue* at the Ship's ſide in *Plymouth* Harbour	5	6
1000 of *Efford Large Blue*	9	9
1000 of *Can Pelmel*	7	0
1000 of *Small Blue* of other Quarries	4	0
1000 of *Large Blue*	8	0

3000 of *Small Blue*, accompted two Tuns in Carriage by Water.

1000 of *Large Blue*, 1 Tun.

3000 of *Small* will cover 1 *Pole* of Work at the 5 Pin Plain.

Every *Pole* of Work is either 6 Foot broad and 14 up, on both ſides, or 168 Foot in Length, and one in Breadth.

3000 of *Large*, will cover two *Poles* of Plain Work.

Hewing of all ſorts of plain *Pelmel per* 1000	1	6
Pinning per 1000, 8 *d. Pins per* 1000: 8 *d.*	1	4

Three Buſhels (*Winchester* Meaſure) of good *Lime*, will take 6 Buſhels of freſh Water Sand, and ſerve to lay on one *Pole* of work; tho' much leſs may ſerve the Turn.

300 of Lathes to every *Pole* of Work.

1000 of Lathe Nail to every 300 of Lathes.

An Able Workman may { Lath one *Pole* of Work
Lay on 2000 or more of *Slate*
Hew 1500 plain
Pin 4000 } by the Day.

Chequer-work conſiſts in Angles, Circles, and Semicircles, &c. which require no common Skill and Time in hewing and laying.

It is worthy Obſervation, That, if a ſide Wall happen to take Wet by the beating of the Weather, or the like, when nothing elſe will cure it, our *Kerſeying* with *Slate* (which is much uſed in the curious Fronts of Houſes, eſpecially in Towns) will quickly remedy it.

We have ſome ſorts, which by the Conjectures of the more experienced *Helliers*, (or Coverers with Slate), have continued on Houſes ſeveral Hundreds of Years, and yet as firm as when firſt put up.

The beſt Time of Felling of Timber; by Dr. Rob. Plot. n. 192. p. 455. Jan. An. 1691.

III. The *Cuſtom* of *Felling Timber* here in the *South* of *England*, differs from that of *Staffordſhire* in the Time of *Felling*, and Manner of *Barking*. It is *Felled* here in the *Spring*, as ſoon as the *Sap* is found to be fully up, by the Trees putting out, and then *Barked* after the Trees are proſtrate, the *Sap* yet remaining in the Bodies of them: Whereas there it is firſt *Bark'd*, (in the Spring as *here*) but before it is *Felled*, the Trees yet living and ſtanding all the *Summer*, and not *Felled* till the following *Winter*, when the *Sap* is fully in Repoſe.

In the *Spring* Seaſon, and ſome time after, all Trees are pregnant and ſpend themſelves (as Animals do in their reſpective Offſprings) in the Production of Leaves and Fruits, and ſo become weaker than at other times in the Year; their Cavities and Pores being then turgid with Juices, or Sap, which (the Trees being felled at that time) ſtill remain in the Pores, having no manner of Means of being otherwiſe ſpent, and there putrefy; not only leaving the Tree full of thoſe Cavities which render the *Timber* weak; but, *2dly*, breeding a Worm, as both *Pliny* and Mr. *Evelyn* teſtify, that will ſo exceedingly prejudice it, that it becomes altogether unfit for ſtrong Incumbencies, or other robuſt Uſes. *3dly*, All Timber fell'd at this Time of the Year, whether the Juices putrify, or otherwiſe ſweat forth, or dry away, is not only ſubject to rift and gape, but will ſhrink ſo conſiderably, that a Piece of ſuch *Timber* of a Foot ſquare will uſually ſhrink in the Breadth ¼ of an Inch; than which, ſays *Vegetius*, nothing is more pernicious, if us'd for the *Building* of *Ships*. To which, *4thly*, the firſt and greateſt *Roman* Emperor, *Julius Cæſar*, adds, that tho' *Ships* may be made of ſuch moiſt Timber, felled in the Spring, yet they will certainly be Sluggs, not near ſo good Sailers as Ships made of Timber felled later in the Year.

In all which Circumſtances, I find, moſt of the *Antients* ſo very well agree, that none of them adviſe the felling of *Timber*, for any ſort of Uſe, before *Autumn*, at ſooneſt; others, not till the Trees have born their Fruit; which, ſays *Theophraſtus*, muſt always be proportionably later, as their Fruits are ripe later in the Year. A third ſort, not till *Mid-winter*; not till *November*, ſays *Palladius*, nay, not till the *Winter Solſtice*, ſays the wiſe *Cato*; and then too in the Decreaſe or *Wane* of the *Moon*, between the *15th* and *23d* Day of her Age, ſays *Vegetius*, or rather, according to *Columella*, between the *20th* and the *New-Moon*. In general, ſays *Theophraſtus*, the *Oak* muſt be fell'd very late in the *Winter*, not till *December*, as the Emperor *Conſtantine Pogonatus* poſitively aſſerts, the *Moon* too being then under the Earth, as 'tis for the moſt part in the Day-time in the firſt Part of its Decreaſe. And the felling of *Oak* within thoſe Limits they call *Tempeſtiva Cæſura*, *Felling Timber* in Seaſon, which they all unanimouſly pronounce (if thus felled) will neither ſhrink, warp, nor cleave, nor admit of Decay, in many Years; it being tough as *Horn*, and the whole Tree in a manner (as *Theophraſtus* aſſerts) as hard and firm as the Heart: with whom alſo agrees our Countryman Mr. *Evelyn*, if you fell not *Oak* (ſays he) till the Sap is in Repoſe, as 'tis commonly about *November* and *December*, after the Froſt has well nipped them, the very *Saplings* thus cut will continue without Decay, as long as the Heart of the Tree.

And the Reaſon of this is given in ſhort by *Vitruvius*, *quia Aëris* Hyberni *vis comprimit & conſolidat Arbores*, becauſe the *Winter* Air doth cloſe the Pores, and ſo conſequently conſolidates all Trees, by which means, the *Oak*, (as he and *Pliny* both expreſs it) will acquire a ſort of *Eternity* in its Duration; and much more will it ſo, if it be Bark'd in the Spring, and left ſtanding all the Summer, expoſed to the Sun and Wind, as is uſual in *Staffordſhire*, and the adjacent Countries; whereby they find, by long Experience, the Trunks of their Trees ſo dried and harden'd, that the ſappy Part in a manner, becomes as firm and durable as the Heart itſelf.

Which way of *Barking* and *Felling* of *Timber*, tho' it were unknown to the Ancients (as perhaps it is to all the world beſides theſe few Counties) yet they ſeem not unacquainted with the Rationality of the Practice, The great *Vitruvius* prefers the *Timber* on the *South-ſide* the *Apennine* (where it winds about, and incloſes *Tuſcany* and *Campania*, and ſtrongly reflects the conſtant *Heat* of the *Sun* upon it as it were from a Concave) incomparably before that which grows upon the *North-ſide* of the ſame Hill, in the ſhady moiſt Grounds: Of which his Opinion he renders us this Reaſon, for that the Sun does not only lick up the ſuperfluous Moiſture of the *Earth*, whence the *Trees* are ſupplied in ſuch ſhady Places with too great a Quantity, but in great meaſure exhales the remaining Juices (after the Production of Leaves and Fruits) out of the *Trees* themſelves, rendering the Timber of them the more cloſe, ſubſtantial and durable; which certainly it would do alſo much more effectually, if the *Bark* were taken off in the Spring of the Year, as is accuſtomed in *Staffordſhire*, where the People are content to uſe this Method in their Proviſion of Timber, tho' but for private Uſes.

Much rather then ſhould it be done in ſo publick a Concern as the *Building* of *Ships*, were tough and ſolid Timber is much more neceſſary than in ordinary *Buildings*. There is indeed an *Act* of *Parliament*, 1 *Jac.* I. *Chap.* 22. which forbids the *Felling* of *Timber* for ordinary Uſes (in conſideration of the *Tan*) at any other time but between the 1ſt of *April* and the laſt of *June*, when the Sap is up, and the *Bark* will run; made on Suppoſition, (I gueſs) that ſhould they have admitted *Felling Timber* in any other Seaſon, the *Tanners* would have wanted a Supply of *Bark*. To which I readily anſwer, that I fear the *Legiſlators* that preſſed the making that Act were ignorant that the *Bark* might be taken off in the Spring, and that the Tree would notwithſtanding live and flouriſh till the *Winter* following, as I have ſeen many in *Staffordſhire*: So that tho' the Tree be not *Fell'd* till the *Winter Solſtice*, or *January* following, yet the *Tanner* is not at all defeated of his *Tan*, but has it here in as due Seaſon, as in any of the Southern Counties. The *Legiſlators*, I ſay, were ignorant of this, otherwiſe they would never have made an Act ſo pernicious to the whole Kingdom, as *Felling Timber* at this Seaſon is, for the ſake of a few *Tanners*.

But notwithſtanding this Ignorance, yet then they were ſo wiſe as to except in that Act the *Timber* to be uſed in building of Ships, which may be *Fell'd* in *Winter*, or any other Time; as I am told all the ancient Timber remaining in the *Royal Sovereign* was, it being ſtill ſo hard, that 'tis no eaſy matter to drive a Nail into it.

'Tis true indeed, that the *barking* and *peeling* the Trees ſtanding is ſomewhat more troubleſome, and therefore ſomewhat more chargeable, than when they are proſtrate; and that 'tis likely, People therefore have uſually fell'd their Timber, as well for *Shipping* as other uſes, in the Spring of the Year, for the ſake of the more eaſy and cheap *Barking* it only, rather than any thing elſe. 'Tis too true, that *Timber* is harder to *Fell* in the *Winter*, it being now ſo compact and firm, that the Ax will not make ſo great Impreſſion as it doth in the *Spring*, which will alſo increaſe the Price of the *Felling* ſome ſmall Matter and its *Sawing* afterwards; but how inconſiderable theſe things are in compariſon of the great Good of this manner of *Felling*, I think is ſelf-evident.

The greateſt Objection, that I can foreſee will be urged here in the *South* againſt this Practice, is, that if the *Timber* be not *fell'd* till *Mid-winter*, or *January*, where it grows in *Copſes* and *Woods*, they cannot perhaps incloſe their young *Sprigs* ſo ſoon as ſome may imagine needful, and therefore will be backward to fell their Timber (ſo growing) at that Seaſon. To which I anſwer, That the *Timber* ſo fell'd in *Woods* or *Copſes* may be eaſily carried off before the ſecond Spring, and ſo the Prejudice ſmall, and the firſt it muſt be there, wherever it is fell'd. But *Secondly*, That which will quite remove this inconſiderable Difficulty, is, that perhaps it may be expedient that no Timber whatſoever growing in *Woods* or *Copſes*, be at all bought in the *King*'s *Yards*, for that *Timber* growing in ſuch ſhady Places, and ſo fenced from the *Sun* and *Wind*, as *Timber* in *Woods* for the moſt part is, cannot be ſo good as that which comes from an expoſed Situation, ſuch as it uſually has in *Foreſts*, *Parks*, *Hedge-rows*, and open Fields; whereto it is indifferent at leaſt if not better for the Proprietor, that it be fell'd in *Winter*, (when the Graſs and Corn is gone) than in the *Spring* it ſelf: and the Officers deſigned for that purpoſe may buy all their *Timber* under ſuch Conditions as to be fell'd in *Winter*, enjoining the Proprietor, to take off the Bark in the Spring in due Time, making him ſome ſmall Allowance for the Trouble he will have in peeling it ſtanding.

The Difference of Timber *in different* Countries, *and Fell'd at* different Seaſons; *by M.* Ant. Van. Leuwenhoek. n. 213. *p.* 224. Sep. An. 1694.

IV. It is the common Opinion, That *Timber* which is fell'd in *Winter*, is ſtronger and more laſting, as being more cloſe and firm than that which is fell'd in *Summer*: But, M. *Leuwenhoek*'s Sentiment is, that there is no Difference, except in the *Bark*, and outermoſt Ring of the Wood, which in the *Summer* are ſofter, and ſo more eaſily pierced by the *Worm*; *Wood* conſiſting of hollow Pipes, which in the *Summer* and *Winter* both, are full of Moiſture, they do not ſhrink in the *Winter*, and therefore the *Wood* cannot be cloſer at one time than another, for otherwiſe it would be full of Cracks and Clefts. The ſudden and unexpected rotten of ſome Timber, he conceives to proceed from ſome inward Decay in the Tree before it was fell'd; having obſerved all Trees to begin to decay at firſt in the Midſt, or Heart of the Tree, tho' poſſibly the Tree may ſtand and grow for near an Hundred Years afterwards, and increaſe in Bigneſs all along.

2. He ſays, he was once of Opinion, that Trees growing in good Ground, but increaſing ſlowly, were the beſt and ſtrongeſt Timber; and that thoſe Trees, which in few Years grew large were the ſofteſt and brittleſt; the contrary to which, upon Enquiry of experienced Workmen, he found to be true, and inſtances of an *Elm* of 80 Years Growth, which was 11 Foot in Circumference, and proved excellent tough Timber.

3. The *Age* of *Trees* is to be known by the Number of Rings to be ſeen when the *Tree* is cut athwart, in each of which Rings is one Circle of large open *Pipes*; now the fewer of theſe large *Pipes* the ſtronger the *Timber* is: wherefore by Conſequence theſe *Trees* that make the largeſt Growth, in a Year, muſt be the Cloſer and ſtronger, and therefore thoſe *Trees* that grow in warm Countries grow faſteſt, and are the beſt and tougheſt Timber; which he confirms by *Riga* and *Dantzick* Oak, which is of ſlow Growth, and proves ſpongy and brittle *Timber*, whereas the contrary is obſerveable in *Engliſh* and *French* Oak, which grows faſter, and is excellent *Timber*.

The Bridge at St. Eſprit *in* France; *by Dr.* Tankred Robinſon, n. 160. *p.* 584.

Fig. 215.

V. 1. This Famous *Roman Bridge* at *Pont St. Eſprit*, is very crooked, bowing in many Places, and making ſeveral unequal Angles, eſpecially in thoſe Places where the Torrent runs ſtrongeſt, as where the *Turret* ſtands, 4. In which Place the Angle is moſt unequal, and the greateſt; the *Arches* are very wide, and have their Feet ſecured by two *Pedeſtals* that encompaſs them. Both the *Pedeſtals* have their ſeveral Degrees or Ranks of *Jettings* out, like ſo many Rows of Stairs or Steps, the lowermoſt Order puſhing out moſt, the others being leſs, and going gradually more in; the Second or uppermoſt *Pedeſtal* is much leſs than the firſt or lowermoſt, being built a little within its Lines of *Circumference*; 1, 2. Between the great *Arches* there are *Windows*, or, (as it were) ſmall *Arches*; 3. that come down to the very Plane of the ſecond, or uppermoſt *Pedeſtal*, dividing the Feet of the great Arches. From this my rude Deſcription it appears to me, that the *Romans* have here contrived all poſſible ways to break gradually the mighty Force of the *Rhoſne*, and to render its Paſſage eaſy, and inoffenſive to the Feet of the great *Arches*; for here we ſee ſo many ſeveral *Paliſadoes* and *Sluices*, as may be ſufficient to defend this wonderful Fabrick againſt all Storms of the *Torrent*; the ſeveral Ranks of *Stairs* jetting from the *Pedeſtals* (for the moſt part Triangularly built, and faced well with Free-ſtone) oppoſing and breaking the Stream ſeverally, I mean, not all together, or at the ſame time, by reaſon of their various Inequalities in ſtanding out: in caſe the Flood ſhould ſwell ſo high (as it frequently does) as to cover both the *Pedeſtals*, then the ſmall Arches, dividing the Feet of the great ones, help to convey the Water thro', which otherwiſe might endanger the great *Arches*.

By Dr. Liſter. *ib.* 585.

2. That which ſeems the Foot of the *Arch* is an *Horizontal Arch* gradually contracted, every Stone being of vaſt Length and Wedge-like, laid level with the Water. This I ſpeak by Memory.

Compared with ſome *other* Bridges; *by Dr.* Tank. Robinſon. n. 163. *p.* 712.

3. The ſtately *Modern Bridge* at *Avignon* hath yielded in many Places to the extreme Rapidity and Violence of the *Rhoſne*. Its Fall, in my Opinion, may be aſcribed to three Defects, Firſt, It was not ſo multangular, as that at St. *Eſprit*: Secondly, it wanted in three or four Places, the little *Arches* dividing the Feet of the great ones, and in thoſe Parts it hath ſuffered moſt; for where thoſe uſeful Sluices are, there I obſerved the Bridge to ſtand ſtill the moſt intire. Thirdly, The *Pedeſtals* (or as you very properly call them *Horizontal Arches*) were not ſo Geometrically and exactly laid, as thoſe of *Pont St. Eſprit*; their Jettings out were few, and they not gradually contracted; ſo that the Force of the Stream muſt be greater upon the Fabrick.

Tho' the *Tiber* be not ſo ſwift as the *Rhoſne*, yet it is ſubject to greater Inundations, as many Inſcriptions aſſure us. No River ever had ſo many Bridges built with that Magnificence and Art, as this; and tho' they were more pompous and rich in rare Stones, in Sculpture, *&c.* than that I formerly ſent you a Draught of from *Montpelier*; yet they had the like Proviſion for their Security and Preſervation, and their Deſign was much the ſame; which may be ſeen at *Rome* this very Day at the old *Pons Mellius* (now *Ponte Nolle*) near the *Via Flaminia*, in the Marble Remains of the *Pons Æmilius*, repaired with rich Materials by *Antoninus Pius*, on the Side of the *Ripa*, or *Traſtavere*, near the Root of the *Aventine Hill*, where firſt the *Pons Sublicius* ſtood; as alſo in the *Pons Fabricius* and the *Ceſtius*, that leads over to the *Inſula Tiberina*; in all which there are ſtill very fair Marks of the old *Roman* Structure and *Deſign*; and if that prodigious City had not been knocked ſo oft to pieces by *Barbarous Sackers*, we might have had ſtill as clear Proofs from the other Bridges, *viz.* the *Pons Triumphalis*, the *Senaeotius*, &c. But *Gothiſh* and *Northern Torrents* broke all before them.

A Bridge *without any Pillar under it; from The Journal of the* Phil. Society *of* Oxford. ib. *p.* 714. *Fig.* 216.

VI. A *Timber Bridge* may be built 70 Foot long, or ſomewhat more, without any *Pillar* under it, which may be uſeful in ſome Places where *Pillars* cannot be conveniently built, after this manner; A C, and B O are Beams 28 Foot long, and A B is 32 Foot long. Under the Angles are ſet two large *Braces*, E L, and S R. At each end is a Wall, on which are laid two *Beams* BH, and AD, each 20 Foot long; under theſe two are two *Braces* DE, and RH. There may alſo be Braces at the End of the Arches, that may lie obliquely croſs the Bridge. It may be laid with Planks and Railed. Behind the Walls are *Cauſeys* FD, and AN. The Length of the Bridge CMO, is 70 Foot; in the Height KM is 19 Foot.

An Aqueduct *near* Verſailles N. 171. *p.* 1016. May, An. 1685.

VII. 1. The *Aqueduct* which is to be made near *Maintenon*, for the carrying the River *Bure* to *Verſailles*, will have in Length 7000 Fathom; 462 whereof will be 35 Fathom and 4 Foot high, the reſt will be lower, according to the Difference of the Ground, but no leſs than 5 Foot and 6 Inches high. There will be to the ſaid *Aqueduct* 861 *Arches*, which, where they are higheſt, will have 12 Fathom in Breadth, and 8 Fathom in Thickneſs, diminiſhing to 14 Foot at the Top. The other *Arches* will be leſſer in Breadth as well as Thickneſs, according to the Nature of the Ground. The ſaid *Aqueduct* will have 15 Inches Fall to every Thouſand Fathom in Length; ſo that for the 7000 Fathom, there will be 8 Foot 8 Inches Fall. The River is to paſs by *Maintenon*, *le Parc Eſpernon*, *Gageran*, *Rambouillet*, *les Eſſars*, *le Perrey*, *Cognieres*, and from thence to *Verſailles*. There are 14000 Soldiers that work there, under the Command of the Marqueſs *d'Uxelles*, with three Commiſſaries of War for their Conduct.

N. 176. *p.* 1206. 2. A *Magazine* for the Waters upon the Mountain *Mantboron*, is already cut, which will have 2200 Perches of *Surface* (each Perch being 8 *French* Foot) and 12 Foot in Depth. In another Place, much lower, will be another *Magazine*, to receive the Waters of many *Pools*, the moſt part of which, as yet have no Water in them. In the Valley of *Buc* will be an *Aqueduct*, the Middle whereof will be raiſed 22 Fathom high, for conveying the Pools of *Sarle*, which, it's ſaid, contain much Water, tho' there be nothing but Rain to fill them. This *Aqueduct* is 300 Fathom long, and paſſes thro' two Mountains which have been cut thro' upon that Account. The Valley alſo on both ſides of the *Aqueduct* is raiſed 11 Fathom high, to make Paſſages.

An *Aqueduct* is alſo making near the Tower of *Stone*, where the Mills raiſe the Water, which will now paſs without Force to the Top of the Mountain: and there be part of it diſtributed into ſeveral great *Ciſterns*, which are making about *Marli* for that Place.

The Elevation of the *Aqueduct* of *Maintenon* is now ſet forth at but 2560 Fathom; whereas it was deſigned to be carried on more than 8000 Fathom, and the Remainder will be made of Earth, which muſt be brought thither; this Opinion prevails, in regard it gives a quicker Diſpatch, tho' it may be doubted, it will not be for the better.

Theſe 2560 Fathoms contain 242 *Arcades*, whoſe *Aperture* is 6 Fathom and ½, and the *Face* of each *Pillar* ſuſtaining the *Arches*, 4 Fathom; there will be then on the ſide of *Maintenon* 33 *Single Arches*, afterwards 71 *Double* ones; (as having one *Arch* upon another) then 46 *Treble* ones; which will generally be 216 Foot 6 Inches high, (*viz.* up to the Floor of the *Channel*) afterwards 72 *Double* ones; then 20 *Single*, which will reach to the Mount of *Earth*, that is to be 50 Foot high.

From the Ground up to the *Second Arcade*, are 16 Fathom; from the *Second* to the *Third*, or upper *Arcade*, are 14 Fathom, (which *Arcades* are Double in Number to thoſe they ſtand upon) and 6 Fathom 6 Inches more to the Floor of the *Channel*, which will at leaſt be 7 Foot high, beſides the *Parapet*.

The *Pillars* by the Ground are 8 Fathom thick; but, what with the *Slopes* and *Shortnings*, which are made in every Story, the Top, where the *Channel* goes, will be but 20 Foot broad. There will likewiſe be at each Pillar a *Buttreſs* jetting out one Fathom, and two Fathom wide.

The intelligent Obſerver, tho' well ſkill'd in things of this Nature, as being no Stranger to the *Writings* of the Ancients, or the famous Ruins and Remainders of their *Fabricks* in *Italy*, and other Places, profeſſes himſelf ſurpriz'd with the Greatneſs of this Undertaking at *Verſailles*, and *Maintenon*; for the Magnificence of the Deſign, the Number of Labourers, the Exceſſiveneſs of the Expence, and the admirable Beauty of the Work.

A very large Stone Chimney, with a peculiar ſort of Arch-Work; by Dr. J. Wallis. n. 166. *p.* 800. *Fig.* 217.

VIII. Having been lately at *Edgecot* in *Northamptonſhire*, at the Houſe of *Tobias Chancy*, Eſq; he ſhewed me in an Antient Kitchen (now diſuſed) two *Chimneys*, vaſtly large, of *Stone-Work*: Which I took the more Notice of, becauſe of a peculiar Way of *Arch-Work* in the Front of them; whereby, without the Advantage of a *Diſcharge* of *Timber* (which is uſual, in ſuch Caſes, to defend the *Arch-Work* from being overburden'd) an *Arch* of maſſy Stone (in each of them) ſuſtains itſelf at a great Length, tho' almoſt upon a Flat, being very little raiſ'd in the middle. Over this Arch (after ſome walling interpoſed) there is another Arch (to defend the former) more raiſed from the Flat. The Dimenſions of all, I have thought fit here to ſubjoin.

AB, The Breadth between the *Jambs*, from Inſide to Inſide, 18 Foot.

CD, The Depth of the *Stones* in the Lower Arch, 22 Inches; locked one into another, with a crooked Joint.

DE, The Diſtance in Walling, between the Arches, 2 Foot and 7 Inches.

EF, The Depth of the *Stones* for the Upper Arch, 15 Inches: With a ſtreight Joint.

GH, The Place of two vaſt *Tunnels* of Stone.

K, a *Window* between them.

A new Kind of Stairs; by M. Weighelius. n. 74. *p.* 2212.

IX. M. *Weighelius* hath lately invented an odd *Bridge*, or kind of Stairs, by which a Man ſhall deſcend, and yet really be raiſed upward; and going as 'twere upon a Plain ſhall from a lower, by gently ſubſiding, arrive to an upper Story.

Preſerving of Ships from being Worm-eaten; by . . . . n. 11. *p.* 190. Apr. An. 1666.

X. In the *Indian Seas*, there is a kind of ſmall *Worms* that faſten themſelves to the *Timber* of the Ships, and ſo pierce them that they take Water every where; or if they do not altogether pierce them thro', they ſo weaken the Wood, that it is almoſt impoſſible to repair them. Some have employed *Deal*, *Hair* and *Lime*, &c. and therewith Lined their Ships; but beſides that, this does not altogether affright the *Worms*, it retards much the Ship's Courſe. The *Portuguſe* ſcorch their Ships, inſomuch that in the *Quick Works* there is made a Coaly Cruſt of about an Inch thick. But as this is dangerous, it happening not ſeldom that the whole Veſſel is burnt; ſo the Reaſon why the *Worms* eat not thro' *Portugal* Ships, is conceived to be the exceeding Hardneſs of the *Timber* employed by them. There is in *Holland* a Man that pretends to have found an admirable Secret to remedy this Evil. And a very worthy Perſon in *London* ſuggeſts the *Pitch*, drawn out of *Sea-Coals*, for a good Remedy to ſcare away thoſe noiſome *Inſects*.

An Account of Lead Sheathing; by Mr. J. Bulteel. n. 100. *p.* 6192. Jan. An. 1674.

XI. Some few Years ſince, Sir *Phil. Howard* and Major *Watſon*, with great Charge and Induſtry found out a new Way, by a Manufacture of our own, to preſerve the Hulls of Ships from the Worms, *&c.* which is much ſmoother and conſequently better for Sailing, and more cheap and durable than the Way of *Boards*, *Pitch*, *Tar*, *Roſin*, *Brimſtone*, or any *Sheathing* or *Graving* hitherto uſed. The *King* and *Parliament* being ſatisfied, upon Examination, of the great Benefit that might redound thereby to his *Majeſty* and Subjects in general, for the Inventors Encouragement to make the ſame publick, were pleaſed, almoſt 4 Years ſince, to grant them an *Act* of *Parliament* for the ſole Uſe of this their Invention, with Penalty and Prohibition to all others. In Proſecution whereof, Experiments have been made upon ſeveral of his

Majeſty's Ships, *viz.* the *Phœnix*, done three Years ago, has made two Voyages into the *Streights*, &c. and when ſhe was lately taken into the *Dock* at *Woolwick* to be repaired, upon View of the *Maſter Shipwright* and others, her Sheathing was found to be in as good Condition, as at the firſt doing; and the Ship ſo tight during the whole time, that they were forced to heave in Water to keep her ſweet. The *Dreadnought*, a Third Rate, done in *June* 1671; the *Henrietta*, *Lyon*, and *Mary*, all three of the Third Rate, and done a Year and half ſince, being lately laid on Ground at *Sheerneſs* and *Portſmouth*, are found to be all in as good Condition, and the Sheathing to continue as firm and as well as at the firſt doing; as the *Maſter-Builder* and *Aſſiſtant* at *Portſmouth*, and others, have certified.

The *Bread Rooms* alſo of ſome of theſe, and many others of his *Majeſty*'s Ships, have been lined within, almoſt in the ſame manner the *Sheathing* is without; which has prov'd a great Preſervation of the Bread, as ſeveral of the *Purſers* and *Officers* of the ſaid Ships have certified; and by Reaſon of its Duration muſt be much cheaper and better than *Tin*, which is ſo liable to ruſt, or any Way yet uſed.

Alſo the Lead itſelf (which is the principal thing uſed herein) they make ſo cloſe preſſed, ſmooth, and equal, or of what Thickneſs or Thinneſs deſired, that great Uſe may be made thereof about ſeveral other things relating to *Shipping*.

XII. *A Paper of leſs General Uſe* Omitted, *viz.*

Directions for Inquiries concerning *Stones* and other *Materials* for the Uſe of *Building*. n. 93. p. 6010.

XIII. Accounts of *Books* Omitted.

1. *Vitruvius* done into *Engliſh*; by Mr. *Chr. Waſe*. n. 72. p. 2190.

Les dix Livres d' *Architecture* de *Vitruve*, corrigez, & traduits nouvellement en *Francois*, avec des Notes & des Figures; par *Claude Perrault*. *Paris* 1637. in Folio. n. 112. p. 279.

2. Cours d' *Architecture*, enſeigne dans l' *Academy Royale* d' *Architecture*, premiere Partie; par M. *Francois Blondel*, à *Paris*, 1675, in Fol. n. 112. p. 549.

3. *Raphaelis Fabretti Urbinatis* de Aquis & Aquæductibus Veteris *Romæ*, Diſſertationes tres, *Romæ* 1680. in 4*to*. n. 155. p. 466. n. 95. p. 6071.

4. Modern Fortification, &c. by Sir *Jonas Moor*, 1673. in 8*vo*. n. 158. p. 586.

5. Nouvelle Maniere de *Fortifier* les Places; par M. *Blondel*. *Hague*, 1684.

6. *Marci Meibomii* de Fabrica *Triremium* Liber. *Amſtelodami* 1671. in 4*to*. n. 79. p. 3071.

7. *Scheeps-Boow en Beſtier*, that is, *Naval Architecture* and *Conduct*; by *N. Witſen*. *Amſterdam* 1671 in *Folio*. n. 77. p. 3006.

8. L' *Architecture Navale*, avec le *Routier* des *Indes Orientales* & *Occidentales*: par le Sieur *Daſſié*, à *Paris*. 1677. in 4*to*. n. 135. p. 879.

CHAP. IX.

Perſpective. Sculpture. Painting.

A Perſpective Inſtrument; by Sir Chr. Wren. n. 45. p. 898. *Fig.* 218. Mar. An. 1669.

I. A Is a ſmall Sight with a ſhort Arm *B*, which may be turned round about, and moved up and down the ſmall *Cylinder* C D, which is ſcrewed into the Piece E D, at D: This Piece E D moving round about the *Center* E; by which means the Sight may be removed either towards R, or F.

EF is a Ruler faſtened on the two *Rulers* GG, which *Rulers* ſerve both to keep the Square *Frame* S S S S perpendicular, and, by their ſliding thro' the ſquare Holes T T, they ſerve to ſtay the Sight, either farther from, or nearer to the ſaid Frame; on which Frame is ſtuck on with a little Wax the Paper OOOO, whereon the *Picture* is to be *drawn* by the *Pen* I. This *Pen* I is by a ſmall Braſs Handle V ſo fix'd to the *Ruler* H H, that the Point I may be kept very firm, ſo as always to touch the Paper. HH is a *Ruler*, that is always, by means of the ſmall Strings *aaa*, *bbb*, moved *Horizontally*, or Parallel to itſelf; at the End of which is ſtuck a ſmall Pin, whoſe Head P is the Sight, which is to be moved up and down on the Out-Lines of any *Object*.

The Contrivance of the *Strings* is this. The two *Strings a a a*, *b b b*, are exactly of an equal Length. Two Ends of them are faſten'd into a ſmall Leaden Weight Q Q, which is moved in a Socket on the backſide of the Frame, and ſerves exactly to counterpoiſe the *Ruler* H H, being of equal Weight with it. The other two Ends of them are faſten'd to two ſmall Pins H H, after they have rolled about the ſmall Pulleys N, M M, L L, K K; by means of which Pulleys, if the Pen I be taken hold of, and moved up and down the Paper, the Strings moving very eaſily, the Ruler will always remain in an Horizontal Poſition.

The Manner of uſing it is this: Set the *Inſtrument* upon a Table, and fix the Sight A, at what Height above the Table, and at what Diſtance from the Frame S S S S, you pleaſe. Then, looking thro' the Sight A, and holding the *Pen* I in your Hand, move the Head of the *Pin* P up and down the Out-Lines of the Object, and the Point I will deſcribe on the Paper O O O O, the Shape of the Object ſo traced.

A new way of Delineating by Parallel Viſual Rays, exactly obſerving the Symmetry; by Mr. St. Clare, n. 96. p. 6080. *Fig.* 219. July, An. 1673.

II. A B C D is the *Proſopographick Parallelogram*, H F the Central Style, L C the deſcribing Quill, K A the Index, or an oblong Ruler adapted at right Angles to the Plain of the *Parallelogram*, by means of the winding braſs Pin according to the Pattern E. To this Ruler are fixt two Sights P R, S V; in the Middle of P R is a Hole bored at O, and in the Middle of S V a Thread is erected perpendicular to the Ruler R A, in the Middle of which is a certain little Globe or Bead, through which and the Hole O a

Ray is extended from the Object to the Eye, which while you delineate ſhould not be fixt, but free and at liberty.

1. It is to be obſerved, that a Ray extended through the Hole O and the Bead, will always be perpendicular to the Plain of the *Parallelogram*, or to its Diameter, which is a right Line extended through the deſcribing Style L C, and the fixt Center F, and the ſaid little Globe or Bead, in which Line the Bead will always be found, however the *Parallelogram* may be moved.

2. The ſenſible Delineatory Plain, upon which the Point L moves about, which is the Nib of the drawing Style L C, deſcribing the Image exactly according to the Motion of the Index K A, and upon which the Central Style H F is fixt, is the Plain Q Y X T; but the mere Rational or Mathematical Plain, being a Continuation of the former, is εδβγ.

3. That all the Rays extended from the Object through the Bead and the Hole O to the Eye, (which is placed in ſo many Points of a tranſparent Medium, by means of the Index K A, as there are Points in the viſible Superficies of the Object to be deſcribed, which are infinite,) will always be parallel to one another.

Perhaps ſome may object, that in Objects at a great Diſtance there can be no uſe of the Sights. But which way will that concern us, ſince this Method of ours is deſign'd only for removing thoſe Difficulties in Delineation, which hitherto have attended the *Parallelogram* of *Scheiner*. For I have often found by experience, (tho' for this the Artiſt does not think the worſe of his Inſtrument,) that by no means there is that Proportion between the Parts of the Copy in the Plain, as there is in the Parts of the Original at a Diſtance.

A Method of caſting Statues of an extraordinary Thinneſs; by M. John Weichard Valvaſor. n. 186. p. 259. *Fig.* 220. Jan. An. 1687.

III. I here ſend you a Method of *Caſting Statues* in *Metal*, in Obedience to the Commands of the *Royal Society*; it is as follows. Firſt, I form out of good Clay, that will endure the Fire, and not crack either in drying or burning, ſuch a Figure or *Statue* as I deſire to caſt; when this is well dry, I make, all over the Figure, little Holes of no great Depth (but both Size and Depth proportionate to the Bigneſs of the *Statue*) into which I let ſmall Pieces of *Metal*, and with ſome of the ſame *Clay* fix them firmly in the *Holes*; the Uſe of theſe Bits of *Metal*, *a a a a a*, is to keep the *Core* and *Mould* from touching one another, or falling together when the *Wax* runs out; and that they may remain conſtantly in the ſame fixt Poſture. This done, I ſcrape away with ſome proper Inſtrument, as much of the Clay in thickneſs as I deſign for the thickneſs of my *Statue*; and then laying it in a *Furnace*, I burn the *Core* till it be red hot: (by the *Core* is meant always the *Statue* firſt made in *Clay*,) when it is cold, I rub the *Core* all over with that ſort of *Earth* or Colour, which our *German Potters* uſe to colour the Joints of the Tiles, when they ſet *Stoves* of Tiles or (*Kachel-Ofens*:) this Colour much reſembles *Black Lead* which is uſed to *Deſign* on Paper, and eaſily wipes out with Bread, but it is not the ſame. This Colour I mix with Water, and daub all over the *Core*, becauſe the Metal is found to run freely upon it. There are other Subſtances proper for this Uſe, but I have always made uſe of this, eſpecially for thin *Statues*. This done, I lay upon the *Core* as much yellow Wax mixed with Pitch or Roſin, as will make the Thickneſs of the intended Statue, which I form in the Wax, with all the Exactneſs poſſible.

Here note, that the Particles of *Metal* mentioned to be ſet into the *Core*, to keep it a Diſtance from the *Mould*, muſt be ſo ſet as to fall in with the Surface of the Wax exactly; and that the Reaſon of mixing Pitch or Roſin with the Wax is, becauſe that when it is burnt out, it makes a great Smoke, and that Smoke adhering to the *Mould*, occaſions the *Metal* to run more freely; as I have experienced it. Next I put all over upon the Surface of this *Statue* of Wax, little Pieces of Wax, which I call the little *Channels c c c c c c*, (all which muſt be contrived ſo as to enter the great *Channels d d d*.) This done, I cover the *Core* and Wax all over with the ſame ſort of Clay, that will endure the Fire without cracking; and ſo I have my *Concave Statue* or *Mould* made. Upon this I lay the great *Channels* marked *d d d d*, both upright and tranſverſe, formed likewiſe in Wax, and placed according to Judgment, ſo as beſt to receive the Ends of the little *Channels c c c c c c*, for the more eaſy diſtribution of the *Metal*. The great *Channels* muſt all meet at the top of the *Statue*, ſo as to come out by one Hole, as at E, where the Metal is to be poured in: It is alſo neceſſary to have a *Channel* or two to let out the Air as the *Metal* enters, as thoſe marked *f f*, and there muſt be a Hole or two left at the Foot, as *g g*, where the great *Channels* and *waxen Statue* join; and whereat, when the Mould is burnt, the Wax as well of the *Statue* as of the *Channels* may run out. The great *Channels* being thus placed, the Mould muſt be again laid over with the ſame ſort of Clay; (I uſe conſtantly to bind about the Mould with Iron Wire, and then lay on more Clay) and when this Mould is well dry, then I heat it red-hot; as I did before the *Core*, ſo now both together.

I burn the *Core* firſt, that there may not need ſo ſtrong a Fire to burn the Mould as will melt the ſmall bits of Metal: but for ſmall manageable *Statues*, of not above a Foot or two high, they may be both burnt together, and there is no need of the Holes *g g*, but the Mould may be inverted, and the Wax run out by the *Channels f f*, and E.

The Mould being thus burnt, I ſtop with the ſame Clay the two Holes *g g*, and then I bury it in a Pit, and proceed as is uſual in *Caſting* of *Bells* and the like; but care muſt be taken, that the *Metal* be very well in Fuſion.

If it be a ſmall *Statue*, not above a Foot or two high, whoſe *Mould* may be managed in one's Hands; then I make me a *Concave Statue* of Wax, of the Thickneſs I deſire, and then place upon it all thoſe great and leſſer Channels, as afore: Which done, I put it all together, into a liquid Subſtance made of Plaiſter and Tile, or Brick Duſt tempered with Water.

If the *Statue* be intended very thin, then I take Copper, and when it is well in Fuſion, I mix with it a good quantity of *Zinc*, without obſerving any certain Proportion of Weight; the more *Zinc* the better the *Metal* runs. I have ſometimes for ſmall and thin *Statues* put in above a third part of *Zinc*. I have found by Experience, that this Mineral makes the *Metal* run moſt freely, and gives it a fair Golden Colour.

The Statue being caſt, I take off the *Mould*, and cut off all the little *Channels*; all which, both great and ſmall, are filled with *Metal*, which may be kept for farther Uſe: In theſe there is much more *Metal* than in the whole Statue; for, if the Statue be very thin, there muſt be more and bigger *Channels*, and ſo, the cheaper the Statue, the more weighty the *Channels*, and the more *Metal* remaining.

To know the Quantity of *Metal* requiſite for any intended Work, I take a Lump of the ſame mixture of Wax and Pitch, with which I make the *Mould* of my Statue; and having weighed it, I make a *Mould* upon it, and caſt in the ſame a Lump of *Metal* of the ſame Size; which I weigh, and thereby compute the Proportion of the Weight of the *Metal* and *Wax*; and then, obſerving how many Pounds of Wax I uſe about the *Figure* and Channels, I can calculate to a ſmall matter how much *Metal* I need to melt.

Hitherto I have caſt no Statue above 9 Foot high, but I doubt not but I could, by the ſame Methods, caſt one of any Bigneſs deſired.

IV. 1. *Spaniſh White* is made of Chalk and Allum burnt together.

A Deſcription of ſome Simple Colours; Mr. Rich. Waller. n. 179 p. 26, 30. Jan. An. 1686.

Simple Blues.

2. I take the *Lapis Armenius* to be the *Blue Bice* ſold in the Shops, for it is light and friable; formerly brought out of *Armenia*, now from the Silver *Mines* of *Germany*, called *Melochites*, in *High-Dutch*, *Berghblaw*.

3. *Ultramarine* is made of the blueſt *Lapis Lazuli*, which is freeſt from Gold Veins, by Calcination.

4. *Smalt* is made of *Zaffer* and *Pot Aſhes* calcined together in a Glaſs Furnace.

5. *Litmaſe* or *Litmoſe*, I ſuppoſe the Juice of a Plant.

6. *Indigo*, ſaid by *Pliny* to be brought from *India*; a kind of *Mud* adhering to the *Froth* about *Reeds*, and that when tried with a Coal, the true burns with a purple *Flame*, and ſmells of the *Sea*. *Linſchoten* ſays it is called *Anil*, that it grows in *Cabaia*, and is a Plant like *Roſemary*, which is gathered and dried, then wetted with fair Water, and beaten to a Mud.

7. *India Ink*; its Uſe is known to *Pliny*, tho' not its Compoſition; which is yet undiſcovered, except it ſhould be burnt *Rice*, as hath been thought.

Simple Yellows and Reds.

1. *Ceruſe* is the *Ruſt* of *Lead*, made by a vaporous Calcination. *Pliny* writes thus of it in *Cap.* 34. *Lib.* 18. *Ceruſe Pſimithium* is made in the Plumbers Shops, of ſmall Plates of *Lead* laid upon a Veſſel of ſtrong *Vinegar*; what falls into the *Vinegar* is taken out and dried in the *Sun*: and in *Cap.* 6. *Lib.* 35. he ſays it was made at *Rome* of burnt *Marble Flint* quenched in Vinegar.

2. *Maſticot* is a kind of improper *Calx* or *Tin*.

3. *Gutta Cambæ*, or *Cambodia*, the inſpiſſated Juice of a Plant, not well known; it comes from both the *Indies*. Some think it the Juice of *Euphorbium*; others *Scammony* or *Tithymal*; others *Ricinus*; others refer it to the greater *Catapatia*, *Eſula*, or the *Flowers* of the *Indian Ricinus*, and will have it coloured with *Turmerick*; as *Schroder*.

4. *Oker*, a kind of natural Earth. There are two ſorts thereof, the one *Native*, formerly brought out of *Africa*, now from *Dacia* and *Hungary*, and from many Places in *England*, eſpecially in the Foreſt of *Dean*: The other, a factitious Subſtance of *Lead* burnt and quenched in Vinegar. In *Pliny*'s Time it was made of *Rubrica*, or *Ruddle* burnt.

5. *Orpiment*, a fat inflammable Mineral, juſtly ranked amongſt Poiſon for its extreme corroſive Quality. *Pliny* ſays, it was dug up in *Syria* on the Surface of the *Earth*, and that the Emperor *Caligula* had hopes of getting *Gold* out of it; wherefore he cauſed 14 Pounds of it to be tried, which afforded him very good *Gold*, but in ſo ſmall a Proportion, that he loſt by the Trial.

6. *Umber* is a native Earth.

7. *Red-Lead*, a Colour unknown to the Antients, made of *Litharge*, or burnt *Lead*, by a *Reverberatory Calcination*, or of *Ceruſe* put in a Platter over the Fire, which muſt be continually ſtirred till it has acquired a *Red-Lead* Colour. Dr. *Charleton de Foſ.*

8. *Burnt Oker* is the common *Yellow-Oker* burnt in the open Fire.

9. *Cinnabar*, or *Vermilion*. There are two ſorts, *Native*, or the *Minium* of the *Antients*, which is the Mineral that yields *Quickſilver*; whereof, and of *Sulphur*, it chiefly conſiſts; it is found in the Mines of *Iſtria*. This Colour was amongſt the *Antient Romans* uſed to ſacred Purpoſes, and on Feſtivals *Jupiter*'s Face was painted therewith, as likewiſe the Bodies of thoſe that entred in Triumph. The *Factitious Cinnabar* is that which we now uſe, and is made by a Sublimation of *Mercury* and *Sulphur*.

10. *Carmin*, made of *Cochineel.*

11. *Lake*, thought to be an *Arabick* Word: It is made of *Flocks* dyed, or ſhavings of ſcarlet Cloth, or of the *Cochineel Inſect*, or elſe of *Kermes-Berries*, their Tincture being extracted with a Lye of *Pot-Aſhes*, and then precipitated with a Solution of *Roch-Allom*. After the ſame manner a *Lake* may be made of any Plant or Flower. There is another ſort of *Lake* made of *Gum-lac*, by extracting its Tincture with Urine.

12 *Sanguis Graconis* is the Gum of a Tree, which looks like dried Blood; 'tis brought out of ſeveral Places in the *Eaſt-Indies*.

13. *Engliſh Reddle*, or *Ruddle*, is found in many Places of *England*; amongſt the reſt, near *Witney* in *Oxfordſhire*.

14. *Lamp-black*, by *Pliny* thus deſcribed: 'Tis made of the Soot of Roſin, or Pitch burnt, Houſes being built on purpoſe for it, that keep in the Smoak.

To make China Varniſhes; by *Dr.* Will Sherard. n. 262. *p.* 525. Mar. An. 1700.

V. This way of making ſeveral *China Varniſhes* was firſt ſent from the *Jeſuits* in *China* to the great Duke of *Tuſcany*.

Take of *Crude Varniſh* 60 Ounces, ordinary Water 60 Ounces, mix them well together till the Water diſappears, afterwards put this matter into a wooden Veſſel 5 or 6 Palms long, and 2 or 3 broad, mix them with a wooden *Spatula*, for a whole Day in the *Summer*'s Sun, and for two in the *Winter*. It is afterwards kept in earthen Veſſels with a Bladder over it, and cool. This is the *Varniſh prepared in the Sun.*

Boyling the Oyl of Wood.

Take 20 Ounces of the Oil, called *Oil of Wood*, of that of the *Fruit* 10 Drams; give them 5 or 6 Boils, till it comes to be a little *Yellow*. Let it cool, and put to it 5 Drams of *Quick-lime* powdered.

To give the firſt Grounds called Camiſcia.

Take *Swines Blood* and *Quick-lime* powder'd, mix them well, lay this mixture on the Wood, and, when it is dry, ſmooth it with Pumice-Stones.

To make black Varniſh.

Take of the *Varniſh* prepared in the Sun, 60 Ounces, *Stone-black Allum,* (ſuppoſe to be a ſort of *Copperas*) diſſolved in a little Water, 3 Drams; 70 Drams of *Lamp-Oil,* call'd by the *Portugueze Azeite de Candea.* It is prepared in a wooden Veſſel, as the prepared *Varniſh,* obſerving to put in the *Lamp-Oil* at twice.

Pitch-colour'd Varniſh.

Take of the Oil of *Wood* crude, (called by the *Portugueze Azeite de Pao*) 40 Drams, of the Lamp-Oil, called *de Candea,* Crude, 40 Drams: It is prepared in the Sun in a wooden Veſſel, as the prepared Varniſh.

Red Varniſh.

Take 10 Drams of *Cinnabar,* 20 Drams of *Varniſh* prepared, a little Oil *de Candea,* or Lamp-Oil; mix them well.

Yellow Varniſh.

Take of the *Yellow Colour* 10 Drams, 30 Drams of the prepared *Varniſh,* with ſome *Lamp-Oil.*

Musk-colour'd Varniſh.

Take of the *Red Varniſh* 10 Drams, of the *Black Varniſh* 4 Drams; mix them well.

An Examen of Pictures *propos'd; by M.* Colbert. n. 47. p. 953. May. An. 1669.

VI. M. *Colbert* being pleaſed ſome time ſince to viſit the *Academy Royal* for the Improvement of *Painting* and *Sculpture,* expreſſed himſelf to this Effect, That he thought it proper from time to time that the Works of the moſt *excellent Painters* ſhould be *examined,* and ſuch Obſervations made thereon as would inform others wherein the *Perfection* of a *Picture* conſiſts. Which hath been ever ſince practiſed among them, as the beſt Means to carry the Art of *Painting* to its higheſt Perfection; ſuch an *Examen* of the beſt *Pictures* diſcloſing many Secrets of that Art, for which there are no Rules, and opening a door to debate many important Queſtions, not hitherto treated of.

Wax Work, and a new kind of Maps in Low Relievo, *in* France; *by* .... n. 6. p. 99. Nov. An. 1665.

VII. Here is a Man who makes more lively Counterfeits of Nature in *Wax,* than ever I yet ſaw in *Painting,* having an extraordinary Addreſs in *modelling* the *Figures,* and mixing the *Colours* and *Shadows*; making the Eyes ſo lively, that they ſeem to kill, excelling all things of this Art I ever beheld.

I have alſo ſeen a new kind of *Maps* in *Low Relievo,* or Sculpture; for Example, the Iſle of *Antibe,* upon a Square about 8 Foot, made of Boards, with a Frame like a *Picture:* There is repreſented the Sea, with Ships, and other Veſſels artificially made, with their Cannons, and other Tackle of Wood fix'd upon the Surface, after a new and moſt admirable manner; the Rocks about the Iſland exactly form'd, as they are upon the natural Place; and the Iſland itſelf with all its Inequalities, and Hills and Dales; the Town, the Forts, the little Houſes, Plat-form, and Cannons mounted; and even the Gardens, and Plat-forms of Trees, with their green Leaves ſtanding upright, as if they were growing, in their natural Colours; *in fine,* Men, Beaſts, and whatever you may imagine to have any Protuberancy above the Level of the Sea. This new delightful, and moſt inſtructive Form of a *Map,* or wooden Country, you are to look upon either Horizontally, or ſide long, and it affords equally a very pleaſant Object.

To Colour Marble; by ..... n. 7. p. 125. Dec. An. 1665.

VIII. Whether the way mentioned by *Kircher* in his *Mundus Subterraneus* will ſucceed or not, is much doubted by ſome experienced Men: But 'tis certain, that a *Stone-cutter* in *Oxford*, Mr. *Bird*, hath many Years ſince found out a way of doing the ſame thing, in effect, that is there mentioned; and hath practiſed it for many Years: That is, he is able ſo to apply a *Colour* to the Out-ſide of poliſhed *Marble*, as that it ſhall ſink a conſiderable *Depth* into the Body of the Stone, and there repreſent like Figures or Images as thoſe are on the out-ſide; deeper or ſhallower, according as he continues the Application a longer or leſſer while.

An Extraordinary Tincture given to a Stone; by Dr. Salomon Reiſel. n. 179. p. 22.

IX. A Goldſmith at *Stutgard*, a very ingenious Ingraver, by Name *Chriſtopher Muller*, in the Year 1685. while he was rubbing ſome *Aurum fulminans* in a little Scuttle made of *Chalcedony* of the pellucid *Onyx* Colour, or of a horney Caſt, mixed with red Glaſs prepared for Fuſion, and moiſtened with Spring Water, in order to make Enamel, (of which *Antonius Nerius*, as tranſlated by *Andrea Friſio* treats in his Book upon the Art of making Glaſs) found upon repeating it for the third Time, that the Purple Colour of the Powder, which had remained ſome Days dry in the Veſſel, and had ſpread as far as the Lips of it in rubbing, had penetrated this exceeding hard Stone, which the File can make no Impreſſion upon, ſo deep not only in the Scuttle, but in the Piſtle itſelf, and had mark'd it with Spots or pretty regular Circles, that it could neither be taken out by ſimple Water, nor Lixive, or any other aerid Liquor, and all this without hurting the exquiſite Poliſh of the Stone, leaving however ſome Spots here and there ſtill of the *Onyx* Colour. This was repeated ſeveral times in another Veſſel of the ſame Colour, without ever having the ſame Effect. But what is chiefly to be obſerved in the colouring of this Stone, is, that according to the Texture of the Gem, as appeared both to the naked Eye and a Glaſs, in the external Part of the Veſſel, where the Tincture has juſt reached, you can perceive Fibres or circular Layers, according to which it is probable that it encreaſed in its Bulk, freſh Layers of ſtony Matter being laid upon one another; as *Bezoar* and other Stones are encreaſed by new Lamina, and alſo Trees, in whoſe Trunks the Circles or Rings denote the Number of Years that the Tree was a growing. And this purple Colour appears lighter or darker, according as the tinging Stuff met with larger or leſſer Pores, a harder or ſofter Texture of the Stone, and produces circular Streaks ſurrounding a Kind of Vertex, as you ſee round the Pith, or Cor, as it is called, the Grain, or Chaff in other Stones and Wood; intermixed here and there with darker Spots. As the illuſtrious *Boyle* in his *Specim. de Orig. & Virt. Gemmarum.* Sect. 1. p. 22, 23. has obſerved the Points and Joinings of thin Layers or Planes in *Adamant* and *Granates*; which Artiſts call the *Grain*, or ſomething reſembling the Texture of cleft Wood.

Several Writers have mentioned that *Marble*, *Alabaſter* and *Bones* may be tinged by lixivious and acrid Juices: and this may perhaps be expected of Gems, ſince the above mentioned *Boyle*, Sect. 2. p. 123. ſays that a Tincture may evidently be extracted from them; and elſewhere, p. 43 and 190. that *Rock Cryſtal* may be tinged by Mineral Acids, and p. 45. the Sapphire itſelf by ſubterraneous Vapours.

Seeing then it is plain from this Obſervation, that the Gem called the *Chalcedony* was really tinged, tho' it happened accidentally, nor by repeating the Proceſs could the ſame Effect be produced, it deſerves however to be conſidered, whether it was not owing to the Influence of the Stars, or rather ſome other hidden Quality, and a Trial to be made, whether from a Mixture of Salts and acrid Juices there might not be produced a Tincture of that Kind, and even without the Aſſiſtance of Fire, by which the Splendor and Pellucidity of the Gem ſhall not be deſtroyed and its Hardneſs ſtill remain, and therefore its Value be not only preſerved, but even increaſed by its new acquired Colour.

X. *Papers* omitted.

1. A Deſcription of *Scheiner's Stereographick Parallelogram*, and its *Imper-* n. 96. p. 6085.
fections conſidered; by Mr. *J. St. Clare*, vid. ſup. Sect. II.
2. A Table of *Simple* and *mix'd Colours*, in *Latin*, *Greek*, *French*, and n. 179. p. 24, 29.
Engliſh: with a Specimen of each Colour prefix'd to its proper *Name*; by Mr. *Rich. Waller*.

XI. *Accounts of Books* omitted.

1. Entretiens ſur les Vies & ſur les Ouvrages des plus excellens *Peintres*, n. 21. p. 383.
Antiens & *Modernes*; par M. *Felibien*.
2. An *Idea* of the *Perfection* of *Painting*: Originally written in *French*, n. 39. p. 784.
by *Roland Freart Sieur de Cambray*; and render'd into *Engliſh* by *J. Evelin*, Eſq; F. R. S. *Lond.* 1668. in 8*vo.*
3. A General *Idea* of the *Art* of *Painting*, and Relation of ſeven *Confe-* n. 47. p. 954.
rences held at *Paris* in the *Academy Royal*, for the Improvement of the *Arts* of *Painting* and *Sculpture*.
4. Optique de *Portraiture* & *Peinture*, contenant la *Perſpective*, *Specula-* n. 86. p. 5048.
tive, & *Pratique* Accomplie; &c. Par *Gregoire Huret*, de l' *Academie Royal* de *Peinture* & *Sculpture*. A *Paris*, 1670, *in Fol.*

CHAP.

CHAP. X.

Muſick.

Of the Trembling of Conſonant Strings; by Dr. Wallis. n. 134. p. 839. Mar. An. 1677.

Fig. 221.

I. IT hath been long ſince obſerved, That if a *Viol-ſtring*, or *Lute-ſtring*, be touched with the Bow or Hand, another *String* on the ſame or another Inſtrument not far from it, (if an *Uniſon* to it, or an *Octave*, or the like) will at the ſame time tremble of its own Accord. But I can now add, That not the whole of that other String doth thus tremble, but the ſeveral Parts ſeverally, according as they are *Uniſons* to the whole, or the Parts of that String ſo ſtruck. For Inſtance, Suppoſing A C to be an upper *Octave* to α γ, and therefore an *Uniſon* to each Half of it, ſtopped at β. If, while α γ is open, A C be ſtruck; the two Halves of this other, that is, α β, and β γ, will both tremble; but not the middle Point at β. Which will eaſily be obſerved, if a little Bit of Paper be lightly wrapt about the String α γ, and removed ſucceſſively from one End of the String to the other.

Fig. 222. Fig. 223. Fig. 224.

In like manner, If A D be an Upper *Twelfth* to α δ, and conſequently an *Uniſon* to its three Parts equally divided in β, γ; if α δ, being open, A D be ſtruck, its three Parts α β, β γ, γ δ, will ſeverally tremble, but not the Points β γ. In like manner, if A E be a *double Octave* to α ε, the four Quarters of this will tremble when that is ſtruck, but not the Points β, γ, δ. So if A G be a *Fifth* to α η; and conſequently each Half of that ſtopped in D, an *Uniſon* to each third Part of this ſtopped in β γ; while that is ſtruck, each Part of this will tremble ſeverally, but not the Points β, γ; and while this is ſtruck, each of that will tremble, but not the Point D. The like will hold in leſſer *Concords*; but the leſs remarkably, as the Number of Diviſions increaſes.

Fig. 225.

This was firſt of all, (as I know of) diſcovered by Mr. *Will. Noble*, M. A. of *Merton College*; and by him ſhewed to ſome of our *Muſicians*, about three Years ſince; and after him by Mr. *Tho. Pigot*, A. B. of *Wadham College*, without knowing that Mr. *Noble* had diſcover'd it before. I add this further, (which I took Notice of upon Occaſion of making Trial of the other) that the ſame *String*, as α γ, being ſtruck in the midſt of β, each Part being *Uniſon* to the other, will give no *clear Sound* at all, but very confuſed. And not only ſo (which others have obſerved, that a *String* doth not ſound clear, if ſtruck in the midſt) but alſo, if α δ be ſtruck at β, or γ, where one Part is an *Octave* to the other; and in like manner, if α ε be ſtruck at β or δ; the one Part being a *double Octave* to the other. And ſo if α ζ be ſtruck in γ or δ; the one Part being a *Fifth* to the other; and ſo in other like *Conſonant* Diviſions; but ſtill the leſs remarkable, as the Number of Divi-

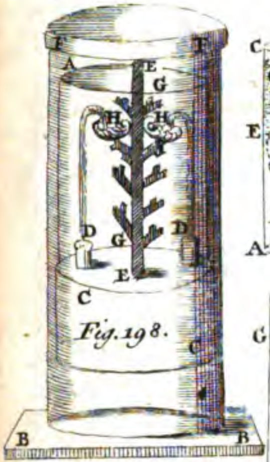
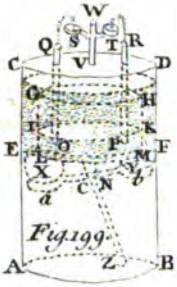


Fig. 198.



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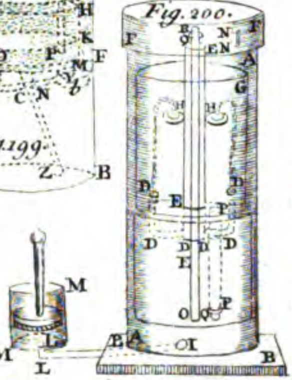

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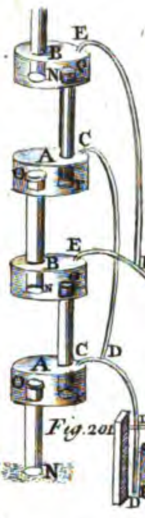

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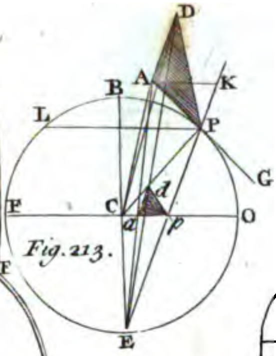

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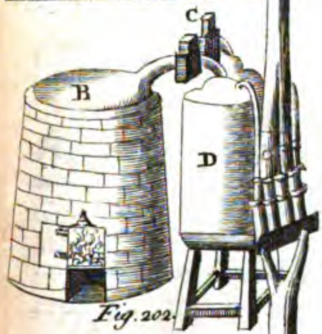

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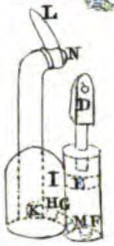

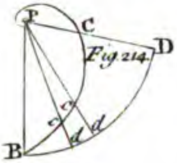

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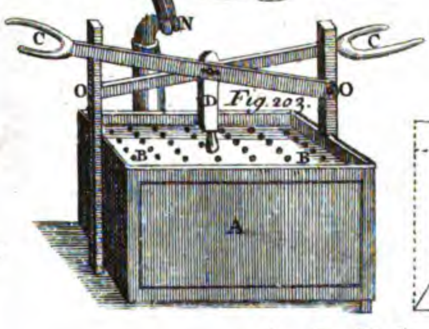

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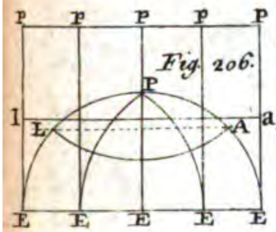

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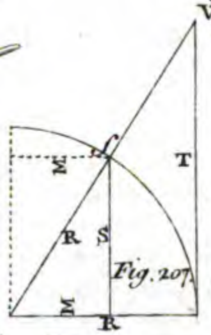

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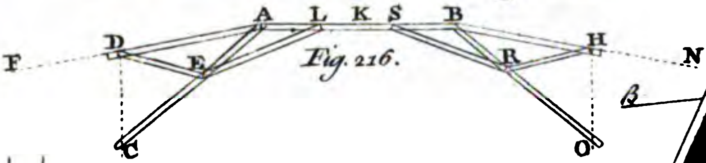

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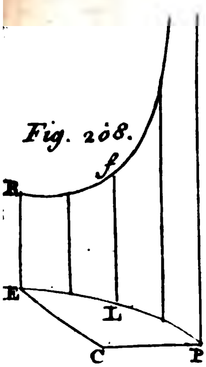

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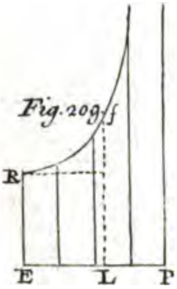

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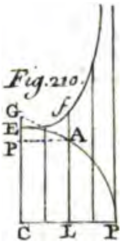

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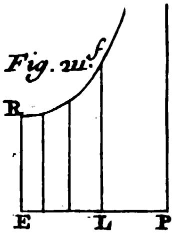

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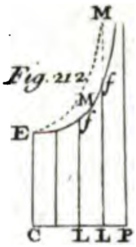

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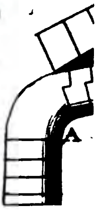
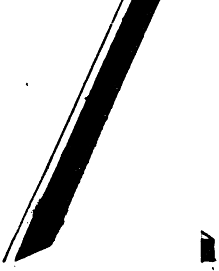




iviſions encreaſeth. This and the former I judge to depend upon one and e ſame Cauſe, *viz.* the contemporary Vibrations of the ſeveral *Uniſon* arts which make the one tremble at the Motion of the other: But, when uck at the reſpective Points of Diviſions, the Sound is incongruous, by eaſon the Point is diſturbed, which ſhould be at reſt.

A *Lute-ſtring* or *Viol-ſtring* will alſo thus anſwer to a *Conſonant Note* in *ind Inſtruments*: But not ſo remarkably to the *Wire-ſtrings* of an *Harpſichord*: nd we feel the Wainſcot Seats, on which we ſit or lean, to tremble con-antly at certain Notes on the *Organ*, or other *Wind Inſtruments*; as well as t the ſame Notes on a *Baſs Viol.* I have heard alſo (but cannot aver it) of thin fine *Venice Glaſs* cracked with the ſtrong and laſting Sound of a *Trum-et*, or *Cornet*, (near it) ſounding an *Uniſon* or a *Conſonant Note* that of the *one* or *Ting* of the *Glaſs*.

Concerning theſe *Phænomena*, an Exquiſite Solution is given by Dr. *Nar-iſſus Marſh*, In Dr. *Plot*'s *Natural Hiſtory* of *Oxfordſhire*. n. 155. p. 872.

The Defects of a Trumpet, and Trumpet-Marine; *by Mr.* Fran. Roberts. n. 195. p. 559. Fig. 226. Oct. An. 1692.

II. The *Extent* of the *Trumpet* cannot be ſtrictly determined; it reaches as *High* as the Strength of the Breadth can force it: But by conſidering its *Notes* within the ordinary Compaſs of the Scale of *Muſick* (from *Double C-fa-ut* to *C-ſol-fa in alt*) the Nature of the higher Notes will plainly appear. Theſe are all ſet down in the Table; only take Notice, that the *Prick'd Notes* are imperfect, not exactly in Tune, but a little *Flatter* or *Sharper* than the Places where they ſtand, according as *f* or *s* is ſet over them.

Here we may make two Enquiries.

1. Whence it comes to paſs, that the *Trumpet* will perform no other *Notes* (in that Compaſs) but only thoſe in the Table, which are uſually called by *Muſicians Trumpet-Notes.*

2. What is the Reaſon that the *7th*, *11th*, *13th* and *14th Notes* are out of Tune, and the others exactly in Tune.

In this Matter we may receive ſome Light from the *Trumpet Marine*, an Inſtrument, tho' as unlike as poſſible to the *Trumpet* in its Frame, one being a *Wind Inſtrument*, the other a *Monochord*, yet has a wonderful Agreement with it in its Effect.

The Sound is ſo like, as not to be eaſily diſtinguiſhed by the niceſt Ear, and, as it performs the very ſame *Notes*, ſo it has the ſame Defects as a *Trumpet*; for, if the *Strings* be ſtopt in any part but ſuch as produces a *Trumpet-Note*: it yields a harſh and uncouth (not a *Muſical*) *Sound.*

Let us therefore proceed to our firſt Inquiry, and examine what is the Reaſon that the *Trumpet Marine* will perform no other but the *Trumpet-Notes.* It is a known *Experiment* of two *Uniſon Strings*, that ſtriking one of them moves the other; which probably proceeds from hence, that the Impulſes of the Air, which are made by one String, do more eaſily ſet another in Motion, which lies in a Diſpoſition to have its *Vibrations Synchronous* to them, than a Third, whoſe Motion would be croſs.

We may improve this a little farther, by obſerving that a String will move not only at the Striking of an *Uniſon*, but an *8th* or *12th*, tho' after a different Manner.

If

Fig. 227. If an *Uniſon* be ſtruck, it makes one intire Vibration in the whole String, and the Motion is moſt ſenſibly in the midſt at *m*, for there the Vibrations take the greateſt Scope.

Fig. 228. If an 8*th* is ſtruck, it makes two Vibrations; and the Point *m* is in a manner Quieſcent, and the moſt ſenſible Motion at *n*, *n*.

Fig. 229. If a 12*th* be ſtruck, then it makes three Vibrations: and the greateſt Motion at *q*, *m*, *q*; and hardly to be perceived at *p*, *p*. So that in ſhort, this *Experiment* holds when any *Note* is ſtruck which is an *Uniſon* to half the String, and a 12*th* to a third Part of it.

Fig. 230. In this Caſe, (the Vibrations of the equal parts of a String being ſynchronous) there is no Contrariety in the Motion to hinder each other; whereas it is otherwiſe, if a *Note* is *Uniſon* to S, that does not divide the String into equal Parts; for then the Vibrations of the Remainder *r*, not ſuiting with thoſe of the other Parts, immediately make a Confuſion in the whole.

Now in the *Trumpet-Marine*, you do not ſtop cloſe, as in other *Inſtruments*, but touch the String gently with your Thumb, whereby there is a mutual Concurrence of the upper and lower part of the String to produce the Sound. This is ſufficiently evident from that, That if any thing touches the String below the Stop, the Sound will be as effectually ſpoiled, as if it were laid upon that Part which is immediately ſtruck with the *Bow*. From hence therefore we may collect, that the *Trumpet Marine* yields no *Muſical* Sound, but when the Stop makes the upper part of the *String* an *Aliquot* of the Remainder and conſequently of the whole Otherwiſe, as we juſt now remarked, the Vibrations of the Parts will ſtop one another, and make a Sound ſuitable to their Motion, altogether confuſed.

Now that theſe *Aliquot Parts* are the very Stops which produce the *Trumpet-Notes*, ſhall be plainly ſhewn in the treating of the ſecond Inquiry, *viz.* What is the Reaſon that the 7*th*, 11*th*, 13*th*, 14*th Notes* are out of Tune; and the reſt exactly in Tune.

All Writers of the Mathematical Part of *Muſick* agree,

That by Shortning a String	Half	the Sound is raiſed	an Eighth
	a Third Part		a Fifth
	a Fourth		a Fourth
	a Fifth		a Sharp Third
	a Sixth		a Flat Third.

From this Foundation all the other *Notes* are derived. The Flat and Sharp Sixth are to be the Flat and ſharp *Third* to the *Fourth*, and the 7*th* the like to the 5*th*: The Second to be a Fifth to the Fourth below, &c. By this Rule let us examine what Notes a *Monochord* fretted in its *Aliquot Parts* will produce.

Fig. 231. Suppoſe the *Monochord* F to conſiſt of 720 Parts, and its Tone *Double C-fa-ut*, the firſt *Note* in the *Table*; then Half of it will be 360, and a third Part 240, &c.

Now I ſay, *Fretting*, (or *Stopping* with the Thumb) at 360 muſt produce *C-fa-ut*; becauſe 360 being half 720, the Sound will riſe an *Eighth* from *double C-fa-ut*. Again 360 being *C-fa-ut*, 240 muſt make *G-ſol-re-ut*, the third Note in the Table; becauſe 240 being juſt a Third-part leſs than 360, the Sound will riſe a *Fifth* from that Note. After the ſame manner proceeding Step by Step it will be evident that,

180	which is leſs than	240	by juſt	a Fourth	produces	*C-ſol-fa-ut*	the fourth	*Note in the Table.*
144		180		a Fifth		*E-la-mi*	fifth	
120		144		a Sixth		*G-ſol-re-ut*	6th	
90		180		Half		*C-ſol-fa*	8th	
80		120		a Third		*D-la-ſol*	9th	
72		90		a Fifth		*E-la*	10th	
60		90		a Third		*G-ſol-re-ut*	12th	
48		60		a Fifth		*B-fa-b-mi*	15th	
45		90		Half		*C-ſol-fa*	16th	

By the ſame Reaſon,

100	which is leſs than	120	by juſt	a 6th	produces	*B-fa-b-mi Flat*,
$67\frac{1}{2}$		90		a 4th		*F-fa-ut*,
54		$67\frac{1}{2}$		a 5th		*A-la-mi-re*,
50		100		Half		*B-fa-b-mi Flat*,

And conſequently,

$102\frac{6}{7}$	the 7th	*Note* in the Table, is a little	*Flatter*	than	*B-fa-b-mi Flat*,
$65\frac{5}{11}$	11th		*Sharper*		*F-fa-ut*,
$55\frac{5}{13}$	13th		*Flatter*		*A-la-mi-re*,
$51\frac{3}{7}$	14th		*Flatter*		*B-fa-b-mi Flat*,

Which anſwers the ſecond Inquiry.

Now to apply this (in a few words) to the *Trumpet*, where the *Notes* are produced only by the different Force of the *Breath*; it is reaſonable to imagine that the ſtrongeſt Blaſt *raiſes* the Sound by breaking the Air within the *Tube* into the ſhorteſt Vibrations, but that no *muſical* Sound will riſe, unleſs they are ſuited to ſome *aliquot Part*, and ſo by Reduplication exactly meaſure out the whole Length of the *Inſtrument*, as in *Fig.* 229. for otherwiſe a Remainder will cauſe the ſame Inconvenience in this Caſe, as in *Fig.* 230. To which if we add, that a *Pipe*, being ſhortened acccording to the Proportions we even now diſcourſed of in a *String*, *raiſes* the Sound in the ſame Degrees, it renders the Caſe of the *Trumpet* juſt the ſame with the *Monochord*.

For a *Corollary* to this Diſcourſe, we may obſerve that the Diſtances of the *Trumpet Notes*, aſcending continually, decreaſed in Proportion of $\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, *in infinitum*. For,

$$\text{The}\left\{\begin{array}{l}\text{Second}\\ \text{Third}\\ \text{Fourth, \&c.}\end{array}\right\}\begin{array}{l}\text{Note in the}\\ \text{Table, differs}\\ \text{from the}\end{array}\left\{\begin{array}{l}\text{Firſt}\\ \text{Second}\\ \text{Third, \&c.}\end{array}\right\}\text{by}\left\{\begin{array}{c}\frac{1}{2}\\ \frac{1}{3}\\ \frac{1}{4}\end{array}\right\}\text{of the } \textit{String}\text{, \&c.}$$

The Diviſion of the Monochord; *by* Dr. J. Wallis. n. 238. *p.* 80. Mar. An. 1698.

III. Any *String* or *Chord* of a *Muſical Inſtrument Open* (or at its full length) will ſound (what we call) an *Octave* (or *Diapaſon*) to that of the ſame *String* ſtopt in the Middle, or at half its Length: Hence it is, that we commonly aſſign to an *Octave*, the *Duple* Proportion (or that of 2 to 1) becauſe ſuch is the Proportion of Lengths (taken in the ſame *String*) which give thoſe Sounds. And (upon a like Account) we aſſign to a *Fifth* (or *Diapente*) the *Seſqui-alter* Proportion (or that of 3 to 2.) And to a *Fourth* (or *Diateſſaron*) the *Seſqui-tertian* (or that of 4 to 3.) And to a *Tone* (which is the Difference of a *Fourth* and *Fifth*) the *Seſqui-octave* (or that of 9 to 8:) Becauſe Lengths (taken in the ſame *String*) in theſe Proportions, do give ſuch Sounds.

And (univerſally) whatever Proportion of Lengths (taken in the ſame *String* equally ſtretched) do give ſuch and ſuch Sounds; ſuch Proportions (of Gravity) we aſſign to the Sounds ſo given.

But, when an *Eighth* (or *Octave*) is ſaid (in common Speech) to conſiſt of 12 *Hemi-tones*, or 6 *Tones*; this is not to be underſtood according to the utmoſt Rigour of *Mathematical* Exactneſs, (of ſuch 6 *Tones*) as what they call the *Diazeutick Tone*, or that of *la*, *mi*, which is the Difference of a *Fourth* and *Fifth*; but, as exact enough for common Uſe. For 6 ſuch *Tones*, (that is, the Proportion of 9 to 8, 6 times repeated) is ſomewhat more than that of an *Octave*, (or the Proportion of 2 to 1.) And conſequently, ſuch an *Hemitone*, is ſomewhat more than the Twelfth-part of an *Eighth* or *Octave*, or *Diapaſon*. But the Difference is ſo little, that the Ear can hardly diſtinguiſh it: And therefore (in common Speech) it is uſual ſo to ſpeak.

And, accordingly, when we are directed to take the Lengths (for what are called the 12 *Hemi-tones*) in *Geometrical* Proportion, it is to be underſtood (not to be ſo in the utmoſt Strictneſs, but) to be accurate enough for common Uſe, for placing the *Frets* on the Neck of a *Vial*, or other *Muſical Inſtrument*, wherein a greater Exactneſs is not thought neceſſary. And this is very convenient, becauſe (thus) the Change of the *Key* (upon altering the Seat of *mi*) gives no new Trouble, for this doth indifferently ſerve any *Key*; and the Difference is ſo ſmall, as not to offend the Ear.

But thoſe who chuſe to treat of it with more Exactneſs, go this way to work.

Preſuppoſing the Proportion for an *Octave* (or *Dia-paſon*) to be that of 2 to 1; they divide this into two Proportions; not juſt equal (for that would fall upon the *Surd* Numbers, as $\sqrt{2}$ to 1;) but near equal (ſo as to be e preſſed in ſmall Numbers.) In order to which, inſtead of taking 2 to 1, th take (the Double of theſe Numbers) 4 to 2; (which is the ſame Pr portion as before;) and interpoſe the Middle Number 3. And of the three Numbers, 4, 3, 2, that of 4 to 3, is the Proportion of a *Fourth* (or *Dia-teſſaron.*) And that of 3 to 2, the Proportion for a *Fifth* (or *Diapente.*)

pente.) And theſe two, put together, make up that of an *Octave* (or *Diapaſon*, that of 4 to 2, (or 2 to 1.) And the Difference of thoſe two, that of a *Tone*, or 9 to 8. As will plainly appear in the ordinary Method of *multiplying* and *dividing Fractions*, That is, $\frac{4}{3}\times\frac{3}{2}=\frac{4}{2}=\frac{2}{1}$. And $\frac{4}{3})\,\frac{3}{2}\,(\frac{9}{8}$.

Thus in the common *Scale* (or *Gam-ut*) taking an *Octave*, in theſe Notes, *la*, *fa*, *ſol*, *la*, *mi*, *fa*, *ſol*, *la*; ſuppoſe from E to *e* (placing *mi*, *B-fa-b-mi*, which is called the *natural Scale*;) the Lengths for the Extremes *la*, *la*, an *Octave*, are as 2 to 1, or 12 to 6. Thoſe for *la*, *la*, (in *la*, *fa*, *ſol*, *la*,) or *mi*, *la*, (in *mi*, *fa*, *ſol*, *la*,) a *Fourth*, as 4 to 3, or 12 to 9, or 8 to 6. Thoſe for *la*, *mi*, (in *la*, *fa*, *ſol*, *la*, *mi*,) or *la*, *la*, (in *la*, *mi*, *fa*, *ſol*, *la*,) a *Fifth*, as 3 to 2, or 12 to 8, or 9 to 6. Thoſe for *la*, *mi*, the *Diazeutick-Tone* (or Difference of a *Fourth* and *Fifth*, as 9 to 8. So have we for theſe four Notes *la*, *la*, *mi*, *la*, their proportionate Length in the Numbers 12, 9, 8, 6.

Then, if we proceed in like manner to divide a *Fifth* (or *Dia-pente*,) *la*, *fa*, *ſol*, *la*, *mi*, or *la*, *mi*, *fa*, *ſol*, *la*, or the Proportion of 3 to 2, into near Equals, (taking double Numbers in the ſame Proportion, 6, 4; and interpoſing the middle Number 5;) of theſe three Numbers, 6, 5, 4, that of 6 to 5, is the Proportion of a *leſſer Third*, (called a *Tri-hemitone*, or *Tone* and half,) as *la*, *fa*, (in *la*, *mi*, *fa*.) And that of 5 to 4, is the Proportion of the *greater Third*, (commonly called a *Ditone*, or two *Tones*,) as *fa*, *la*, (in *fa*, *ſol*, *la*,) which two put together make a *Fifth*, as 3 to 2; that is $\frac{6}{5}\times\frac{5}{4}=\frac{6}{4}=\frac{3}{2}$; and their Difference is, as 25 to 24: That is $\frac{6}{5})\,\frac{5}{4}\,(\frac{25}{24}$. So have we for theſe 3 Notes, *la*, *fa*, *la*, their proportionate Lengths in Numbers, as 6, 5, 4.

In like manner, if we divide a *Ditone*, (or *greater Third*,) as *fa*, *la*, (in *fa*, *ſol*, *la*,) whoſe Proportion is as 5 to 4, (or 10 to 8,) into two near Equals (by help of a middle Number 9;) then have we (in theſe three Numbers 10, 9, 8,) that of 10 to 9, for (what they call) the *leſſer Tone*: And that of 9 to 8, for (what they call) the *greater Tone*.

But, whether *fa*, *ſol*, ſhall be made the *Leſſer* (as 10 to 9,) and *ſol*, *la*, the *Greater*, (as 9 to 8;) or, This the *Leſſer*, (as 10 to 9,) and that the *Greater*, (as 9 to 8,) or ſome time This, ſome time That, as there is occaſion, (to avoid what they call a *Schiſm*;) is ſomewhat indifferent: For, either way, the Compound will be as 5 to 4; and the Difference (which they call a *Comma*,) as 81 to 80. This is $\frac{9}{8}\times\frac{10}{9}=\frac{10}{9}\times\frac{9}{8}=\frac{10}{8}=\frac{5}{4}$. And $\frac{10}{9})\,\frac{9}{8}\,(\frac{81}{80}$.

Laſtly, if from that of the *Tri-hemi-tone* (or *Leſſer Third*) *la, mi, fa,* whoſe Proportion is as 6 to 5; we take that of the *Tone, la, mi,* (which is the Difference of a *Fourth* and *Fifth*) as 9 to 8; there remains for the *Hemi-tone, mi, fa,* (or *la, fa,*) that of 16 to 15. That is $\frac{9}{8}) \frac{6}{5} (\frac{48}{45} = \frac{16}{15}$.

Or, the *Tri-hemi-tone* (or *leſſer Third*) whoſe Proportion is as 6 to 5, may be divided into three near Equals, (by taking triple Numbers, in the ſame Proportion 18, 15; and interpoſing the two Intermediates 17, 16;) which will therefore be as 18 to 17, and as 17 to 16, and as 16 to 15; That is, $\frac{18}{17} \times \frac{17}{16} \times \frac{16}{15} = \frac{18}{15} = \frac{6}{5}$.

Where alſo the *greater Tone*, whoſe Proportion is as 9 to 8 or 18 to 16, is divided into its two near Equals (commonly called *Hemi-tones,*) that of 18 to 17, and that of 17 to 16: That is, $\frac{18}{17} \times \frac{17}{16} = \frac{18}{16} = \frac{9}{8}$.

And the *leſſer Tone*, that of 10 to 9, or 20 to 18, may be in like manner divided into that of 20 to 19, and that of 19 to 18: That is, $\frac{20}{19} \times \frac{19}{18} = \frac{20}{18} = \frac{10}{9}$.

Which Diviſions of the *Greater* and *Leſſer Tone* anſwer to what is wont to be deſigned by *Flats* and *Sharps*.

So that (by this Compoſition) of theſe Eight Notes, *la, fa, ſol, la, mi, fa, ſol, la,* their Proportions ſtand thus; that of *la, fa,* (or *mi, fa,*) is as 16 to 15. That of *fa, ſol,* as 10 to 9, and that of *ſol, la,* as 9 to 8; (or elſe that of *fa, ſol,* as 9 to 8, and that of *ſol, la,* as 10 to 9.) That of *la, mi,* as 9 to 8. And, if either of the *Tones* (*Greater* or *Leſſer*) chance to be divided (by *Flats* or *Sharps*) into (what they call) *Hemi-tones*, their Proportions are to be ſuch as is already mentioned.

There may be a like Diviſion of a *Fourth*, (or *Dia-teſſaron*) into two Near Equals: And of ſome others of theſe, into three Near Equals. Which might be of uſe for (what they were wont to call) the *Chromatick* and *Enarmonick Muſick*. But, thoſe ſorts of *Muſick* having been long ſince laid aſide, there is now no need of theſe Diviſions, as to the *Muſick* now in Uſe.

The Imperfection of an Organ; by Dr. J. Wallis. n. 241. p. 249. July, An. 1698.

IV. I think 'tis evident that the *Pipe* in the *Organ* is intended to expreſs a Diſtinct Sound at ſuch a *Pitch*; that is, in ſuch a Determinate Degree of *Gravity* or *Acuteneſs*; or (as it is now called) *Flatneſs* or *Sharpneſs*; And the Relative or Comparative Conſideration of Two (or more) ſuch Sounds or Degrees of *Flatneſs* or *Sharpneſs*, is the Ground of (what we call) *Concord* and *Diſcord*; that is a ſoft, or harſh Coincidence.

Now concerning this, there were amongſt the Antient *Greeks*, Two (the moſt conſiderable) Sects of *Muſicians*: the *Ariſtoxenians*, and *Pythagoreans*.

They both agreed thus far; that *Dia-teſſaron*, and *Dia-pente*, do together make up *Dia-paſon*; That is (as we now ſpeak) a *Fourth* and *Fifth* do together make an *Eighth* or *Octave*: And, the Difference of thoſe two, of a *Fourth* and *Fifth*, they agreed to call a *Tone*, which we now call a *Whole Note*.

Such is that, (in our preſent *Muſick*,) of *la*, *mi*, (or, as it was wont to be called, *re*, *mi*.) For *la*, *fa*, *ſol*, *la*, or *mi*, *fa*, *ſol*, *la*, is a perfect *Fourth*: And *la*, *fa*, *ſol*, *la*, *mi*, or *la*, *mi*, *fa*, *ſol*, *la*,, is a perfect *Fifth*: The Difference of which is *la*, *mi*, which is what the *Greeks* call the *Diazeutick-Tone*; which doth disjoin two *Fourths* (on each ſide of it; and being added to either of them doth make a *Fifth*; Which was, in their *Muſick*, that from *Meſe* to *Parameſe*; that is in our *Muſick*, from A to B: Suppoſing *mi* to ſtand in *B-fa-b-mi*, which is accounted its Natural Poſition.

Now in order to this, *Ariſtoxenus* and his Followers did take that of a *Fourth*, as a known *Interval*, by the Judgment of the Ear; and that of a *Fifth* likewiſe: And conſequently that of an *Octave*, as the Aggregate of both; and that of a *Tone*, as the Difference of thoſe Two.

And this of *Tone* (as a known *Interval*) they took as a common Meaſure, by which they did eſtimate other *Intervals*: And accordingly they accounted a *Fourth* to contain two *Tones* and a Half; a *Fifth* to contain three *Tones* and a Half, and conſequently an *Eighth* to contain ſix *Tones*; or five *Tones* and two *Half Tones*.

And at this Rate our *Practical Muſicians* talk of *Notes* and *Half Notes* at this Day; ſuppoſing an *Octave* to conſiſt of twelve *Hemi-tones*, or *Half Notes*.

But, *Pythagoras* and thoſe who follow him, not taking the Ear alone to be a Competent Judge in a Caſe ſo nice, choſe to diſtinguiſh theſe, not by *Equal Intervals*, but by *Due Proportions*: And this is followed by *Zarline*, *Kepler*, *Cartes*, and others, who treat of *Speculative Muſick* in this and the laſt Age. Accordingly they accounted that of an *Octave*, to be, when the Degree of *Gravity*, or *Acuteneſs* of the one Sound to that of the other, is Double, or as 2 to 1; that of a *Fifth*, when it is *Seſqui-alter*, or as 3 to 2: that of a *Fourth* when *Seſqui-tertian*, or 4 to 3. Accounting that the ſweeteſt Proportion, which is expreſs'd in the ſmalleſt Numbers, and therefore (next to the *Uniſon*) that of an *Octave*, 2 to 1, then that of a *Fifth*, 3 to 2, and then that of a *Fourth*, 4 to 3.

And thus that of a *Fourth* and *Fifth* do together make an *Eighth*; for $\frac{4}{3} \times \frac{2}{3} = \frac{4}{2} = \frac{2}{1} = 2$, or the Proportion of 4 to 3, compounded with that of 3 to 2, is the ſame with that of 4 to 2, or 2 to 1. And conſequently, the Difference of thoſe two, which is that of a *Tone*, or Full Note, is that of 9 to 8. For $\frac{4}{3})\ \frac{3}{2}\ (\frac{9}{8}$; or, if out of the Proportion of 3 to 2, we take that of 4 to 3; the Reſult it that of 9 to 8. Now,

Now according to this Computation, it is manifeſt, that an *Octave* is ſomewhat leſs than Six *Full Notes*. For (as was firſt demonſtrated by *Euclid*, and ſince by others) the Proportion of 9 to 8, being ſix Times compounded, is ſomewhat more than that of 2 to 1. For $\frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} = \frac{531441}{262144}$, is more than $\frac{524288}{262144} = \frac{2}{1}$.

This being the Caſe; they allowed (indiſputably) to that of the *Diazeutick Tone* (*la*, *mi*,) the full Proportion of 9 to 8, as a thing not to be altered; being the Difference of the *Dia-pente* and *Dia-teſſaron*, or the *Fifth* and *Fourth*.

All the Difficulty was, How the remaining *Fourth* (*mi*, *fa*, *ſol*, *la*,) ſhould be divided into three parts, ſo as to anſwer (pretty near) the *Ariſtoxenians* Two *Tones* and a half: And might, all together, make up the Proportion of 4 to 3, which is that of a *Fourth* or *Dia-teſſaron*,

Many Attemps were made to this purpoſe: And according to thoſe, they give Names to the Different *Genera* or kinds of *Muſick*, (the *Diatonick Chromatick*, and *Enarmonick* Kinds,) with the ſeveral *Species*, or leſſer Diſtinctions under thoſe Generals.

The firſt was that of *Euclid* (which did moſt generally obtain for many Ages:) Which allows to *fa*, *ſol*, and to *ſol*, *la*, the full Proportion of 9 to 8; And therefore to *fa*, *ſol*, *la*, (which we call the *Greater Third*) that of 81 to 64. (For $\frac{9}{8} \times \frac{9}{8} = \frac{81}{64}$.) And, conſequently, to that of *Mi*, *fa*, (which is the Remainder to a *Fourth*) that of 256 to 243. For $\frac{81}{64}) \frac{4}{3} (\frac{256}{243}$; that is, if out of the Proportion of 4 to 3 we take that of 81 to 64, the Reſult is that of 256 to 243. To this they give the Name of *Limma* (λεῖμμα) that is, the Remainder (to wit, over and above two *Tones*.) But, in common Diſcourſe (when we do not pretend to ſpeak nicely, nor intend to be ſo underſtood) it is uſual to call it an *Hemi-tone*, or *Half-Note*, (as being very near it) and the other, Two *Whole Notes*. And this is what *Ptolemy* calls *Diatonum Ditonum*, (of the *Diatonick* kind with *Two Full Tones*.)

Againſt this it is objected (as not the moſt convenient Diviſion) that the Numbers of 81 to 64, are too great for that of a *Ditone*, or *Greater Third*; which is not harſh to the Ear; but is rather ſweeter than that of a ſingle *Tone*, whoſe Proportion is 9 to 8. And, in that of 256 to 243, the Numbers are yet much greater. Whereas there are many Proportions (as $\frac{5}{4}, \frac{6}{5}, \frac{7}{6}, \frac{8}{7}$, in ſmaller Numbers than that of 9 to 8; of which, in this Diviſion, there is no Notice taken.

To rectify this, there is another Diviſion thought more convenient; which is *Ptolemy*'s *Diatonum Intenſum* (of the *Diatonick* Kind, more *Intenſe* or *Acute* than that other) which inſtead of two *Full Tones* for *fa*, *ſol*, *la*, aſſigns (what we now call) a *Greater* and a *Leſſer Tone*; (which by the more Nice *Muſicians* of this and the laſt Age ſeems to be more embraced;) Aſſigning to *fa*, *ſol*, that of 9 to 8, (which they call the *Greater Tone*;) and to *ſol*, *la*, that of 10 to 9, (which they call the *Leſſer Tone*:) And therefore to *fa*, *la*, (the *Ditone* or *Greater Third*) that of 5 to 4. (For $\frac{10}{9} \times \frac{9}{8} = \frac{10}{8} = \frac{5}{4}$.) And conſequently to *mi*, *fa*, (which is remaining of the *Fourth*) that of 16 to 15. For $\frac{5}{4})\ \frac{4}{3}\ (\frac{16}{15}$. That, if out of that of 4 to 3 we take that of 5 to 4, there remains that of 16 to 15.

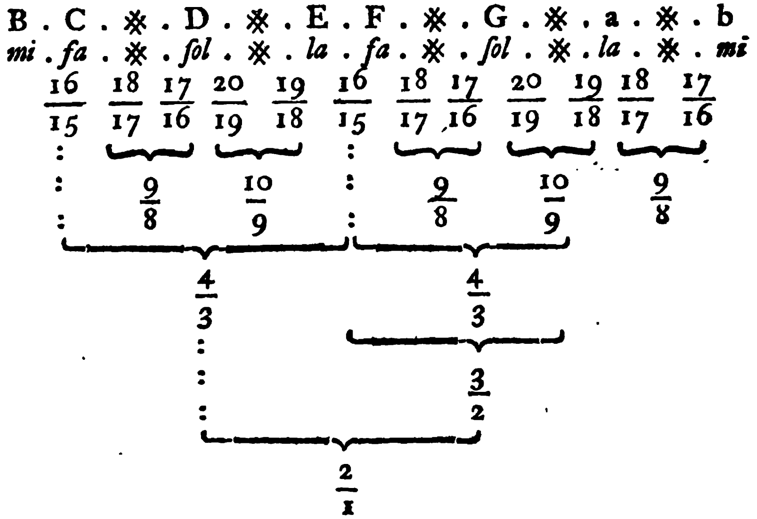
Many other ways there are (with which I ſhould not trouble you at preſent) of dividing the *Fourth* or *Dia-teſſaron*, or the proportion of 4 to 3, into three Parts, anſwering to what (in a looſer way of expreſſion) we call an *half Note*, and *two whole Notes*. But this of $\frac{16}{15} \times \frac{9}{8} \times \frac{10}{9} = \frac{4}{3}$, is that which is now received as the moſt proper.

To which therefore I ſhall apply my Diſcourſe; where $\frac{16}{15}$ is (what we call) the *Hemi-Tone*, or *Half-Note*, in *mi*, *fa*; $\frac{9}{8}$ that of the *greater Tone*, in *fa*, *ſol*, and $\frac{10}{9}$ the *leſſer Tone*, in *ſol*, *la*.

Only with this Addition; that each of thoſe *Tones*, is (upon Occaſion) by *Flats* and *Sharps* (as we now ſpeak) divided into two *Hemi-tones*, or *Half Notes*; Which anſwers to what by the *Greeks* was called *Mutatio quoad Modos* (the Change of *Mood*;) and what is now done by removing *mi* to another *Key*. Namely $\frac{9}{8} = \frac{18}{16} = \frac{18}{17} \times \frac{17}{16}$; and $\frac{10}{9} = \frac{20}{18} = \frac{20}{19} \times \frac{19}{18}$.

Thus by the help of *Flats* and *Sharps* (dividing each *whole Note*, be it the *Greater* or *Leſſer*, into two *Half Notes*, or what we call ſo,) the whole *Octave* is divided into Twelve parts or *Intervals* (contained between Thirteen *Pipes*) which are commonly called *Hemi-tones* or *Half-Notes*; not that each is preciſely *Half* a *Note*, but ſomewhat near it, and ſo called. And I ſay, by *Flats* and *Sharps*; for ſometime the one, ſometime the other is uſed. As for Inſtance, a *Flat* in D, or a *Sharp* in C, do either of them denote a Middling Sound (tho' not preciſely in the Midſt) between C and D; *Sharper* than C, and *Flatter* than D.

Accordingly, ſuppoſing *mi* to ſtand in *B-fa-b-mi* (which is accounted its *Natural* Seat) the Sounds of each *Pipe* are to bear theſe Proportions to each other, *viz.*


B . C . ※ . D . ※ . E . F . ※ . G . ※ . a . ※ . b
mi . *fa* . ※ . *ſol* . ※ . *la* . *fa* . ※ . *ſol* . ※ . *la* . ※ . *mi*
16/15 18/17 17/16 20/19 19/18 16/15 18/17 17/16 20/19 19/18 18/17 17/16
9/8 10/9 9/8 10/9 9/8
4/3 4/3
3/2
2/1

And ſo in each *Octave* ſucceſſively following. And, if the *Pipes* in each *Octave* be fitted to Sounds in theſe Proportions of *Gravity* and *Acuteneſs*, it will be ſuppoſed (according to this *Hypotheſis*) to be perfectly proportioned.

But, inſtead of theſe ſucceſſive Proportions of each *Hemi-tone*; it is found neceſſary (if I do not miſtake the Practice) ſo to order the 13 *Pipes* (containing 12 Intervals which they call *Hemi-tones*) as that their Sounds (as to *Gravity* and *Acuteneſs*) be in continual Proportion, (each to its next following, in *Octave* or *Dia-paſon*, as 2 to 1. Whereby it comes to paſs that each *Pipe* doth not expreſs its proper Sound, but very near it, yet ſomewhat varying from it; which they call *Bearing*; which is ſomewhat of Imperfection in this *Noble Inſtrument*, the Top of all.

It may be aſked, why may not the *Pipes* be ſo ordered, as to have their Sounds in juſt Proportion, as well as thus *Bearing*.

I anſwer, It might very well be ſo, if all *Muſick* were *Compoſed* to the ſame *Key*, or (as the *Greeks* call it) the ſame *Mode*; as for Inſtance, if, in all Compoſitions, *mi*, were always placed in *B-fa-b-mi*. For then the *Pipes* might be ordered in ſuch Proportions as I have now deſigned.

But *Muſical Compoſitions* are made in great Variety of *Modes*, or with great Diverſity in the *Pitch*. *Mi* is not always placed in *B-fa-b-mi*; but ſometimes in *E-la-mi*, ſometimes in *A-la-mi-re*, &c. And (in Sum) there is none of thoſe 12 or 13 *Pipes* but may be made the *Seat* of *mi*. And, if they were exactly fitted to any one of theſe Caſes, they would be quite out of Order for all the reſt.

As for Inſtance; If *mi* be removed from *B-fa-b-mi*, (by a Flat in B) to *E-la-mi*, inſtead of the Proportions but now deſigned, they muſt be thus ordered,

B . ※ . C . ※ . D . ※ . E . F . ※ . G . ※ . a . b
fa . ※ . *ſol* . ※ . *la* . ※ . *mi* . *fa* . ※ . *ſol* . ※ . *la* . *fa*.

$$\frac{18}{17}\quad\frac{17}{16}\quad\frac{20}{19}\quad\frac{19}{18}\quad\frac{18}{17}\quad\frac{17}{16}\quad\frac{16}{15}\quad\frac{18}{17}\quad\frac{17}{16}\quad\frac{20}{19}\quad\frac{19}{18}\quad\frac{16}{15}$$

where 'tis manifeſt, that the Removal of *mi* doth quite diſorder the whole *Series* of Proportions. And the ſame would again happen, if *mi* be removed from E to A (by another *Flat* in E,) and again, if removed from A to D. And ſo perpetually. But the *Hemitones* being made all equal, they do indifferently anſwer all the Poſitions of *mi* (tho' not exactly to any;) yet nearer to ſome than to others. Whence it is, that the ſame Tune ſounds better at one *Key* than at another.

It is aſked, whether this may not be remedy'd by interpoſing more *Pipes*; and thereby dividing a *Note* not only (as now) into *Half Notes*, but into *Quarter-Notes*, or *Half-Quarter-Notes*, &c.

I anſwer; It may be thus remedy'd in Part; (that is, the Imperfection might thus be ſomewhat leſs, and the Sounds ſomewhat nearer to the juſt Proportions:) but it can never be exactly true, ſo long as their Sounds (be they never ſo many) be in continual Proportion; that is, each to the next ſubſequent in the ſame Proportion.

For it hath been long ſince demonſtrated, that there is no ſuch thing as a juſt *Hemitone* practicable in *Muſick* (and the like for the Diviſion of a *Tone* into any Number of equal Parts; three, four or more.) For, ſuppoſing the Proportion of a *Tone* or *Full Note*, to be $\frac{9}{8}$ (or, as 9 to 8) that of the *Half-Note* muſt be as $\sqrt{9}$ to $\sqrt{8}$; that is, as 3 to $\sqrt{8}$. (or 3 to $2\sqrt{2}$) which are *Incommenſurable Quantities*: And that of a *Quarter-Note*, as $\sqrt[4]{9}$ to $\sqrt[4]{8}$, which is yet more *Incommenſurate*. And the like for any other Number of equal Parts; which will therefore never fall in with the Proportions of Number to Number.

So that this can never be perfectly adjuſted for all Keys (without ſomewhat of *Bearing*) by multiplying Pipes; unleſs we would for every *Key*, (or every different Seat of *mi*) have a different *Set* of *Pipes*, of which this or that is to be uſed according as (in the Compoſition) *mi* is ſuppoſed to ſtand in this or that Seat. Which vaſt Number of *Pipes*, (for every *Octave*) would vaſtly increaſe the Charge; and (when all is done) make the whole impracticable.

A New Tuning of the Lyra Viol; *by S.* Salvetti. n. 87. *p.* 5064. Aug. An. 1672.

V. *S. Salvetti*, about 4 Years ago, invented a *New Tuning* of the *Antient Lyra-Viol* with the uſual 13 *Strings*; by Means of which *Tuning*, it is render'd wholly perfect, ſo that you may expreſs upon it all *Concords*, *Diſcords*, and alſo the *Imperfect Concords*, as *ſevenths*, *ſixths*, *&c.*: as well as upon any *Virginal* that hath the Quarters of Notes upon it. 'Tis true, 'tis only for melancholy and paſſionate Matter, and not for *Diviſion*, as is the proper Nature of the *Lyra*. I ſhall only add, that with the aboveſaid *Tuning*, it aſcends in *Alt* as high as *G-ſol-re-ut*; and deſcends as low as *Double C-fa-ut*; and can make every where the ſame Concords as above.

The Strange Effects reported of Muſick *in Former Times, examined; by Dr.* Wallis n. 243. *p.* 297. Aug. An. 1698

VI. 1. I take it for granted, that much of the Reports concerning the great Effects of *Muſick* in former Times, beyond what is to be found in latter Ages, is highly *Hyperbolical*, and next door to Fabulous; and therefore great Abatements muſt be allowed to the Elogies of their *Muſick*.

2. We muſt conſider, That *Muſick* (to any tolerable Degree) was then (if not a new, at leaſt) a rare Thing, which the Ruſticks, on whom it is reported to have had ſuch Effects, had never heard before; and on ſuch, a little Muſick will do great Feats; as we find at this Day a *Fiddle* or a *Bag-pipe* at a Country Morice Dance.

3. We are to conſider, that their *Muſick* (even after it came to ſome good Degree of Perfection) was much more plain and ſimple than ours now-a-days. They had not *Conſorts* of two, three, four or more *Parts* or *Voices*: But one ſingle Voice or ſingle Inſtrument apart which to a rude Ear, is much more taking than more compounded Muſick. For that is at a Pitch not above their Capacity; whereas this other confounds it, with a great Noiſe, but nothing diſtinguiſhable to their Capacity.

4. We are to conſider, that *Muſick* with the *Antients* was of a larger Extent than what we call *Muſick* now a days: For *Poetry* and *Dancing* (or comely Motion) were then accounted Parts of *Muſick* when *Muſick* arrived to ſome Perfection. Now we know that *Verſe* of itſelf, if in good Meaſures and affectionate Language, and this ſet to a *Muſical Tune*, and ſung by a decent Voice, and accompanied but with *ſoft Inſtrumental Muſick*, if any, ſuch as not to drown or obſcure the *emphatick* Expreſſions (like what we call *Recitative Muſick*) will work ſtrangely upon the Ear, and move all Affections ſuitable to the *Tune* and *Ditty*; (whether briſk and pleaſant, or ſoft and pitiful, or fierce and angry, or moderate and ſedate) eſpecially if attended with a *Geſture* and *Action* ſuitable. For, 'tis well known, that ſuitable *Acting* on a *Stage* gives great Life to the Words. Now all this together (which were all Ingredients in what they called *Muſick*) muſt needs operate ſtrongly on the Fancies and Affections of ordinary People unacquainted with ſuch kind of Treatments. For if the deliberate reading of a Romance (when well penn'd) will produce Mirth, Tears, Joy, Grief, Pity, Wrath, or Indignation ſuitable to the reſpective Intents of it; much more would it ſo do, if accompanied with all thoſe Attendants.

5. You will aſk, perhaps, why may not all this be now done, as well as then? I anſwer, no doubt it may, and with like Effect, if an Addreſs be made in proper Words, with moving Accents in juſt Meaſures, (*Poetical* or *Rhetorical*) with the *emphatick Words*, Words ſet in ſignal Places, pronounced with a good Voice, and a true Accent, and attended with a decent Geſture: all theſe ſuitably adjuſted to the Paſſion, Affection, or Temper of Mind, particularly deſigned to be produced, (be it Joy, Love, Grief, Pity, Courage or Indignation) will certainly *now* as well as *then*, produce great Effects upon the Mind, eſpecially upon a Surprize, and where Perſons are not otherwiſe pre-engaged; and if ſo managed, as that you be (or ſeem to be) in earneſt; and if not over-acted by apparent Affectation.

6. We are to conſider, that the uſual Deſign of what we now call *Muſick*, is very different from that of the *Antients*. What we now call *Muſick* is but what they called *Harmonick*; which was but one Part of their *Muſick*, (conſiſting of Words, Verſe, Voice, Tune, Inſtrument, and Acting;) and we are not to expect the ſame Effect of one Piece, as of the whole.

7. When *Muſick* arrived to great Perfection, it was applied to particular Deſigns of exciting this or that particular Affection, Paſſion, or Temper of Mind; the *Tunes* and *Meaſures* being ſuitably adapted to ſuch Deſigns. But ſuch Deſigns ſeem almoſt quite neglected in our *preſent Muſick*. The chief Deſign now, in our moſt accompliſhed *Muſick*, being to pleaſe the Ear; when by a ſweet Mixture of different *Parts* and *Voices*, with *Cadencies* and *Concord* intermixed, a grateful Sound is produced, which only the judicious *Muſician* can diſcern and diſtinguiſh.

8. 'Tis true, that even this *Compound Muſick* admits of different *Characters*: ſome are more briſk and airy; others more ſedate and grave; others more languid; as the different Subjects do require. But that which is moſt proper to excite particular *Paſſions* or *Diſpoſitions*, is ſuch as is more *Simple*, and *Uncompounded*; ſuch as a Nurſe's *languid Tune*, lulling her Babe to ſleep; or a continual Reading in an *Even Tone*; or even the ſoft Murmur of a little Rivulet, running upon Gravel or Pebbles, inducing a quiet Repoſe of the Spirits: And contrariwiſe, the Briſkneſs of a *Jig*, on a *Kit* or *Violin*, exciting to dance. Which are more operative to ſuch particular Ends, than an *Elaborate Compoſition* of *Full Muſick*.

9. To conclude; If we aim only at pleaſing the *Ear*, by a *ſweet Concert*, I doubt not but our *Modern Compoſitions* may be equal, if not exceed thoſe of the *Antients*; amongſt whom I do not find any Footſteps of what we call ſeveral Parts or Voices, (as *Baſe*, *Treble*, *Mean*, &c. ſung in Concert) anſwering each other, to complete the Muſick. But if we would have our Muſick ſo adjuſted, as to excite particular Paſſions, Affections, or Tempers of Mind, (as that of the *Antients* is ſuppoſed to have done) we muſt apply more ſimple Ingredients, fitted to the Temper we would produce. And this, I doubt not, but a Judicious Compoſer may ſo effect, that (with the Help of ſuch *Hyperboles*, as with which the *Antient Muſick* is wont to be ſet off) our *Muſick* may be ſaid to do as great Feats as any of theirs.

VII. *Account of Books*, Omitted.

n. 143. p. 20. 1. *Claudii Ptolemæi Harmonicorum* Libri tres; Ex Cod. MSS. undecim, nunc primum *Græce* editi. *Jo. Wallis*, S. S. Th. D. Recenſuit, Edidit, Verſione & Notis Illuſtravit, & Auctuarium adjecit. *Oxon* 1682. in *4to*.

n. 231. p. 668. 2. *Porphyrii* Commentarius in Librum Primum *Harmonicorum Claudii Ptolemæi:* atque *Manuelis Bryennii* Commentarius in tres Libros *Harmonicas* ejuſdem *Ptolemæi* (Qui ſoli reſtant ex *Græcis Muſicæ* Scriptoribus nondum editis.) *Græce* ac *Latine.* Curâ *Jo. Walliſii*, S. Th. D. *Fol.*

n. 80. p. 3095. 3. An Eſſay to the Advancement of *Muſick*; by *Tho. Salmon*, M. A. *Lond.* 1672. in 8*vo*.

n. 90. p. 5153. n. 100. p. 7000. 4. *Syntagma Muſicæ*; Treating of *Muſick Philoſophically*, *Mathematically*, and *Practically*; by *J. Birchenſha*, Eſq; This Book was preparing for the Preſs, 1674.

n. 100. p. 6194. 5. *Muſica* Speculativa del *Mengoli*. in *Bologna* 1670. *in 4to*.

n. 133. p. 835. 6. A *Philoſophical Eſſay of Muſick*. *Lond.* 1677. *in 4to*.

n. 208. p. 67. 7. A Treatiſe of the *Natural Grounds* and *Principles* of *Harmony*; by *Will. Holden*, D. D. *Lond.* 1694. *in 8vo*.

F I N I S.